# Adapting Neuron Count During Training

## A Bayesian Nonparametric View


Mark van der Wilk


Invited Talk

14th International Conference on Bayesian Nonparametrics


University of Oxford

Department of Computer Science

🔗 [https://mvdw.uk](https://mvdw.uk)

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PhD candidate

Prof Sarah
Filippi
Co-supervisor

# Based on a True Story

**Adjusting Model Size in Continual Gaussian Processes: How Big is Big Enough?**

**Guiomar Pescador-Barrios** [1] **Sarah Filippi** [1] **Mark van der Wilk** [2]

Spotlight at ICML 2025.

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## A Bayesian Nonparametric View on Adapting Neuron Count During Training

In submission, soon to be on arxiv.

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Reason 2:

- Data can arrive in a streaming fashion.
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> **Can we grow a NN's size as we see more data?**
>
> To avoid poor performance, from constant/restricted model size.

🎯

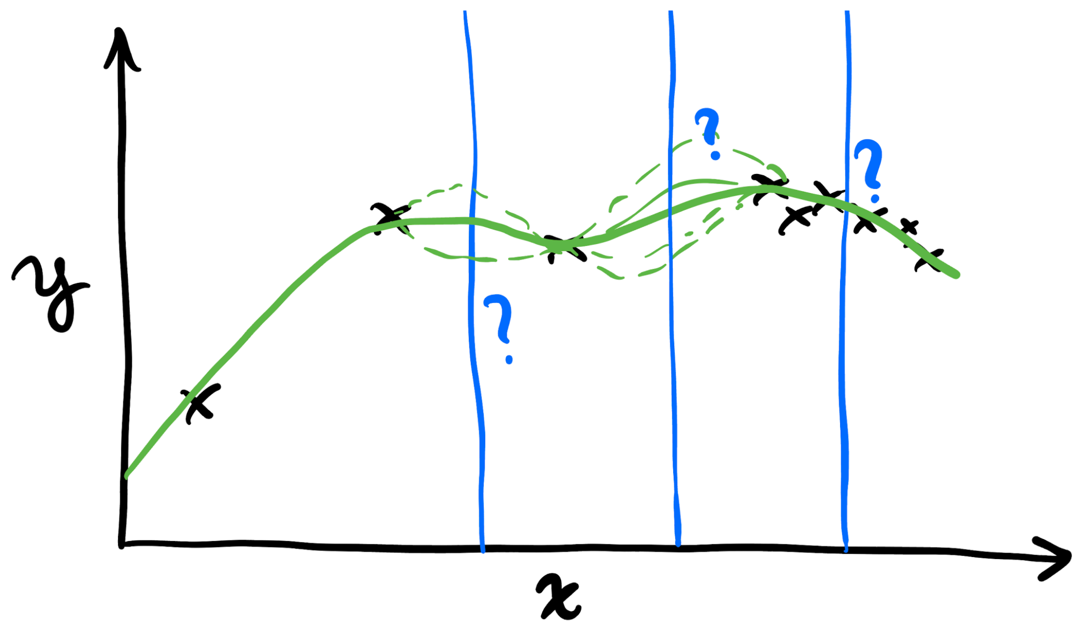
**Minimise model size,**
**while maintaining near-optimal predictions.**

# Most of Machine Learning is just *Curve Fitting*

Dataset: $(x_n, y_n)_{n=1}^N$.

Inputs $x_n \in \mathcal{X}$, outputs $y_n \in \mathcal{Y}$.

Goal: Find $f : \mathcal{X} \to \mathcal{Y}$, that predicts well for new $x$.




Neural networks just parameterise functions $f_w(x)$.

# Designing a Neural Network

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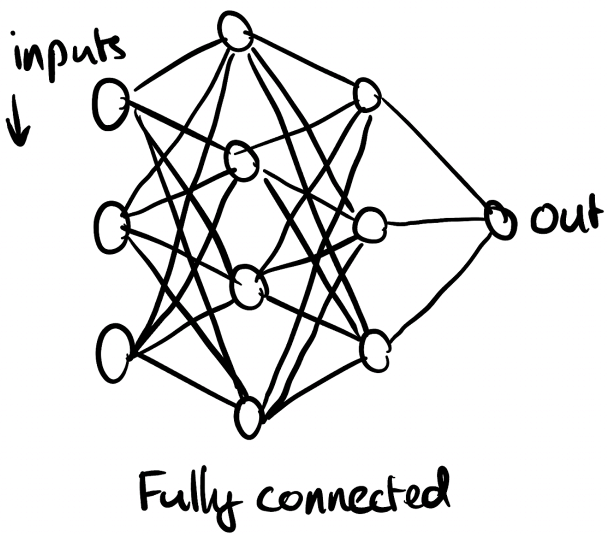
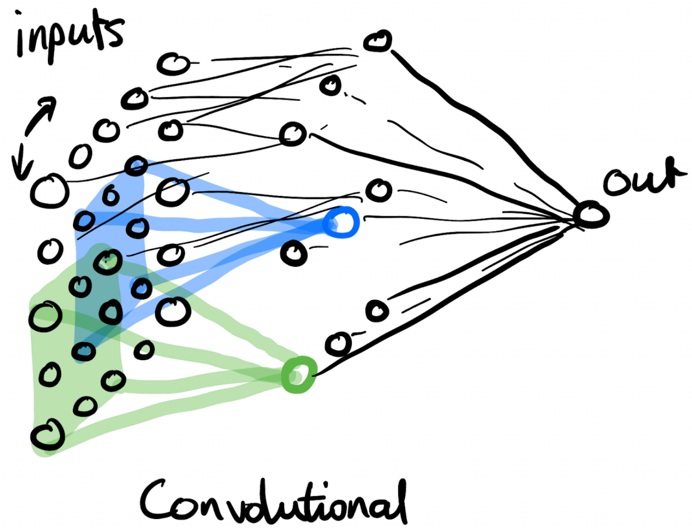
# Designing a Neural Network

- Inductive bias: **connectivity structure** (architecture)

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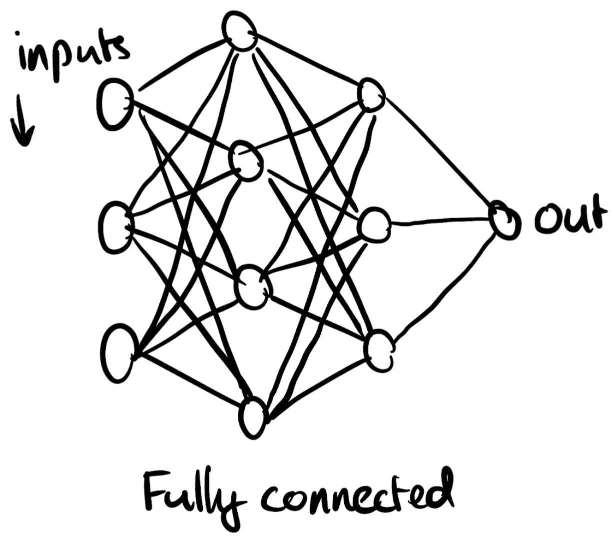


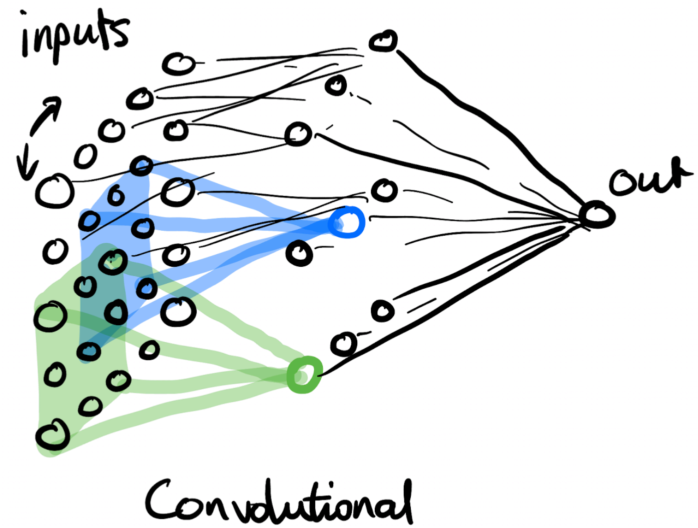


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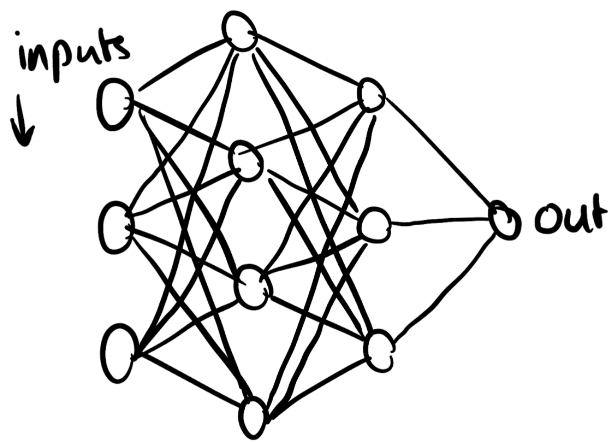
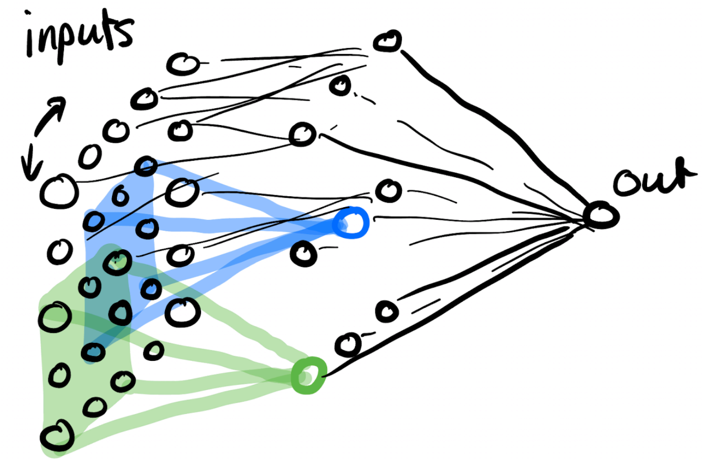



Fully connected




Convolutional

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- Choose **weights**, using *backpropagation*

$$w_{t+1} \leftarrow w_t + \nabla_w \ell(f_w(x_t), y_t)$$

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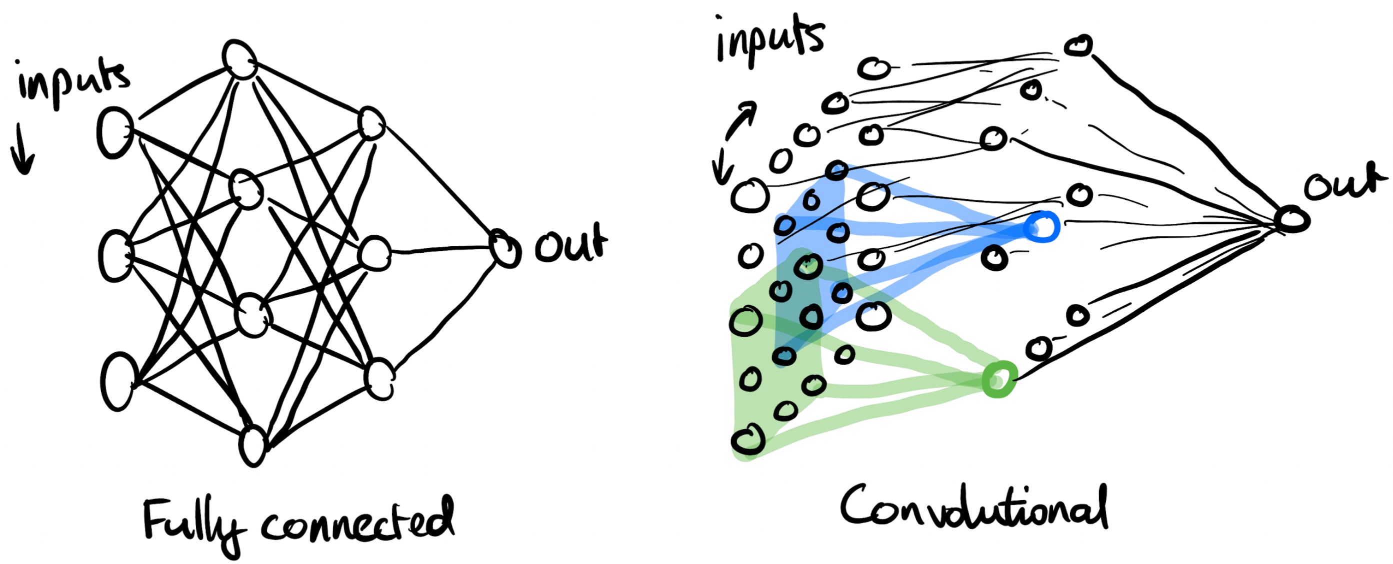


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**These problems should be tackled *together*.**

## **Problem Formulation** (let's walk before we run)

Predictor is a *single layer* neural network:

$$f(x) = \sum_{m=1}^{M} \varphi(x; Z_m, \theta) w_m$$

Goal is to find:

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🎯 **Start by finding *clear* answers for single-layer NNs.**

1. **What is wrong with minimising losses?**
2. Bayesian Model Selection
2. Model Selection over Model Size? Or Nonparametrics?
3. A principle for selecting size

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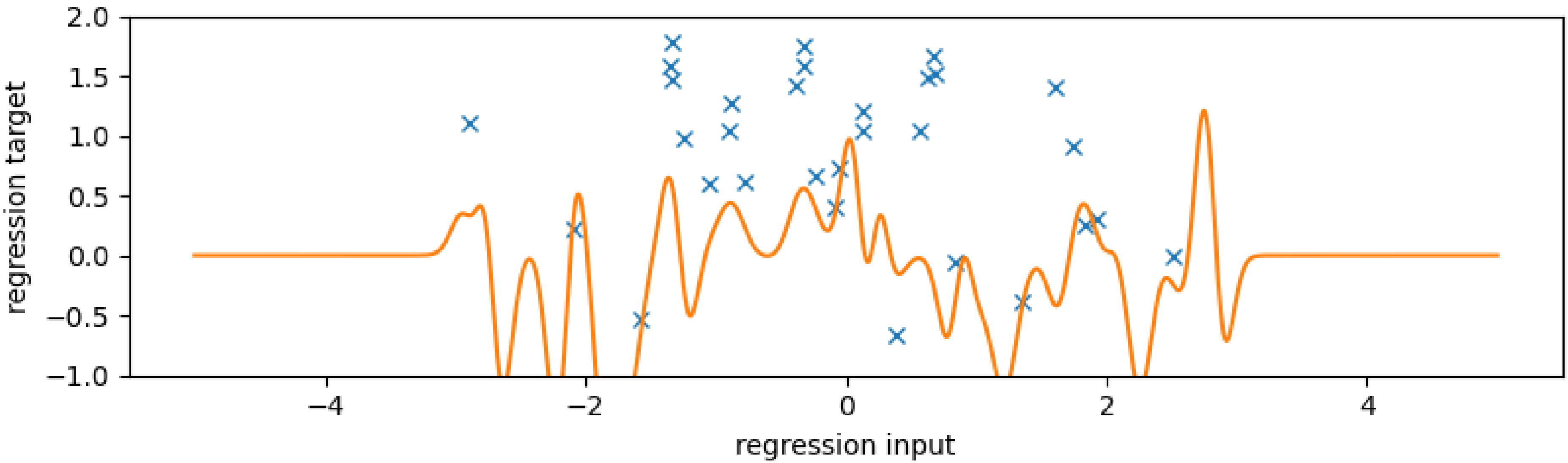

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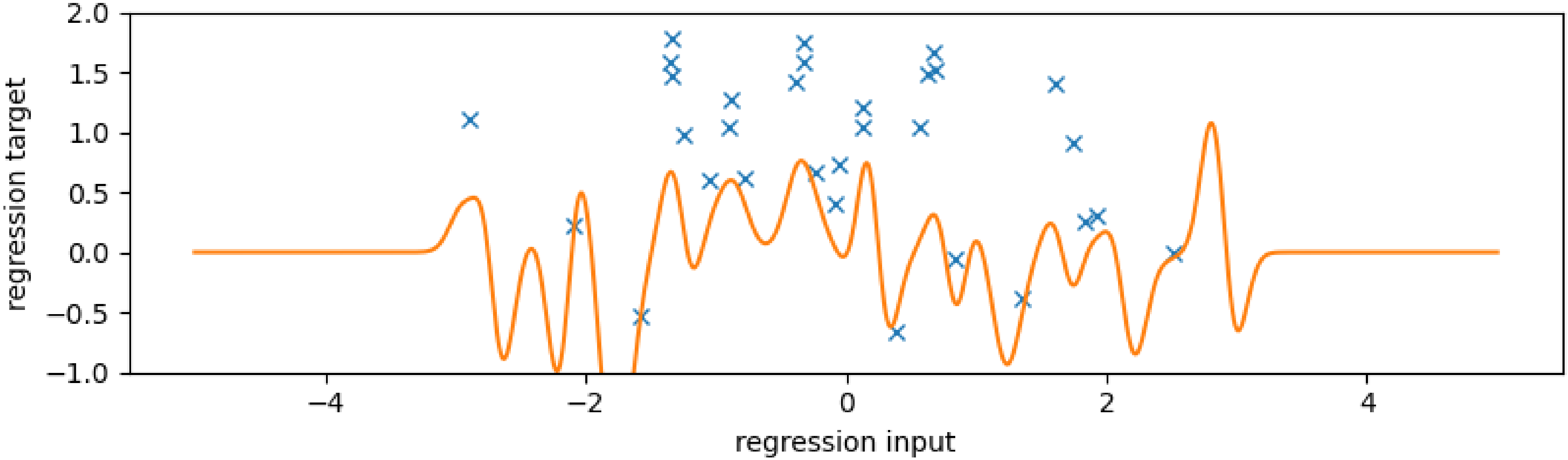

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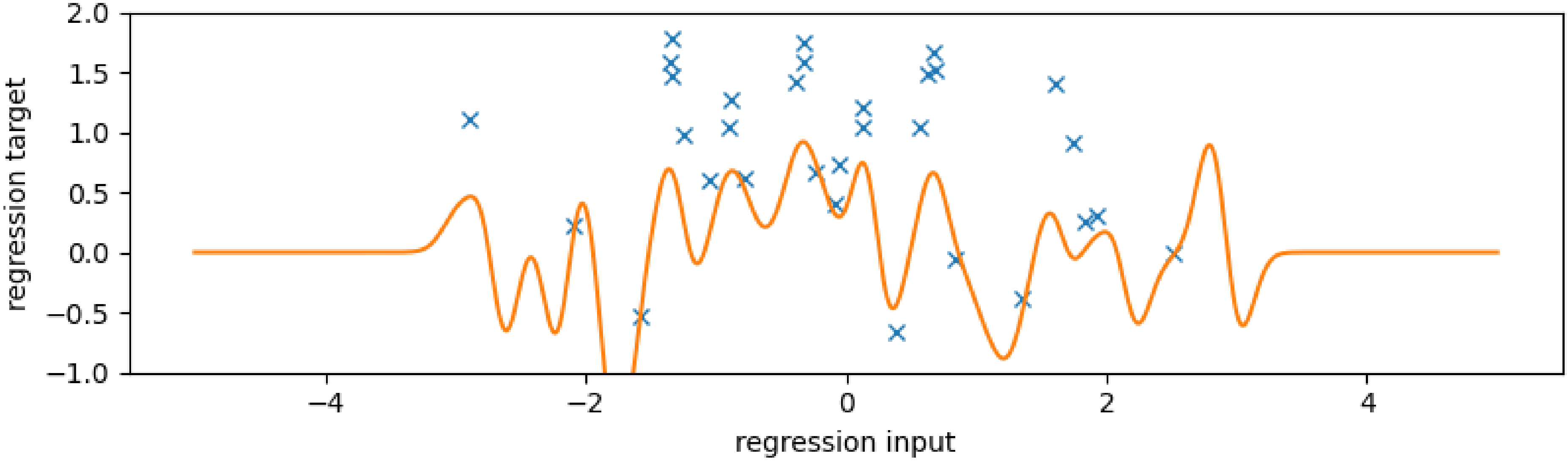

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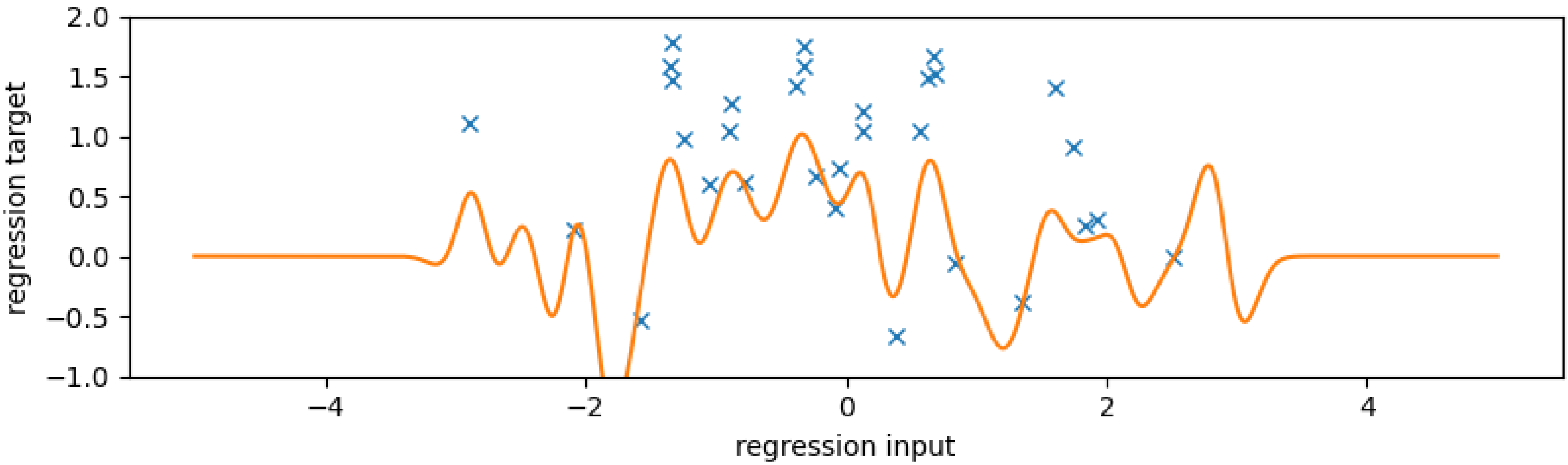

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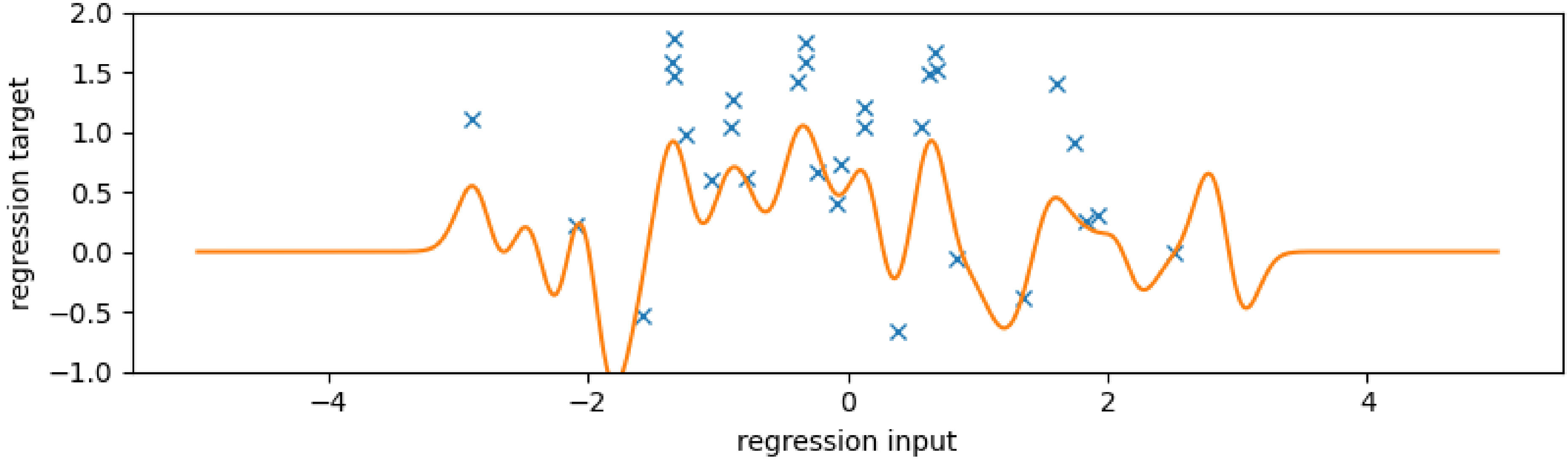

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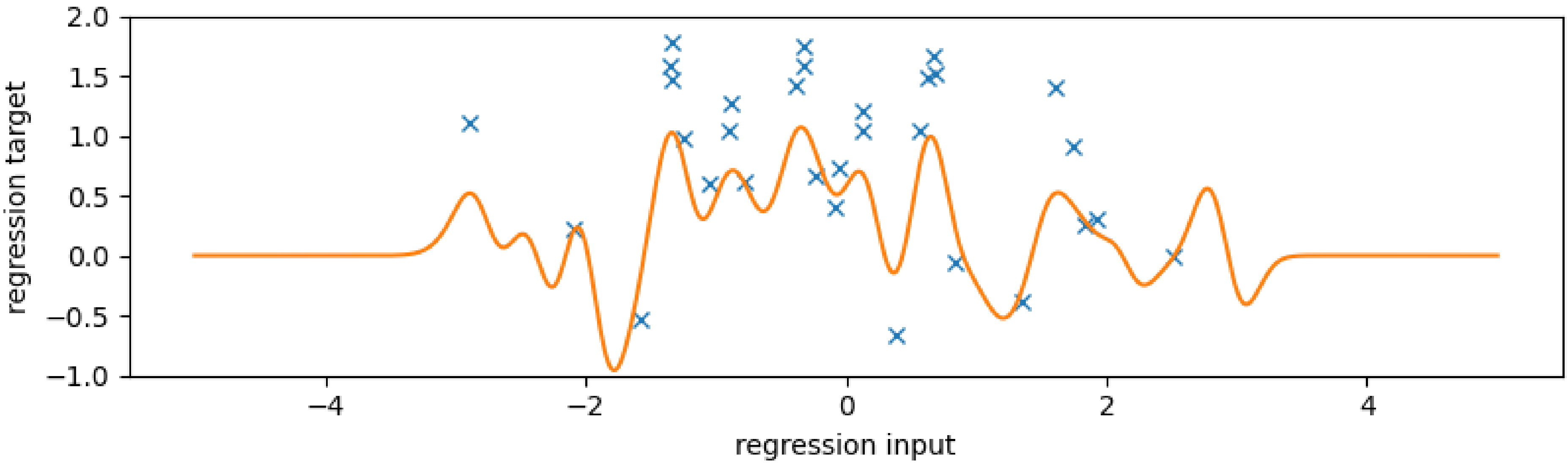

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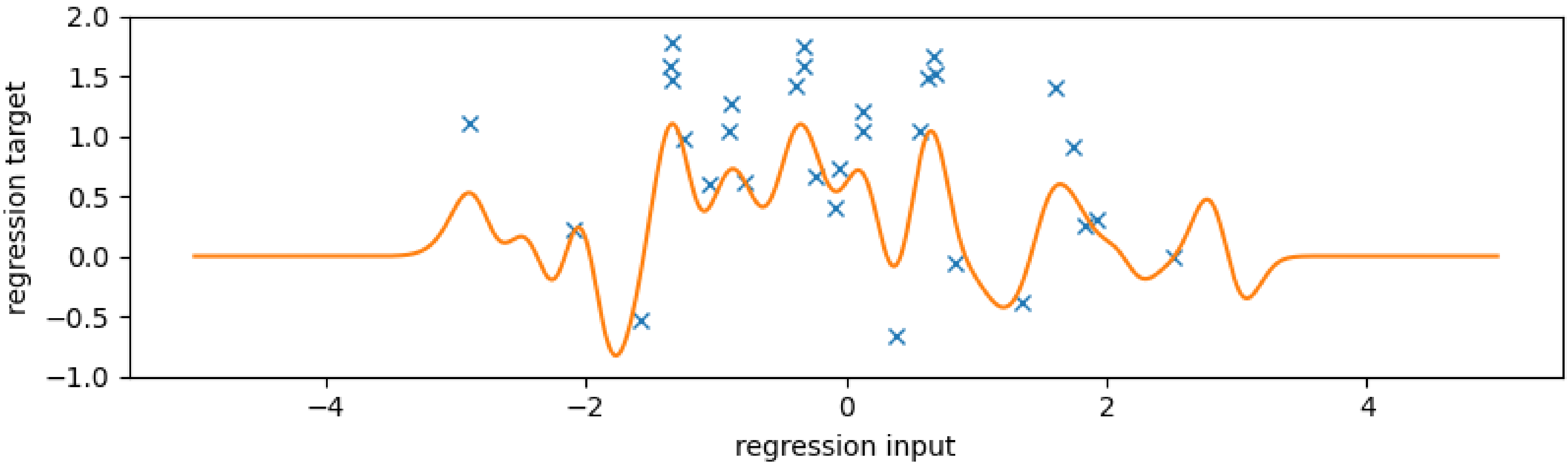

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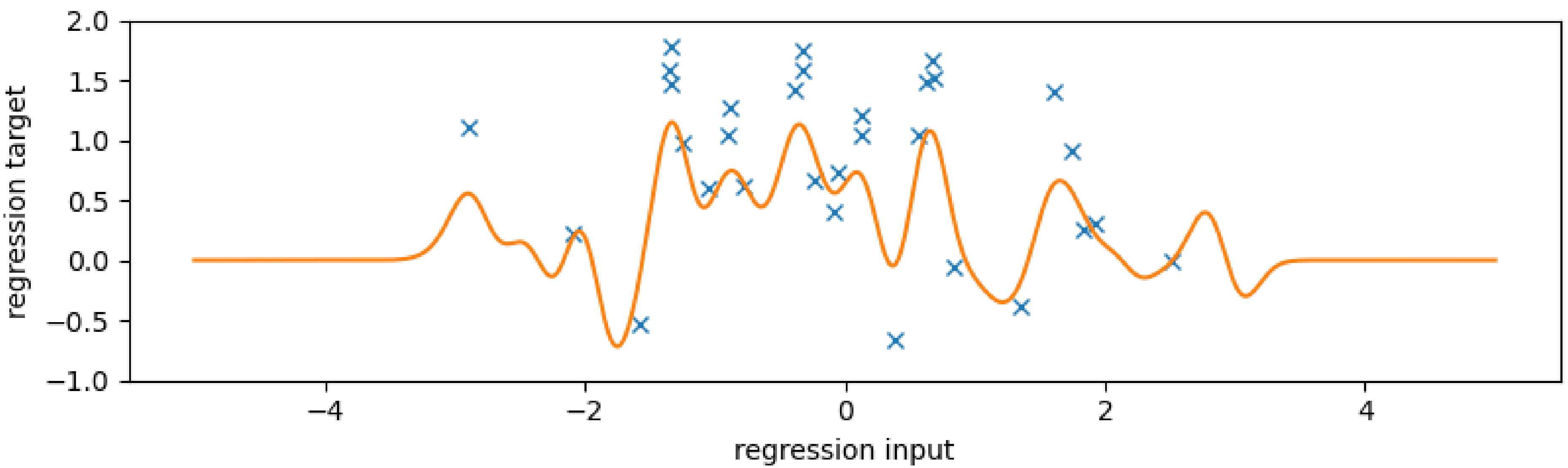

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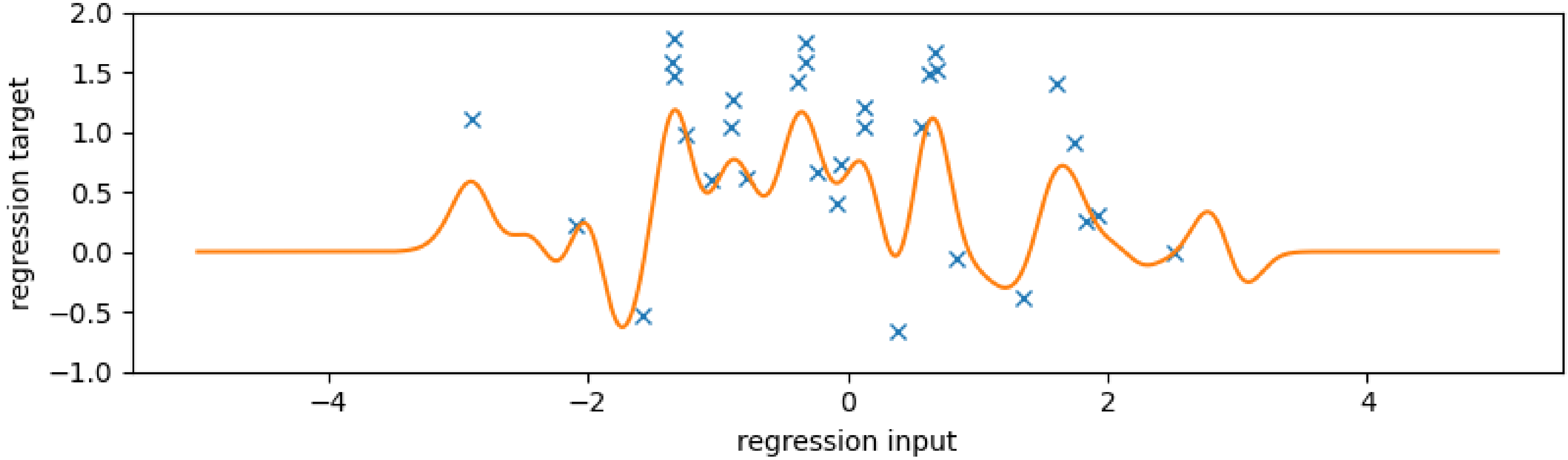

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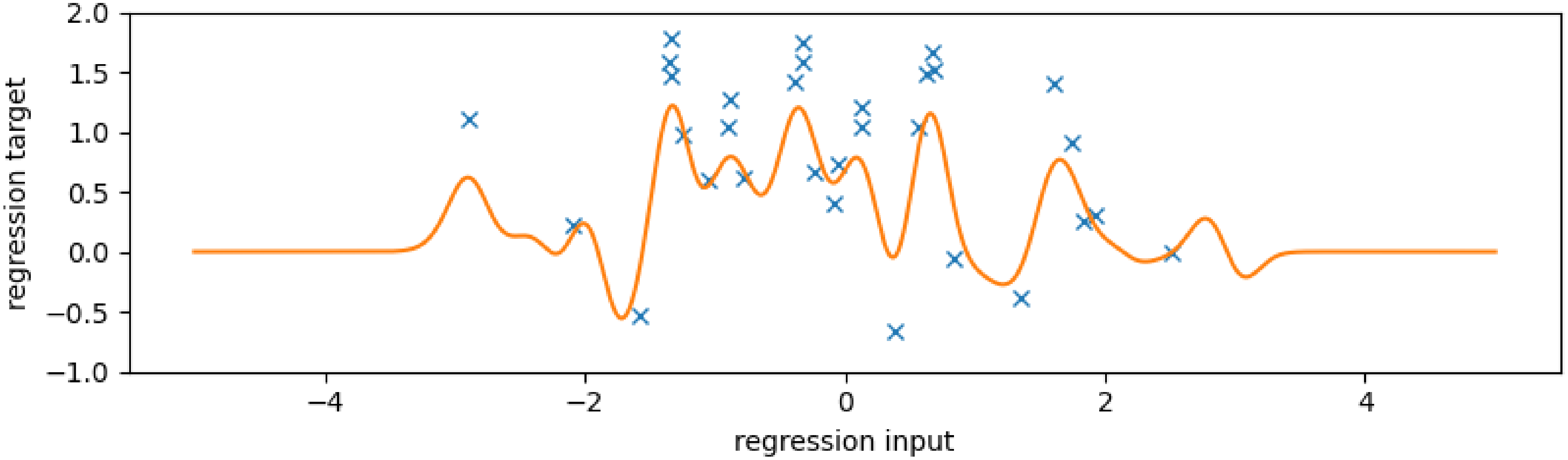

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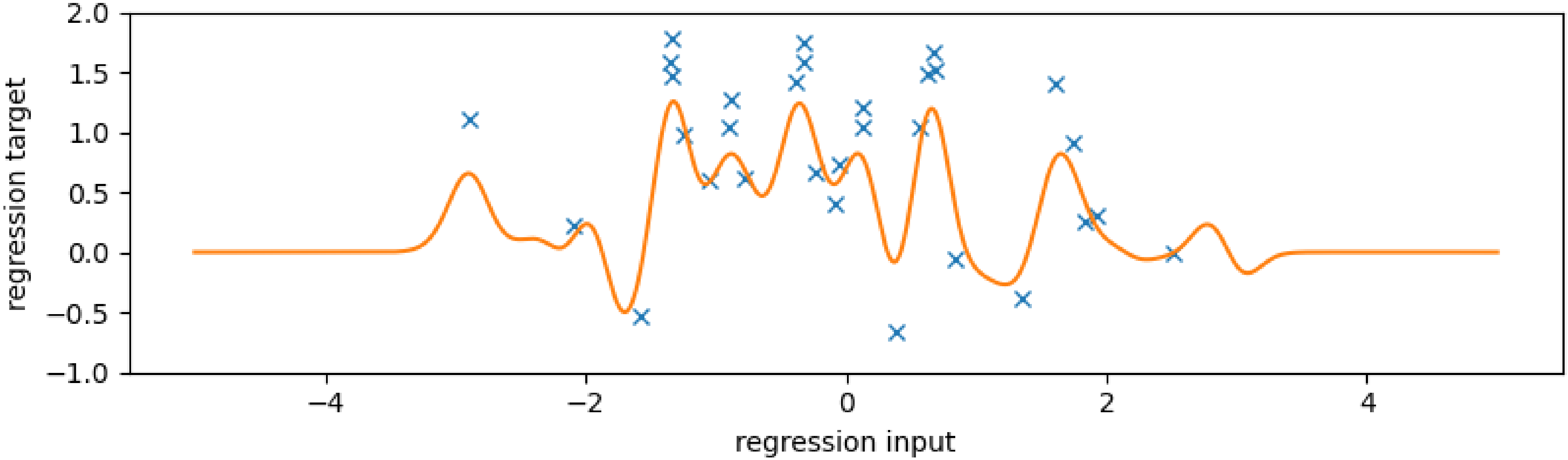

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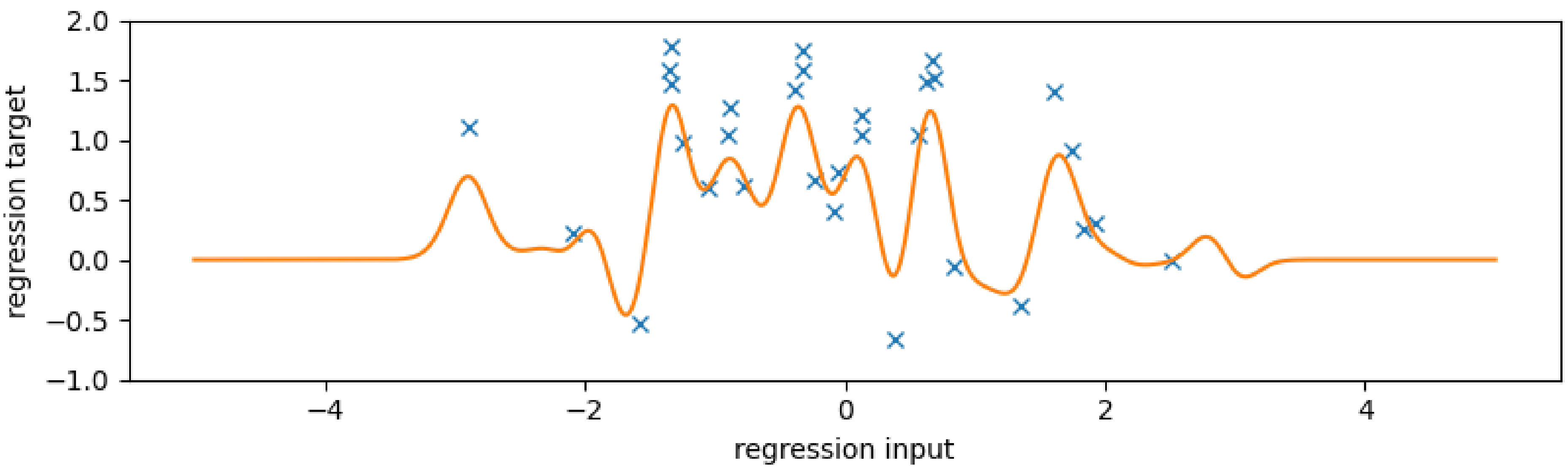

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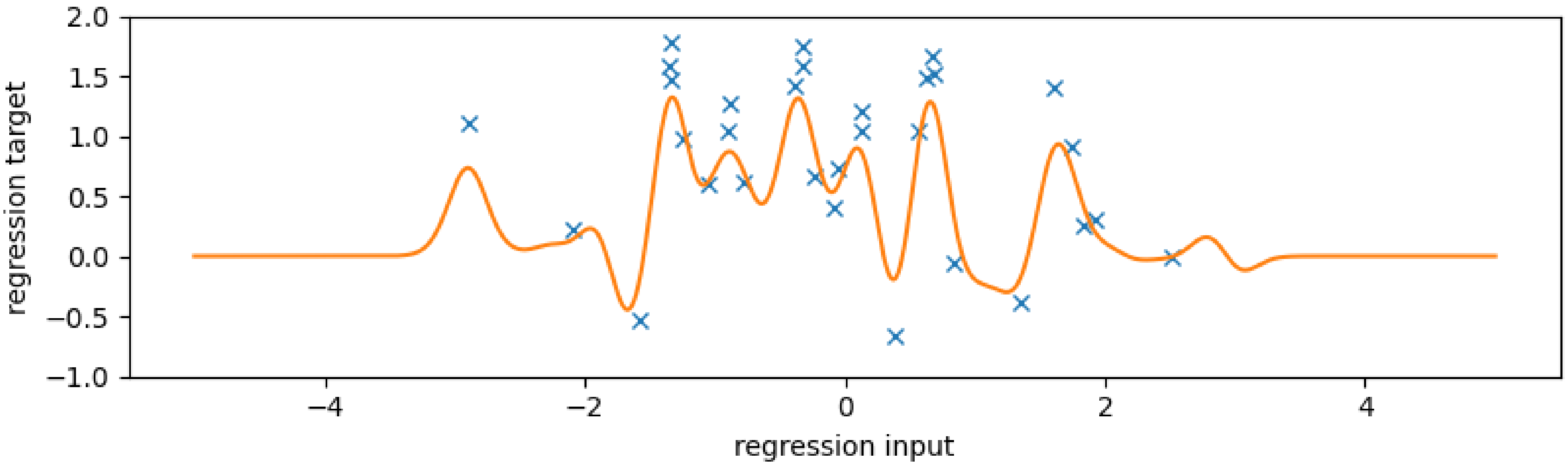

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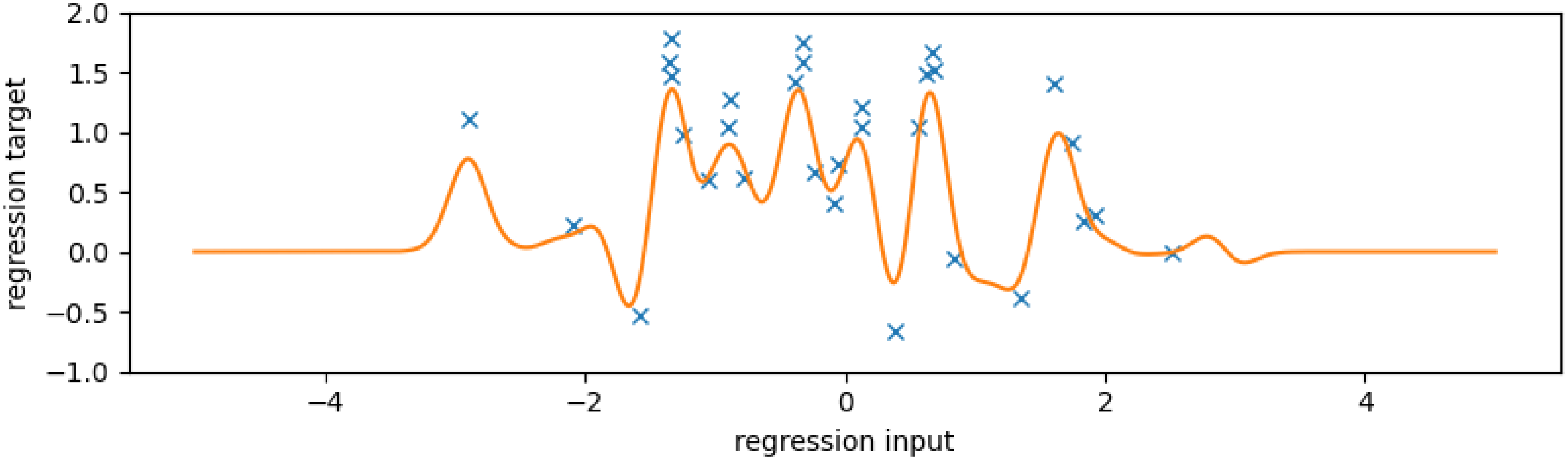

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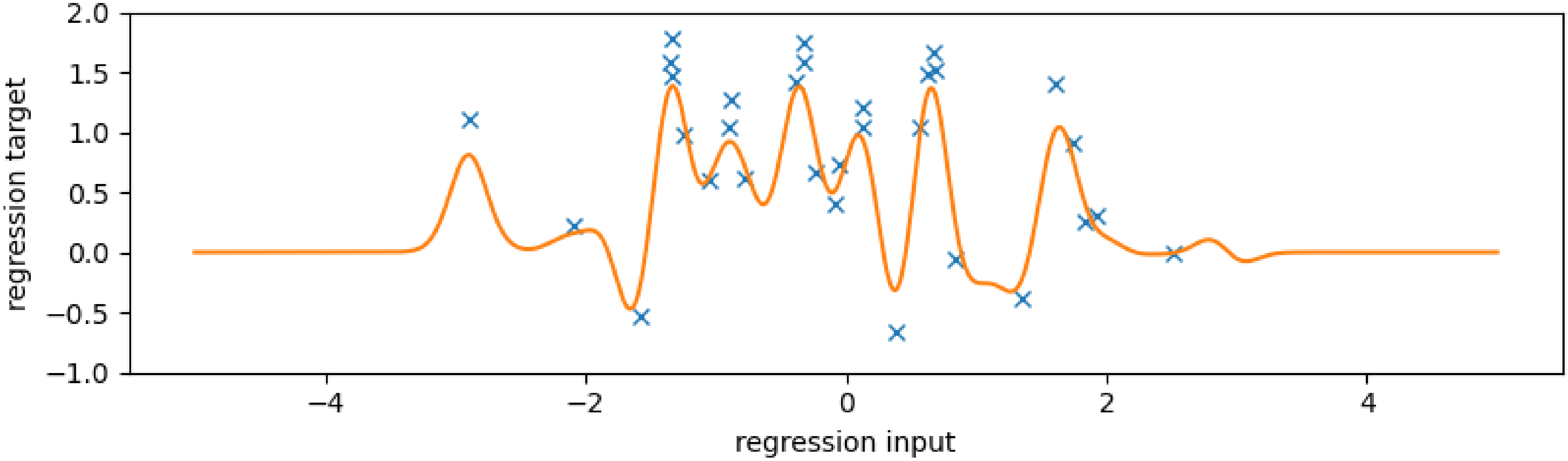

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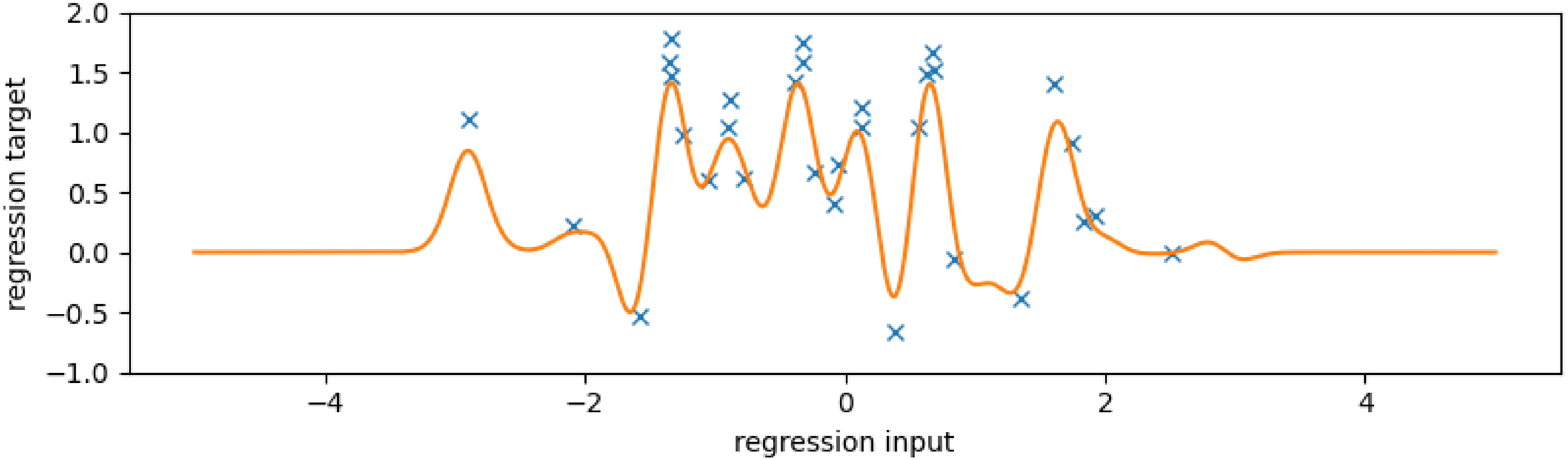

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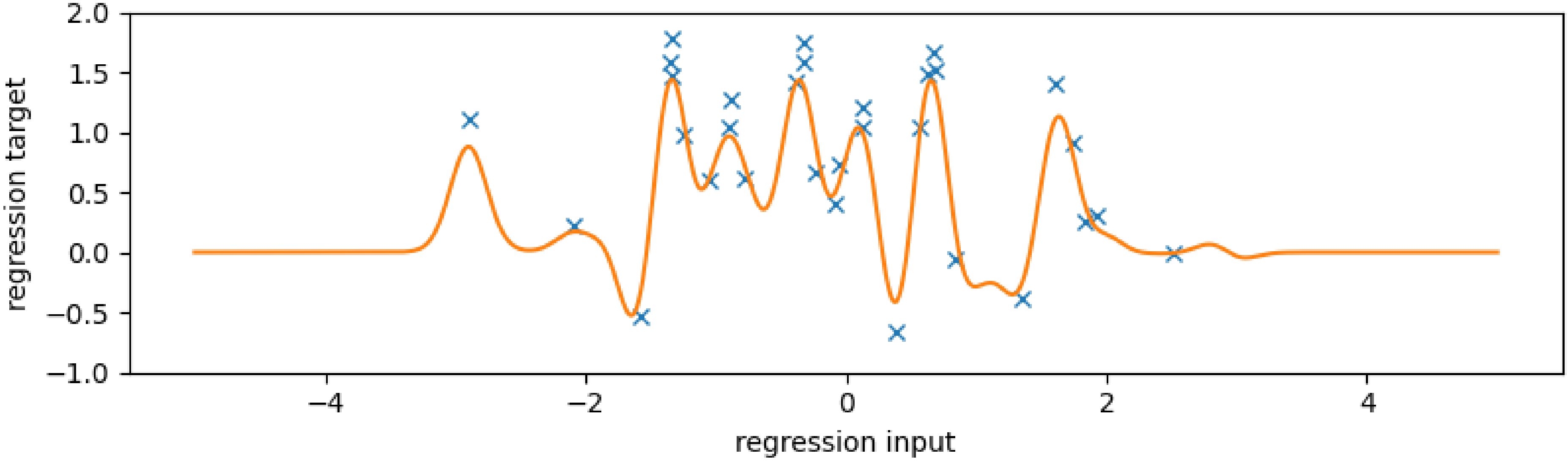

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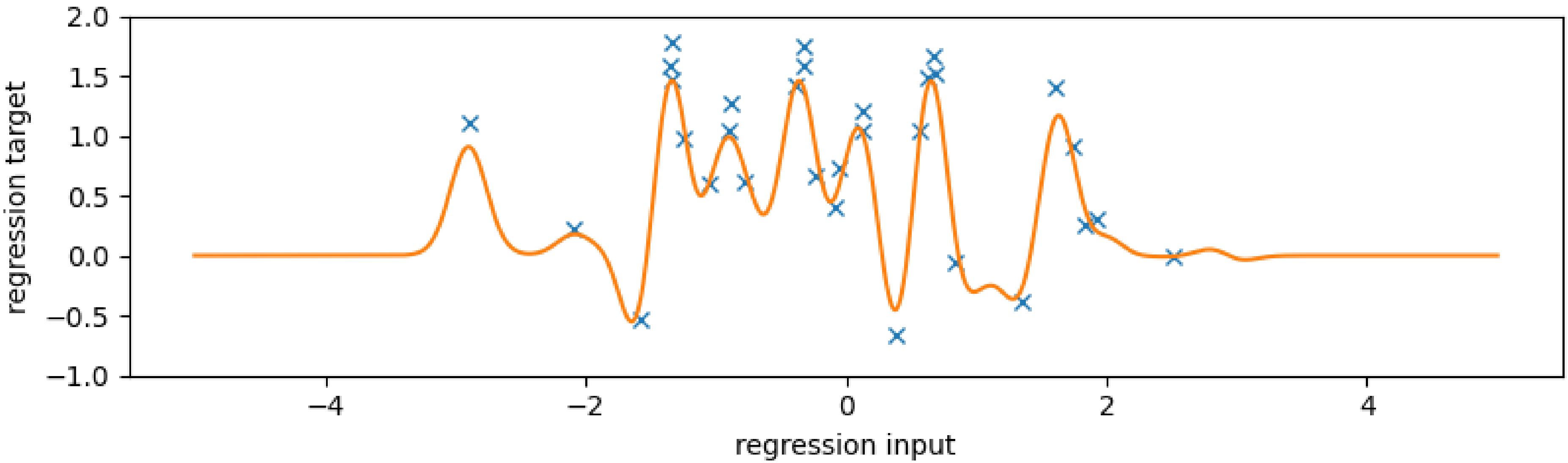

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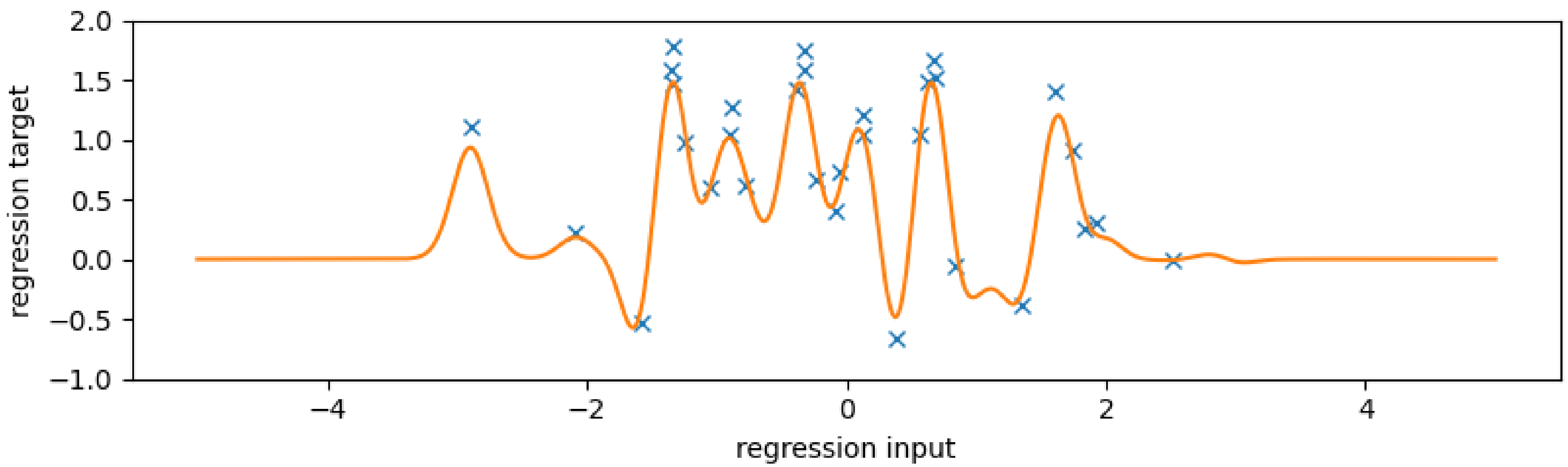

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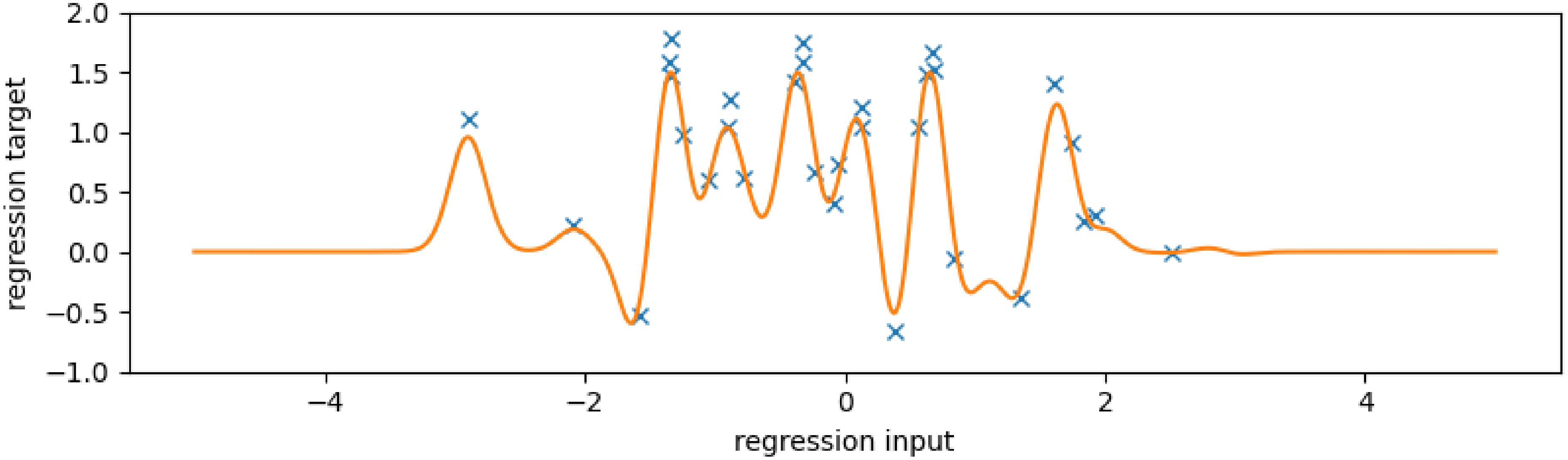

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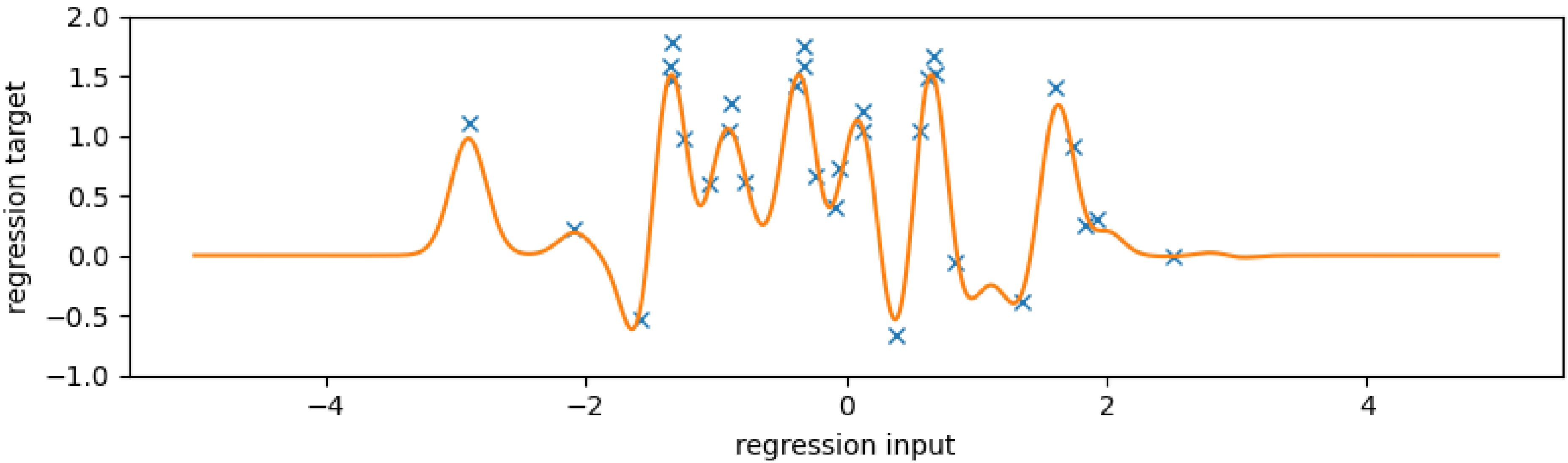

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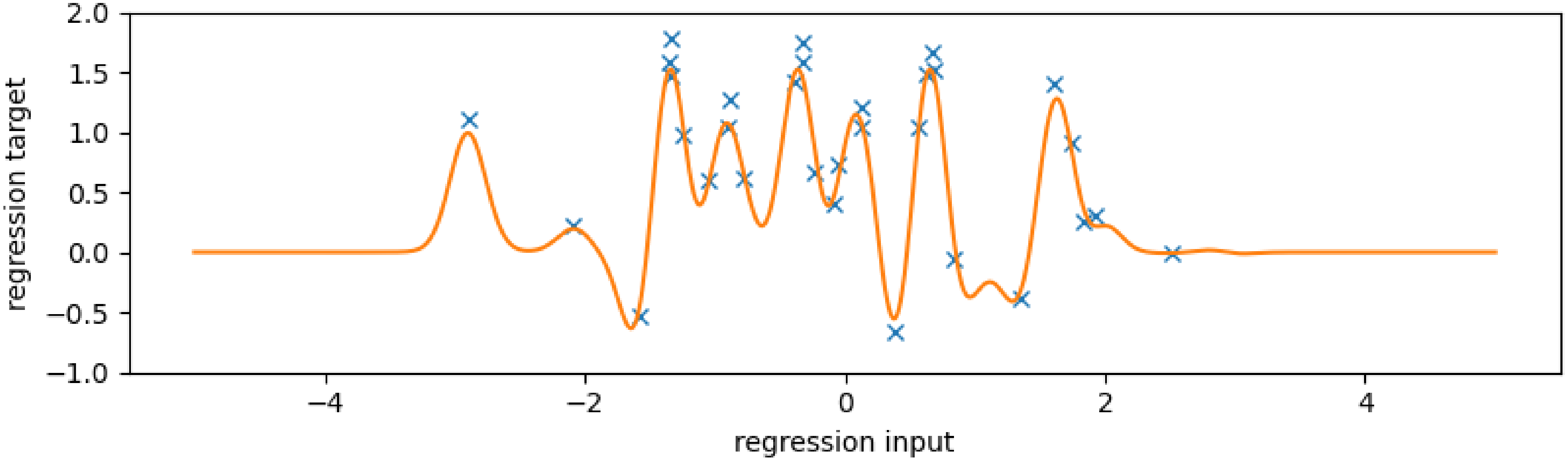

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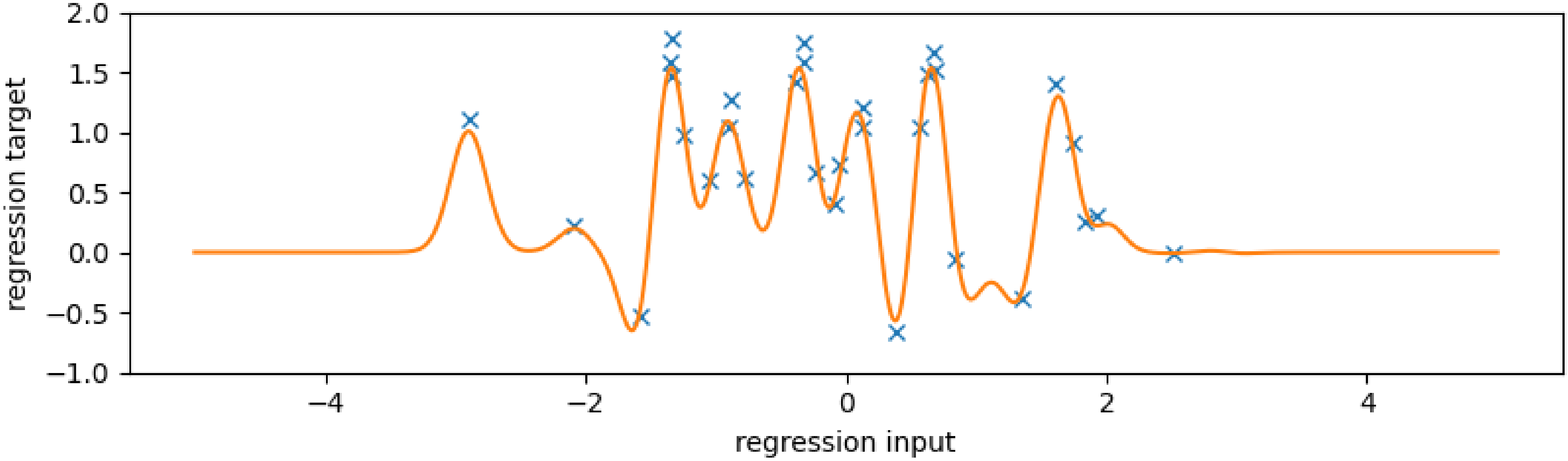

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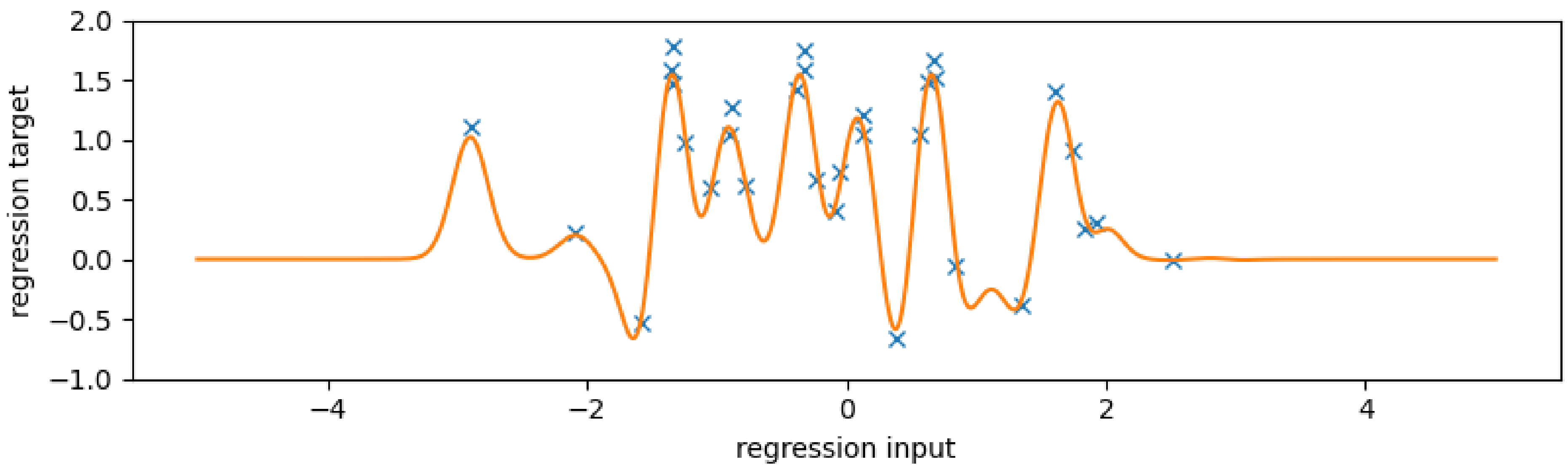

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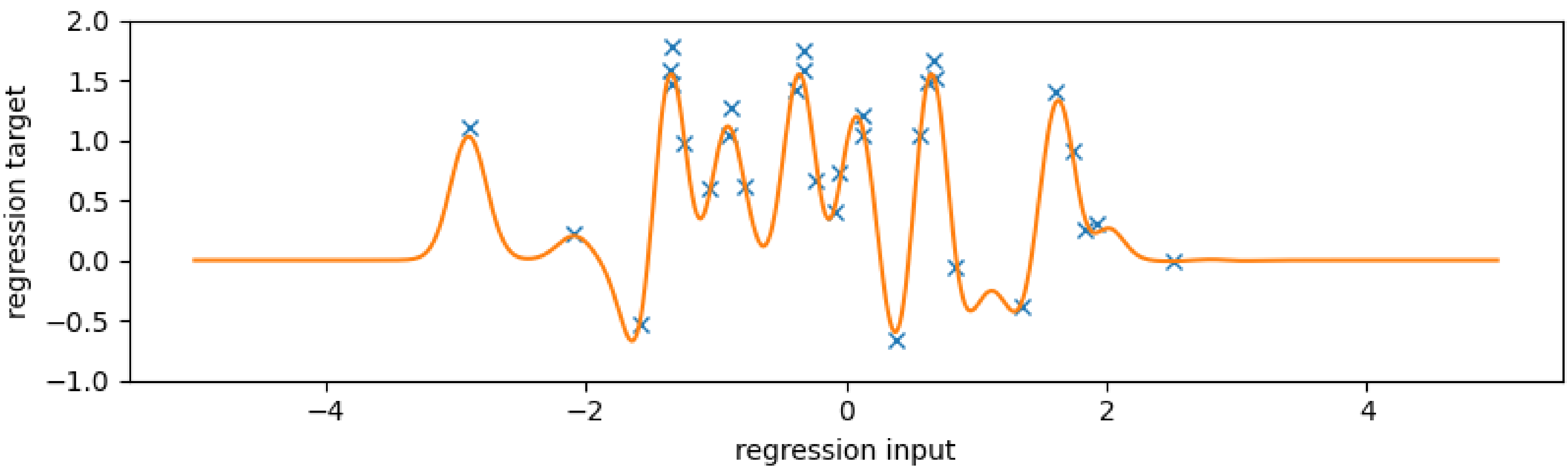

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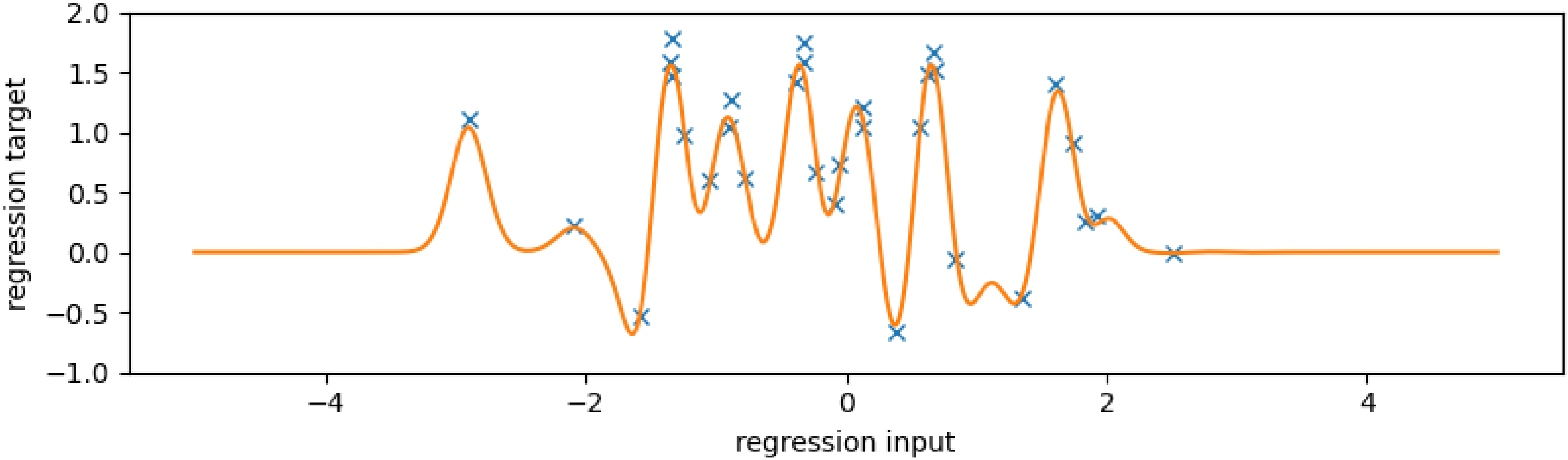

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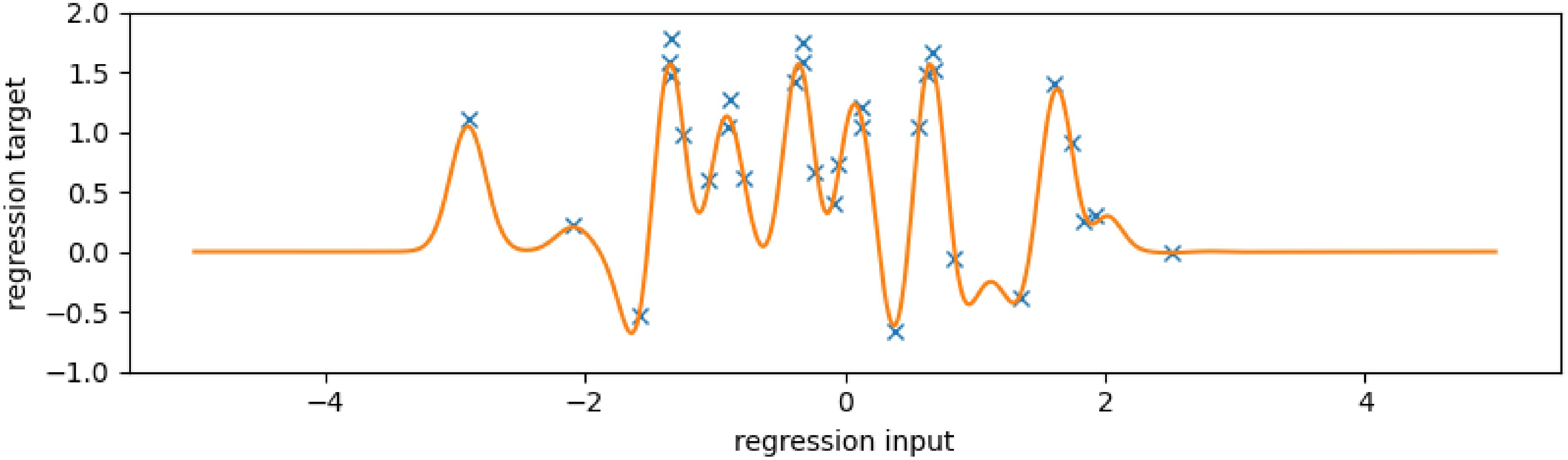

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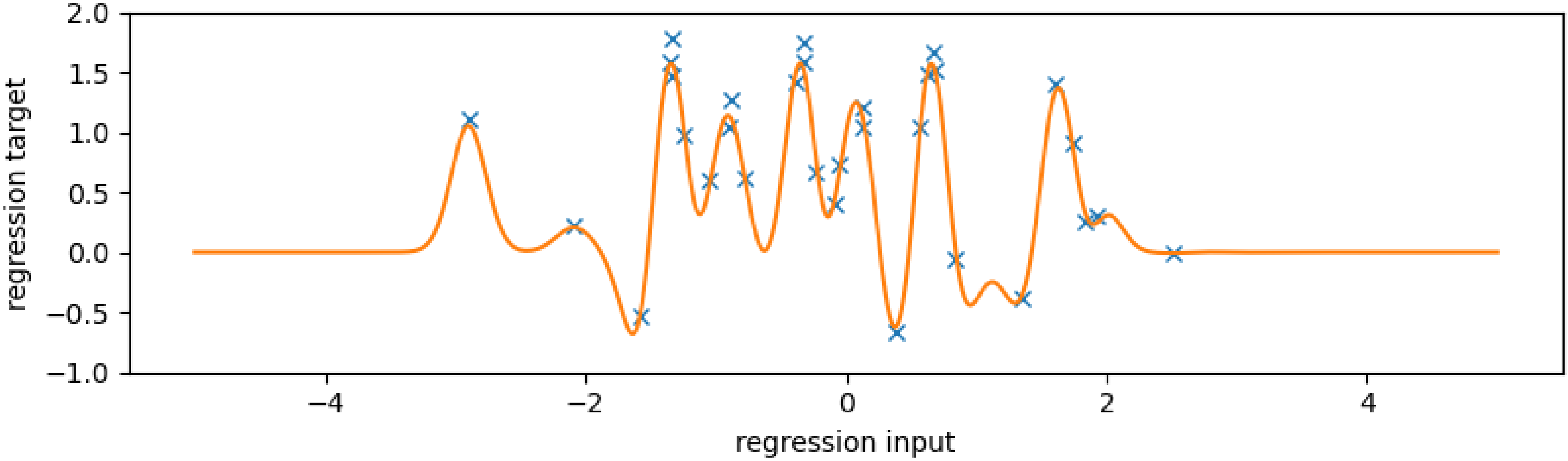

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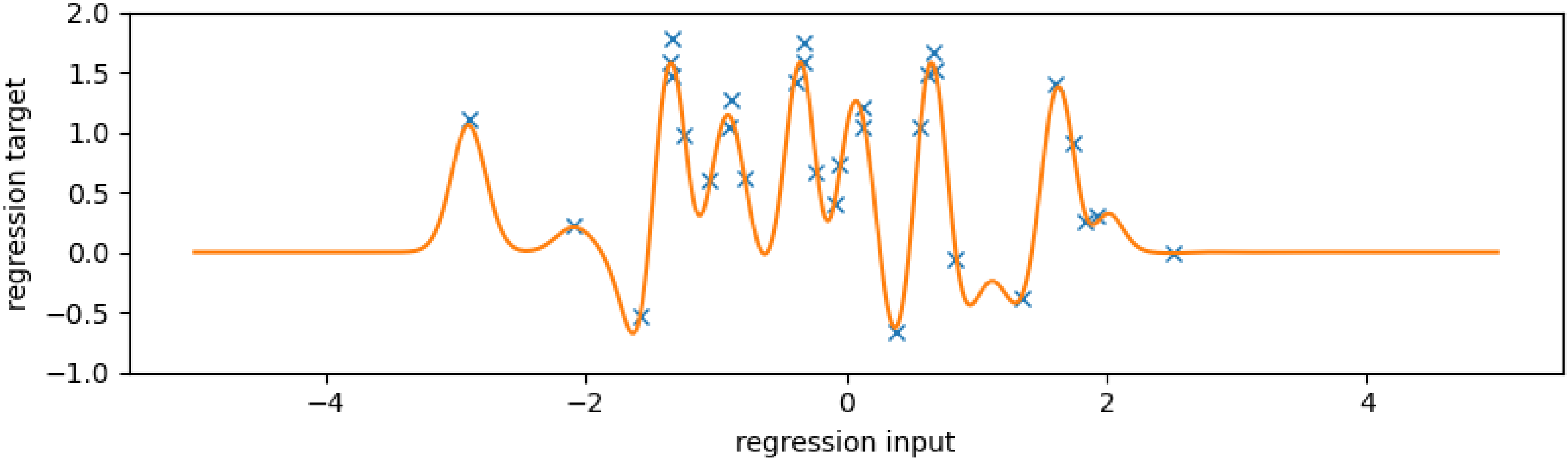

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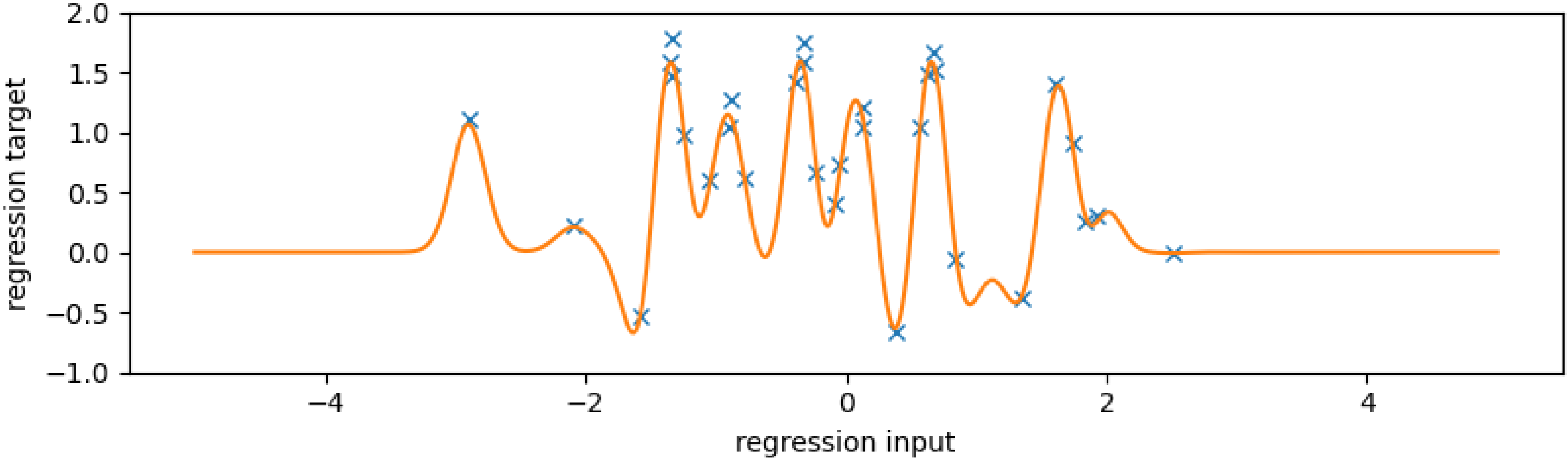

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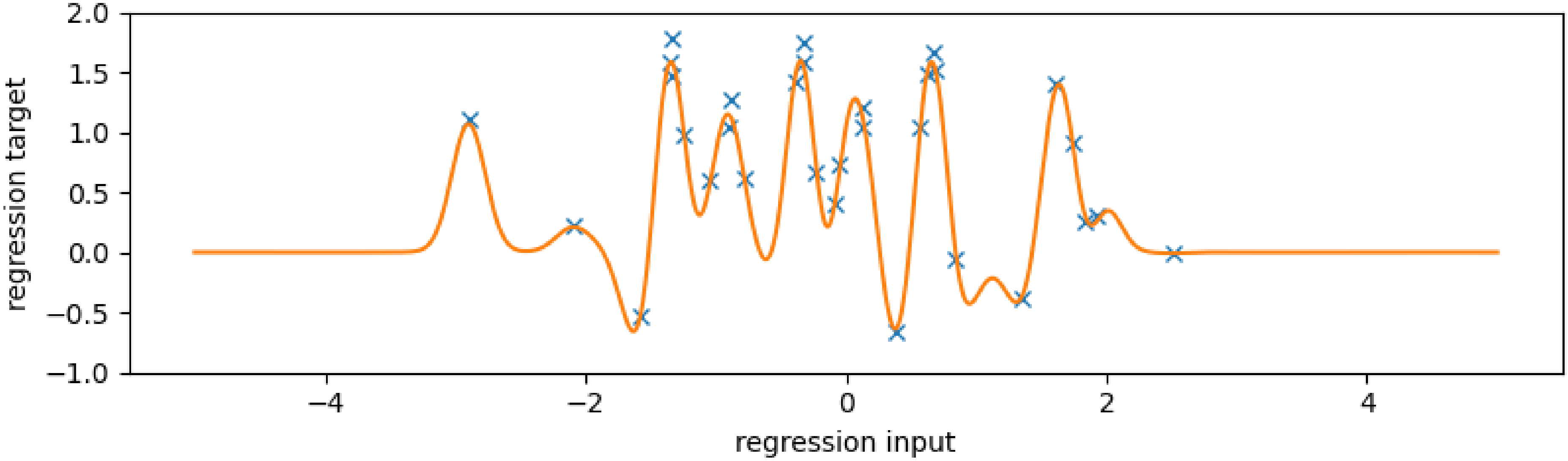

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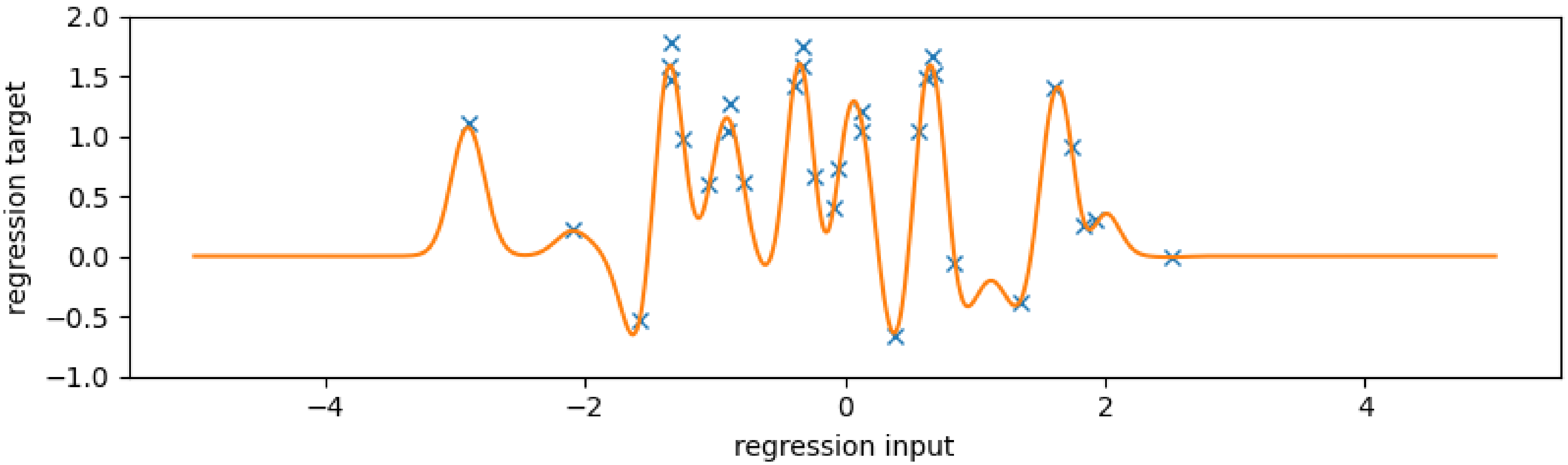

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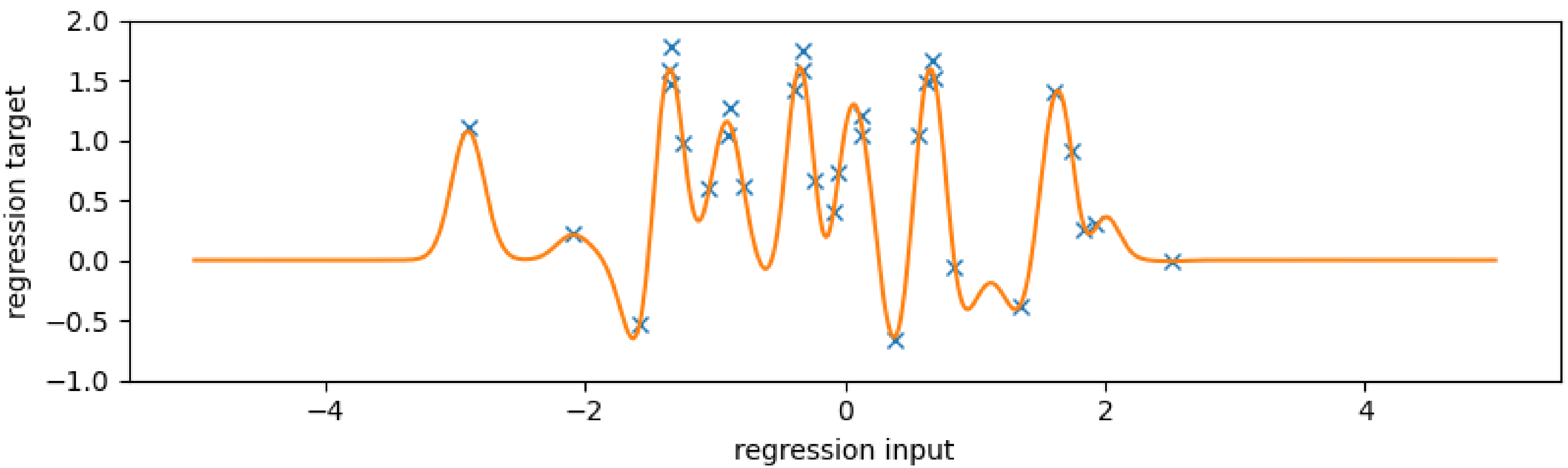

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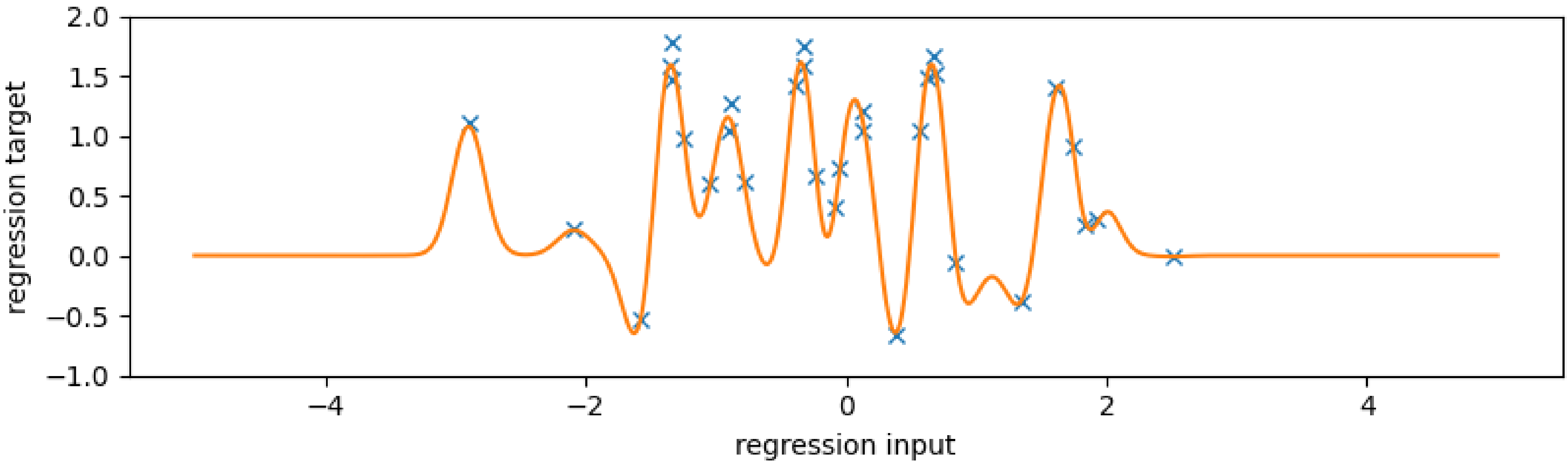

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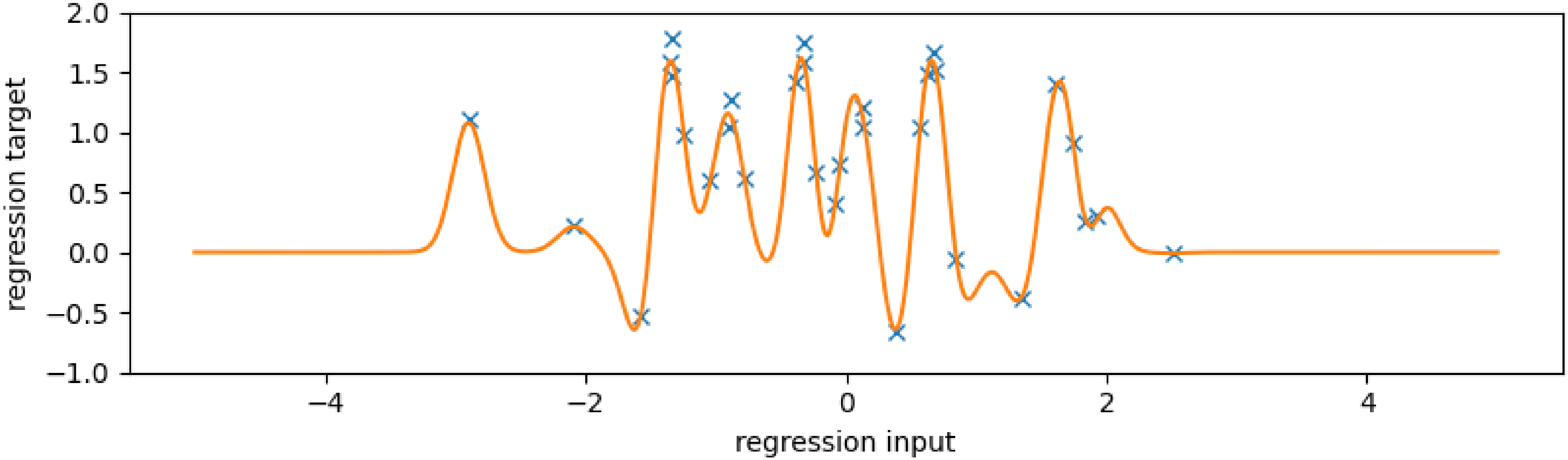

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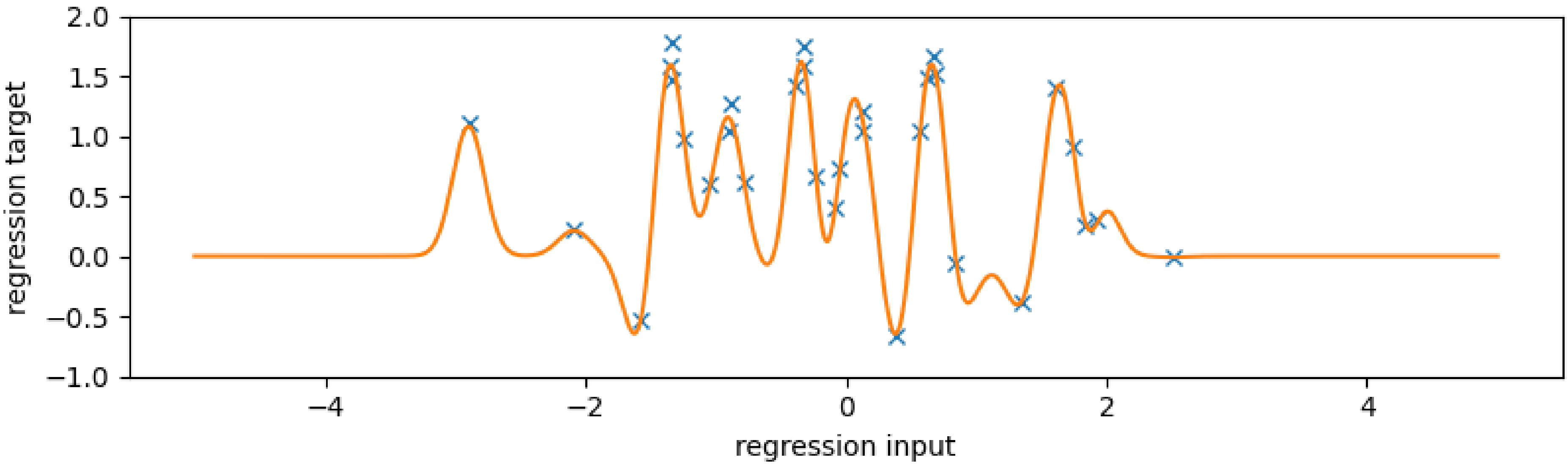

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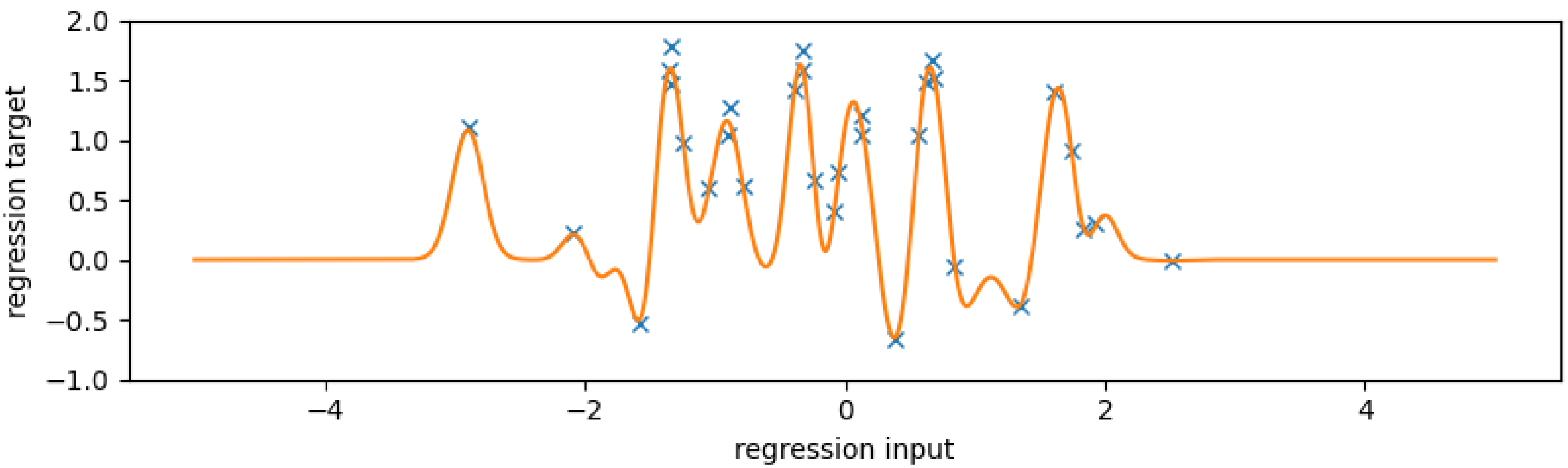

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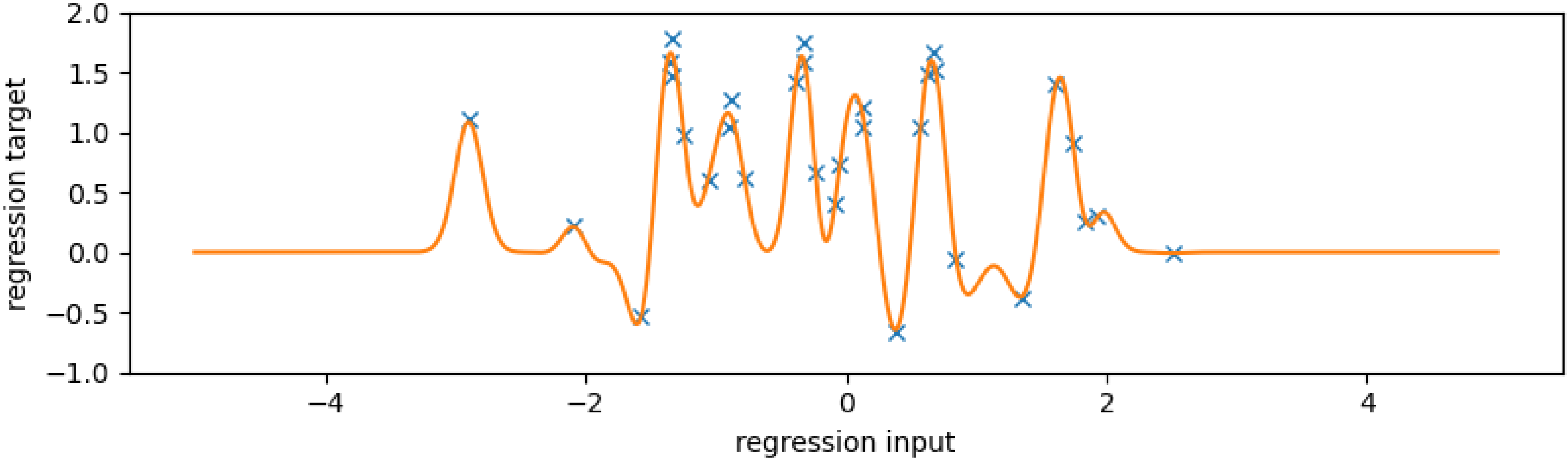

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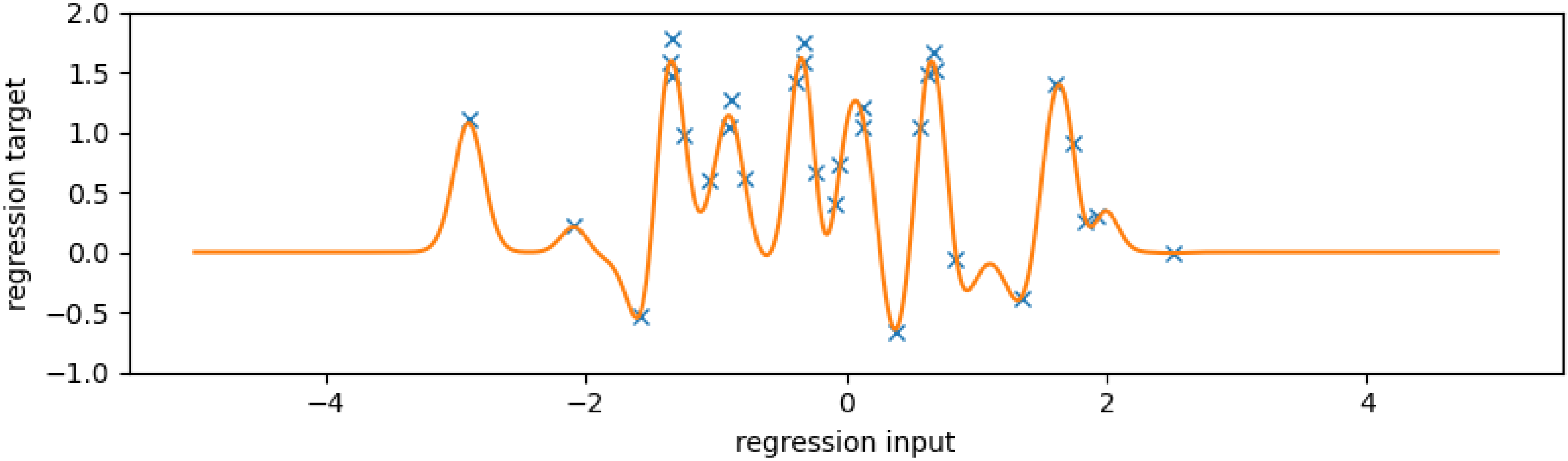

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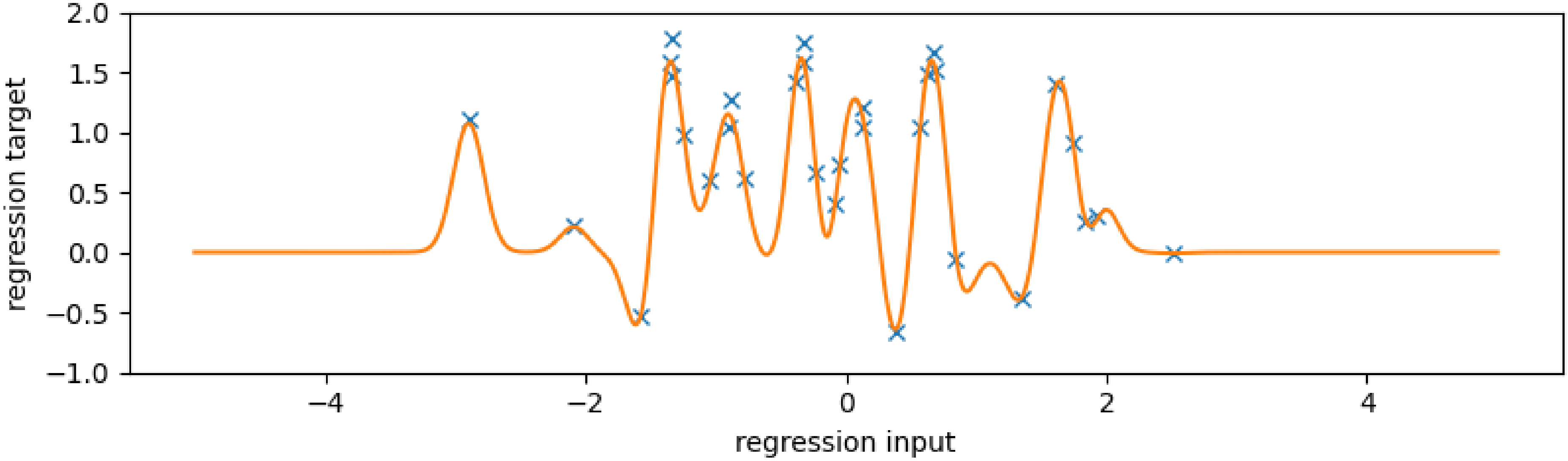

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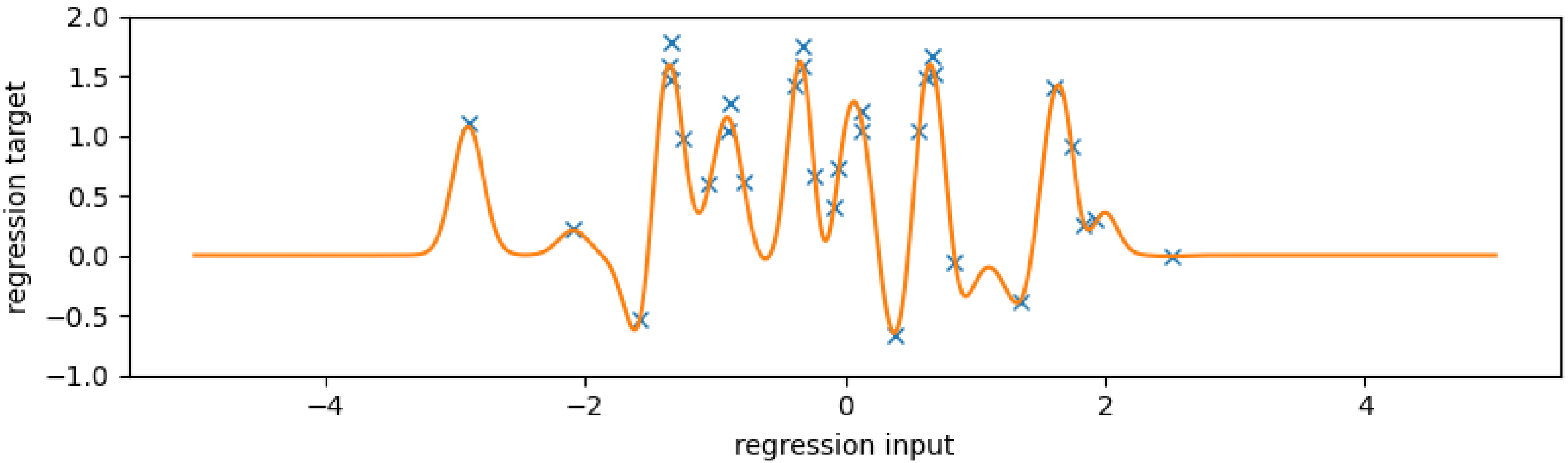

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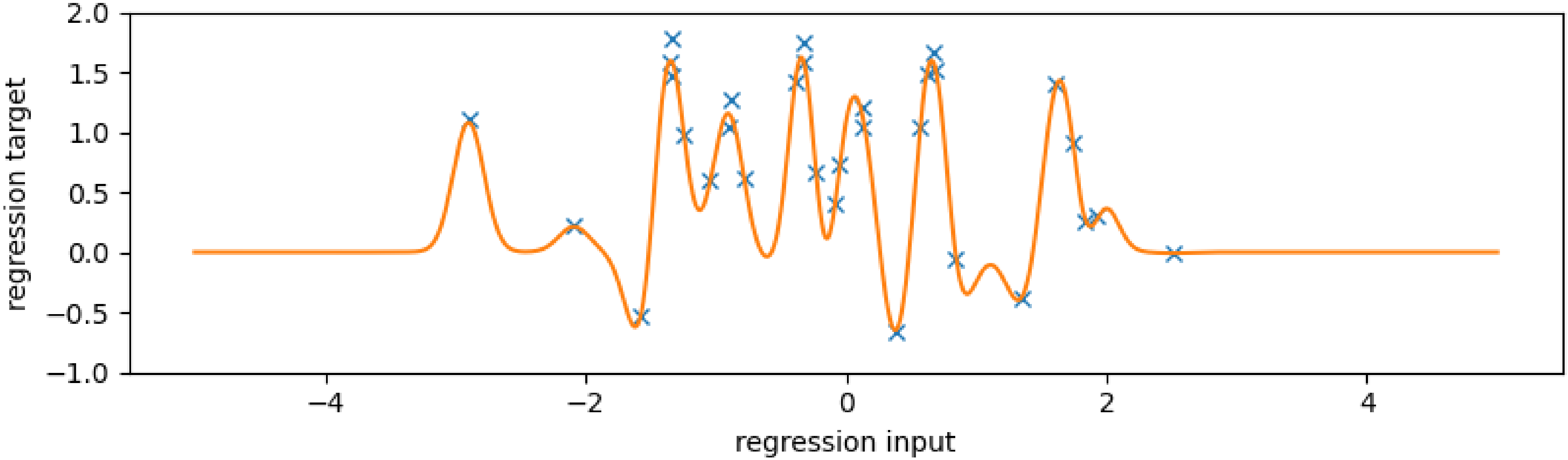

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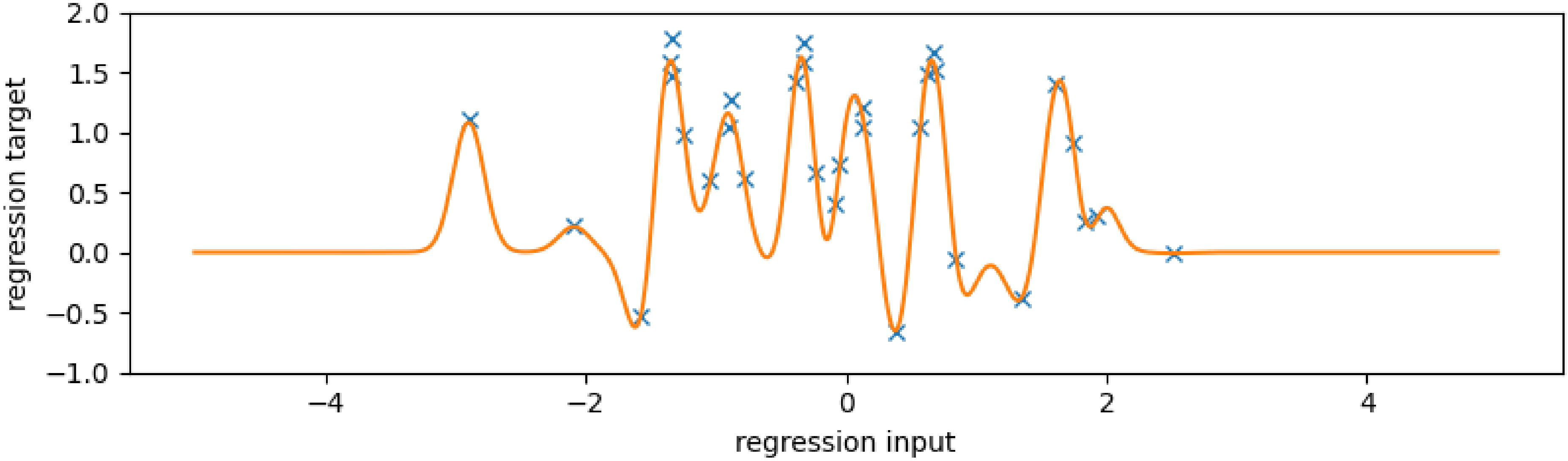

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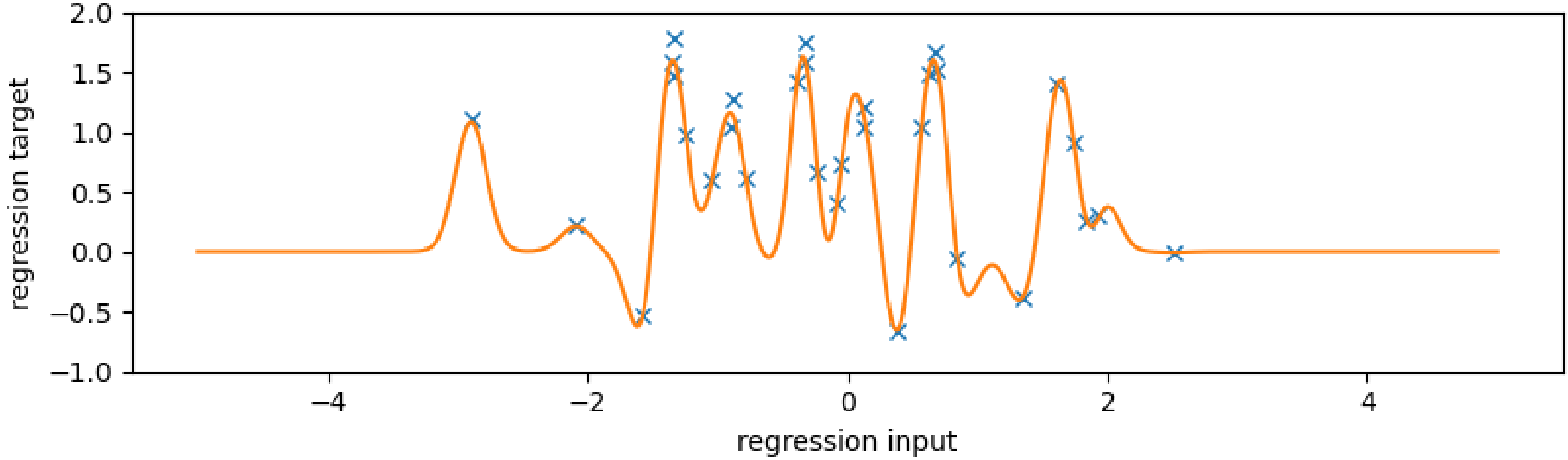

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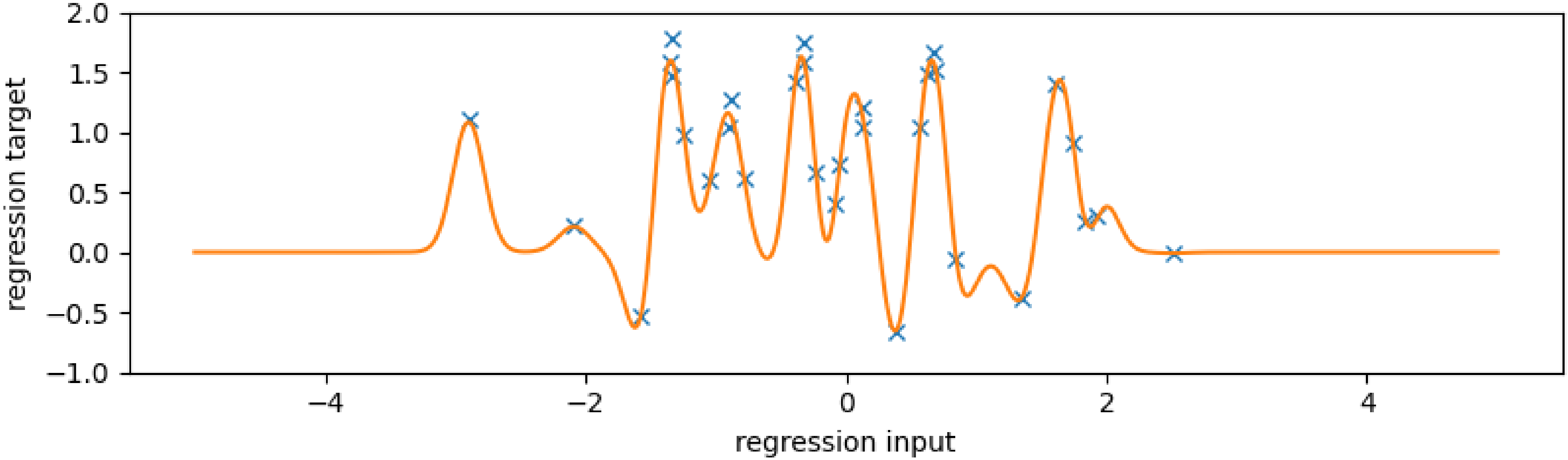

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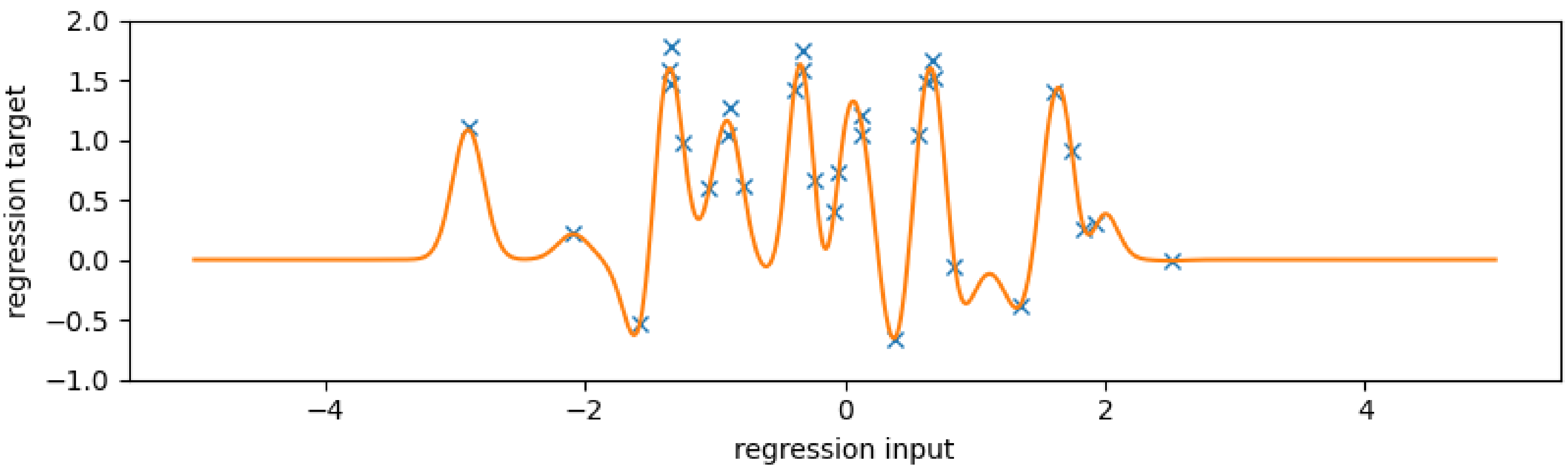

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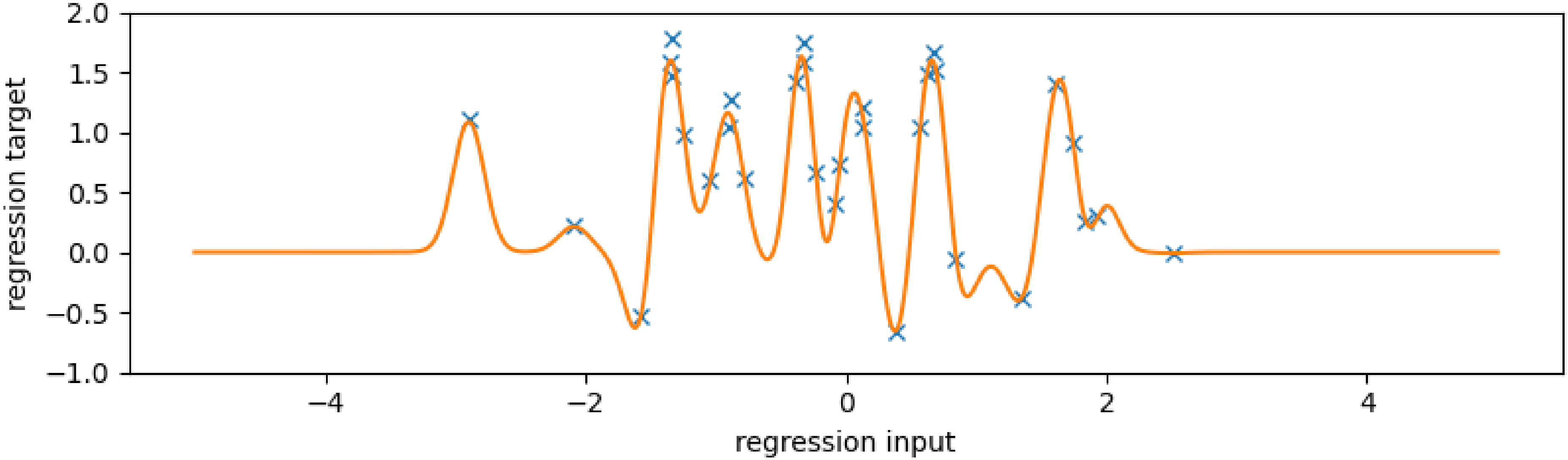

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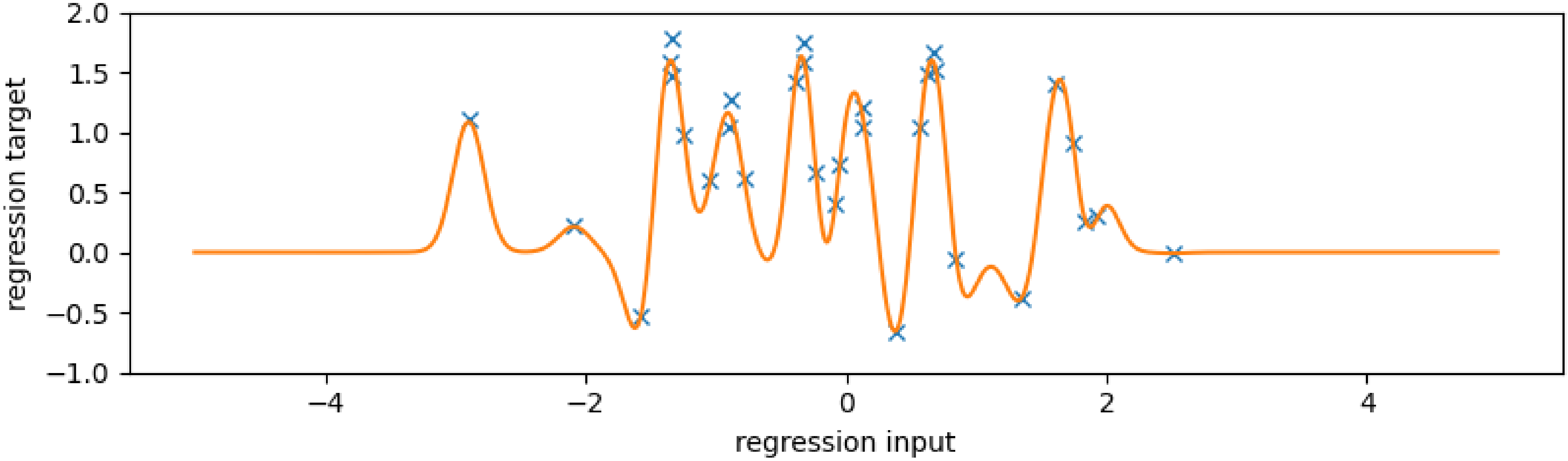

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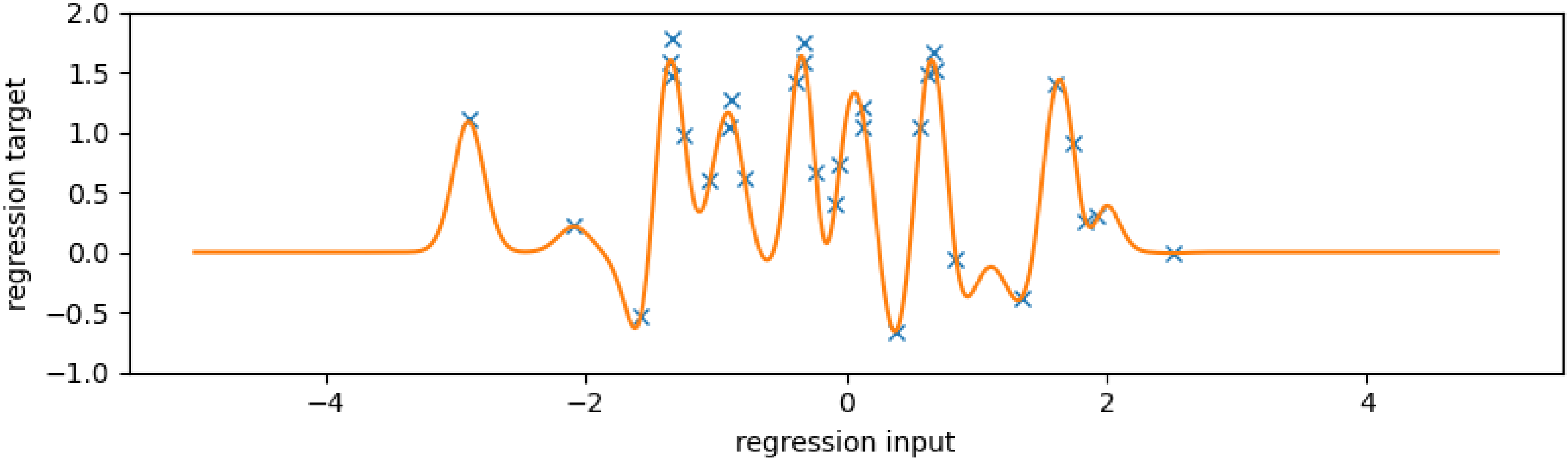

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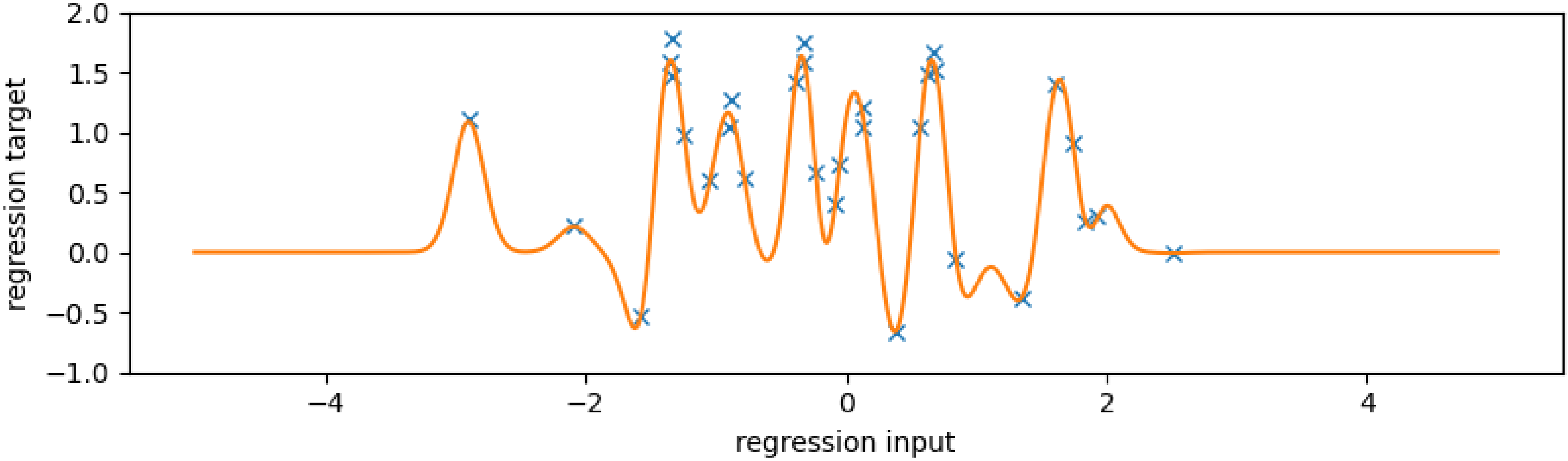

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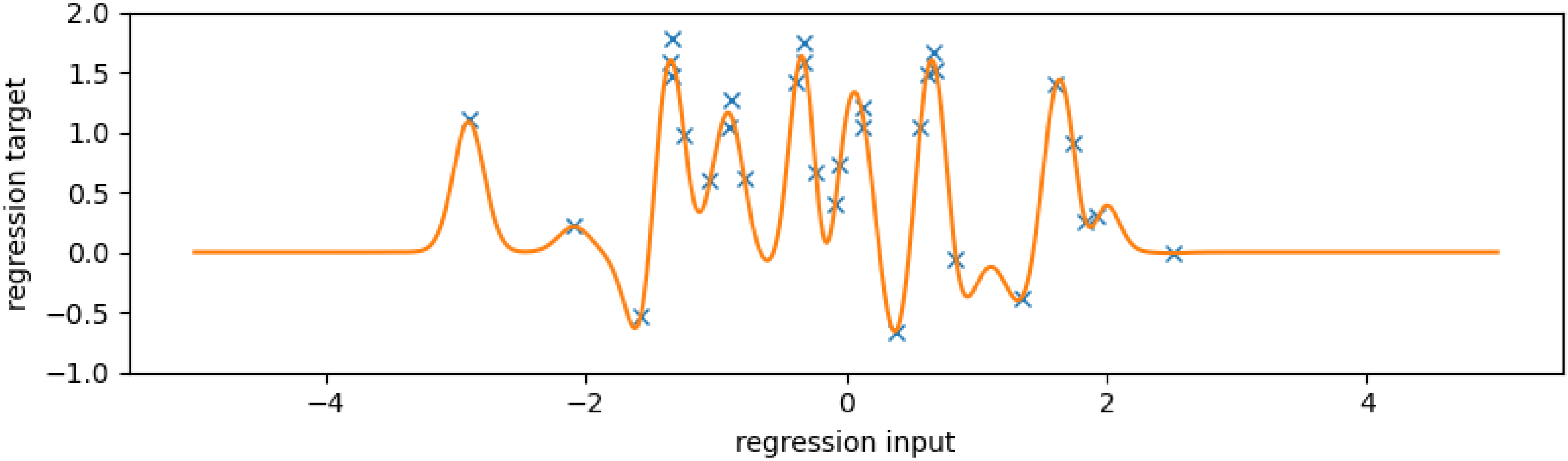

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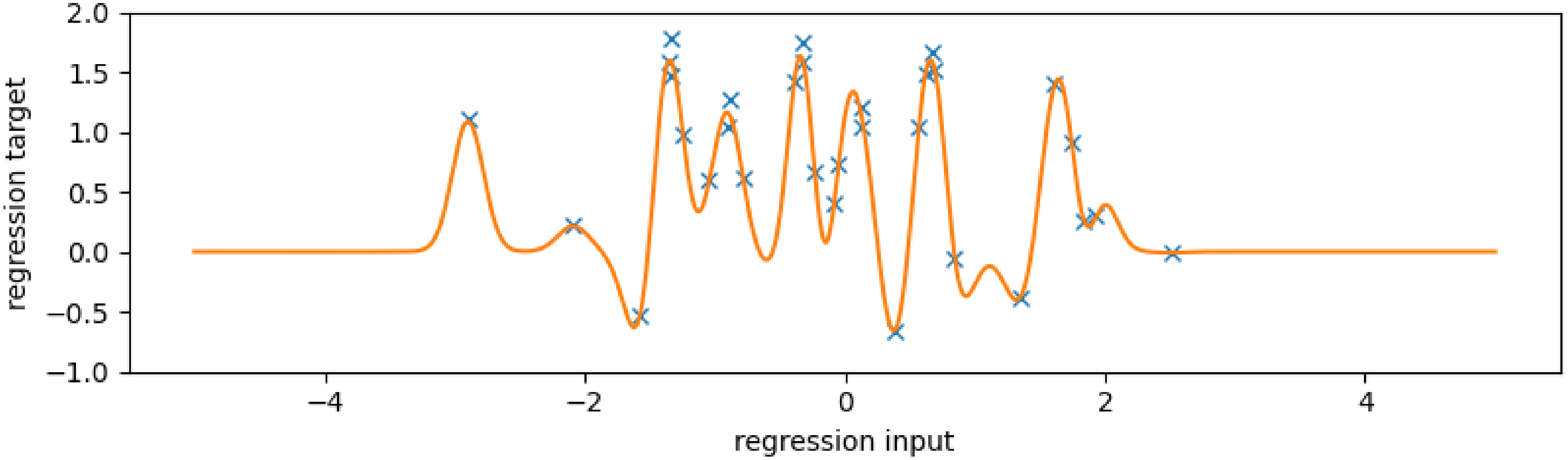

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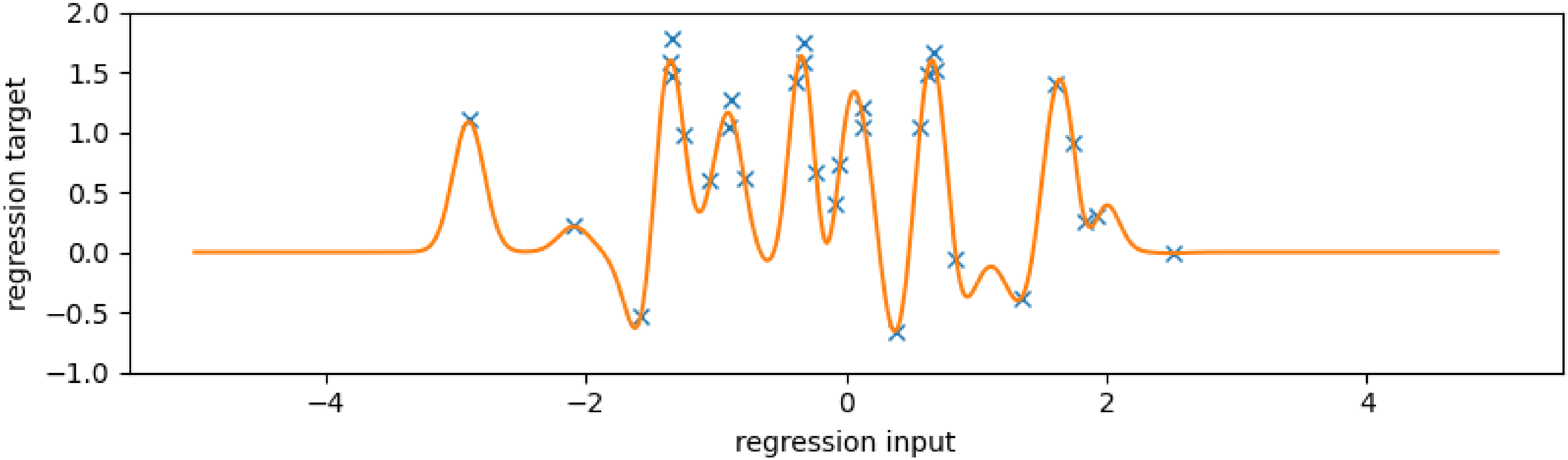

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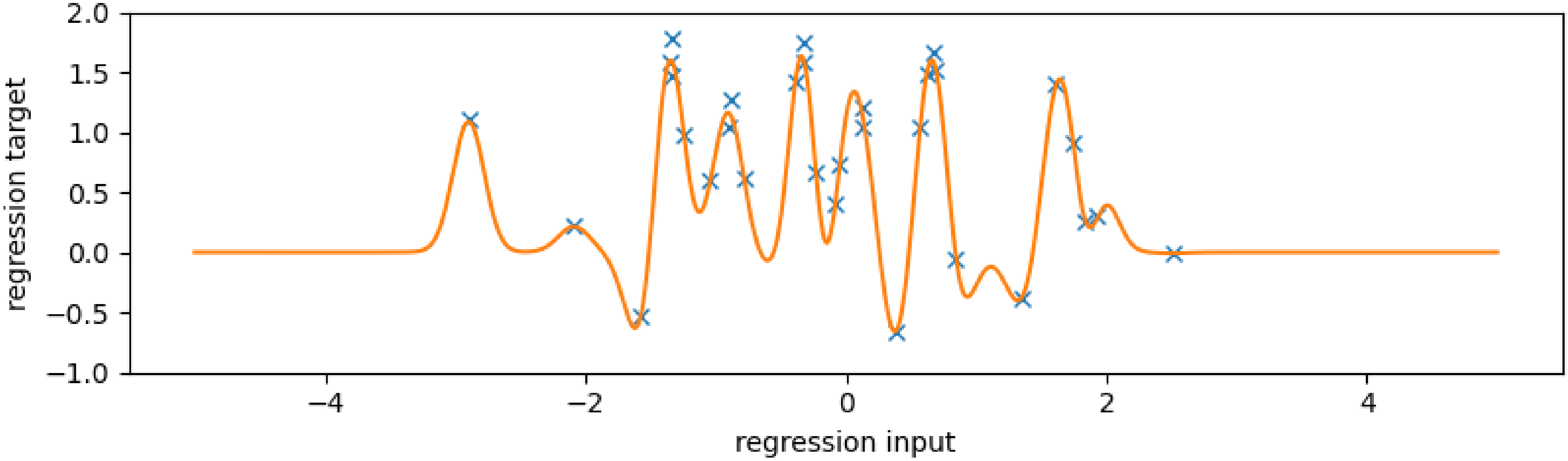

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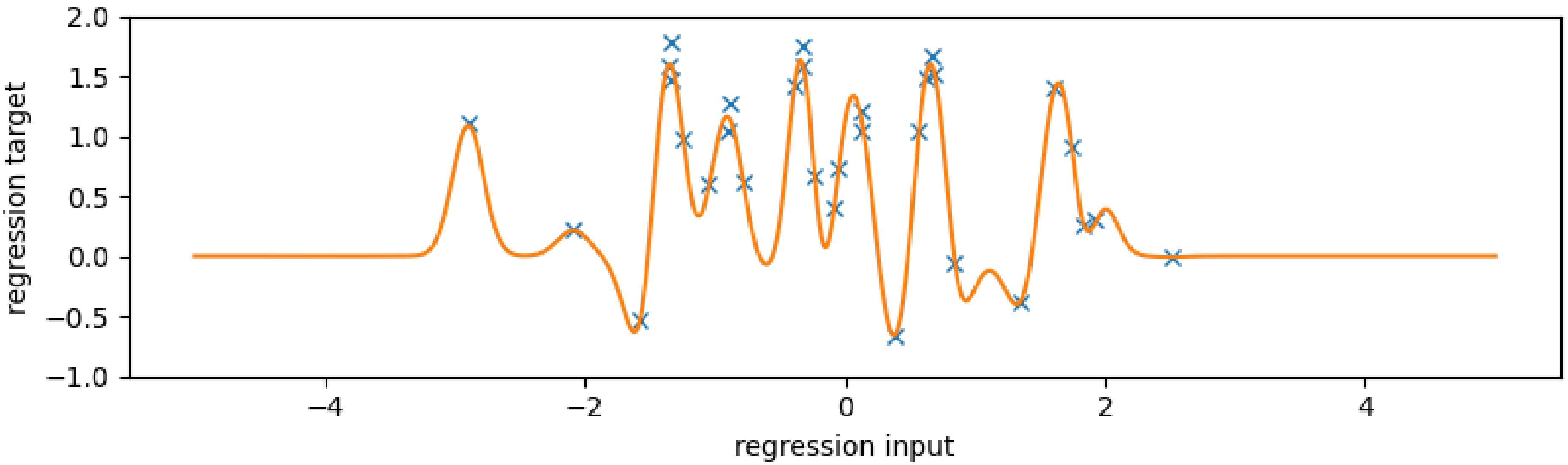

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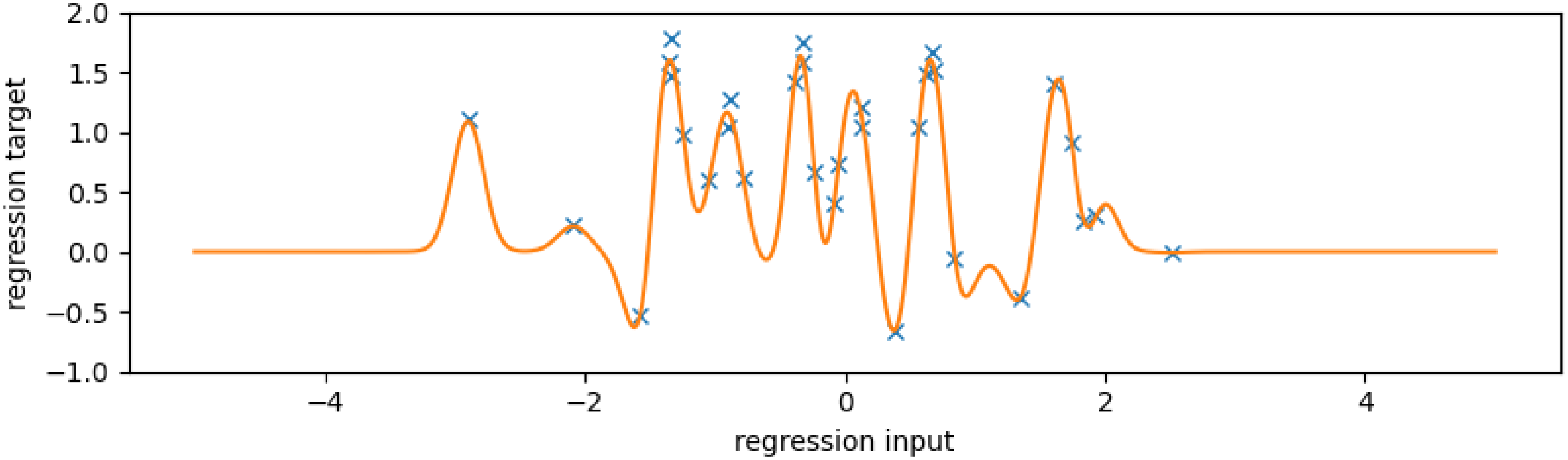

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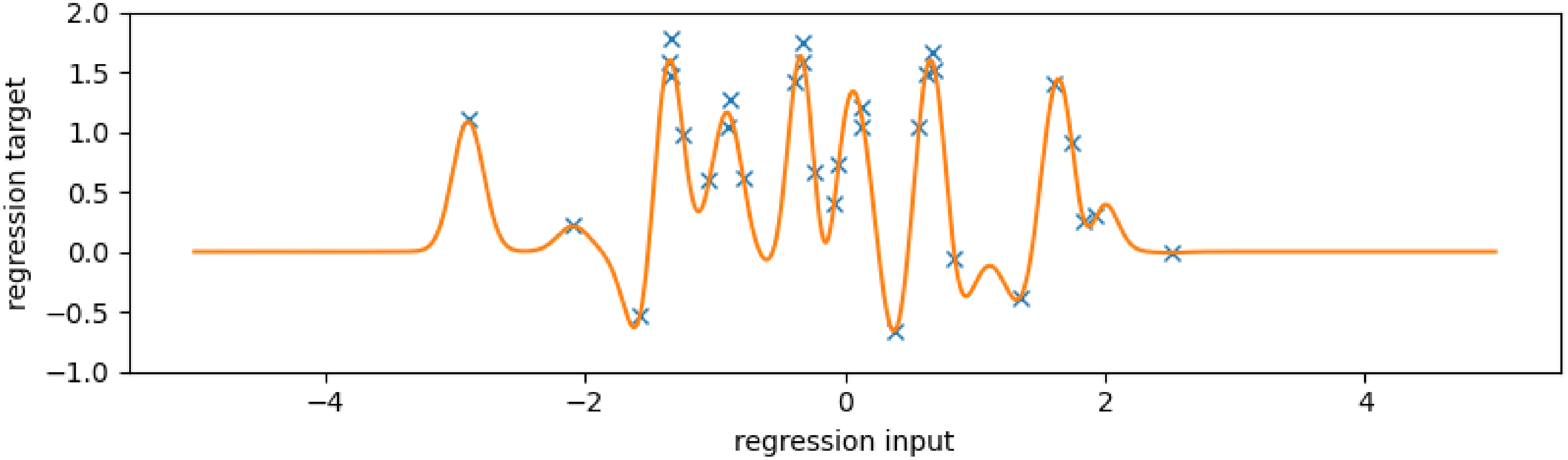

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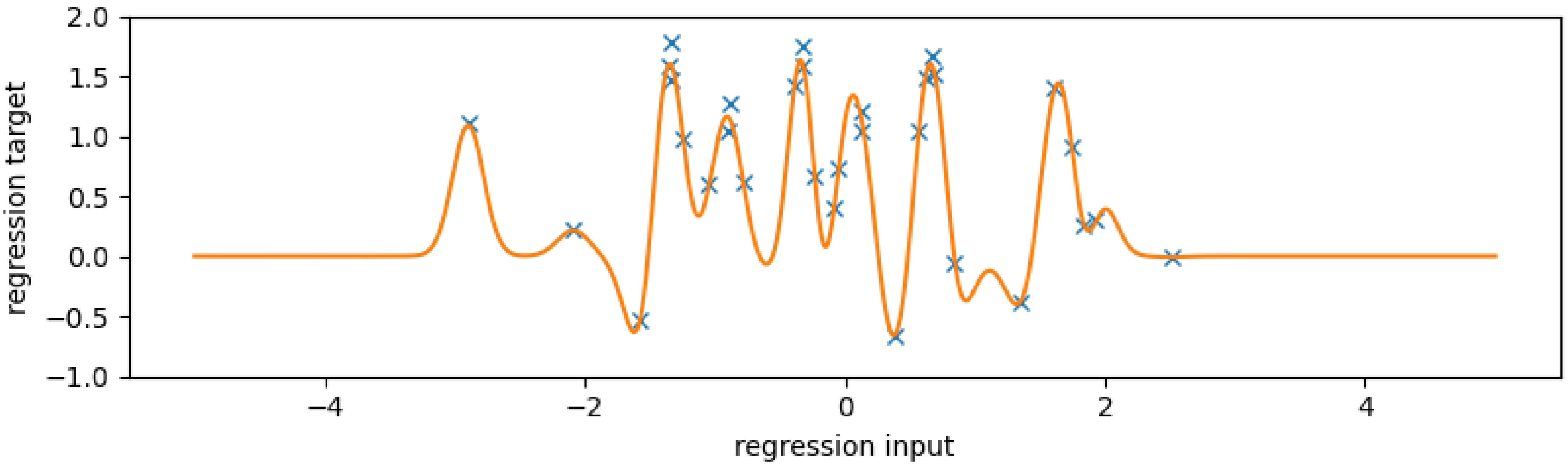

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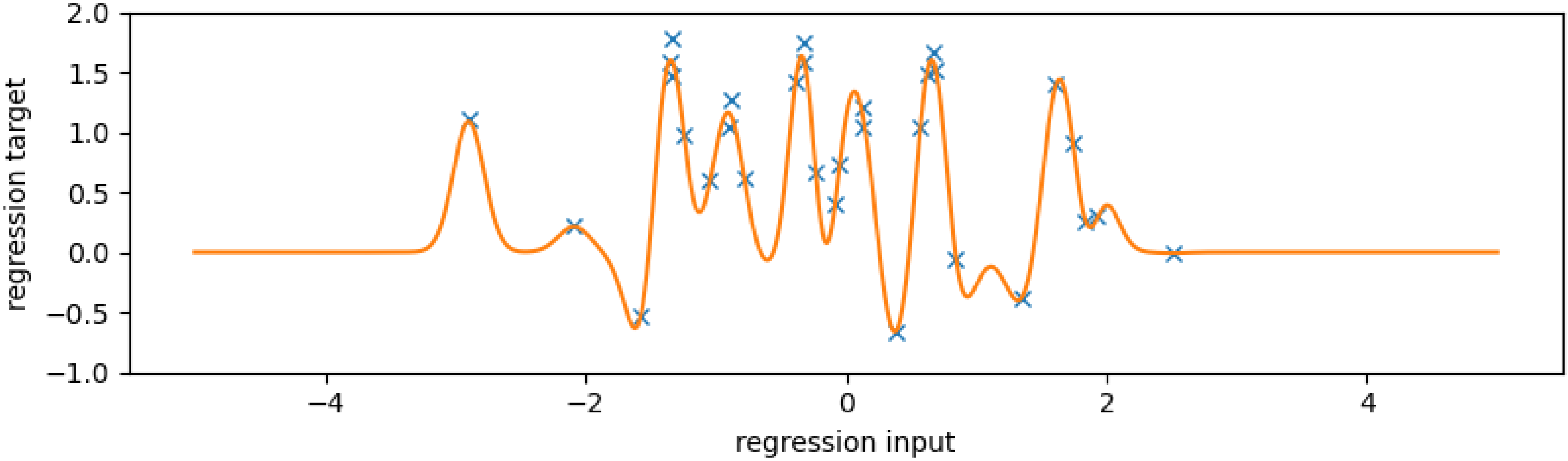

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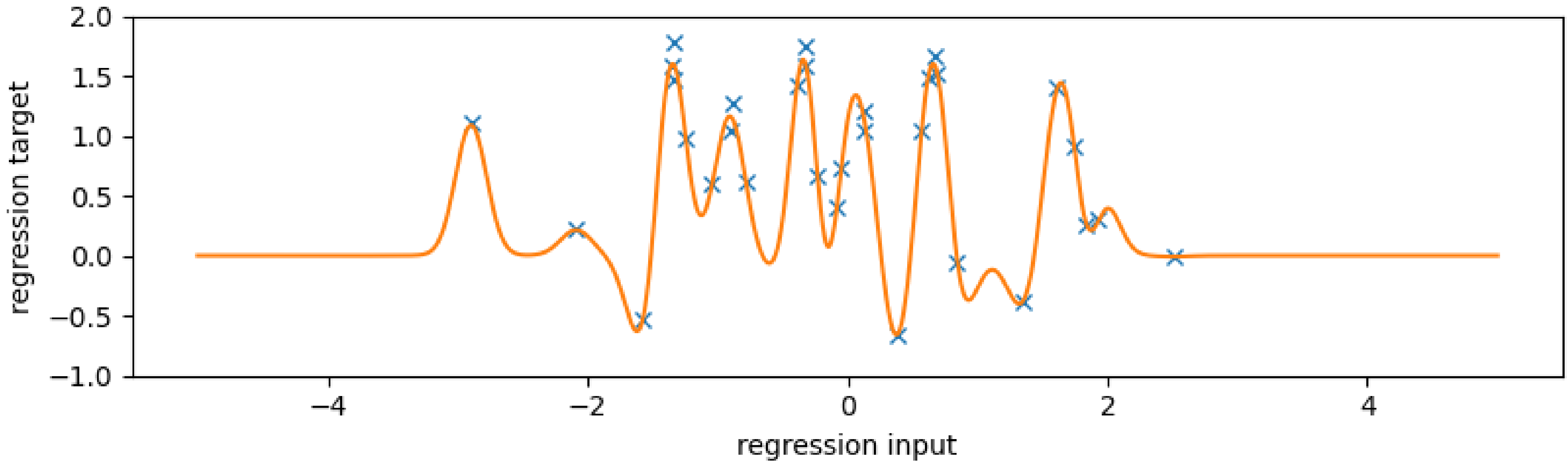

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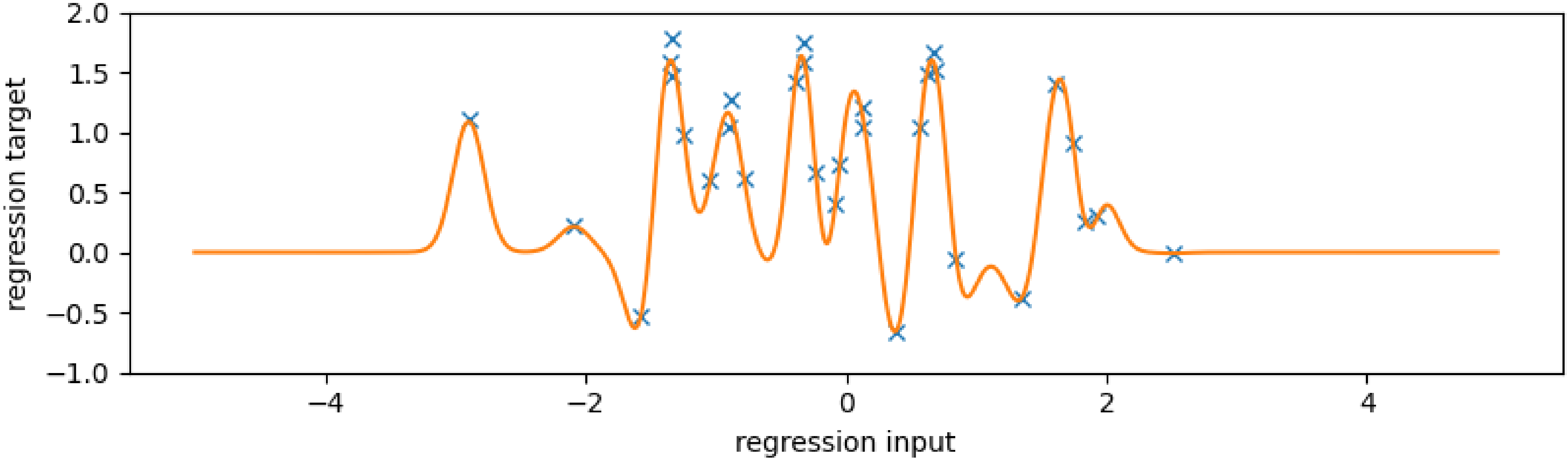

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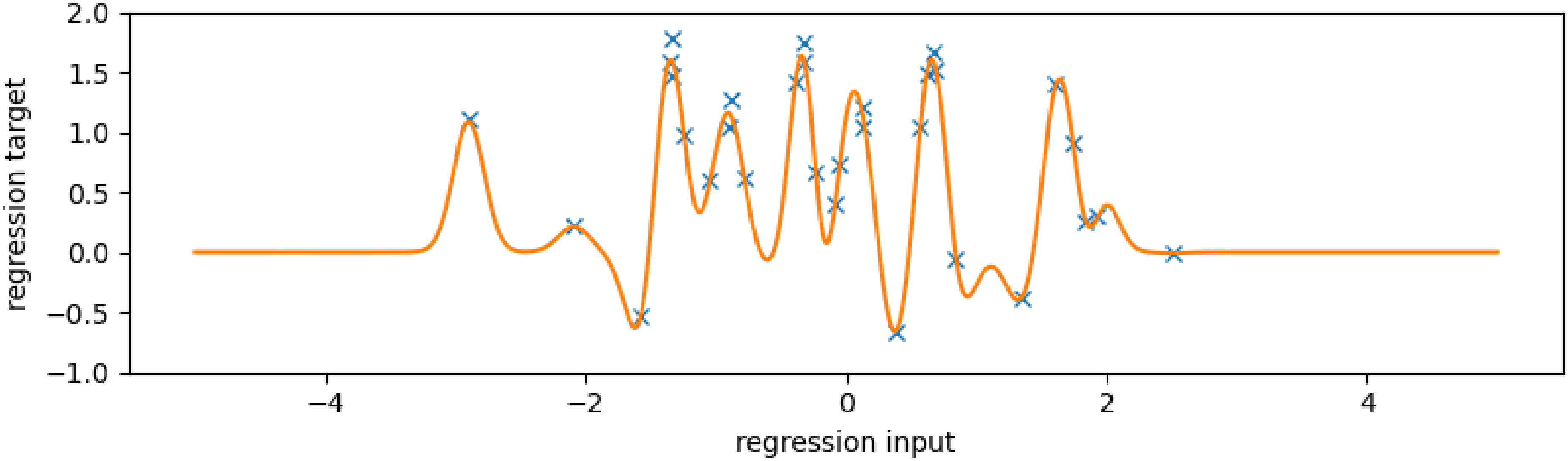

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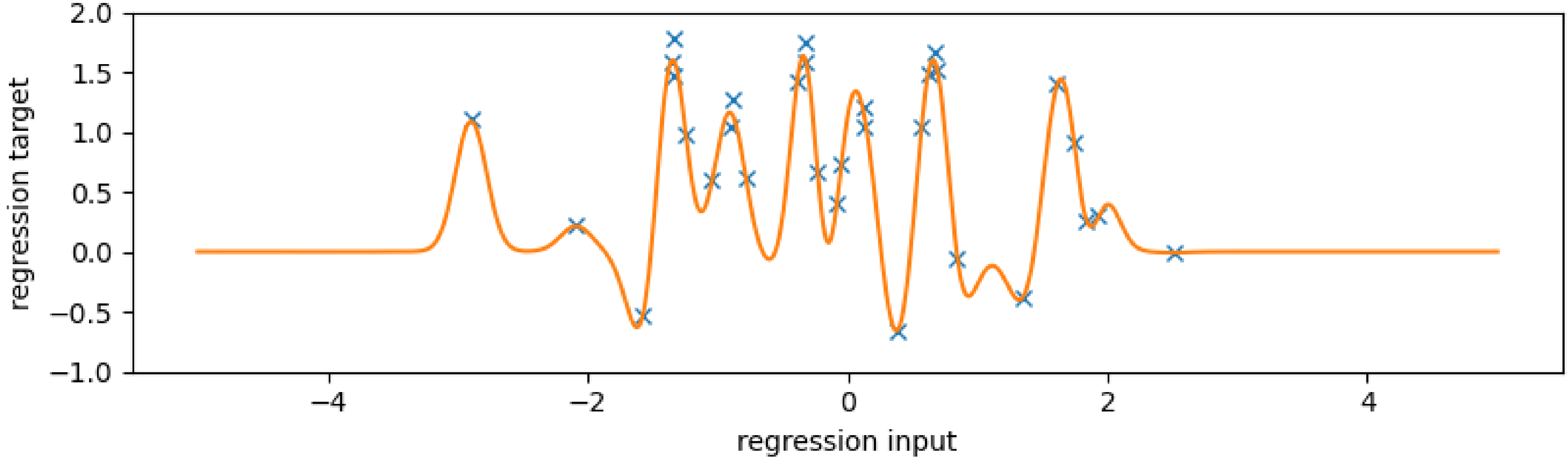

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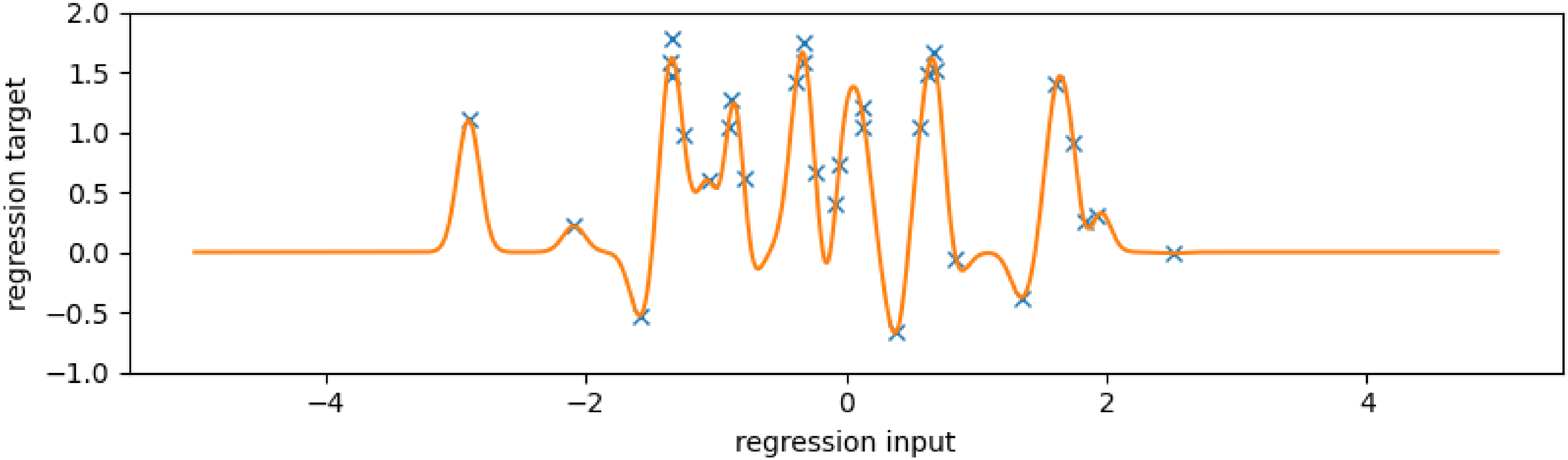

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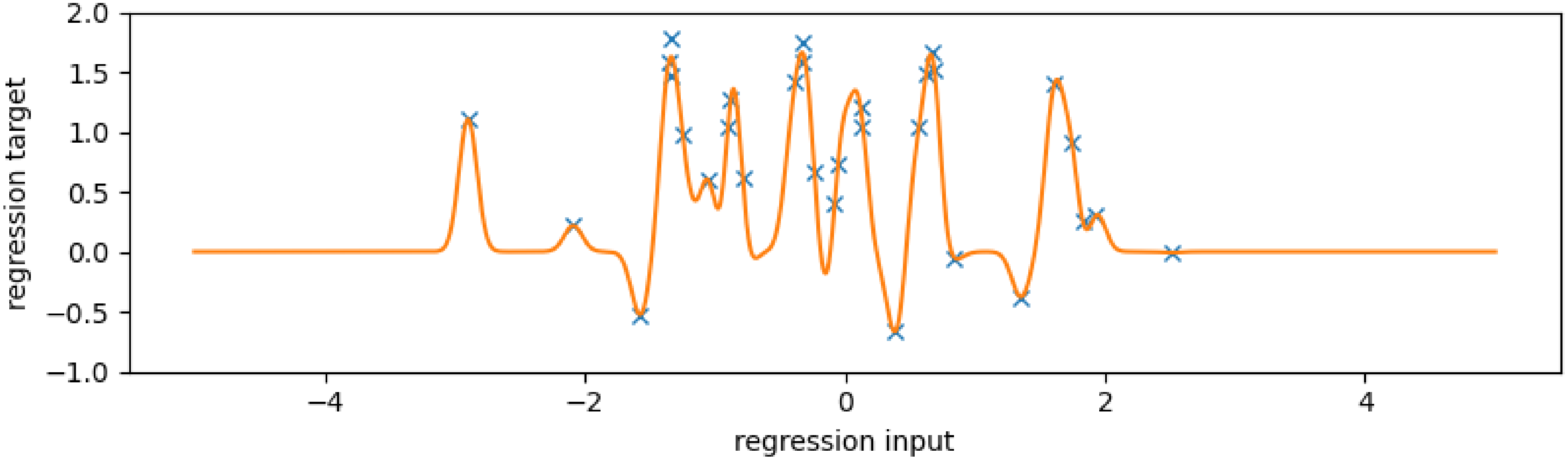

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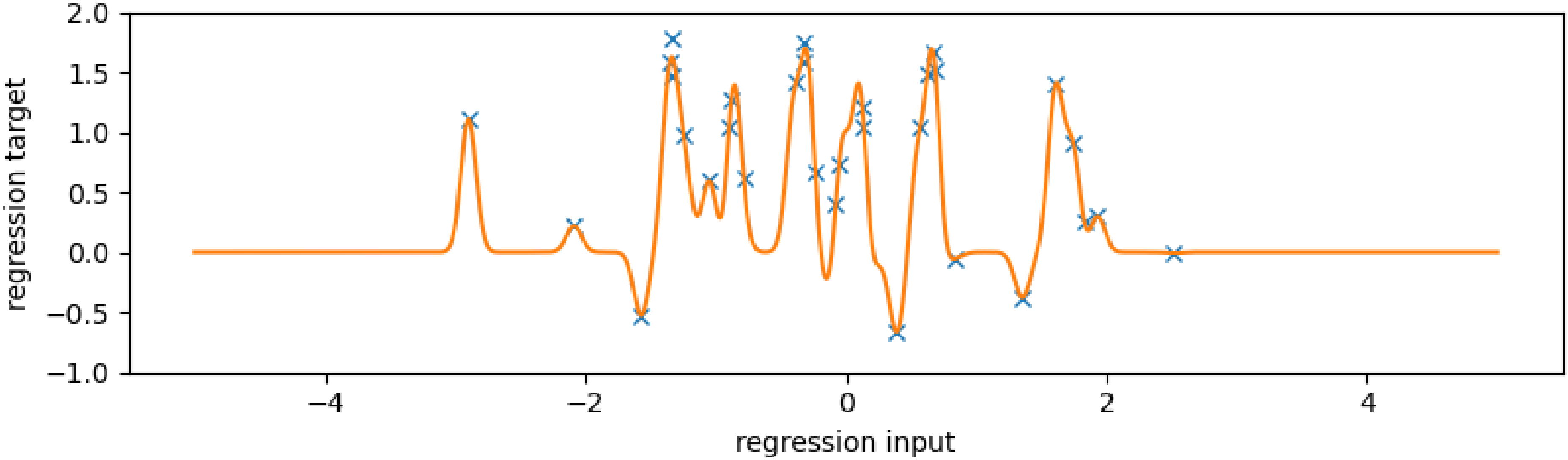

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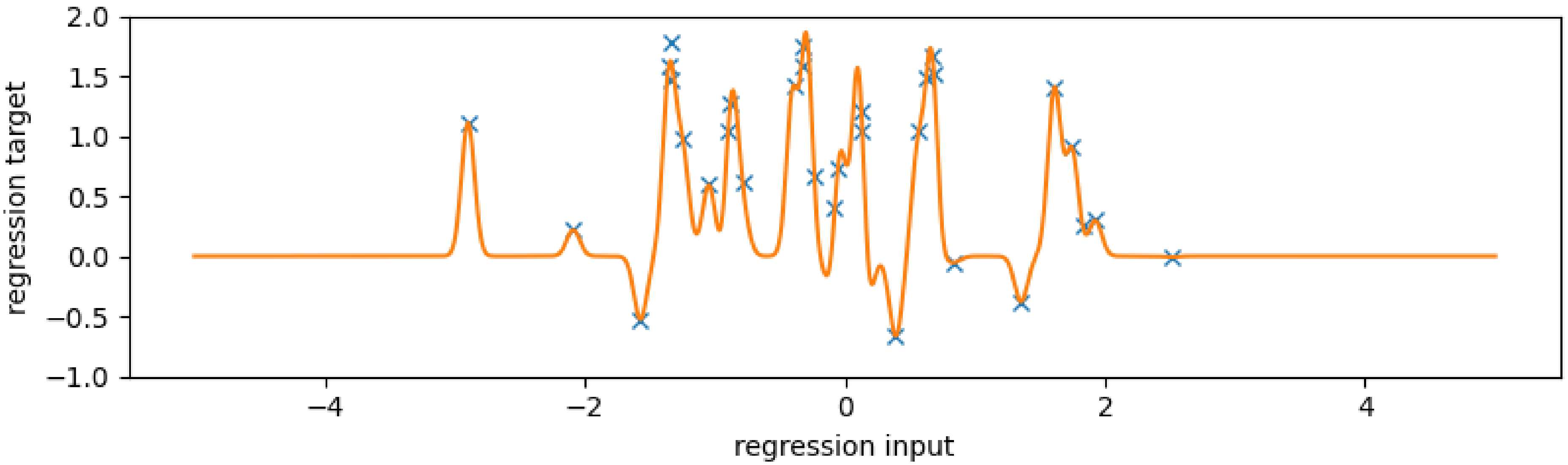

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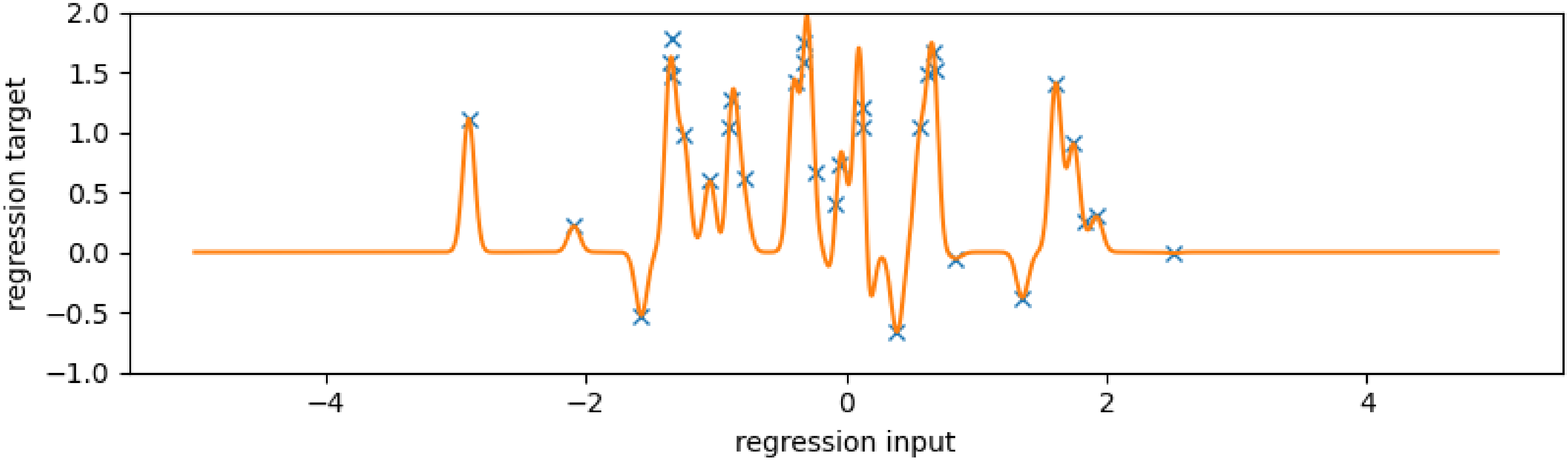

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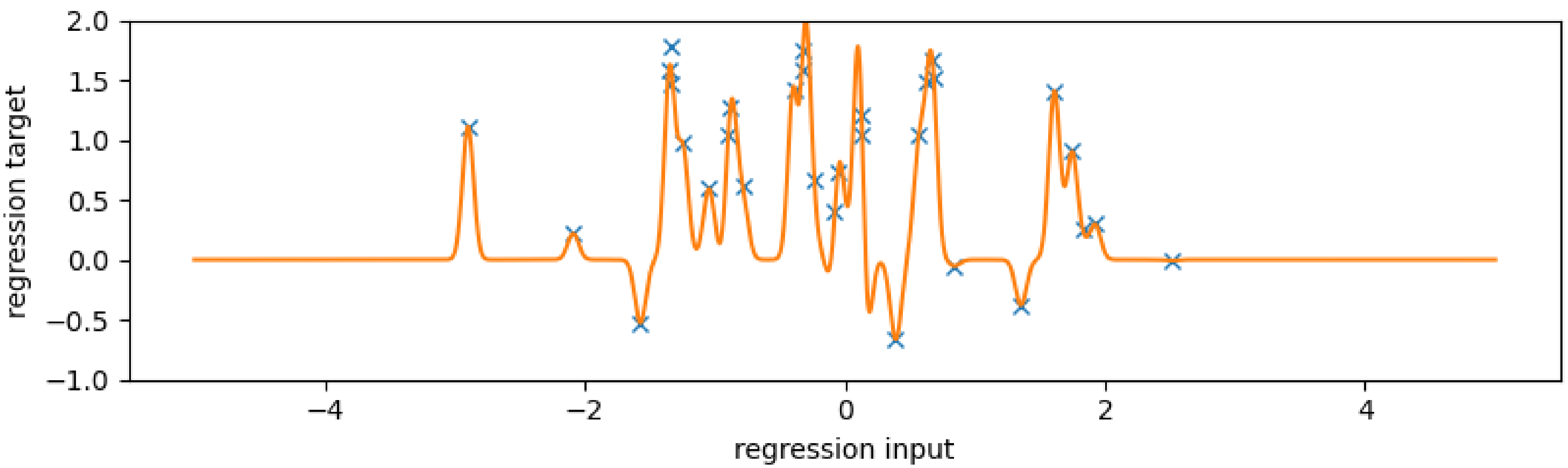

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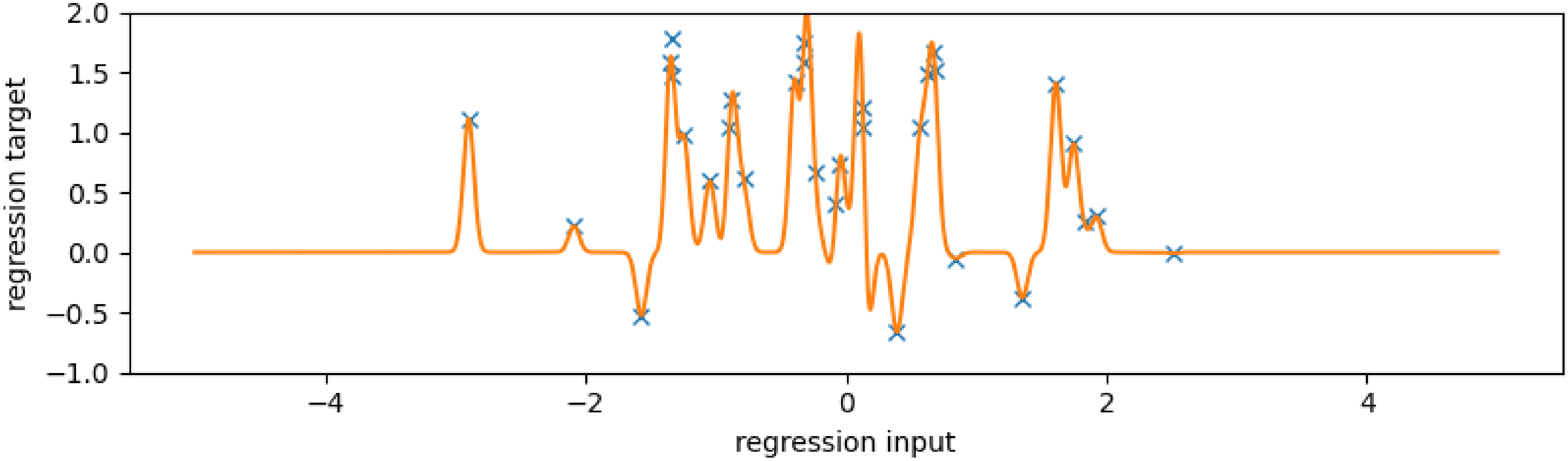

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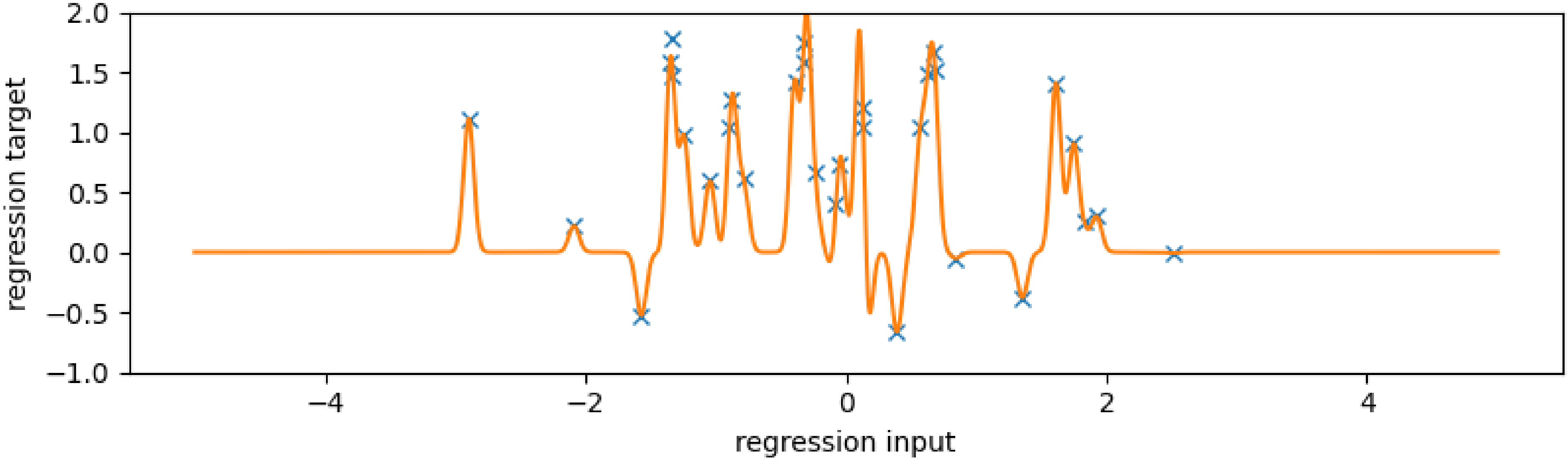

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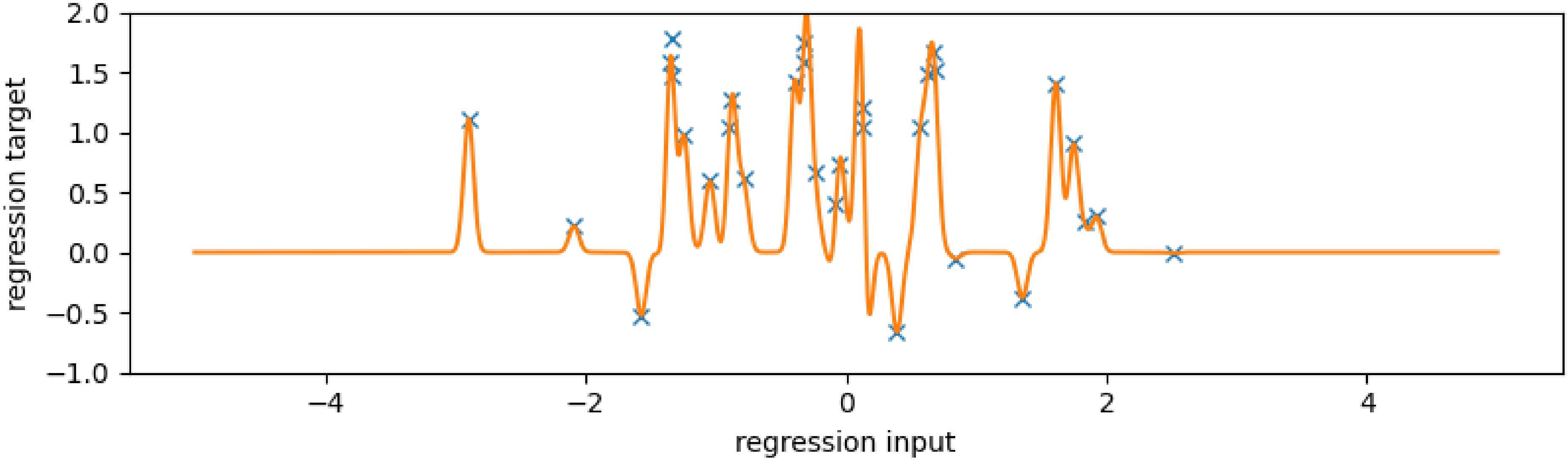

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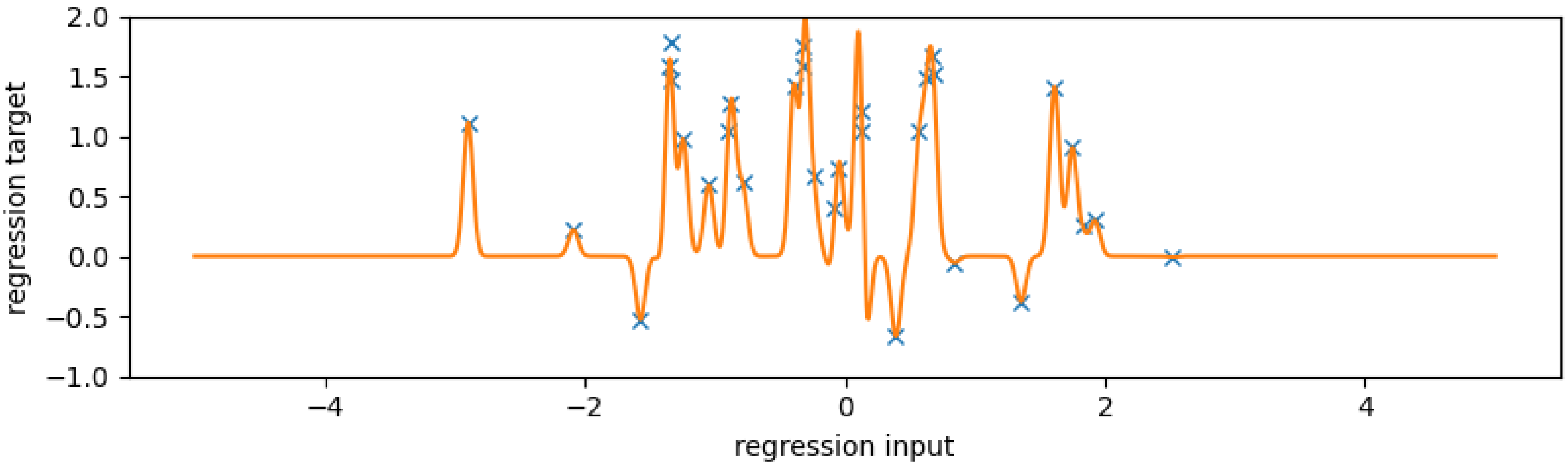

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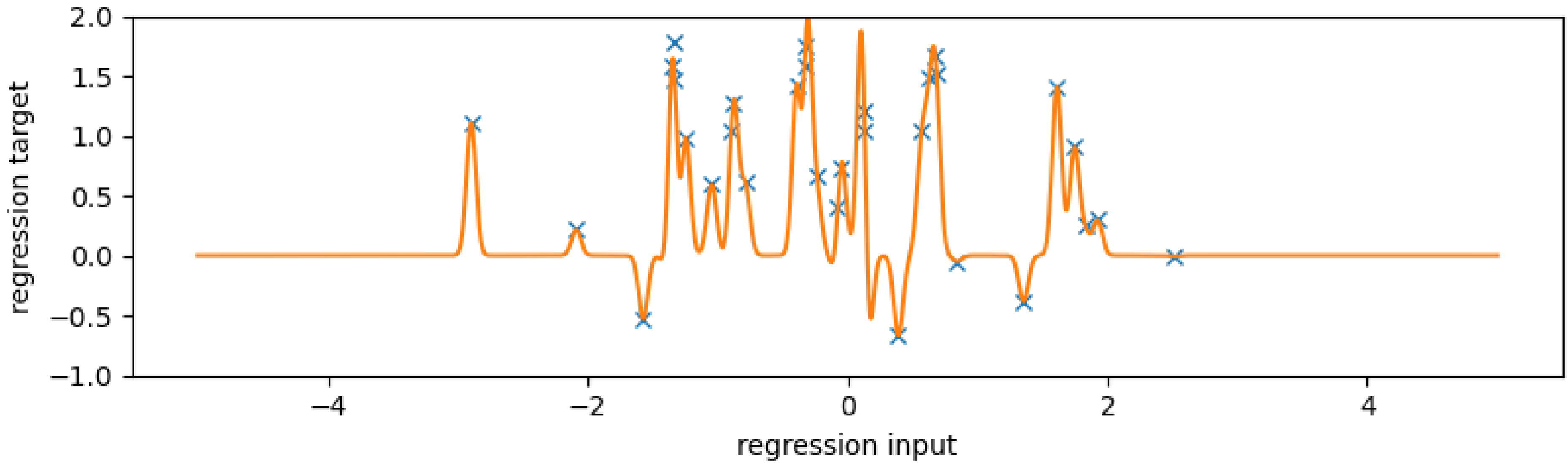

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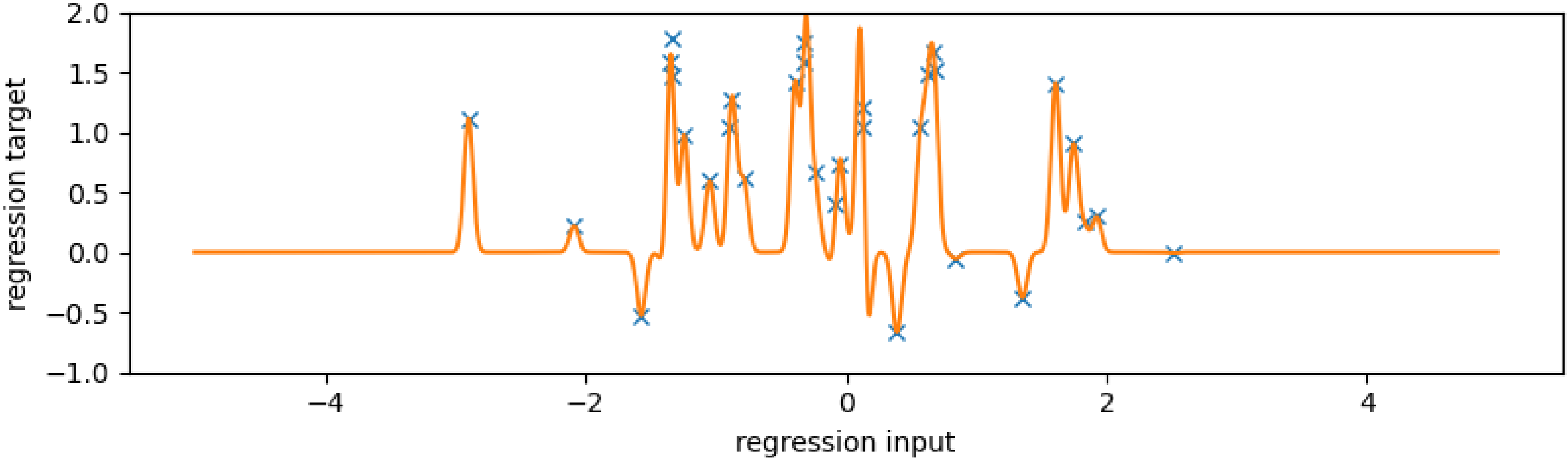

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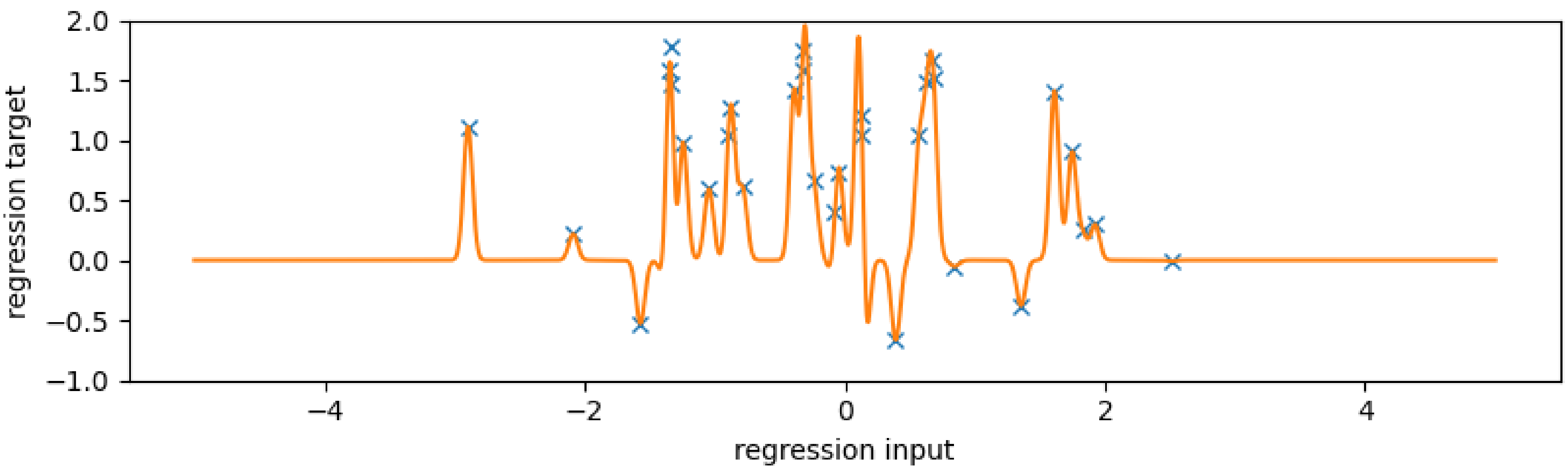

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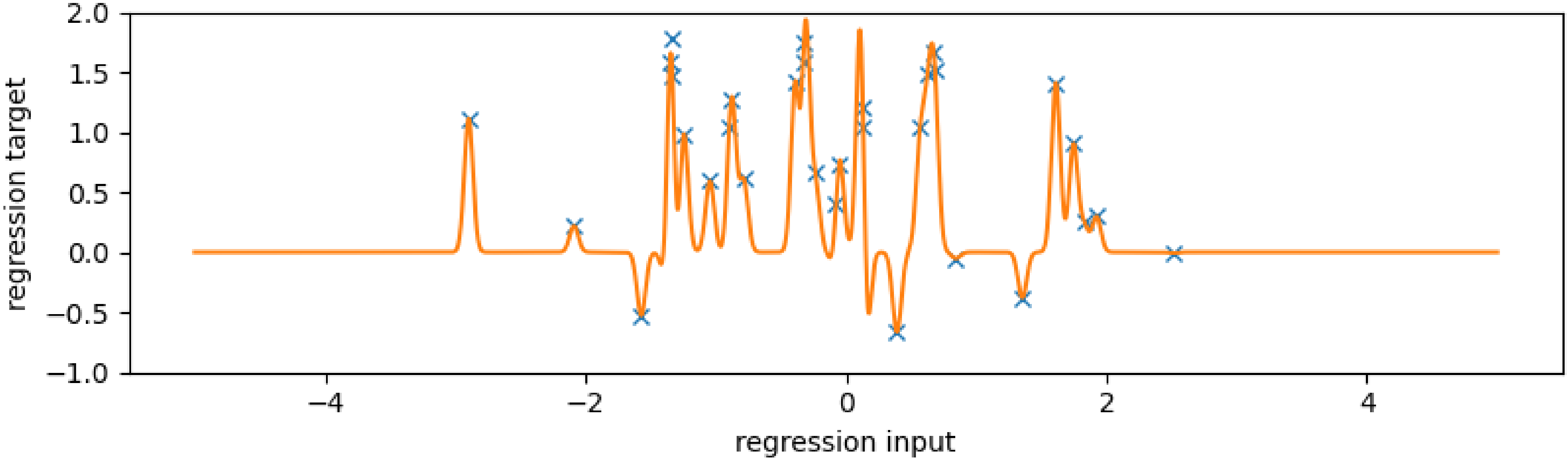

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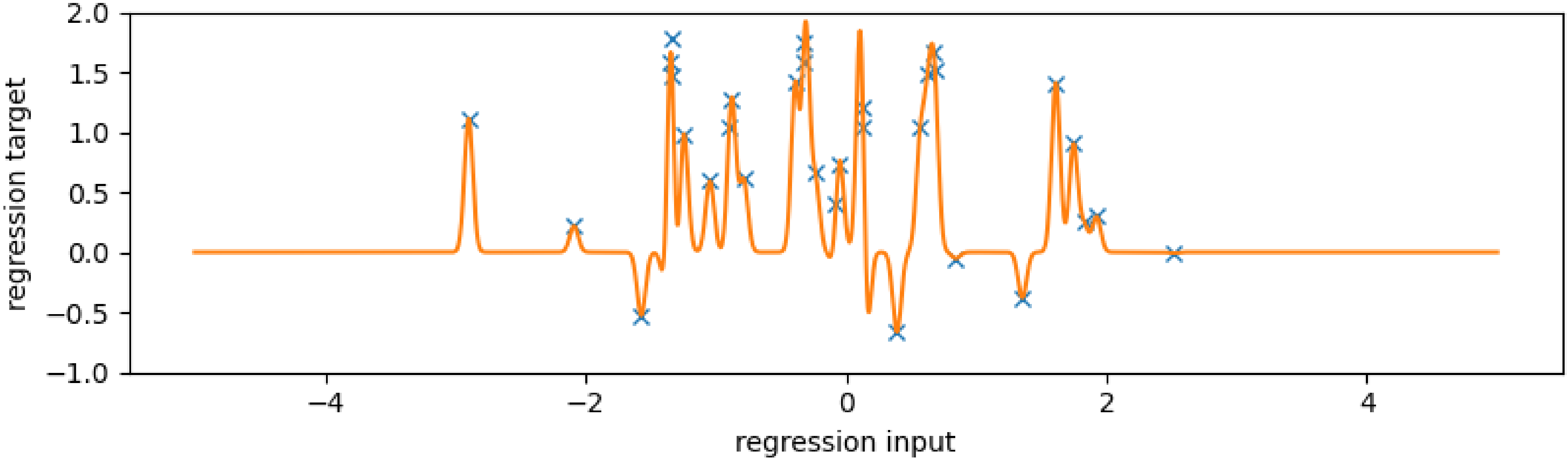

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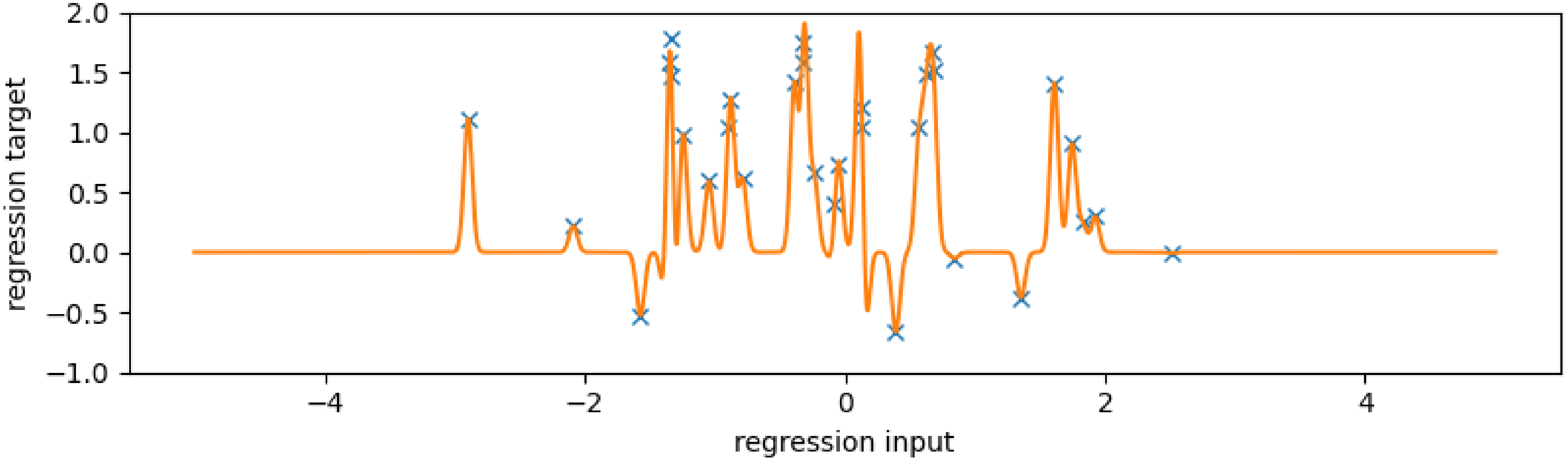

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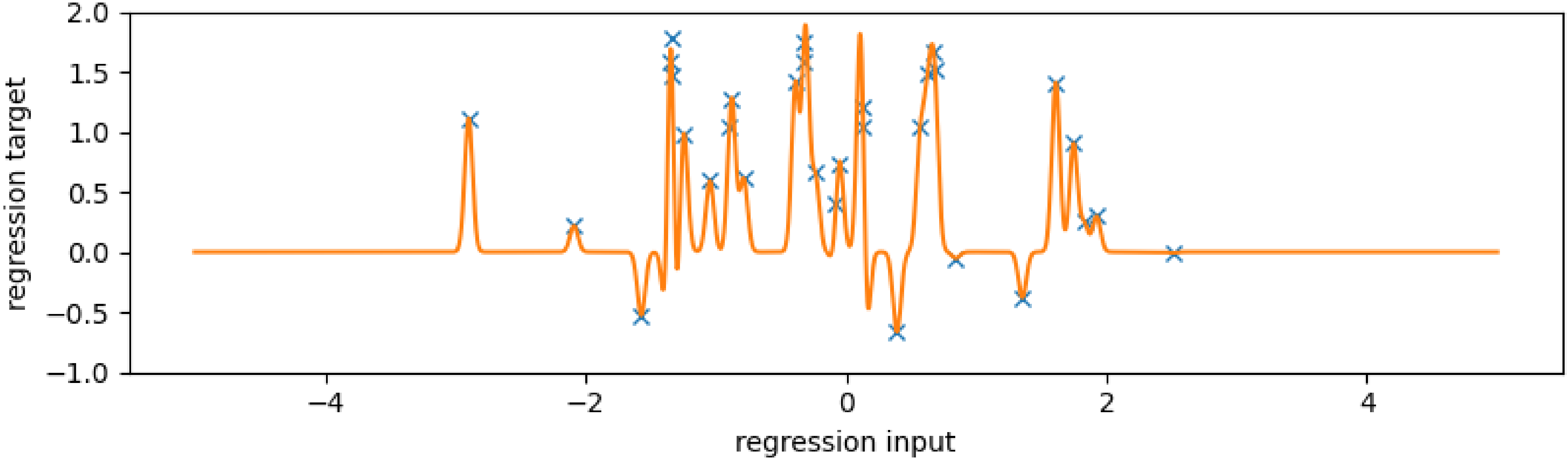

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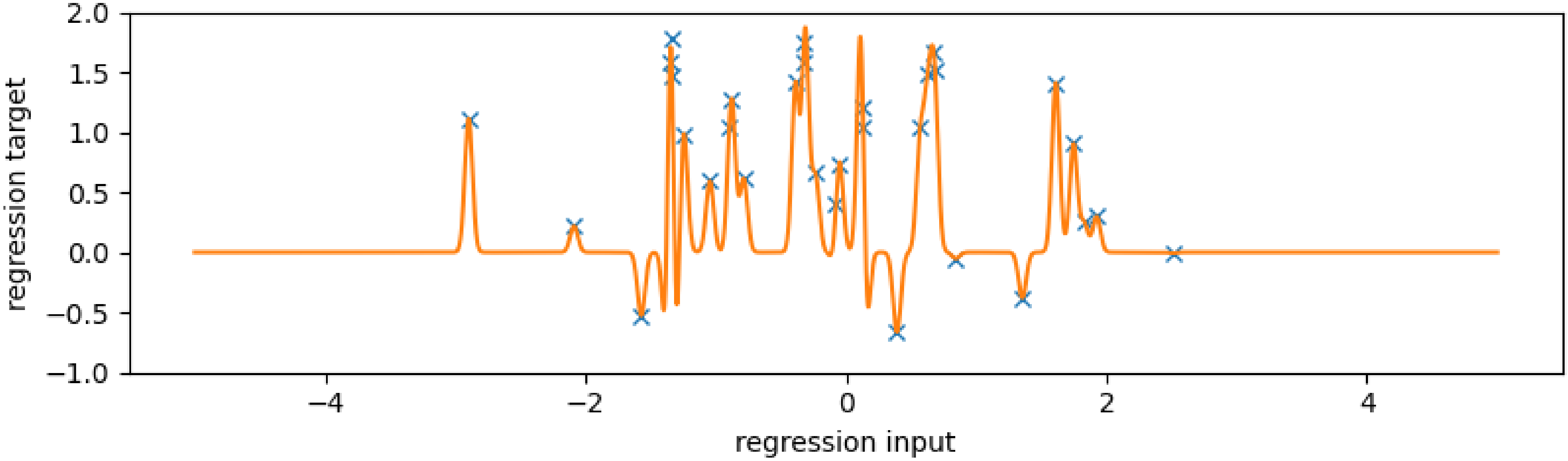

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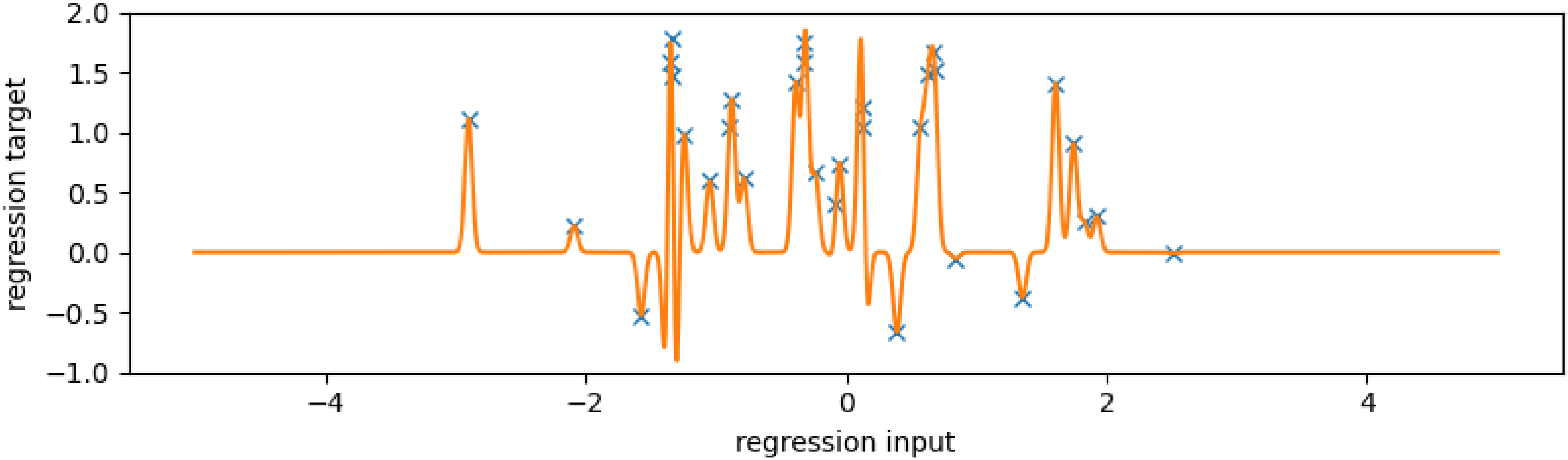

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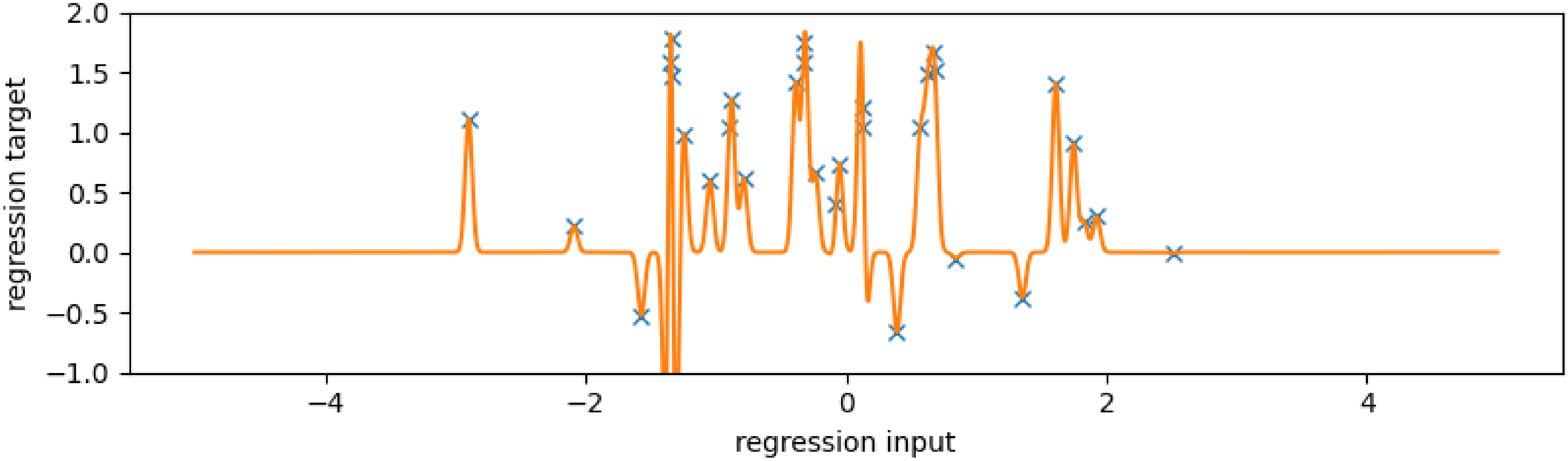

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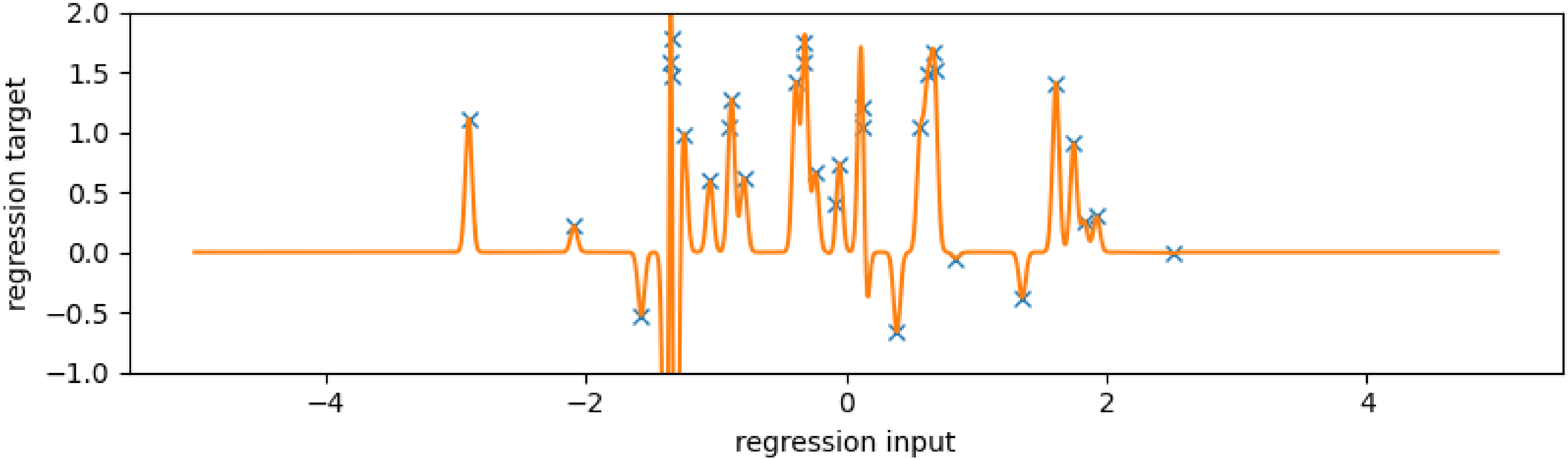

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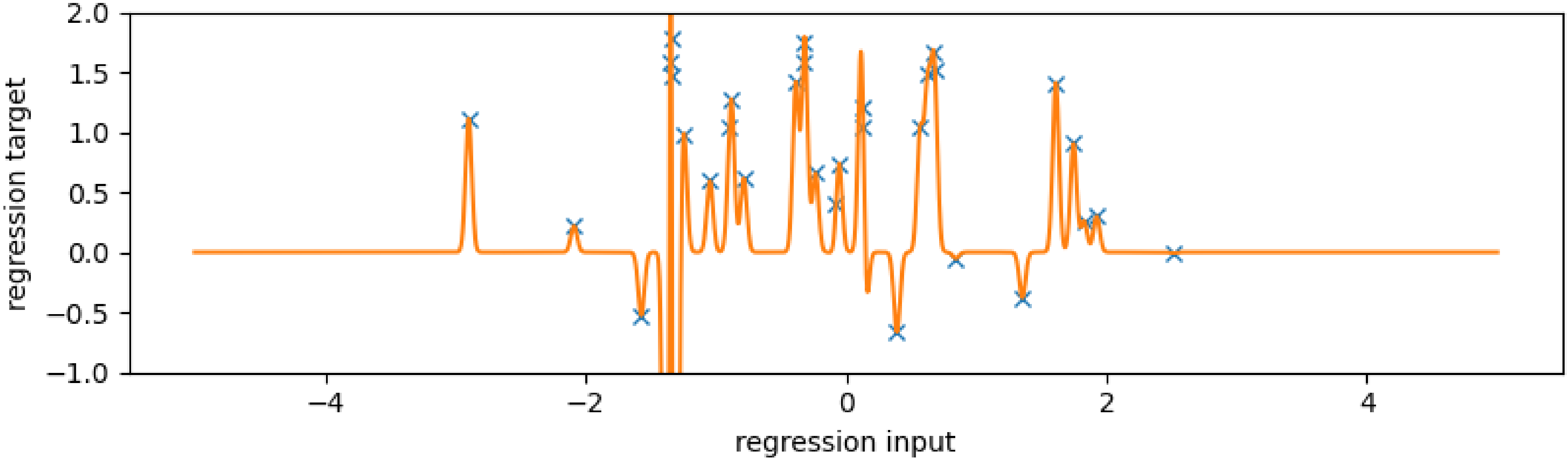

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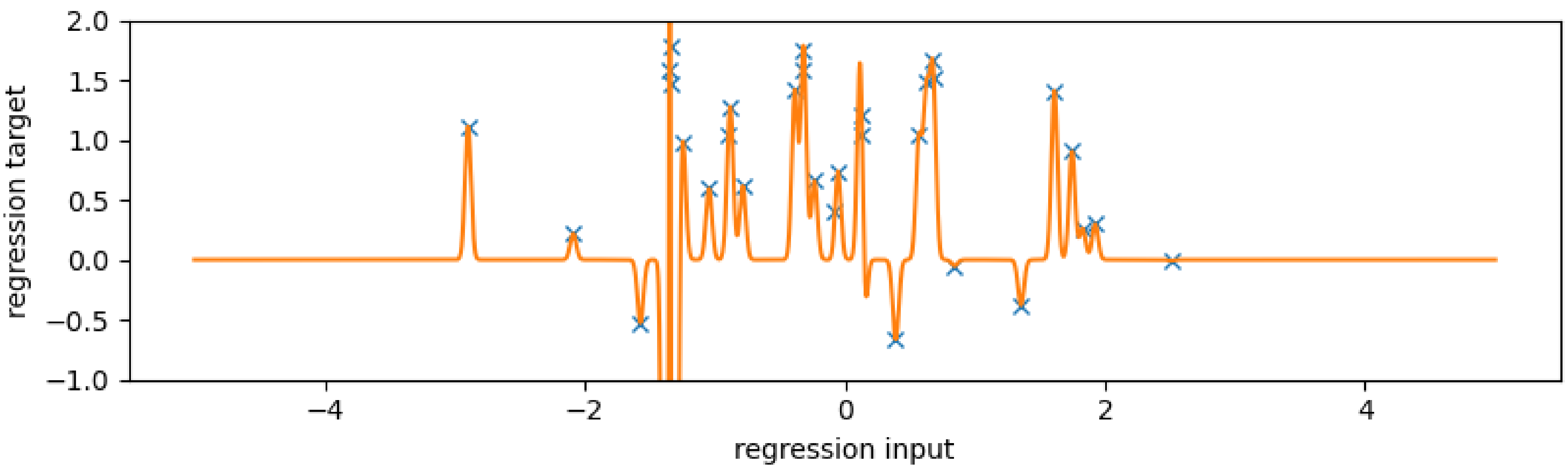

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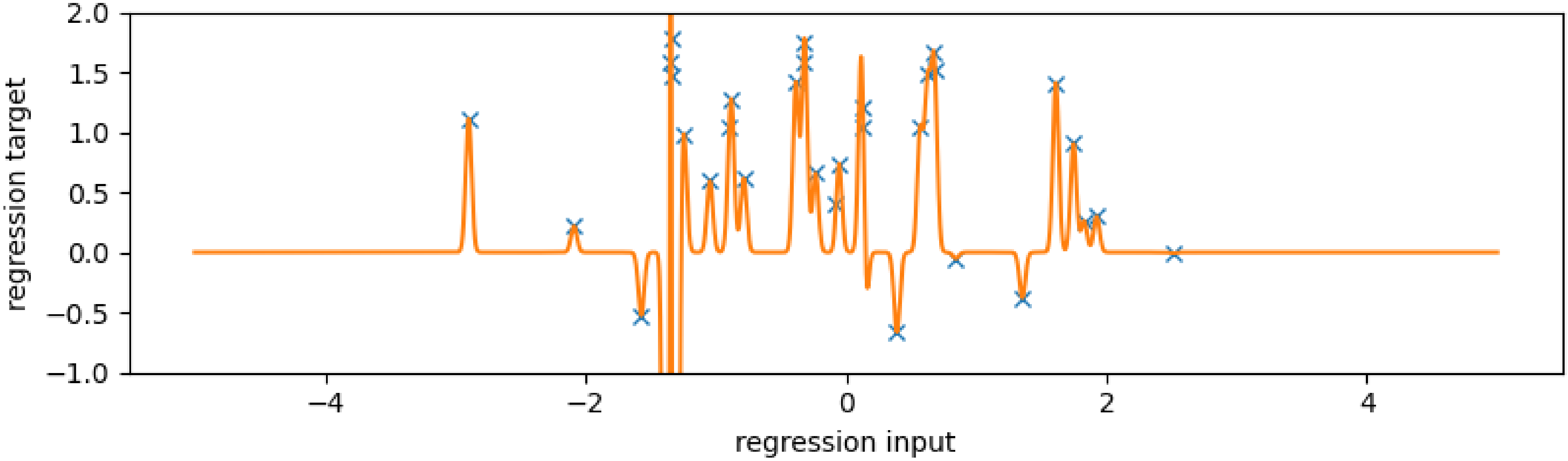

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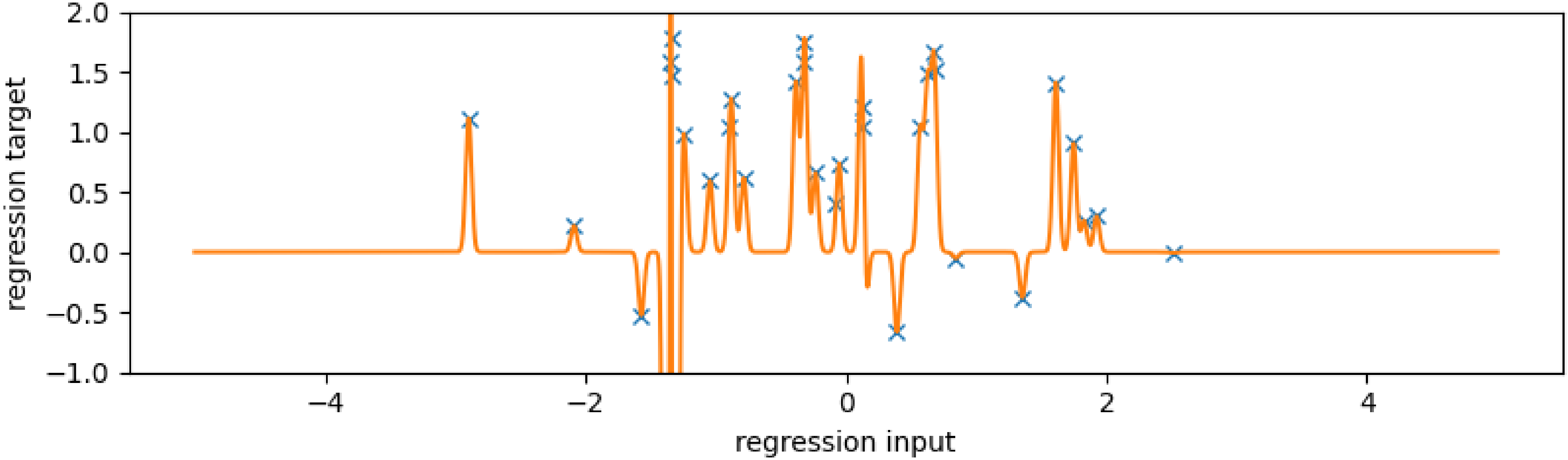

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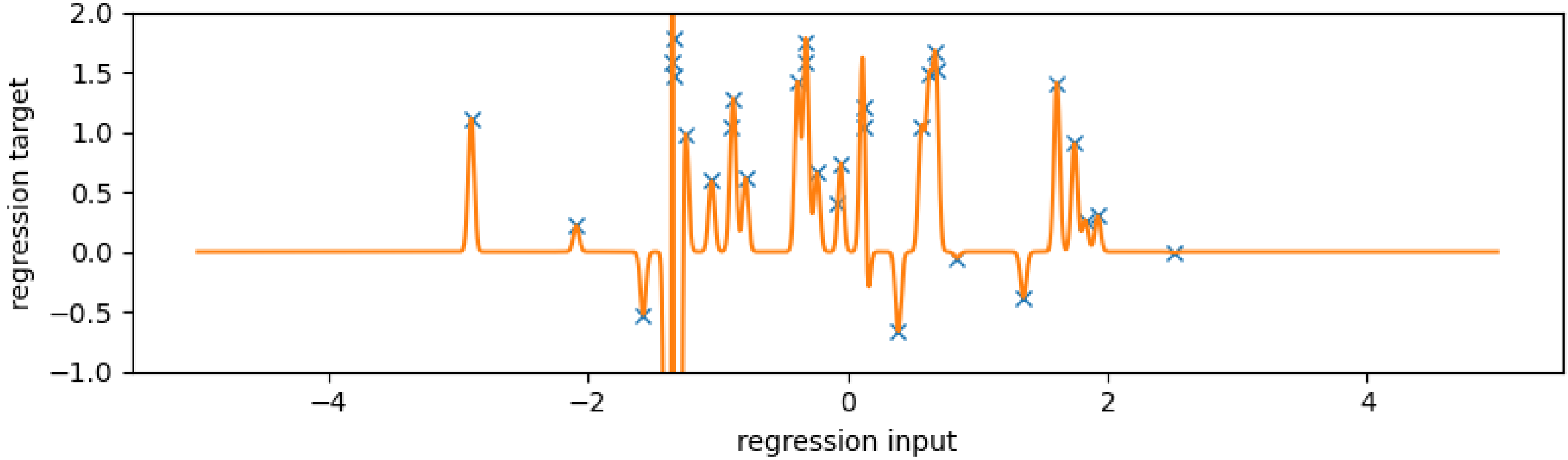

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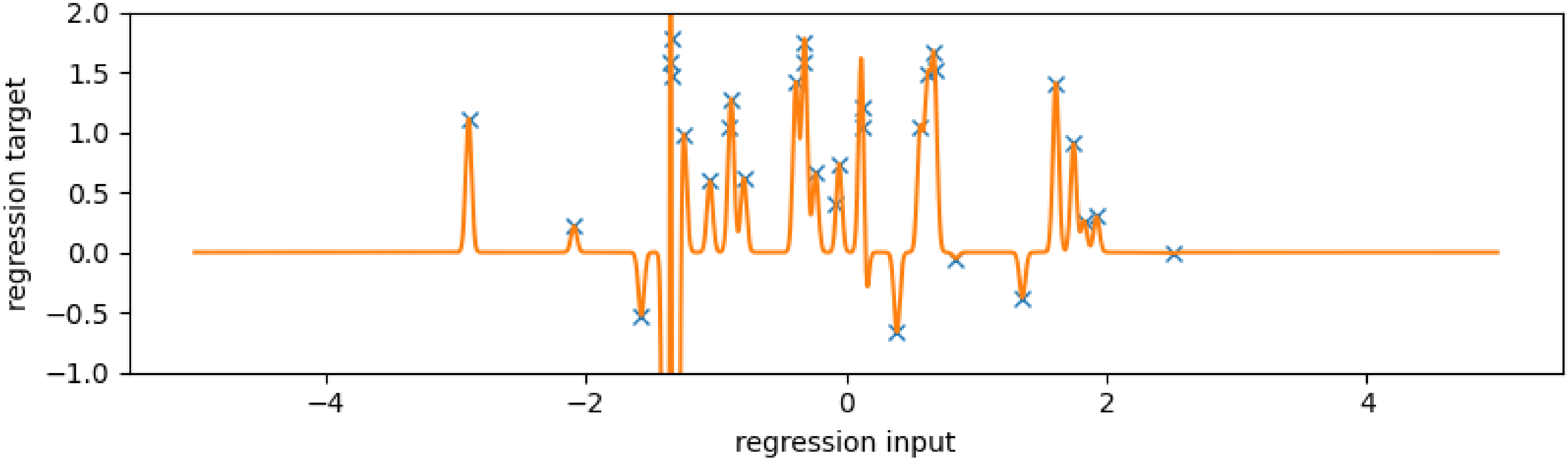

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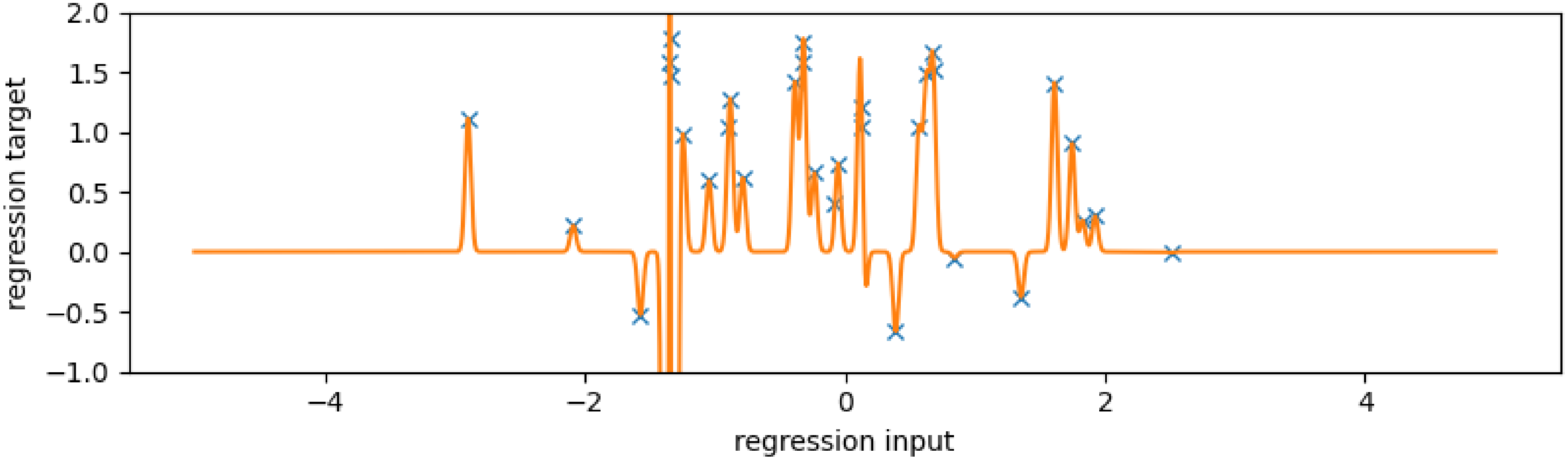

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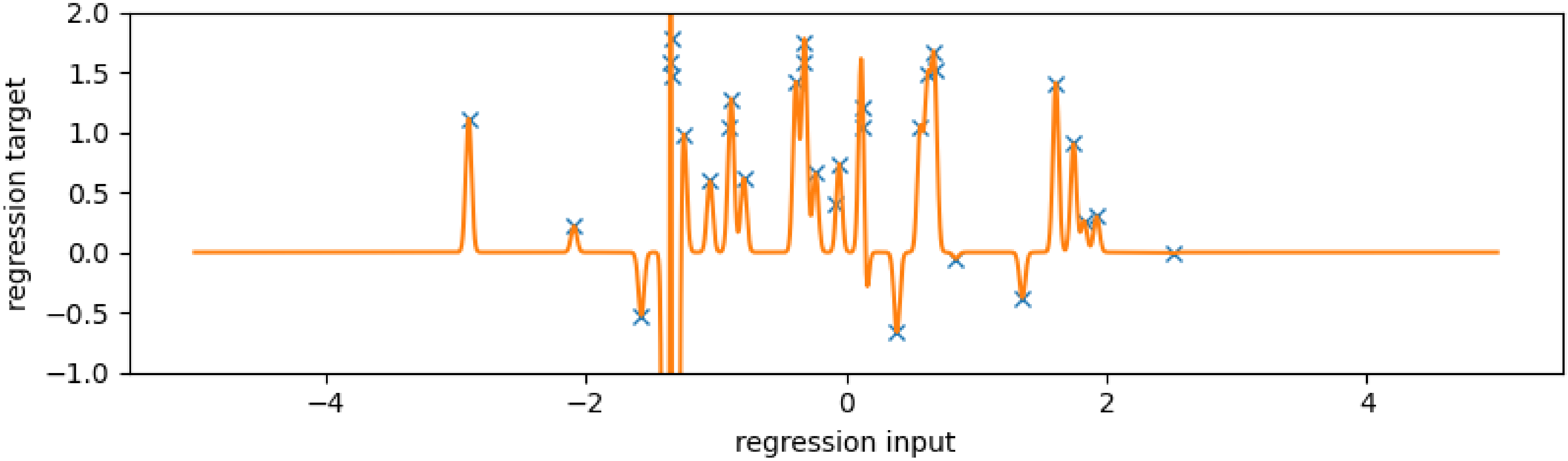

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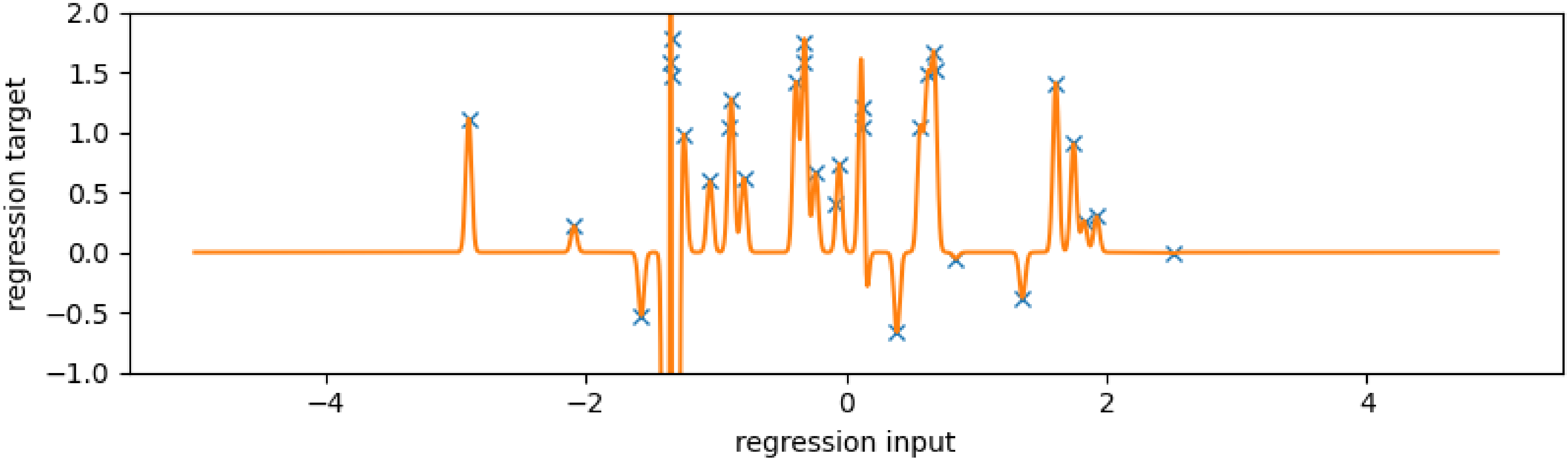

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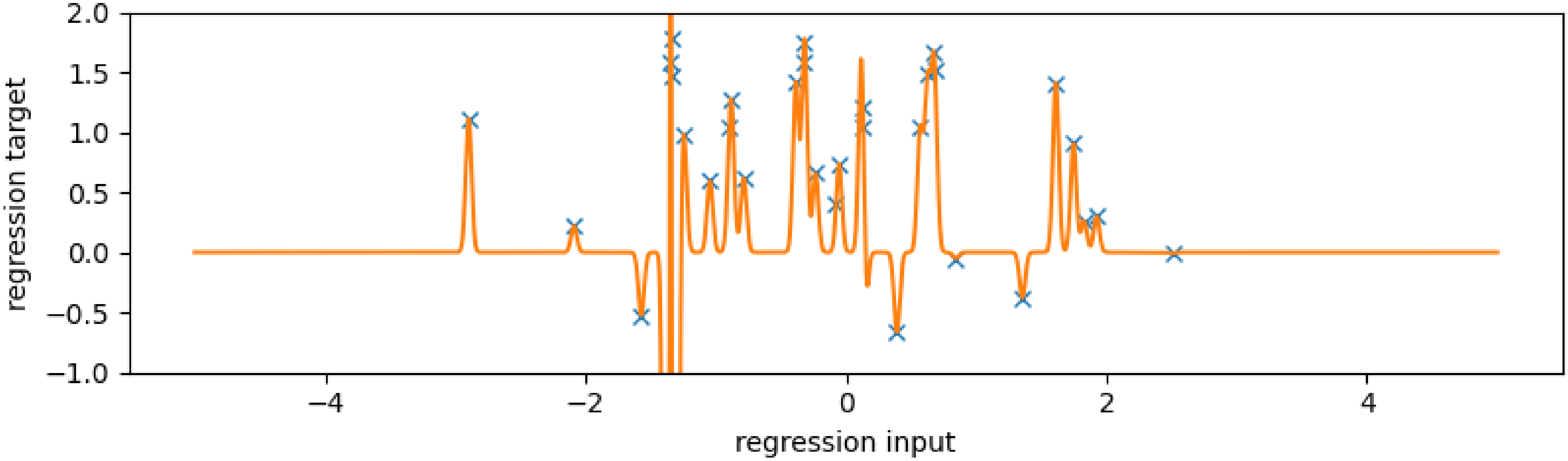

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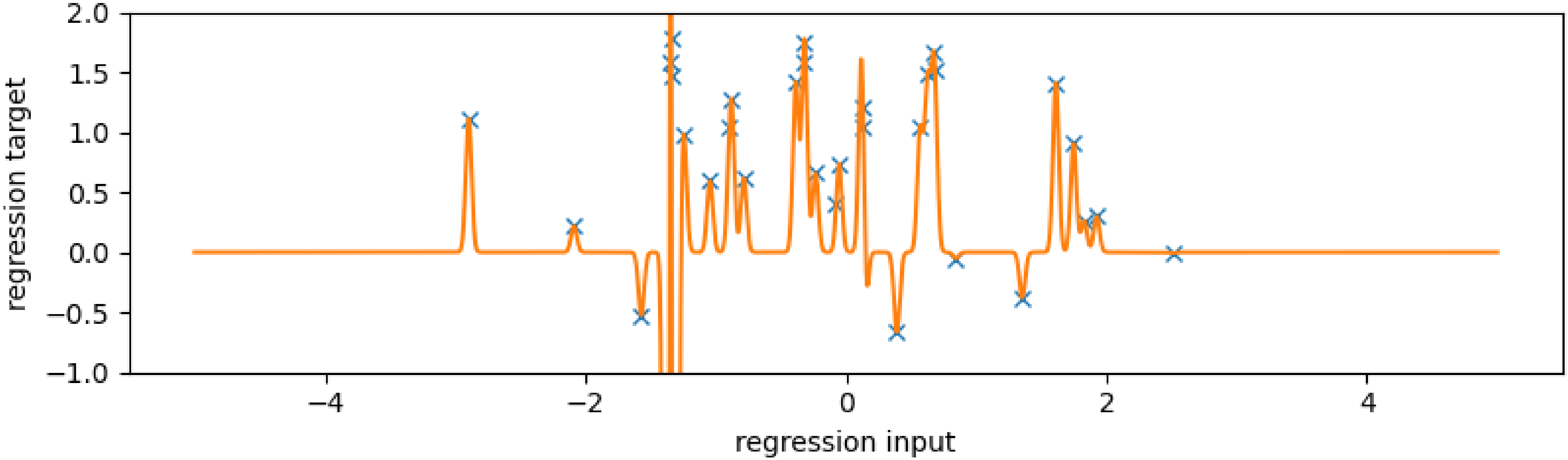

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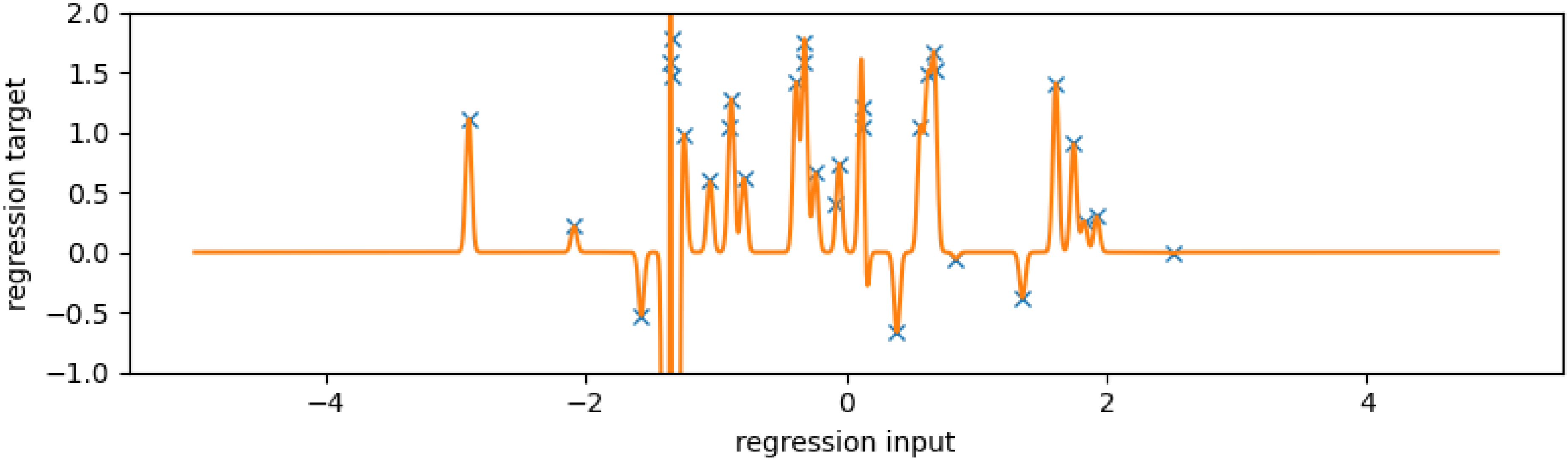

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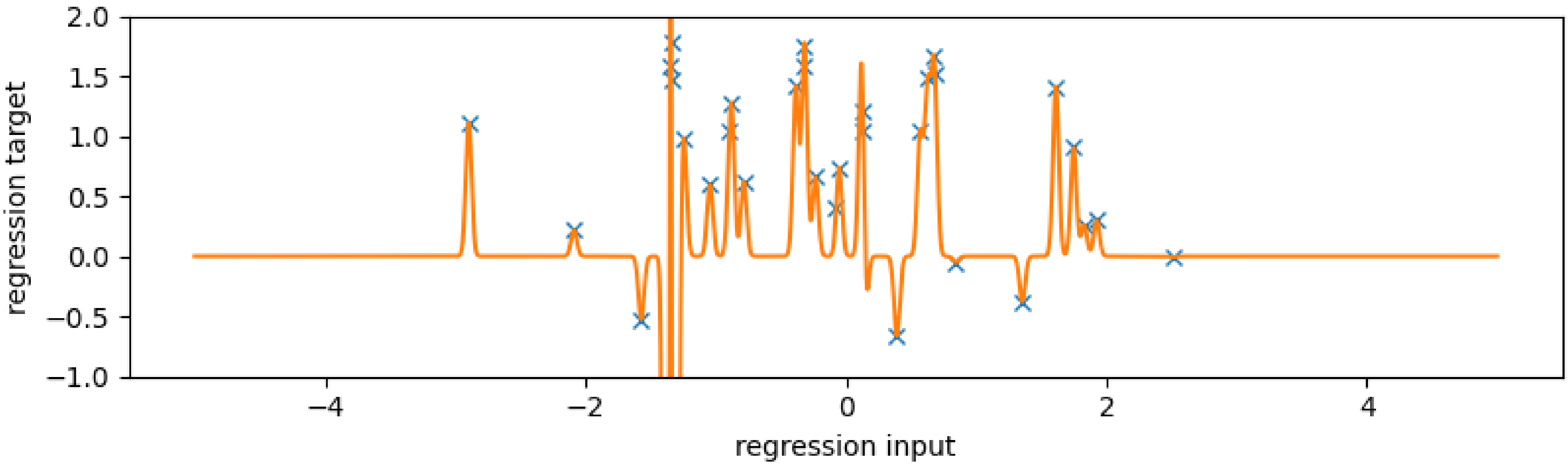

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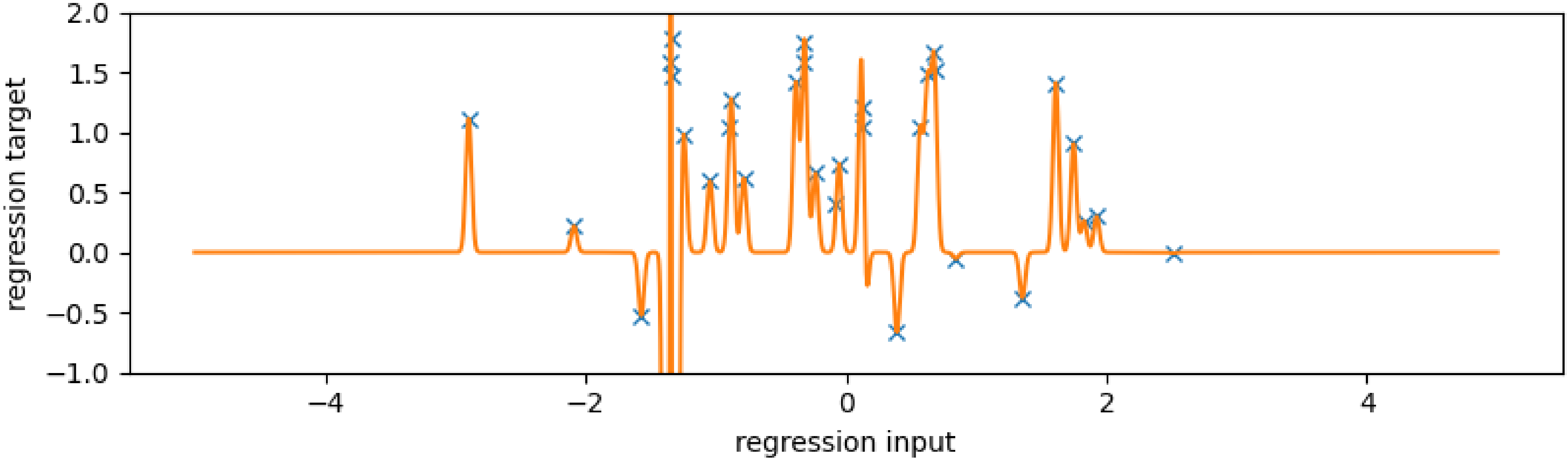

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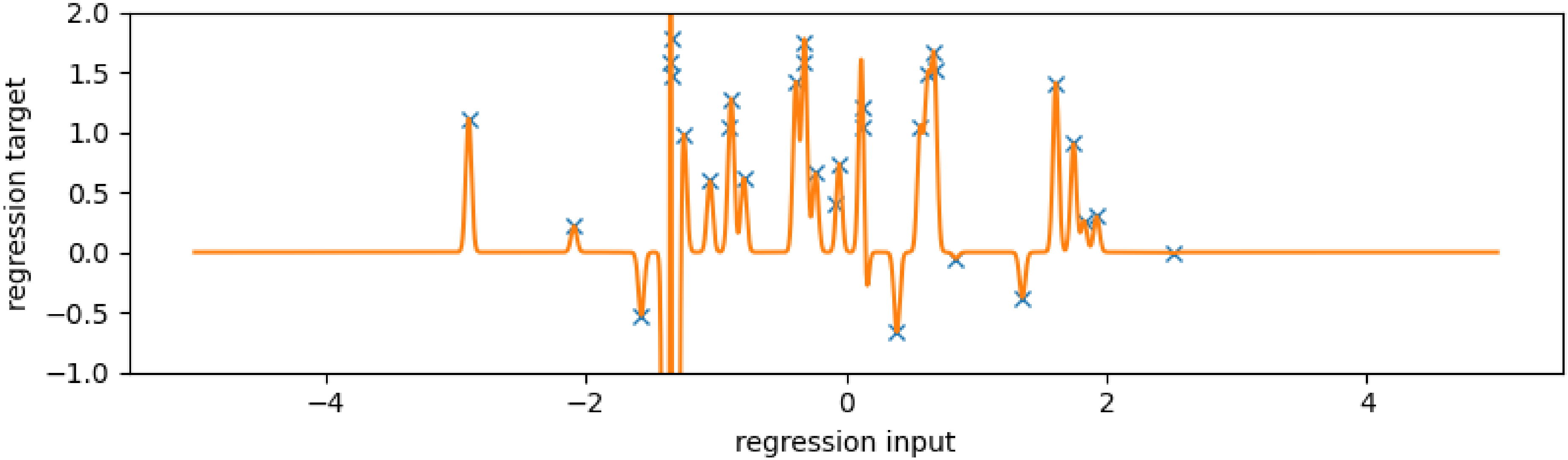

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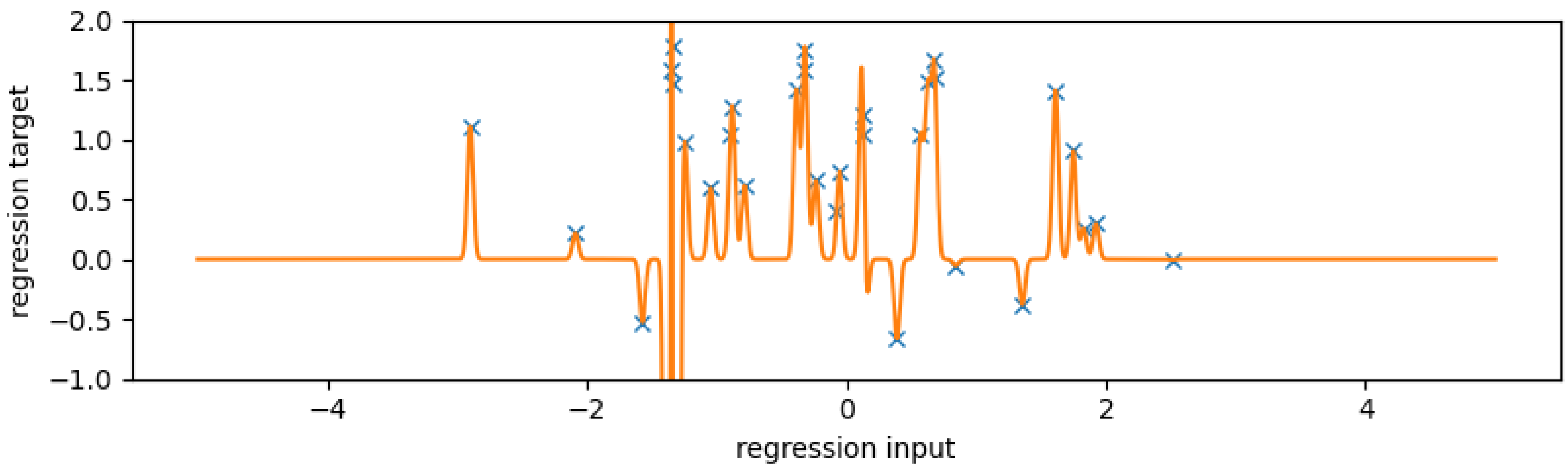

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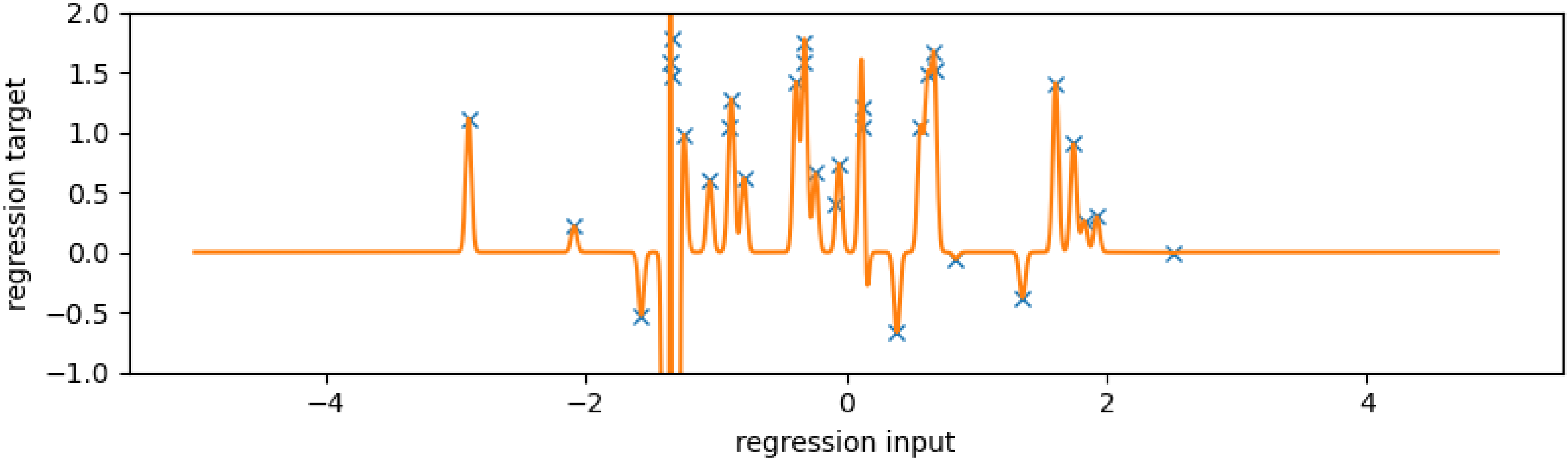

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- Restricting model size never improves loss $\Rightarrow M \to N$
- Narrower basis functions to allow more flexible functions
- “Overfitting”

1. What is wrong with minimising losses.

2. **Bayesian Model Selection?**

2. The Bayesian answer to model size: Nonparametrics.

3. A principle for selecting size

## The Bayesian Answer

Let's accept the "large" number of basis functions for now, and solve the overfitting problem.

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$$p(W, \theta|\mathcal{D}) = \frac{p(\mathcal{D}|W, \theta)p(W|\theta)}{p(\mathcal{D}|\theta)} \frac{p(\mathcal{D}|\theta)p(\theta)}{p(\mathcal{D})}$$

$$p(\mathcal{D}|\theta) = \int p(\mathcal{D}|W, \theta)p(W|\theta)\, \mathrm{d}W$$

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Bayesian computations are often intractable. ⇒ Approximating $p(\mathcal{D}|\theta)$ is hard enough, let alone for many different values of $\theta$!

$$\theta^* = \underset{\theta}{\operatorname{argmin}} \log p(\mathcal{D} \mid \theta)$$

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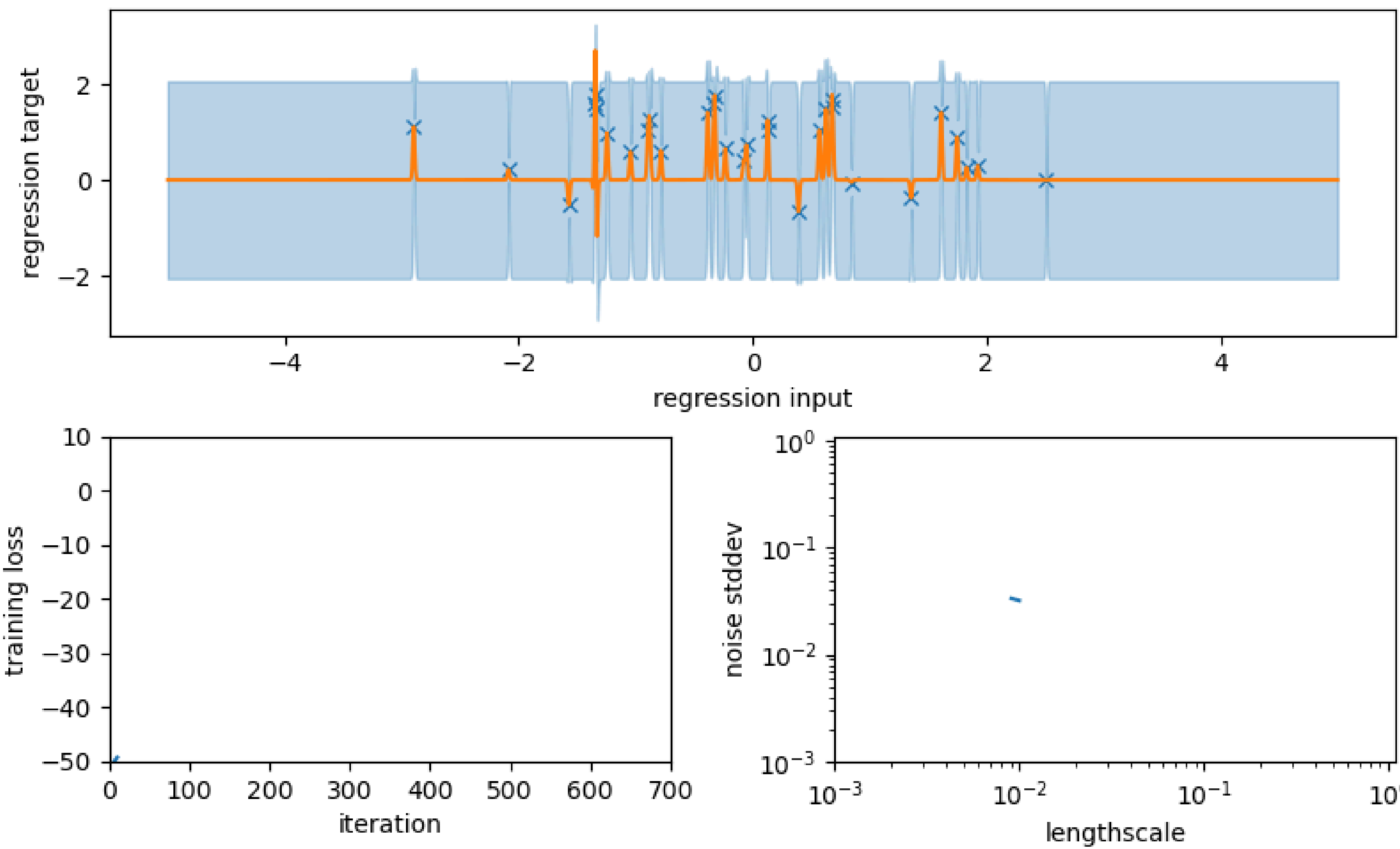

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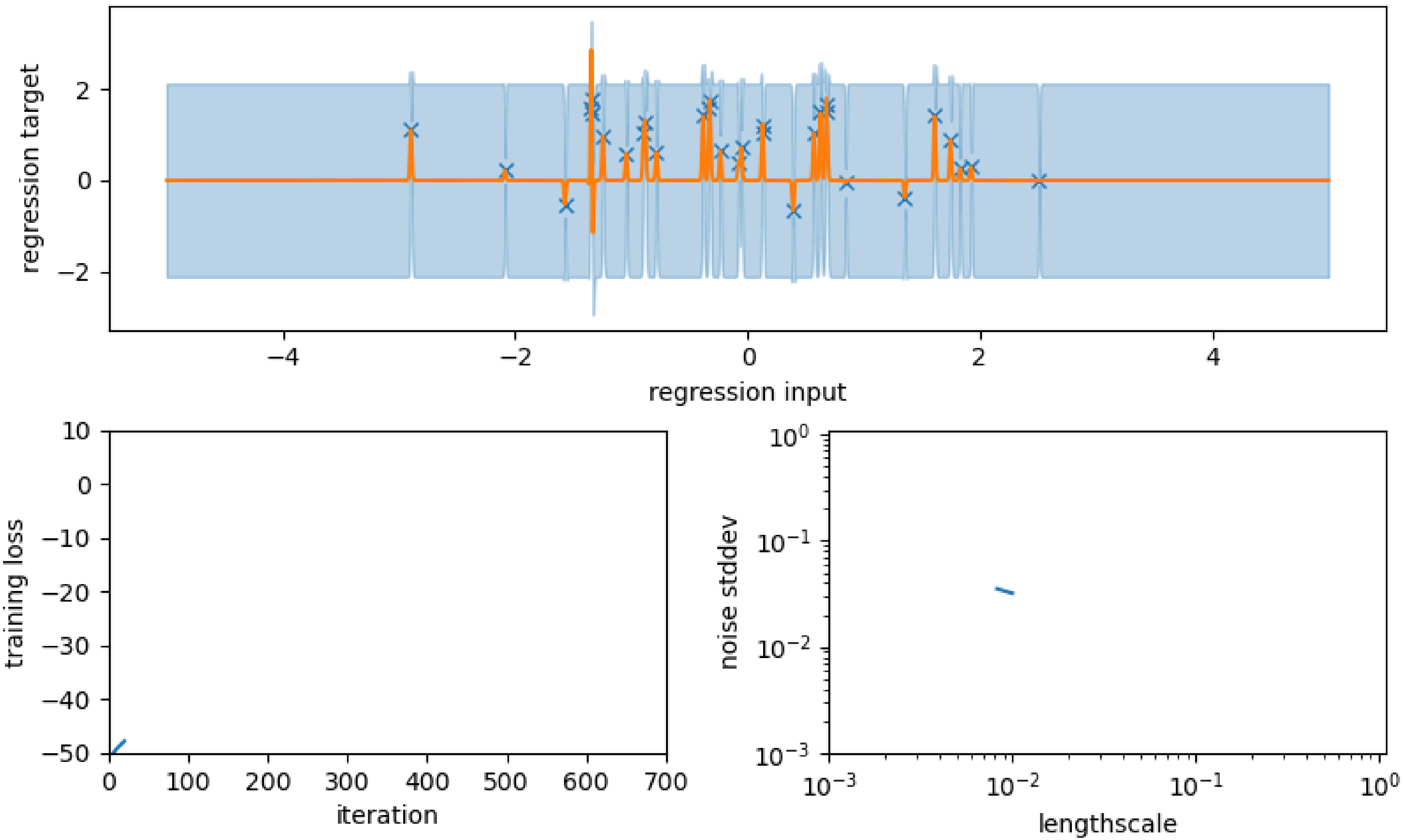

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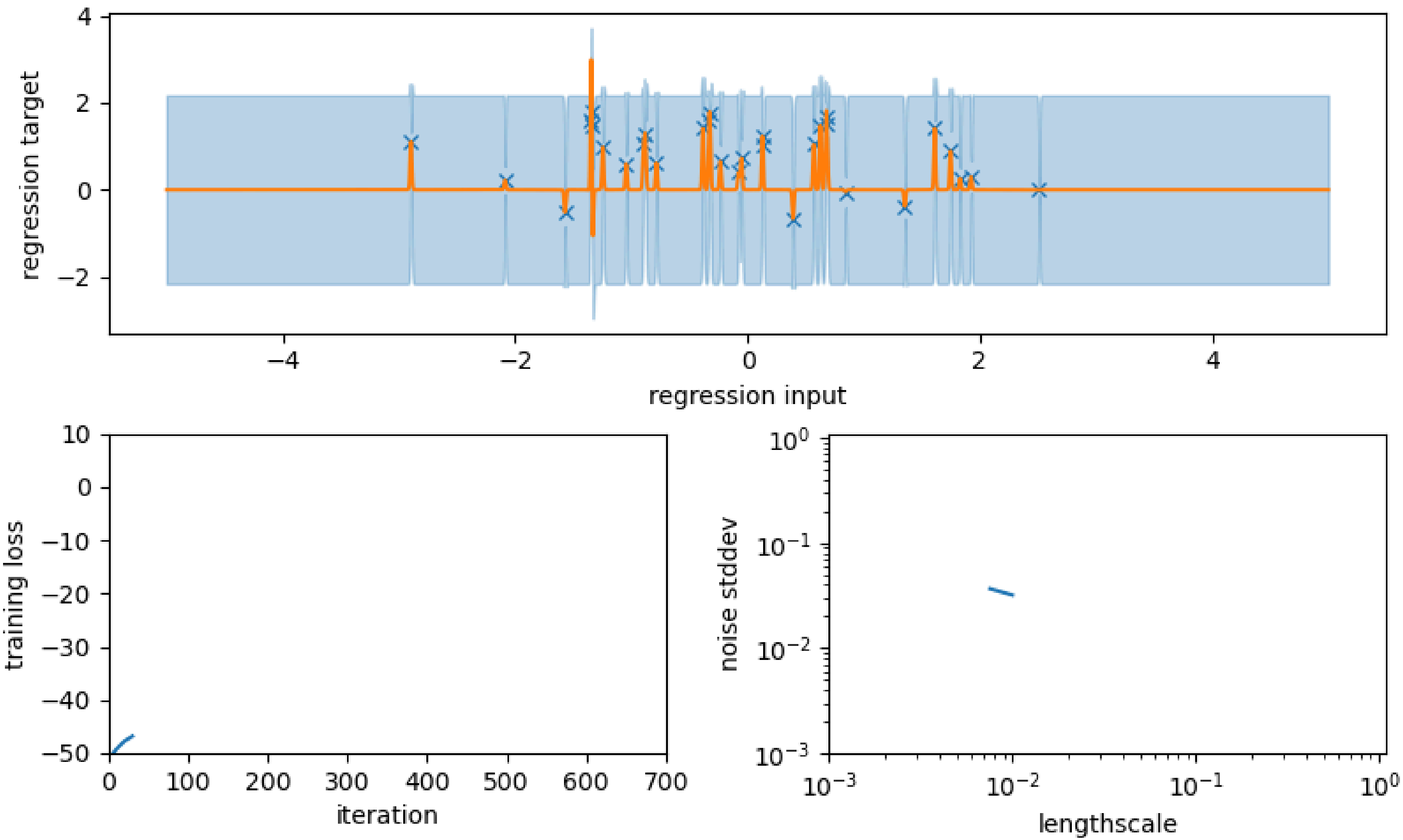

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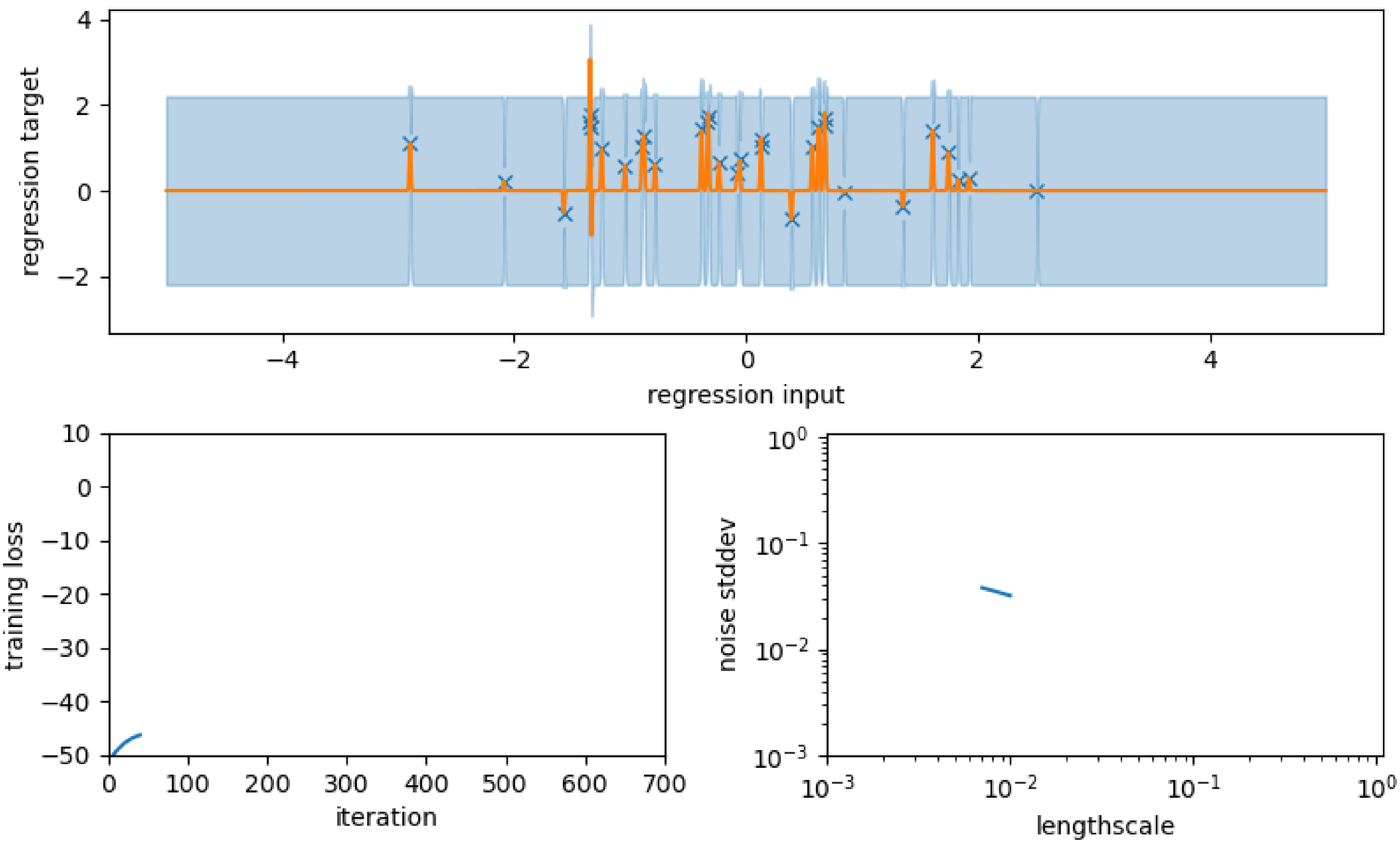

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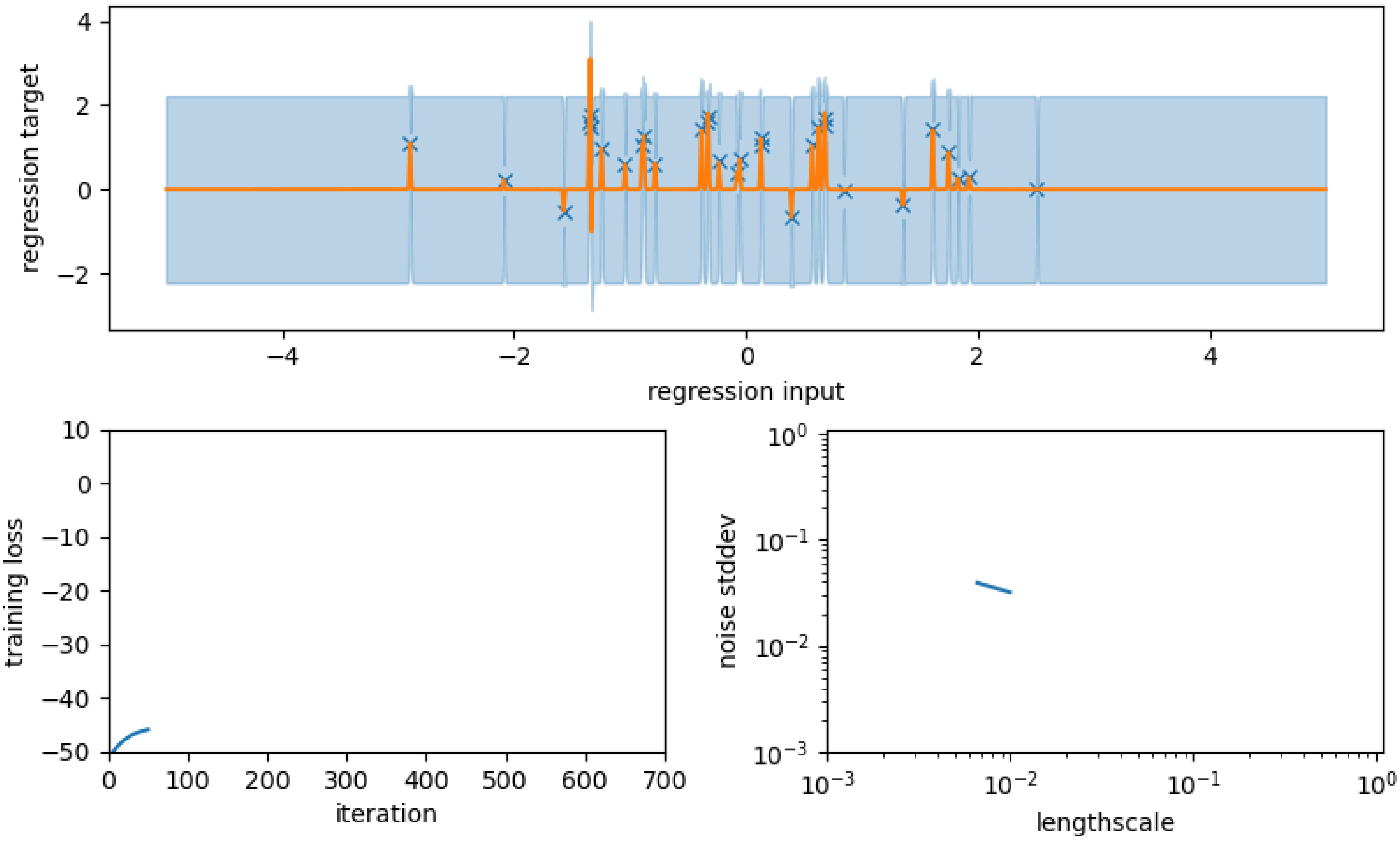

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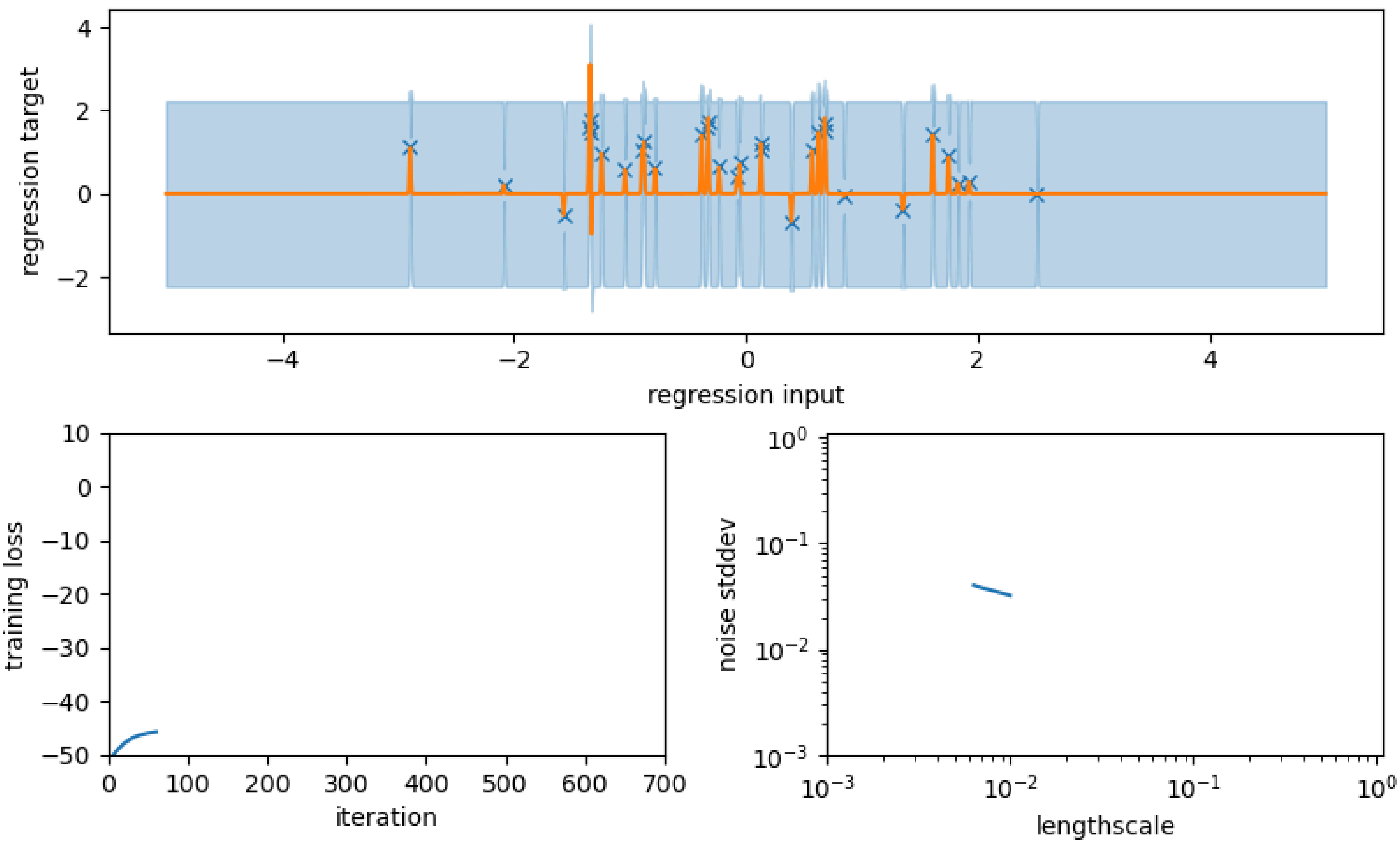

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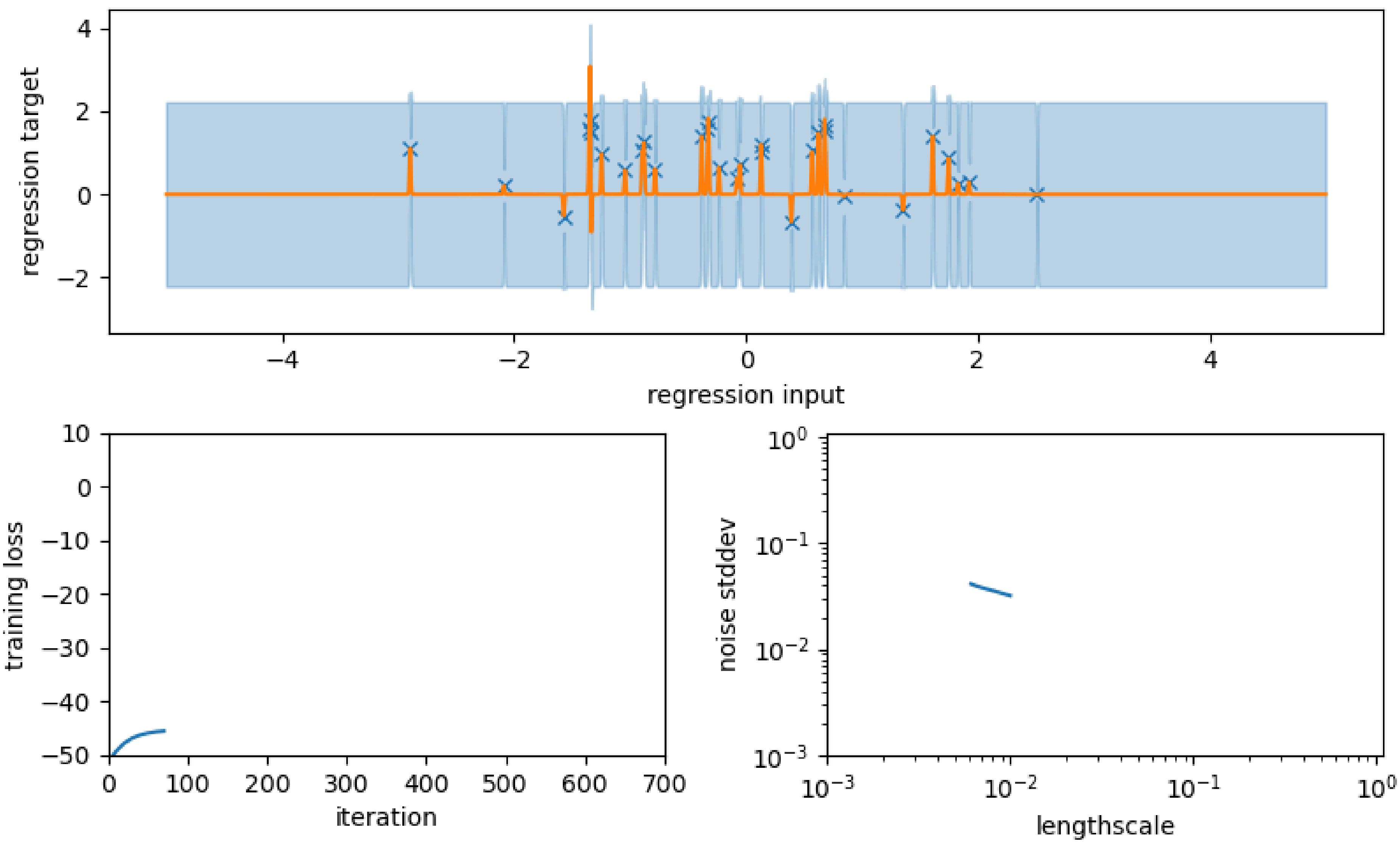

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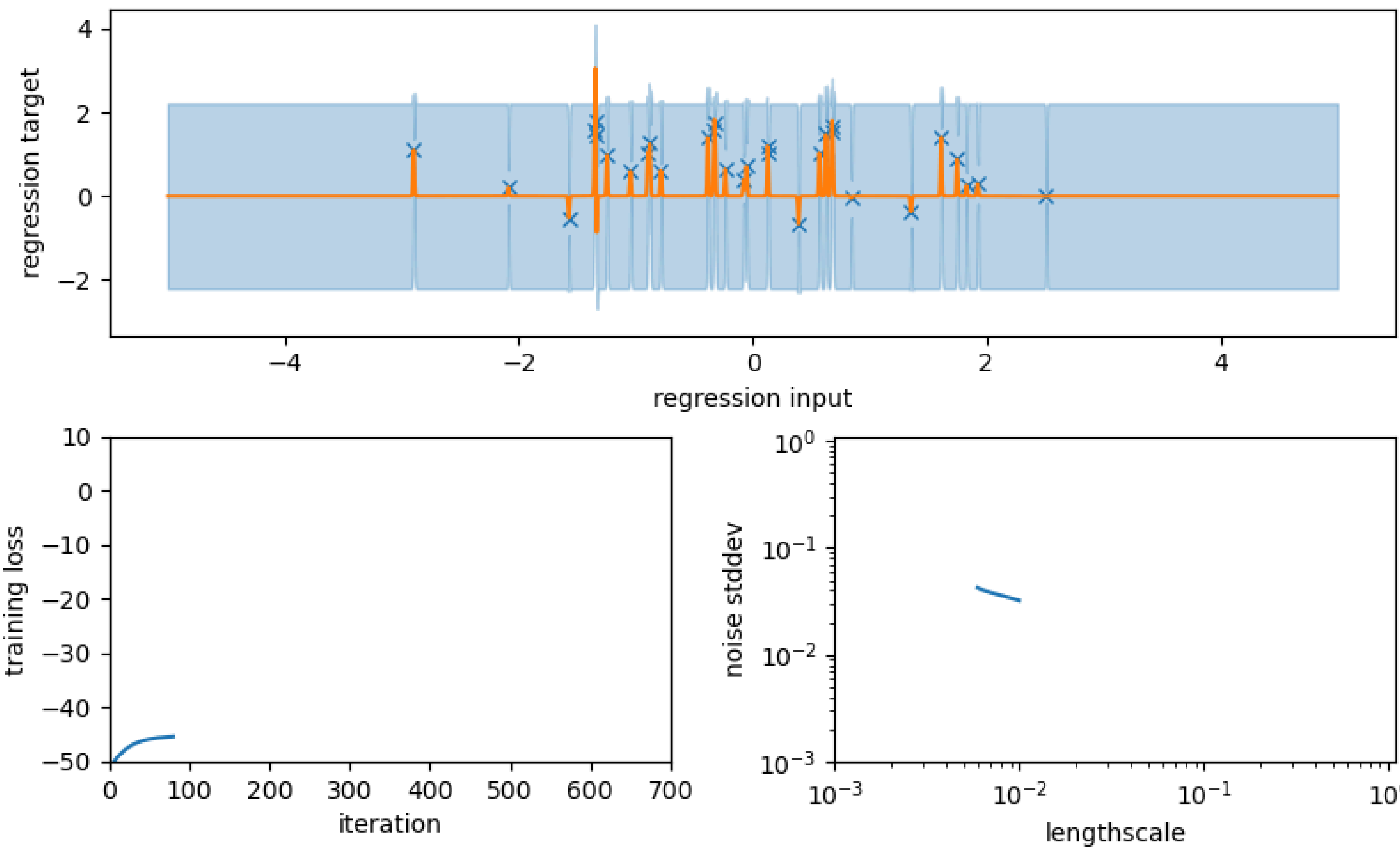

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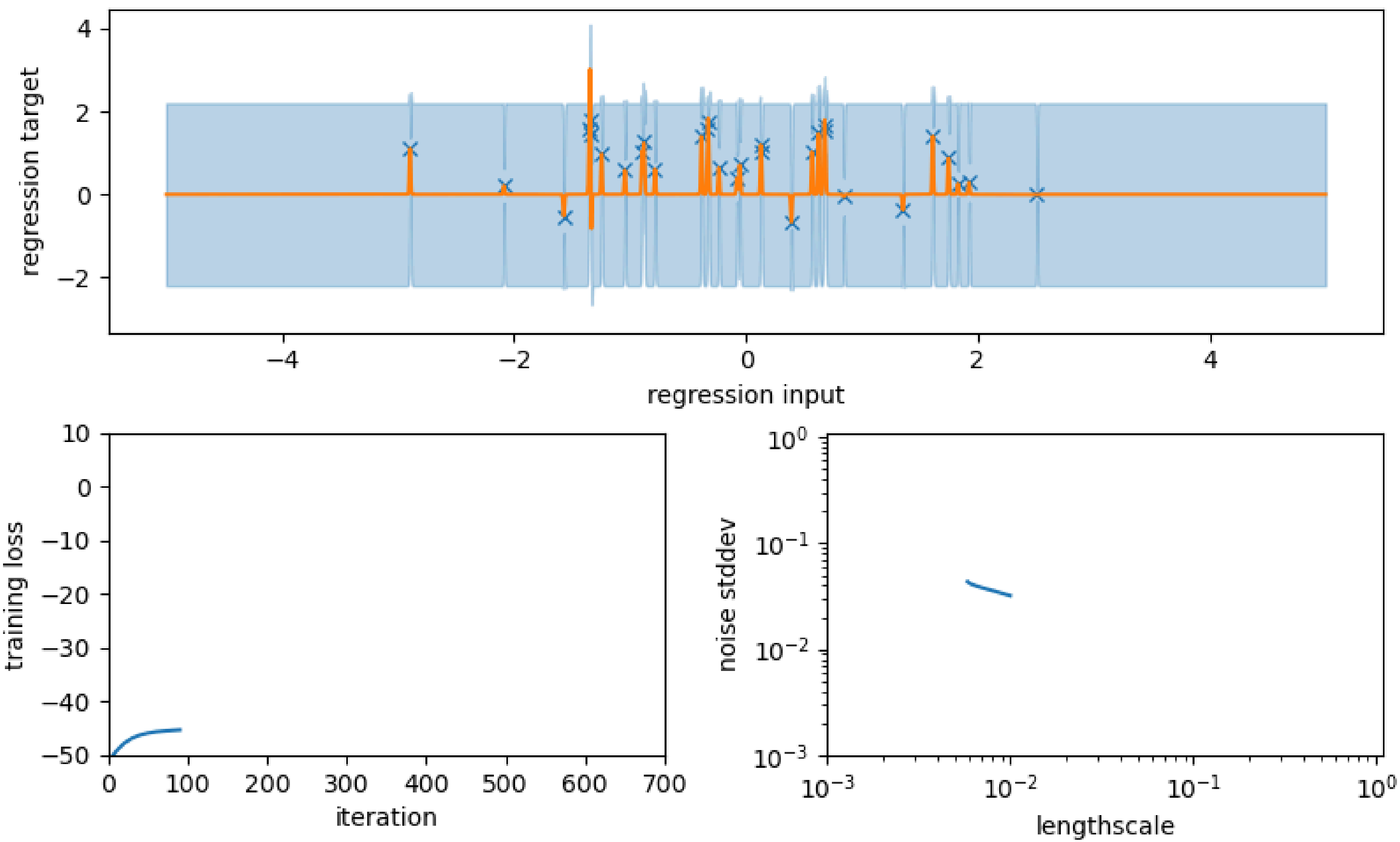

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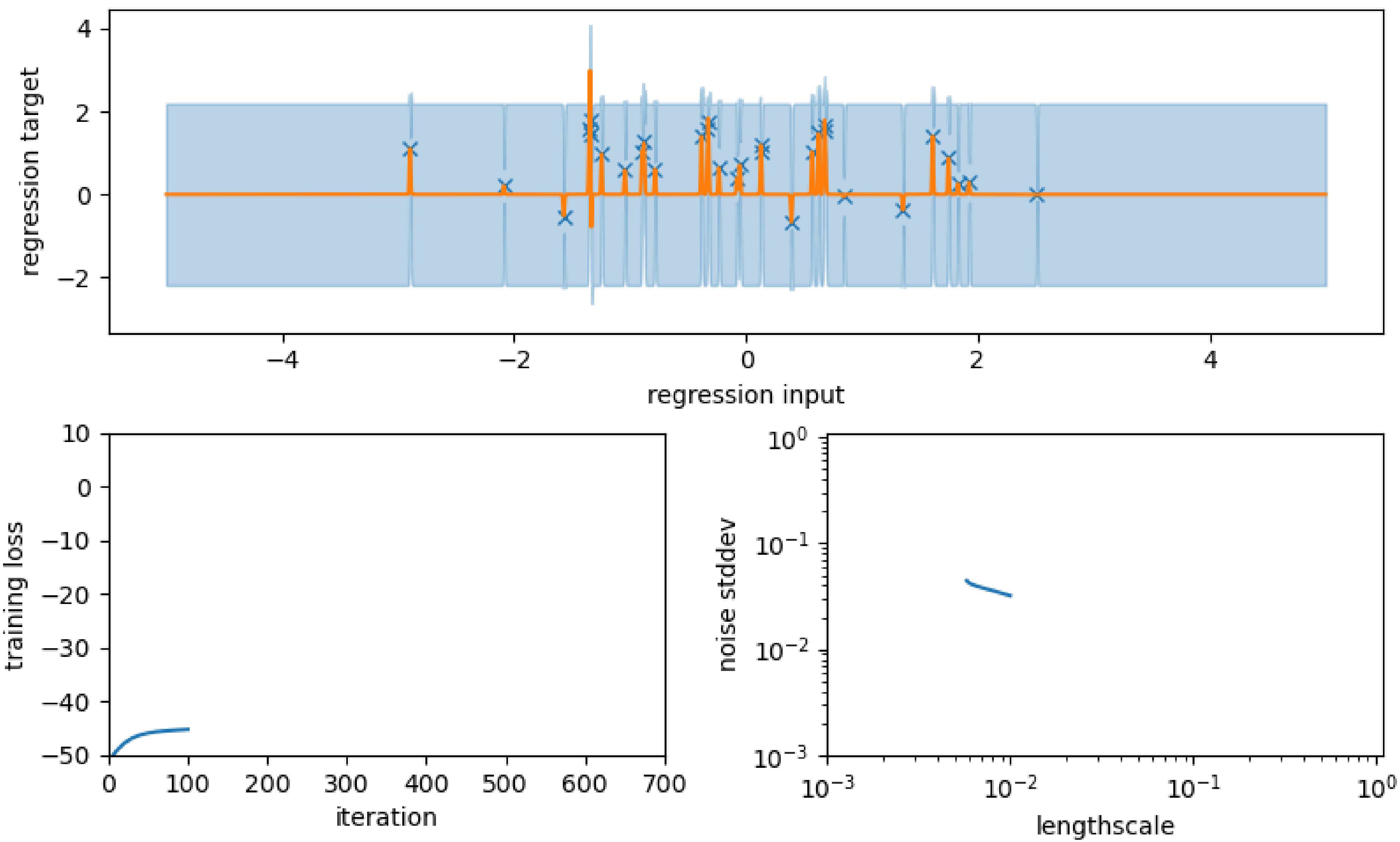

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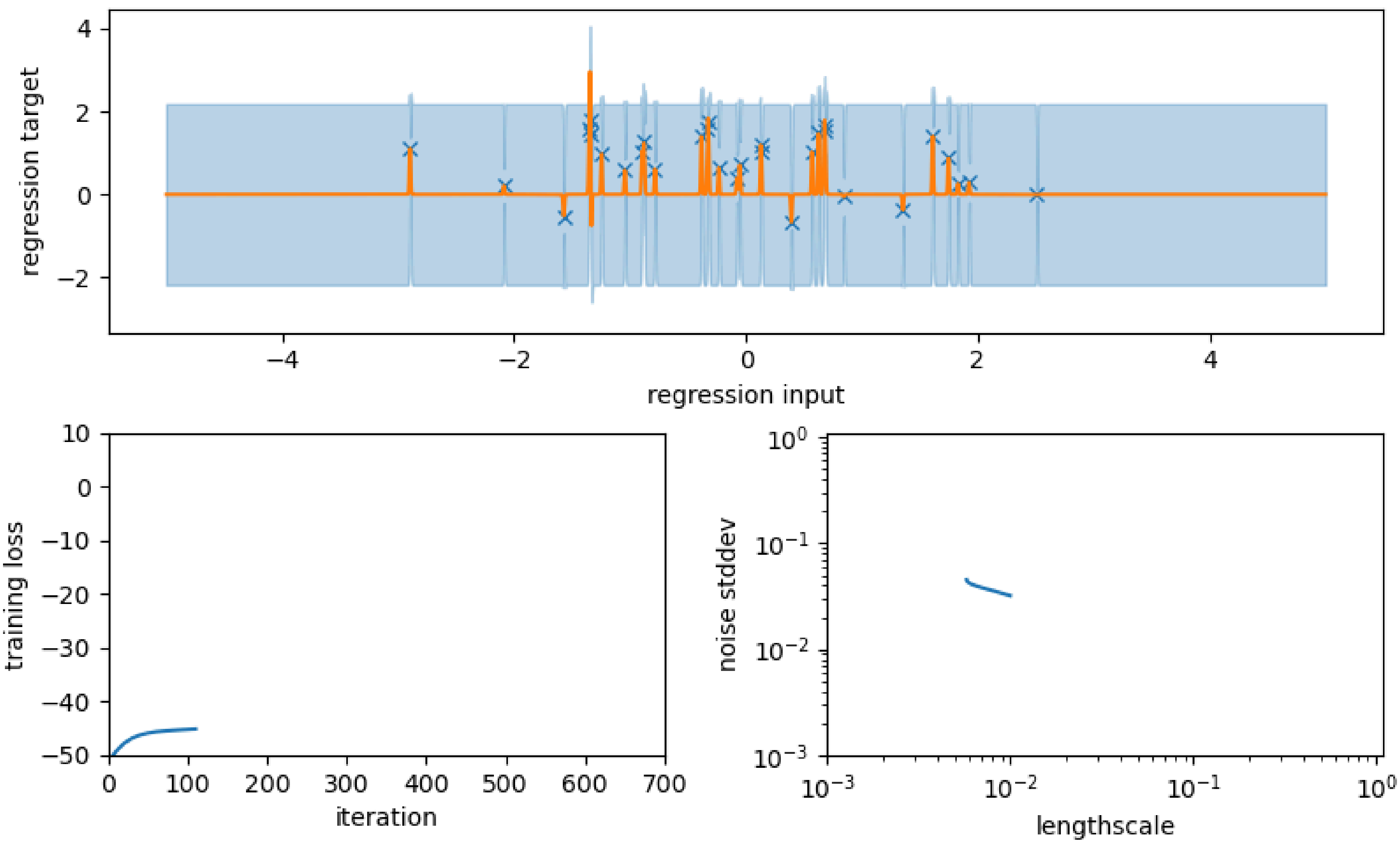

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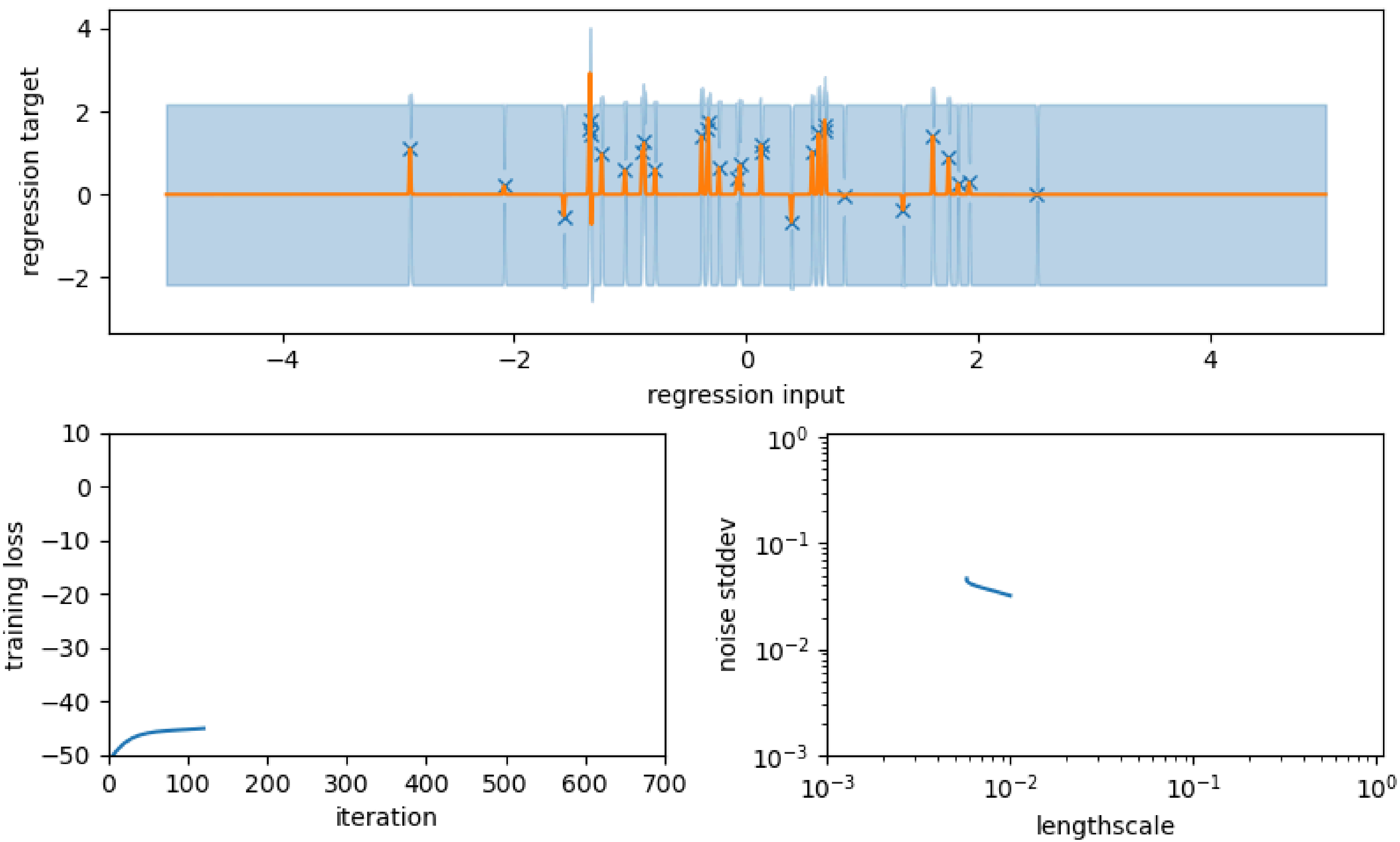

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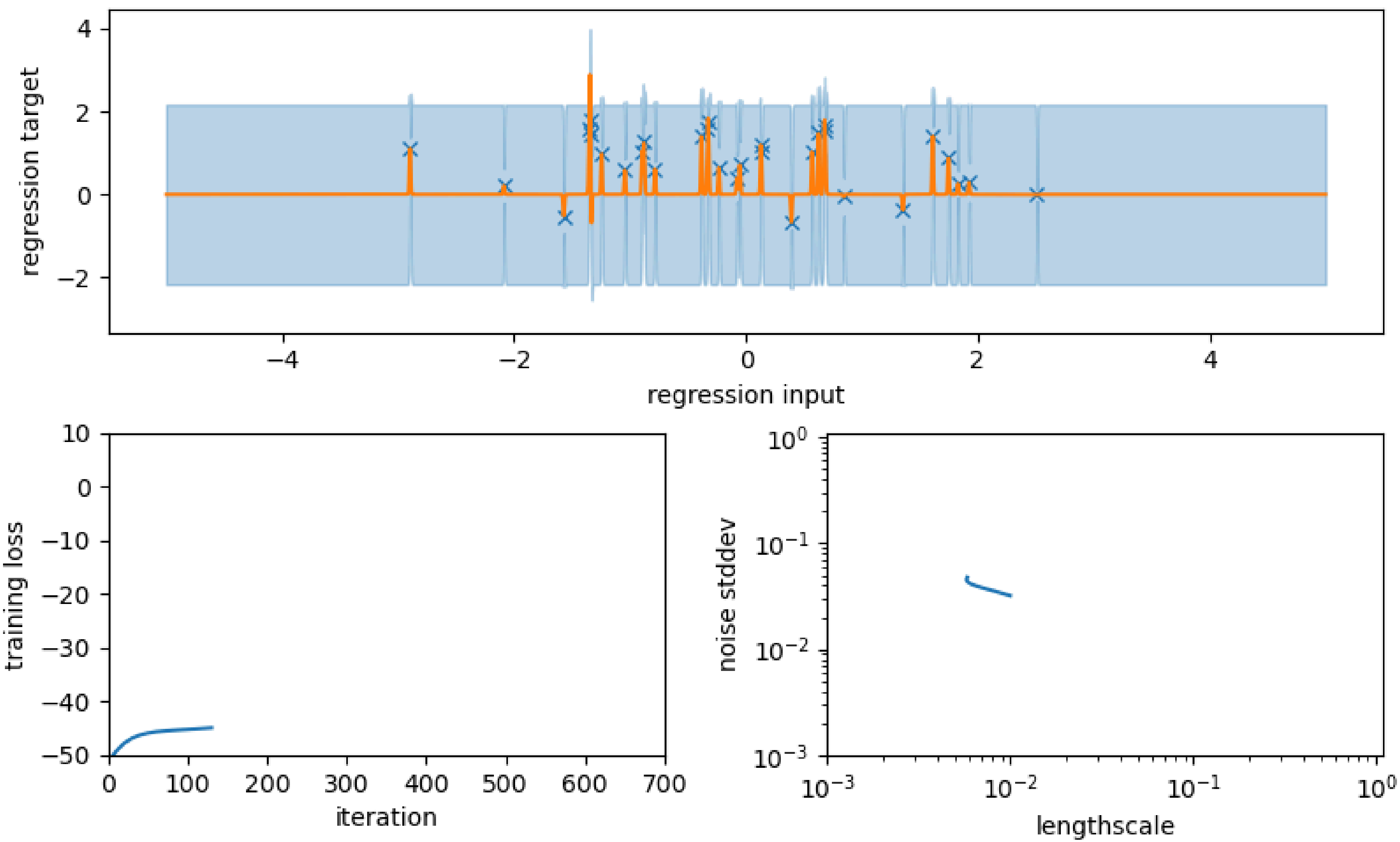

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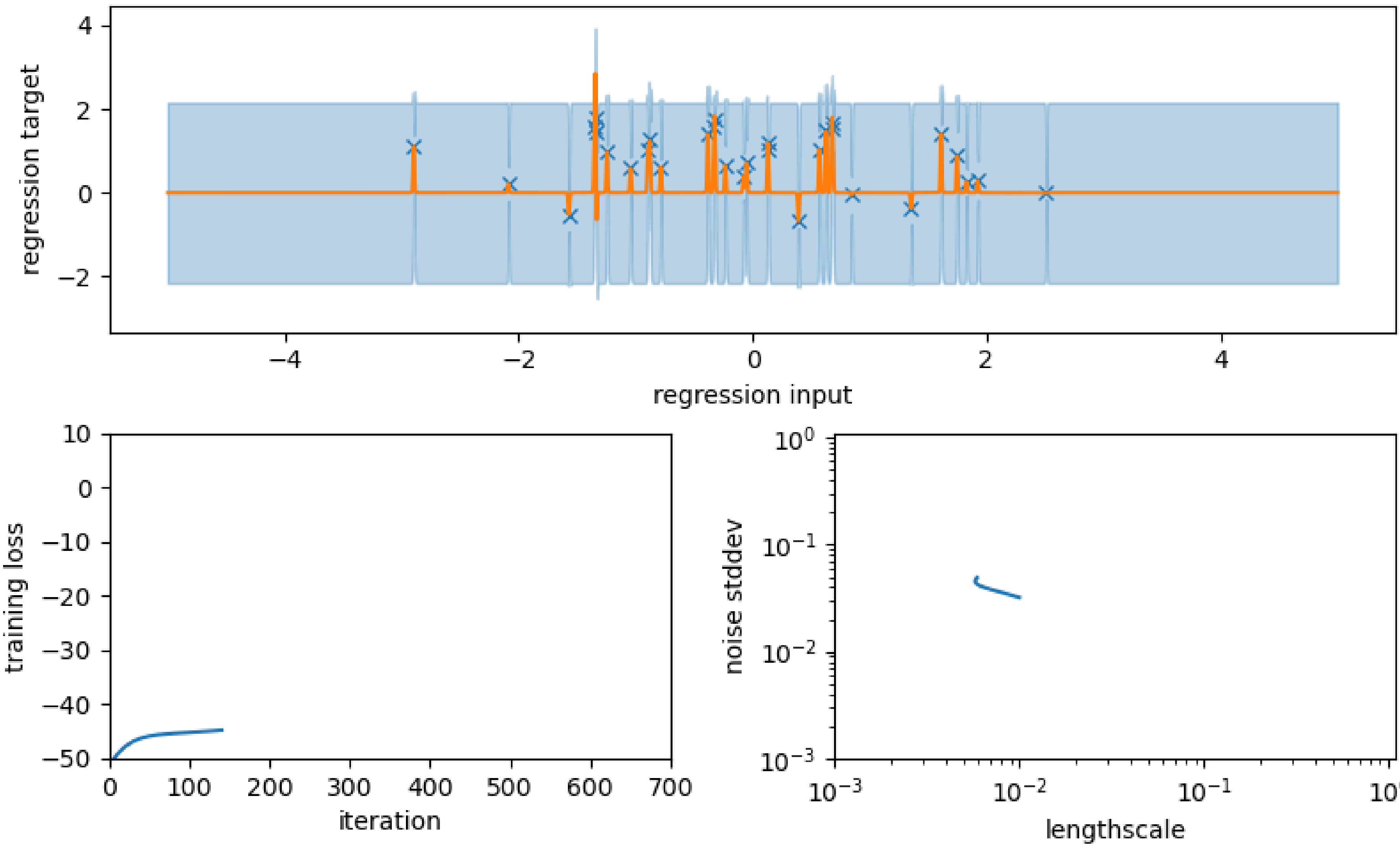

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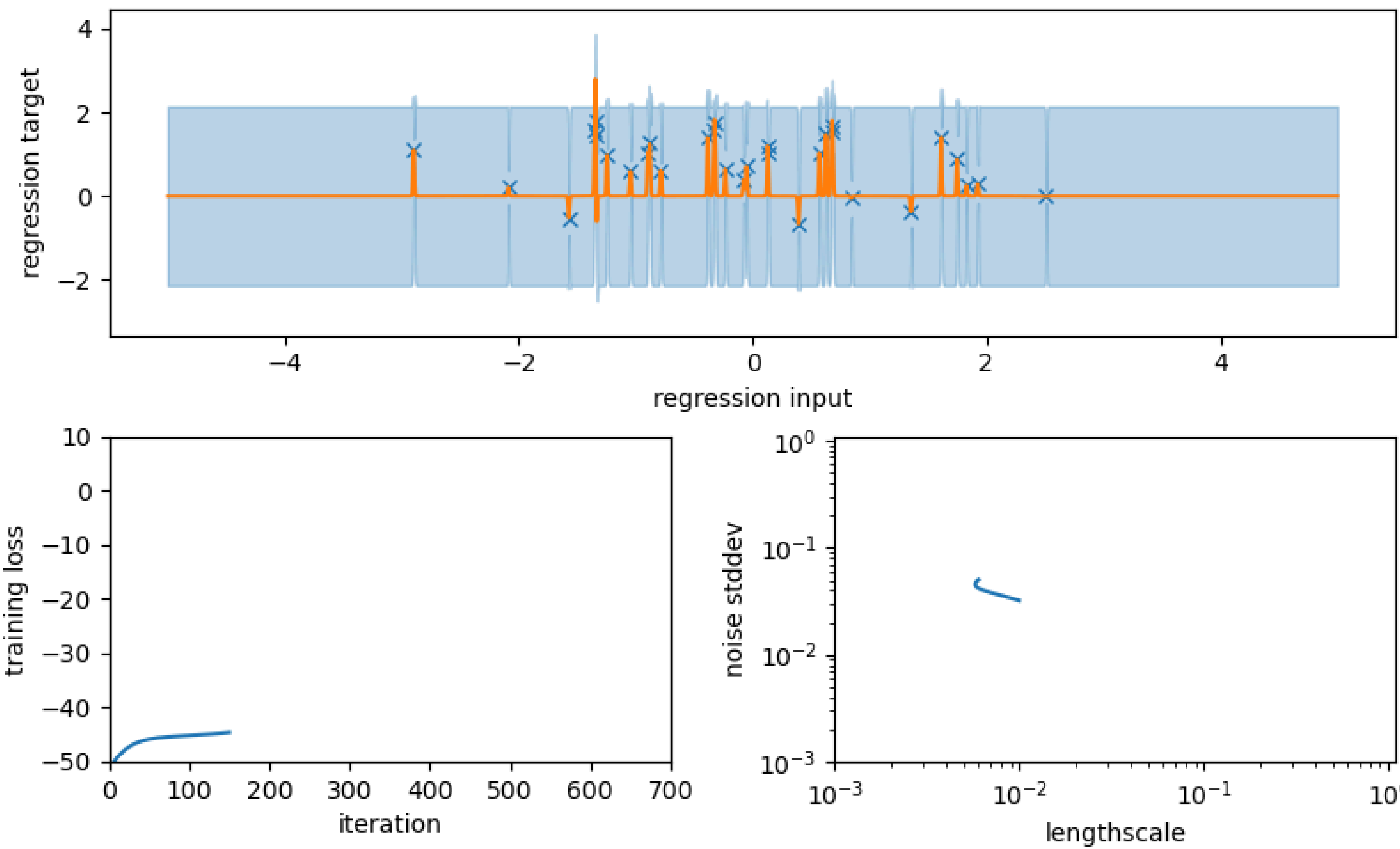

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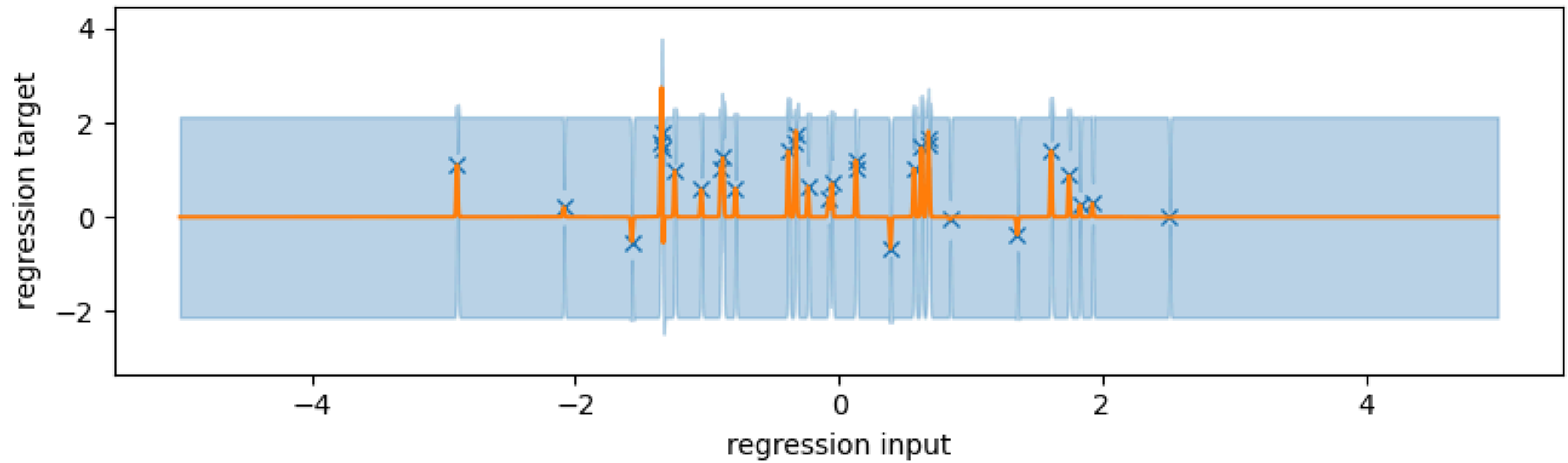


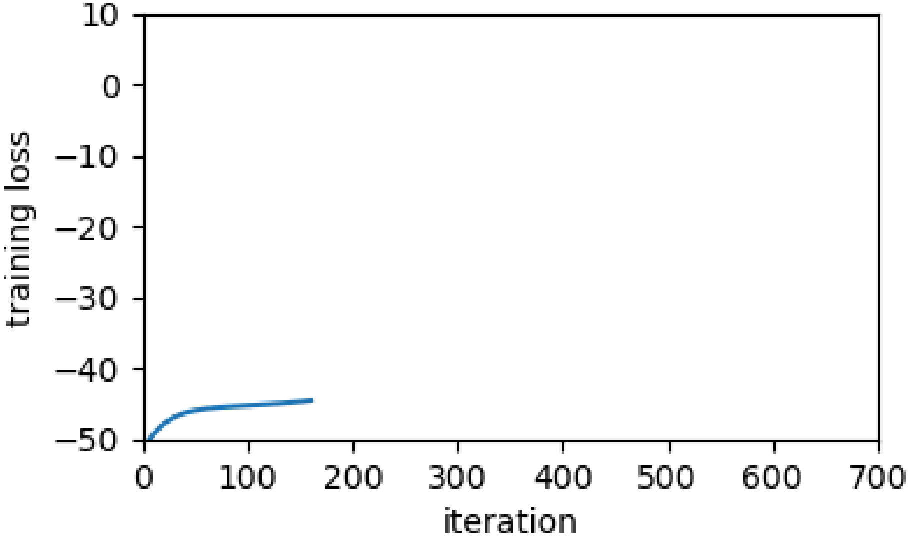


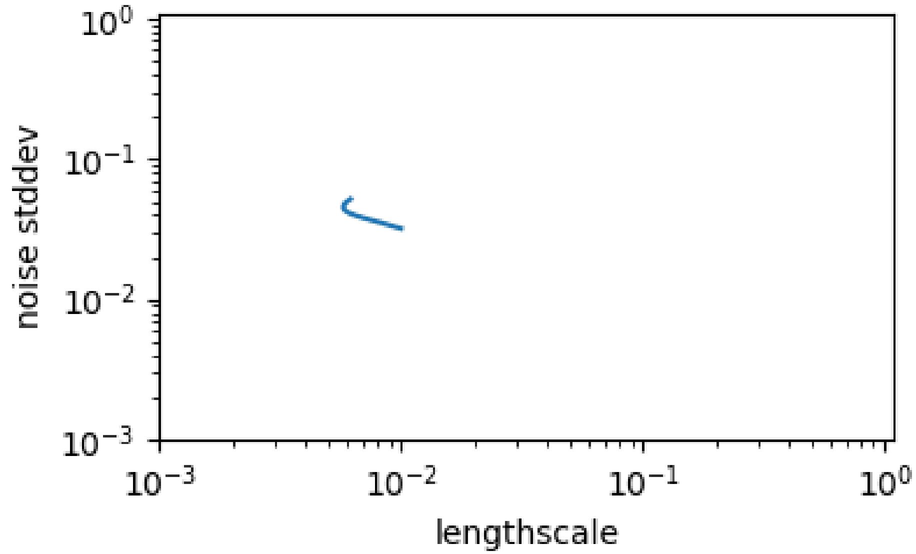

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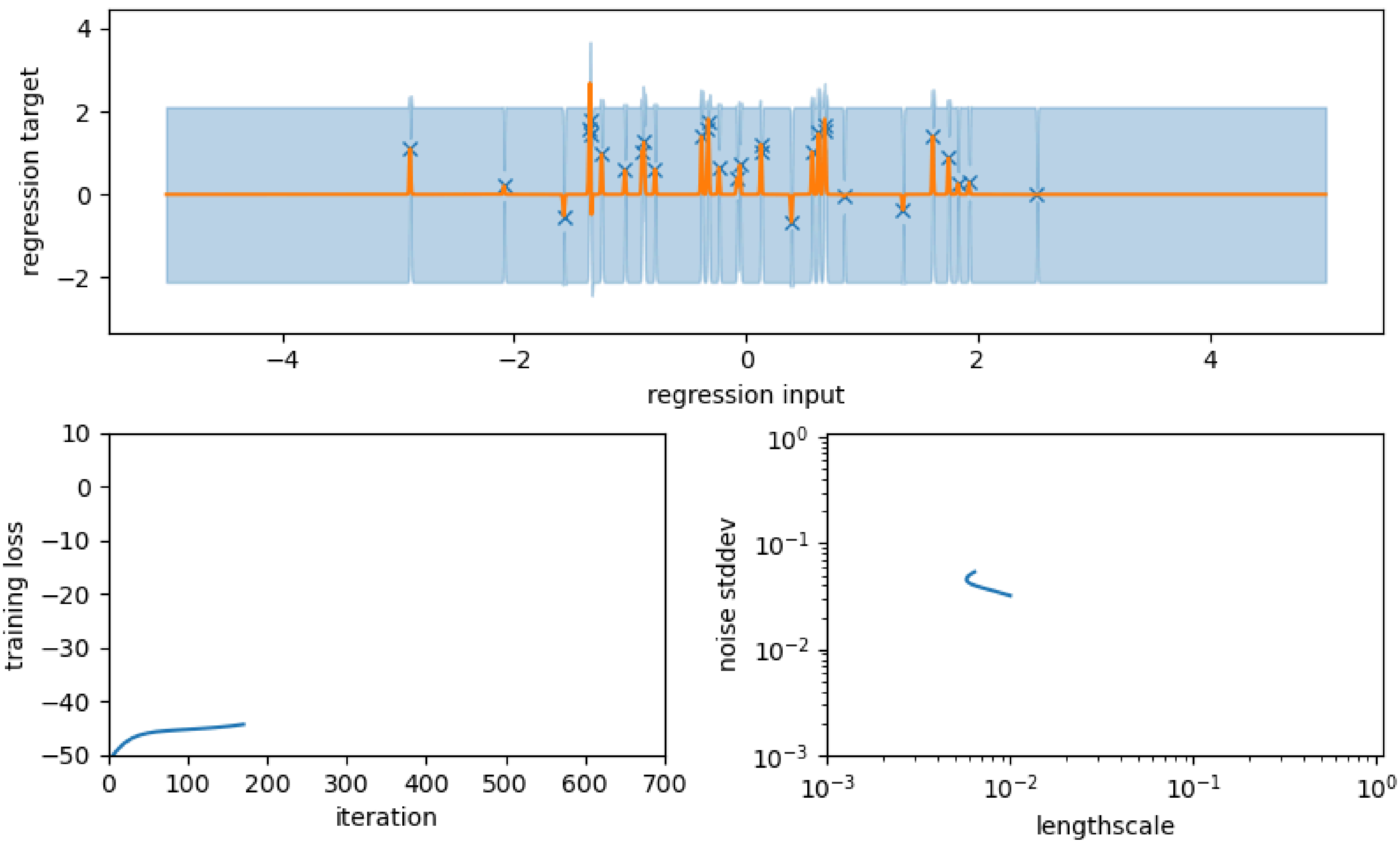

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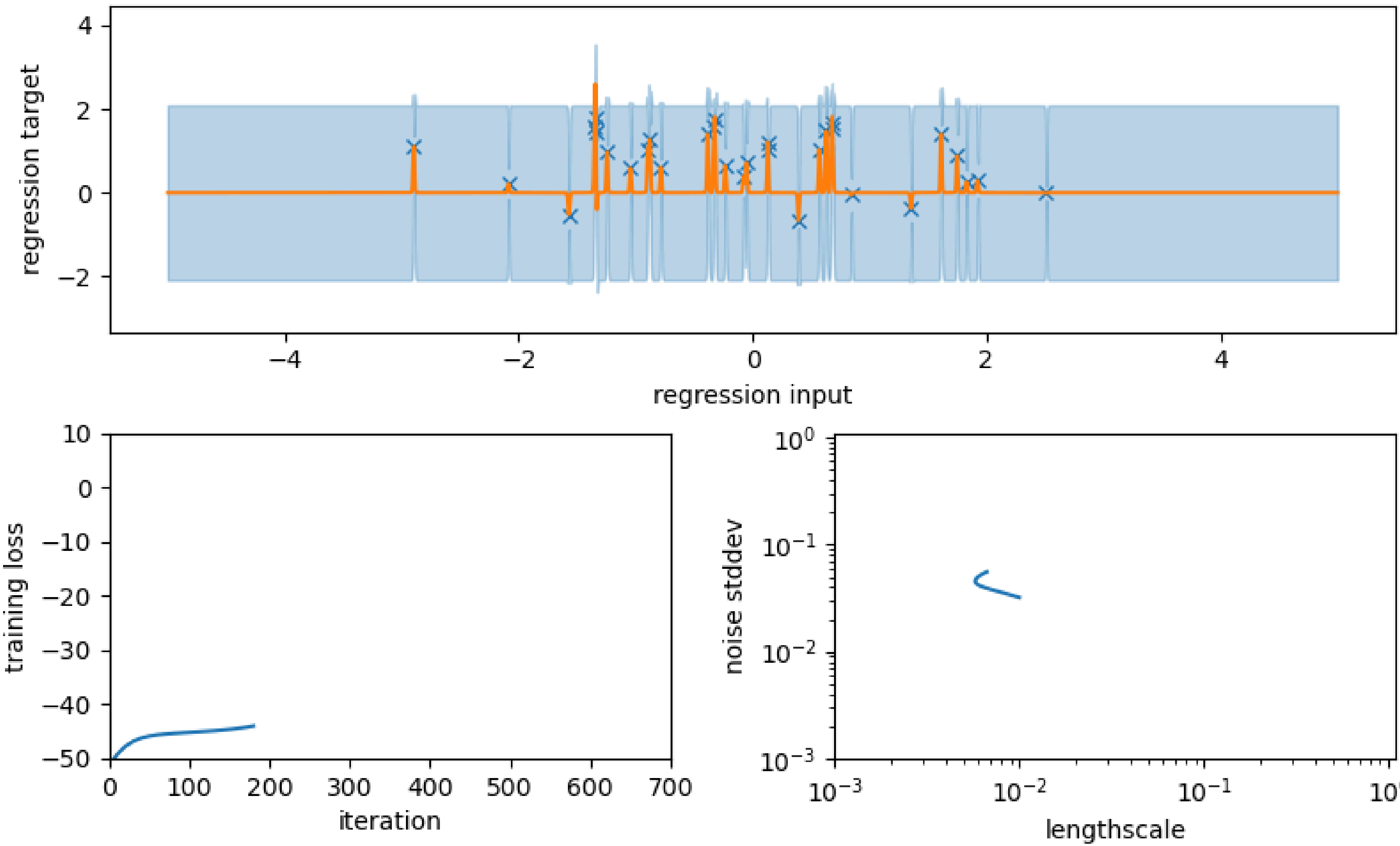

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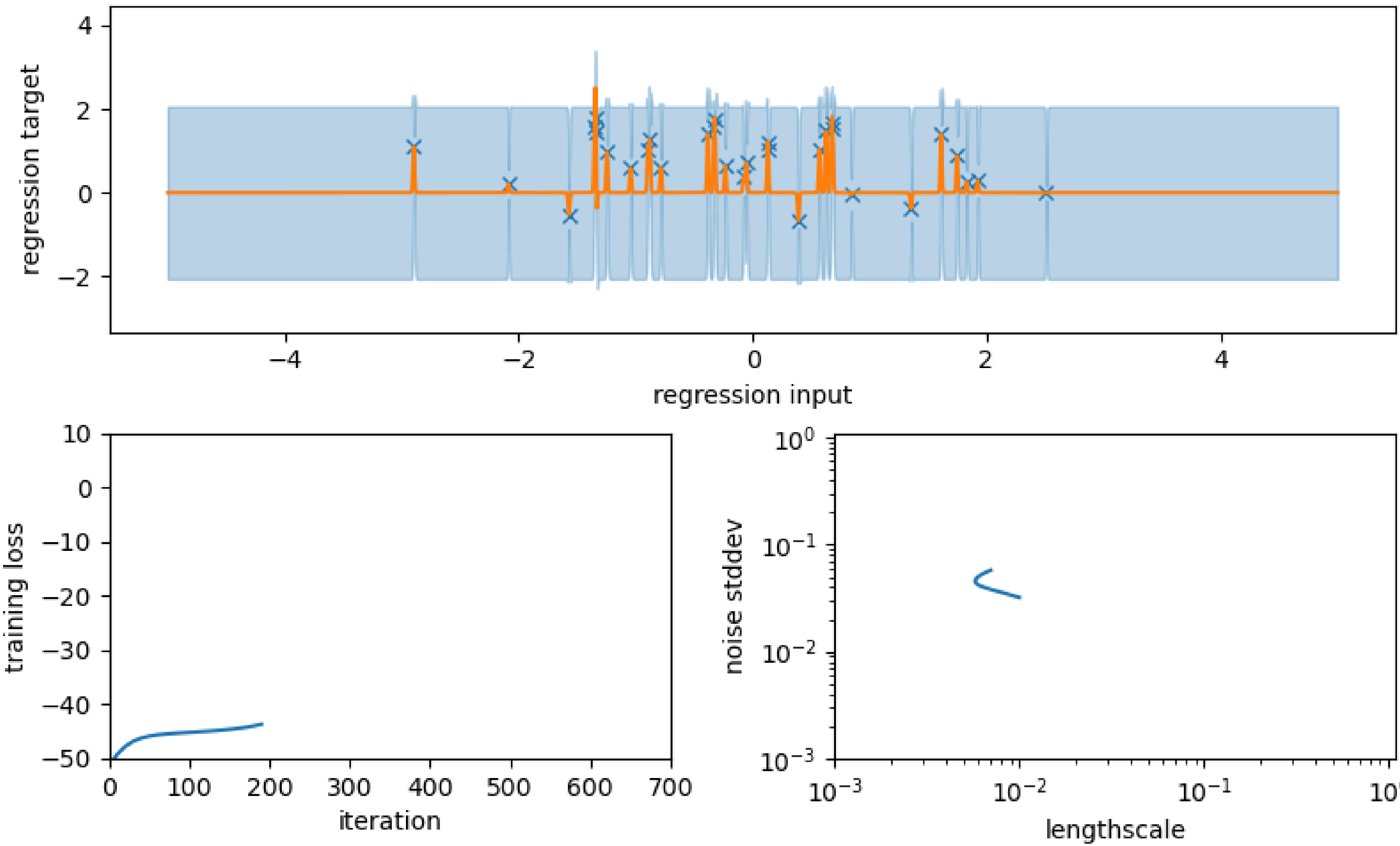

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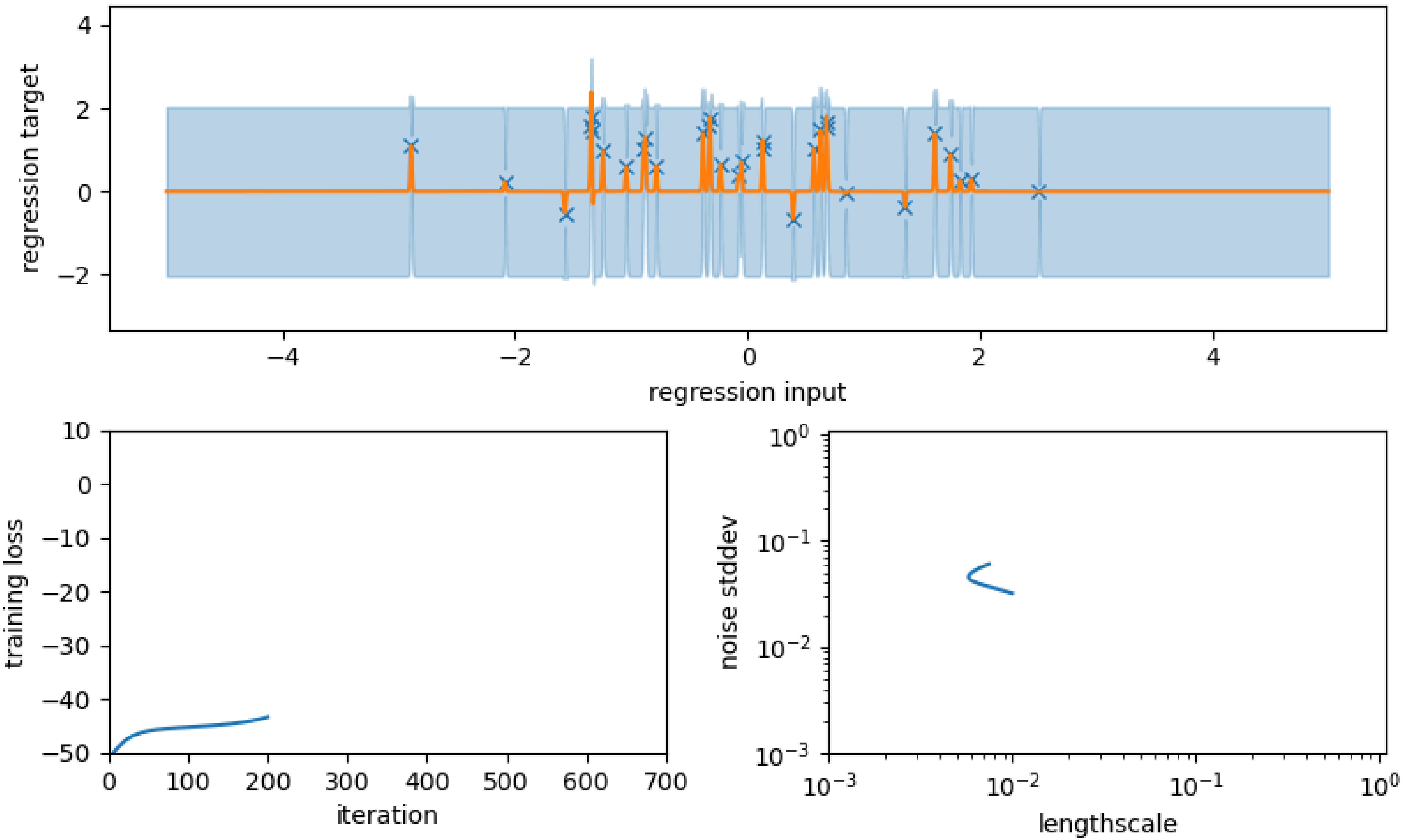

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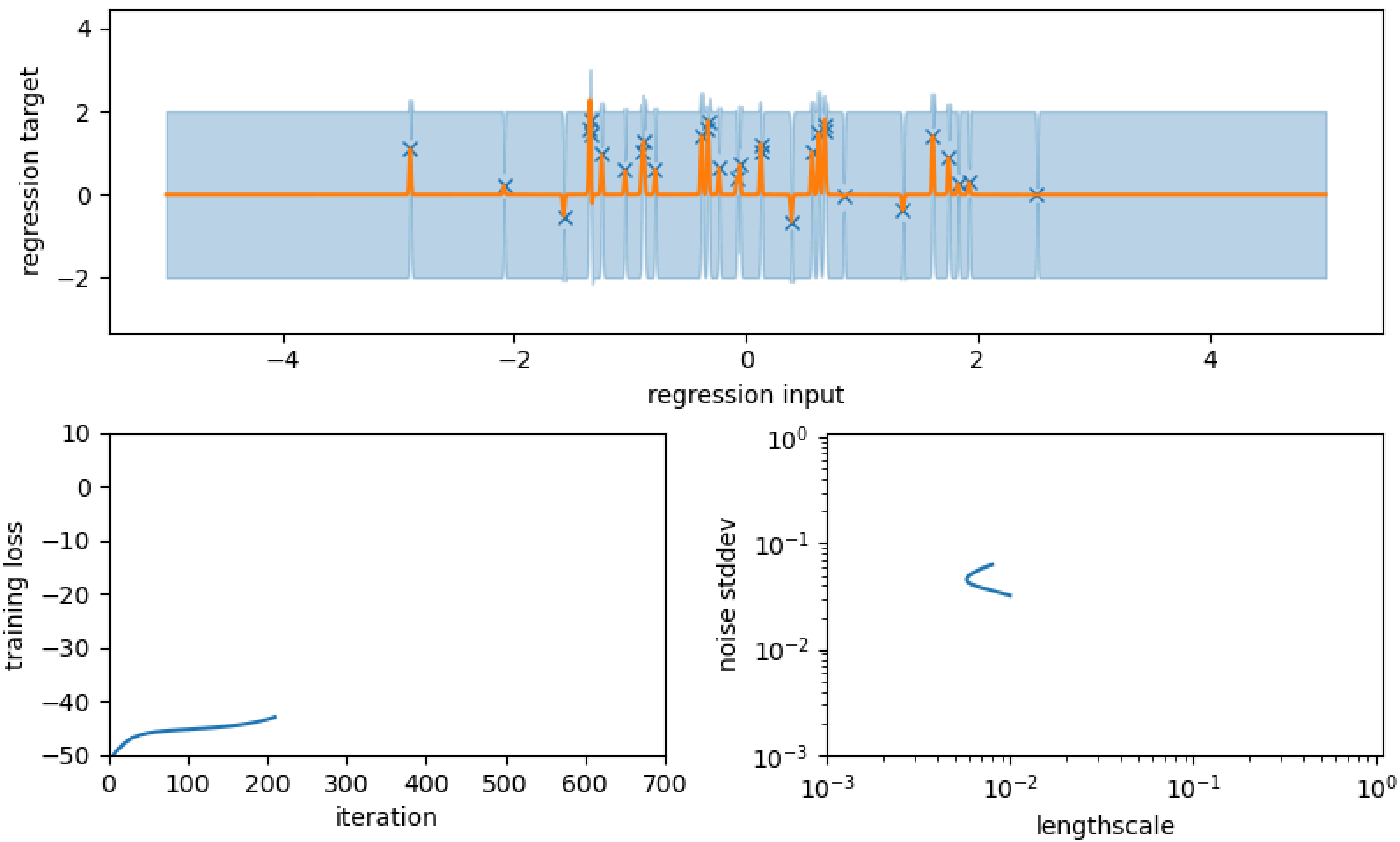

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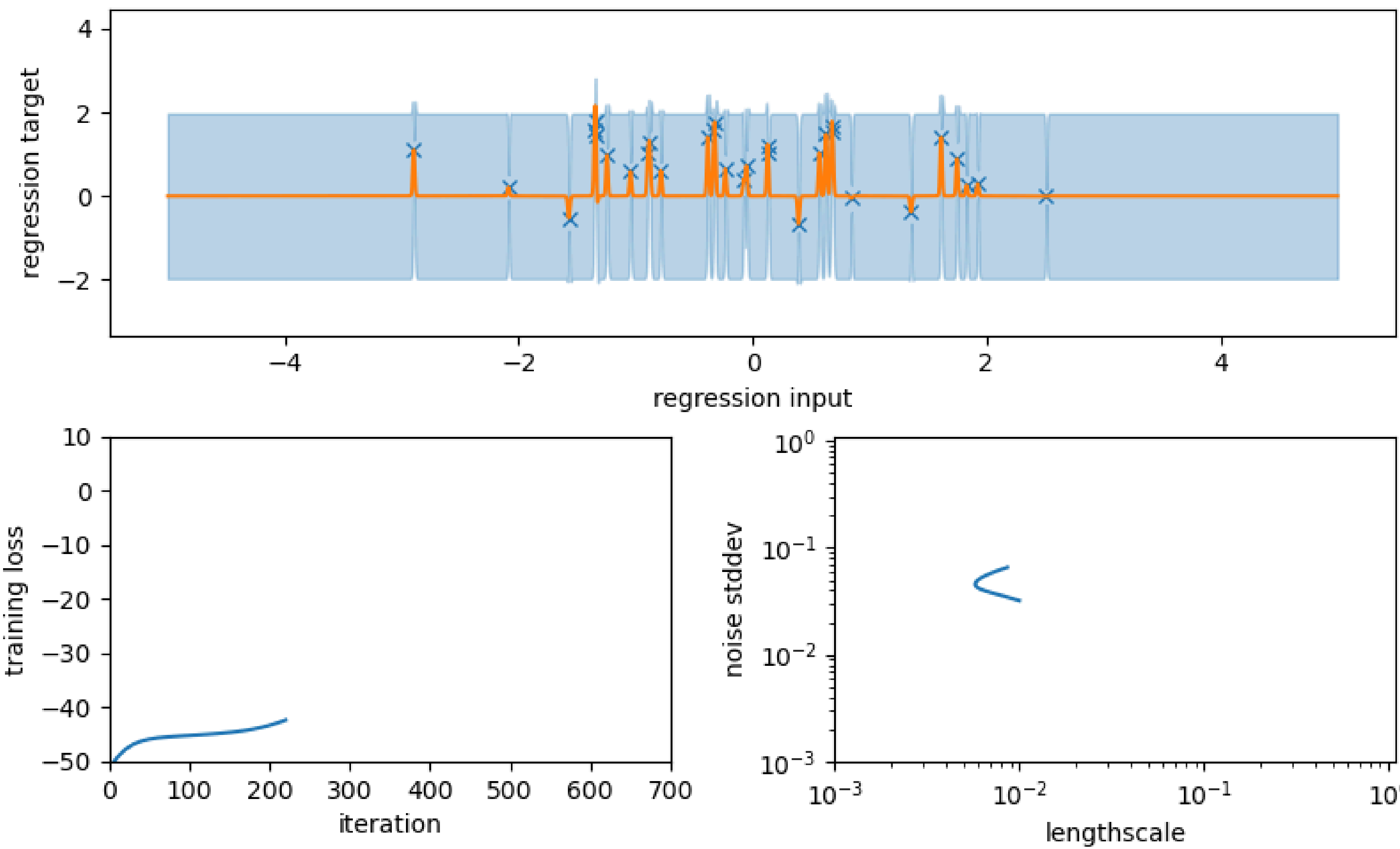

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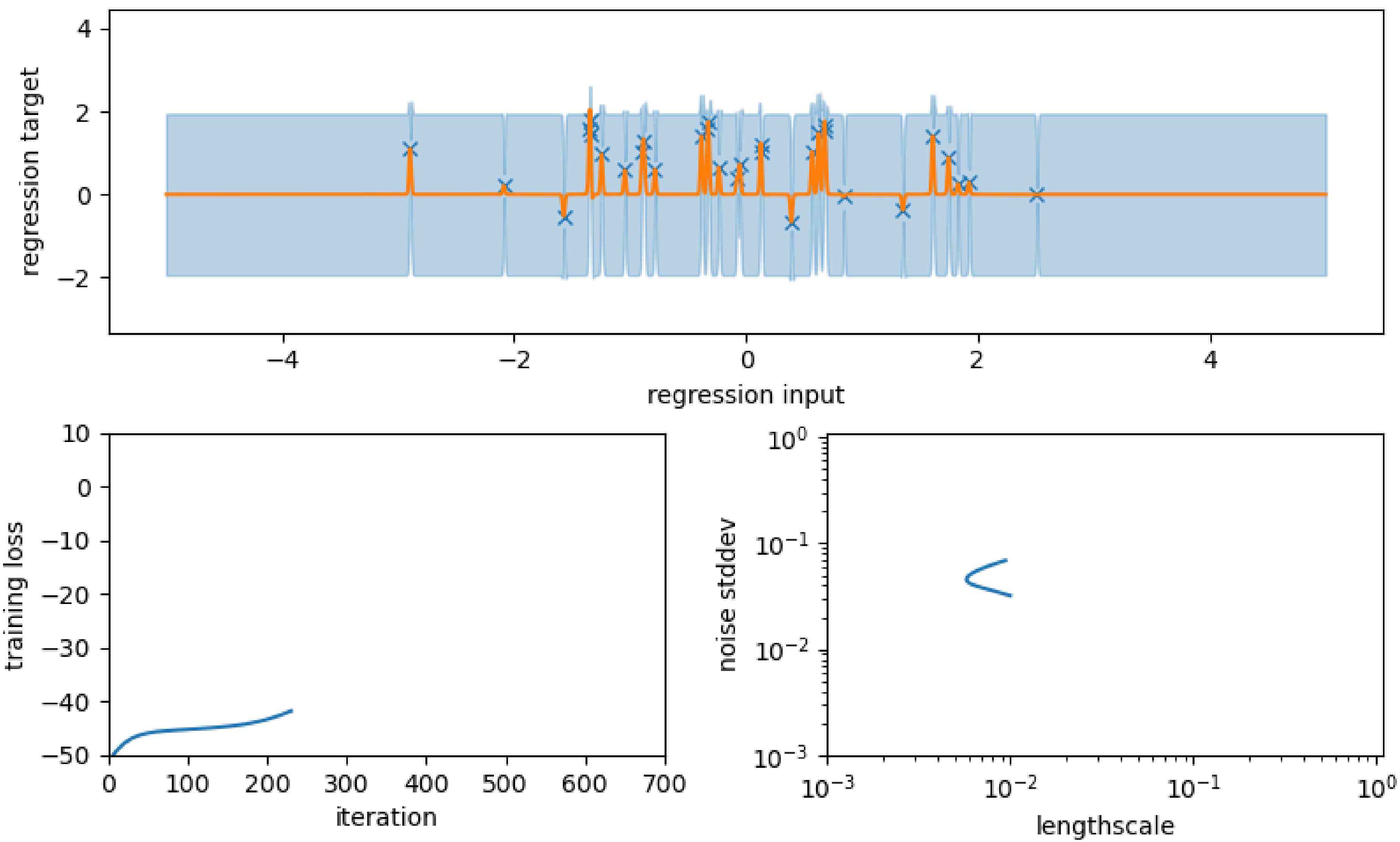

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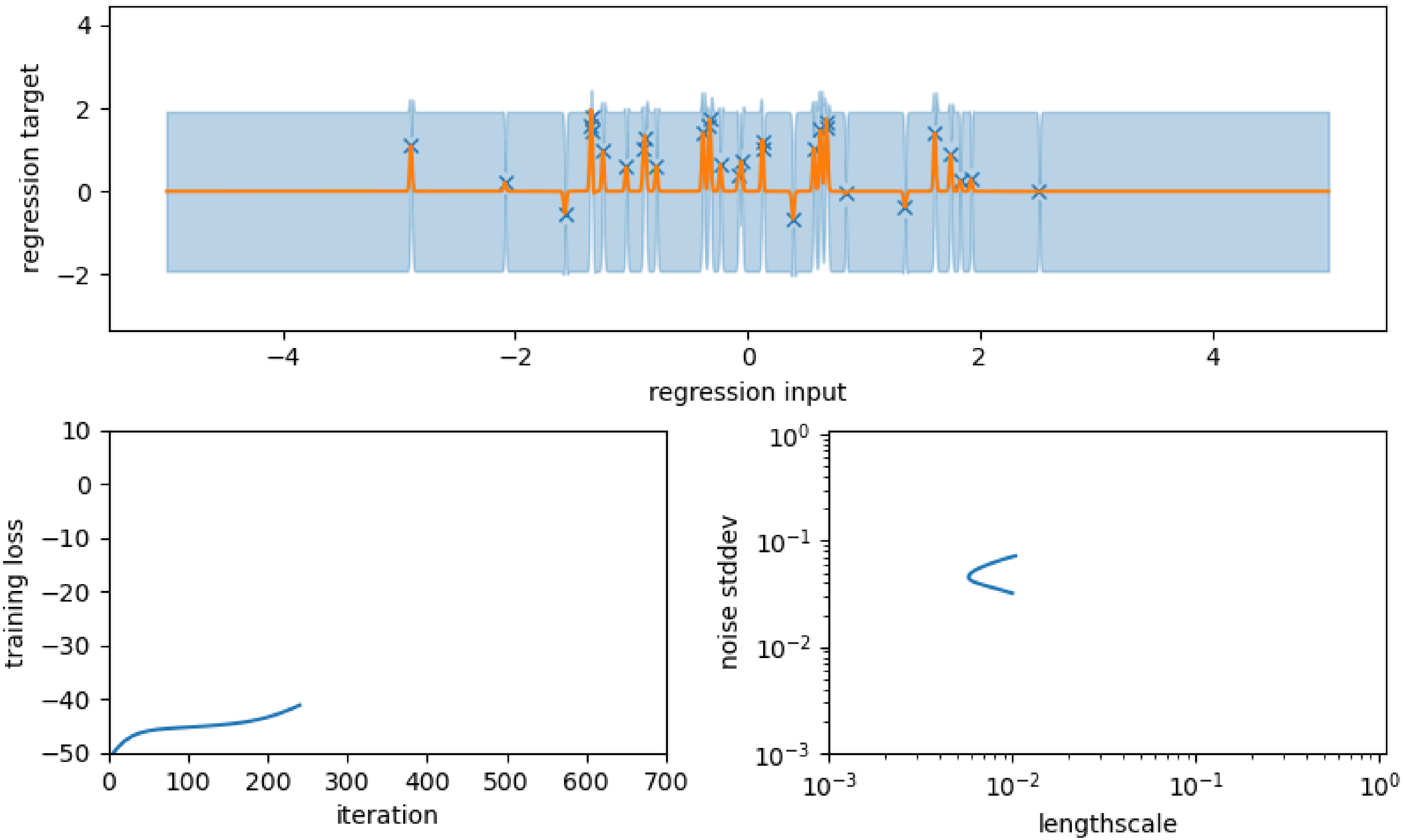

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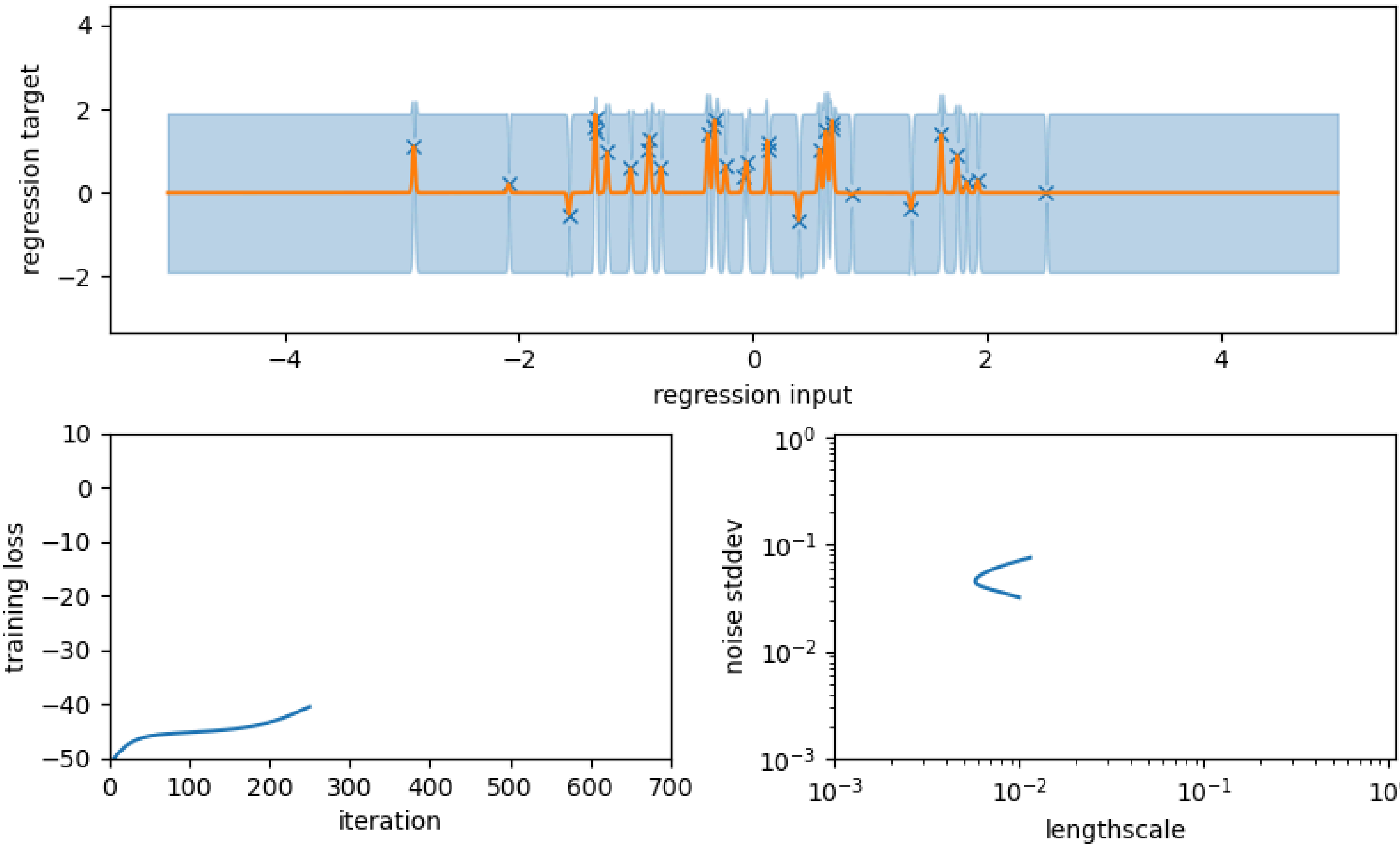

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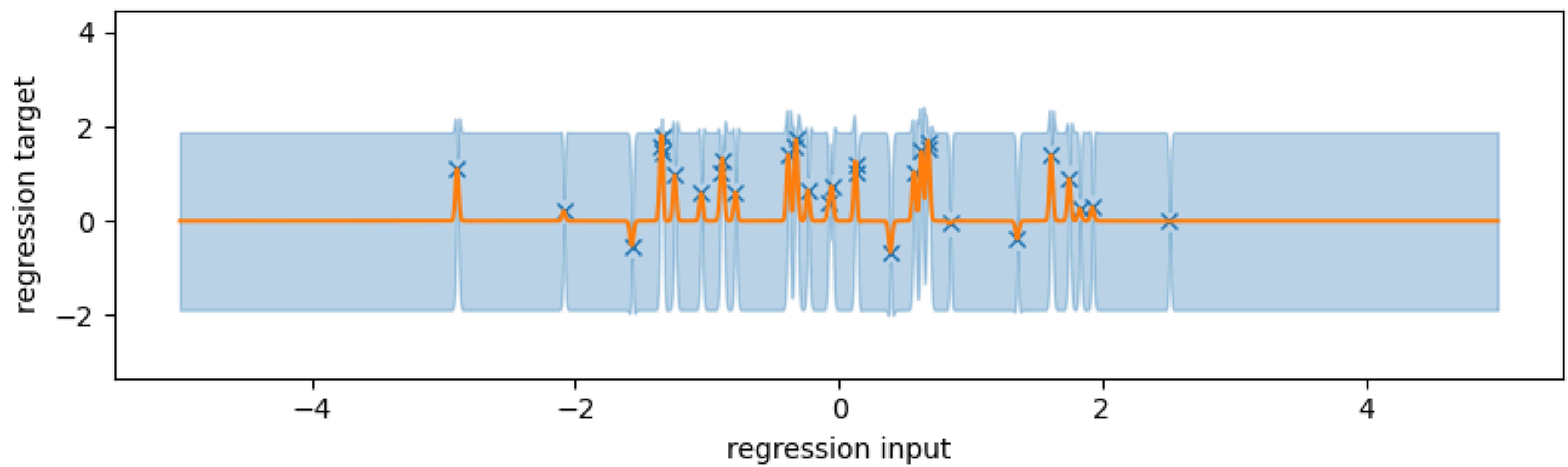


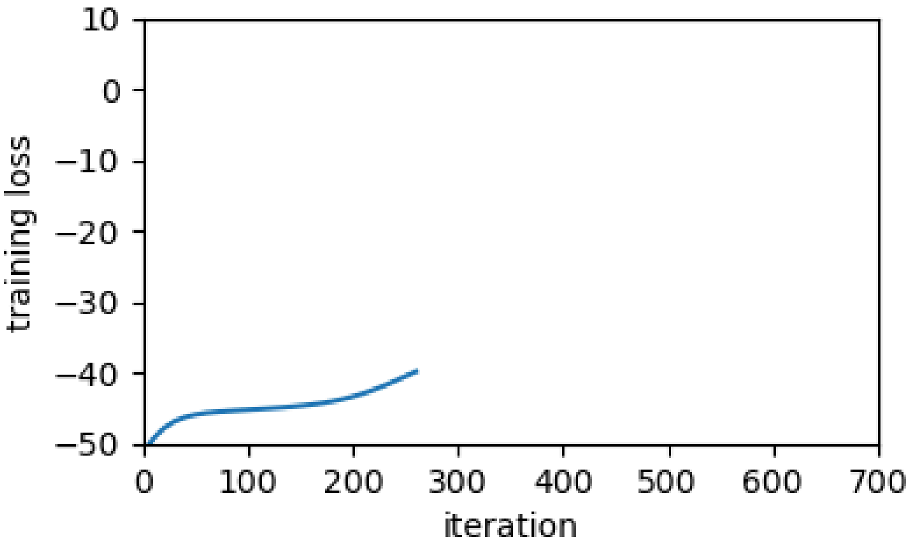


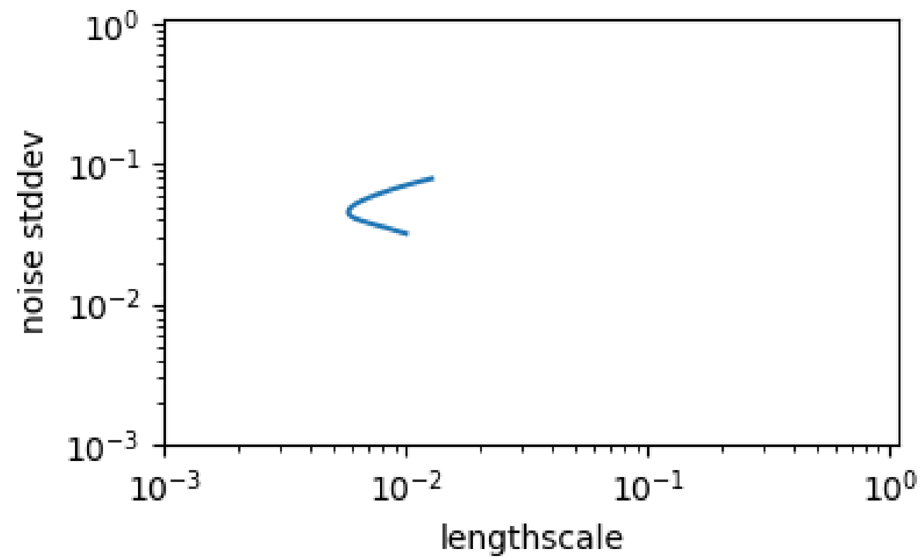

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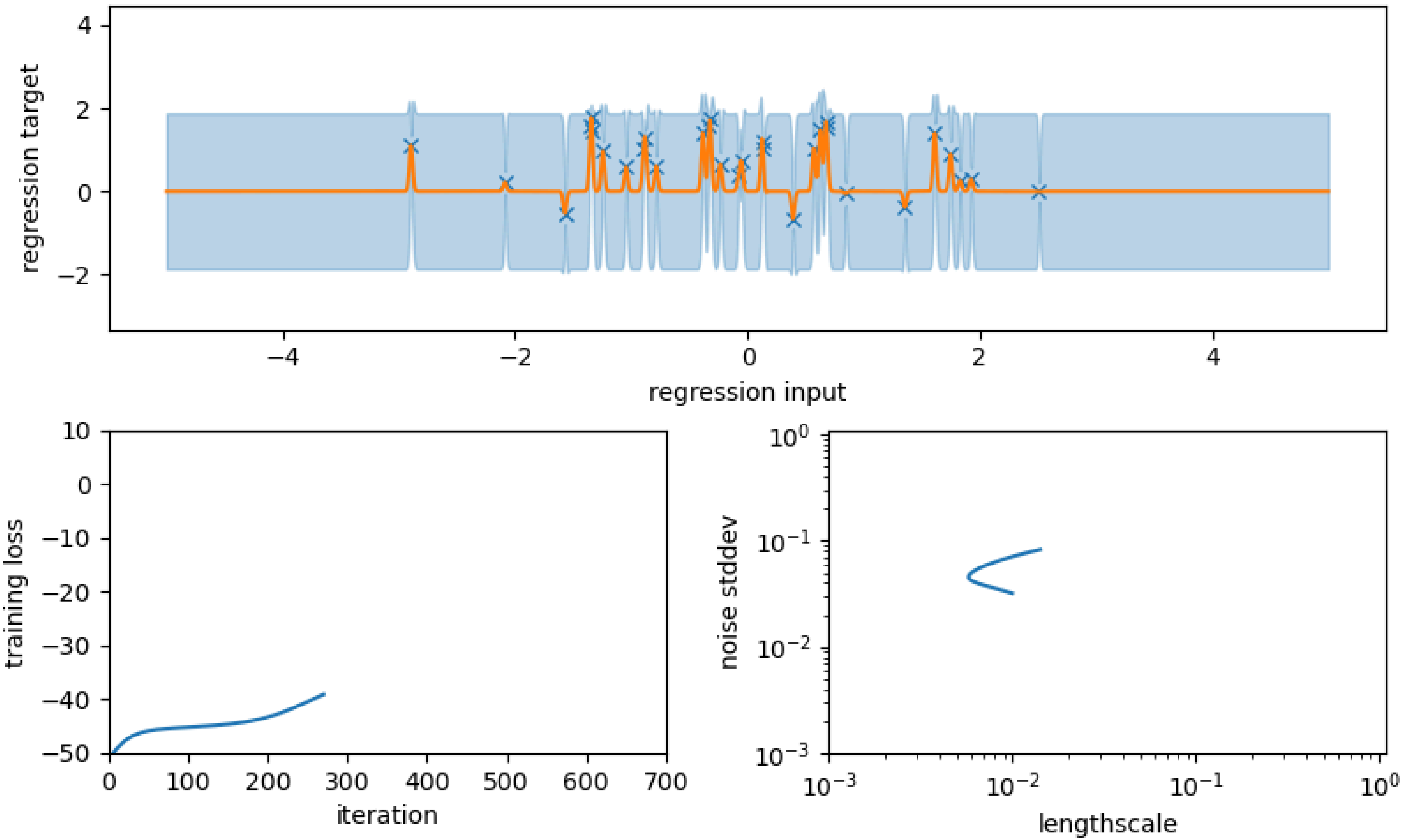

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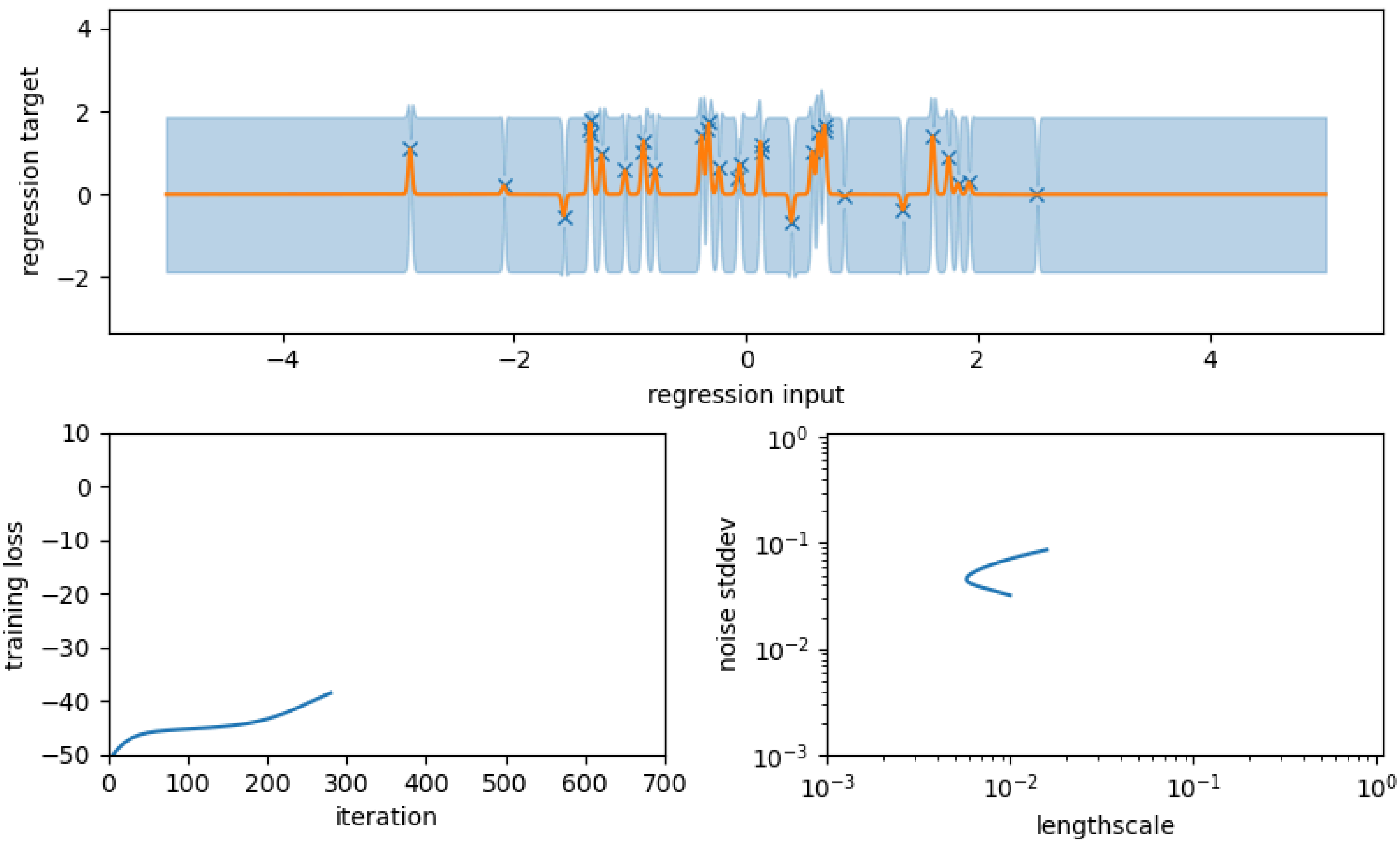

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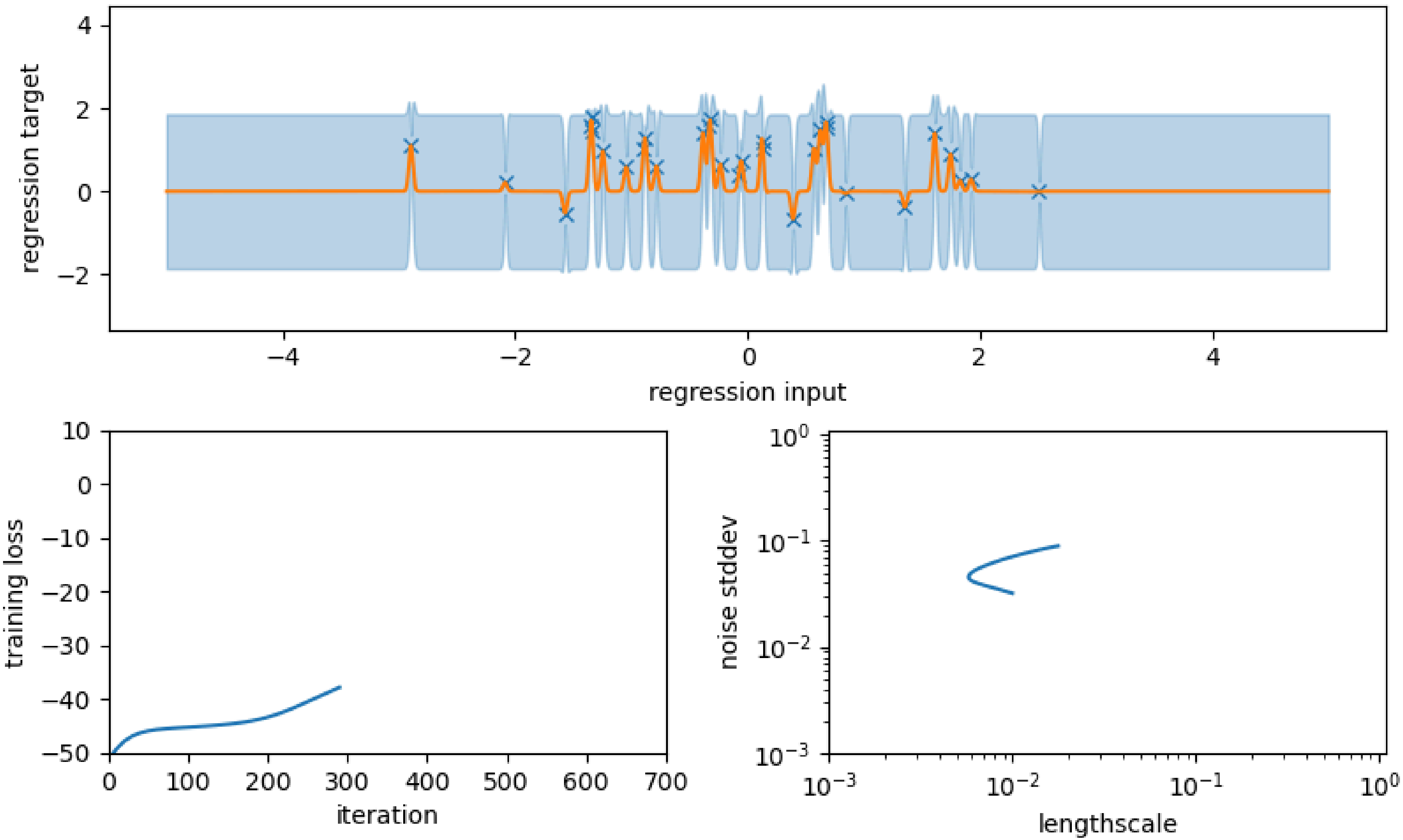

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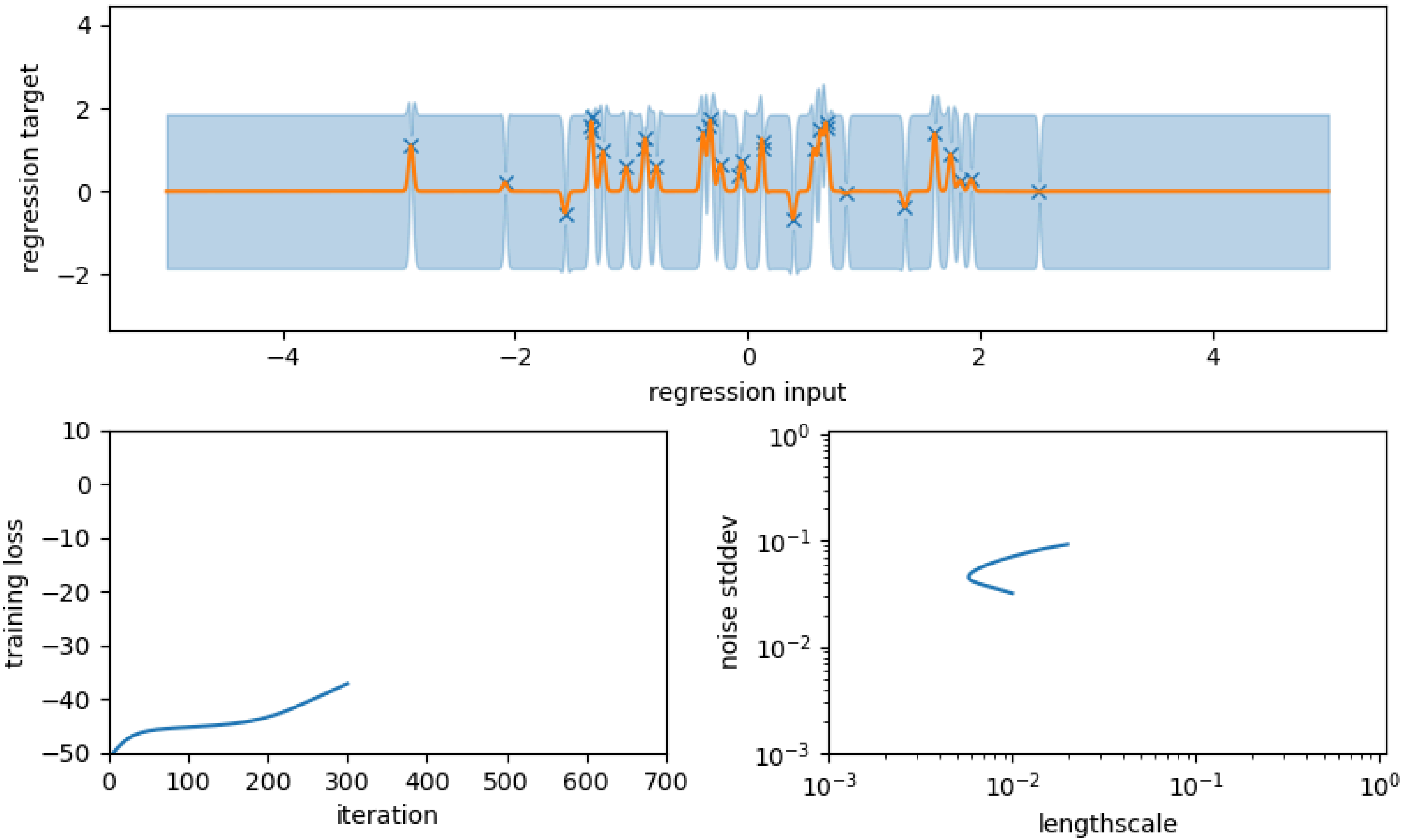

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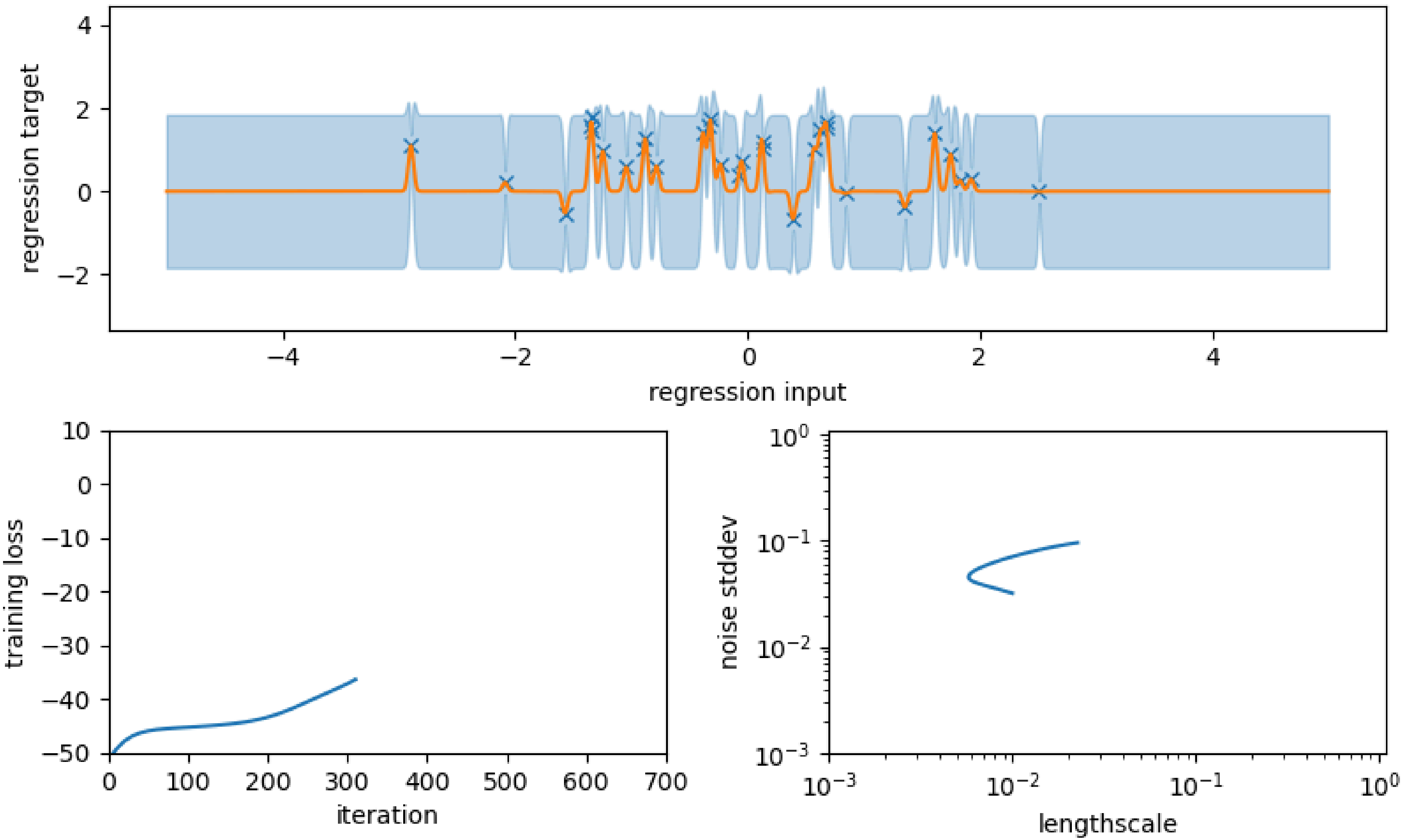

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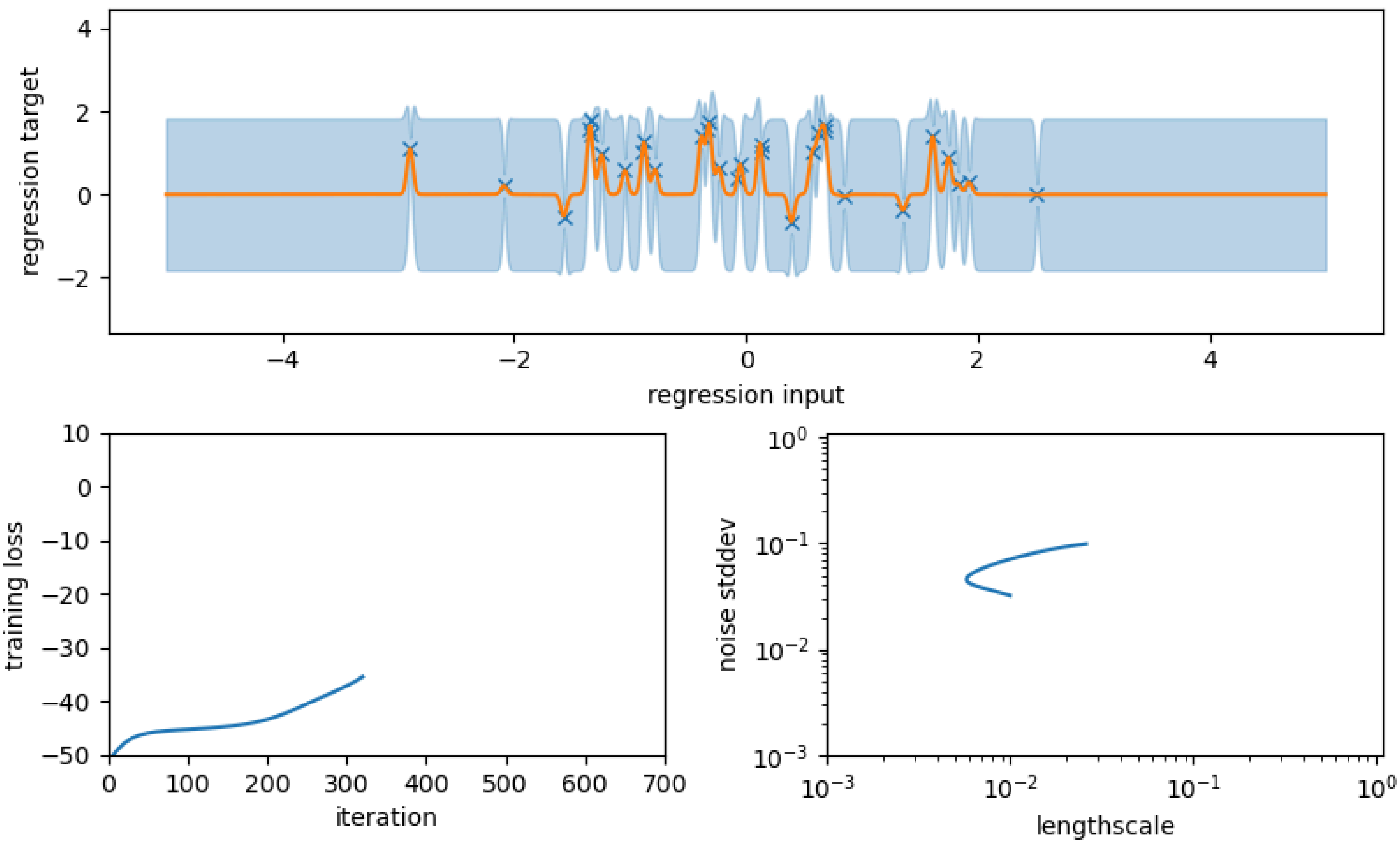

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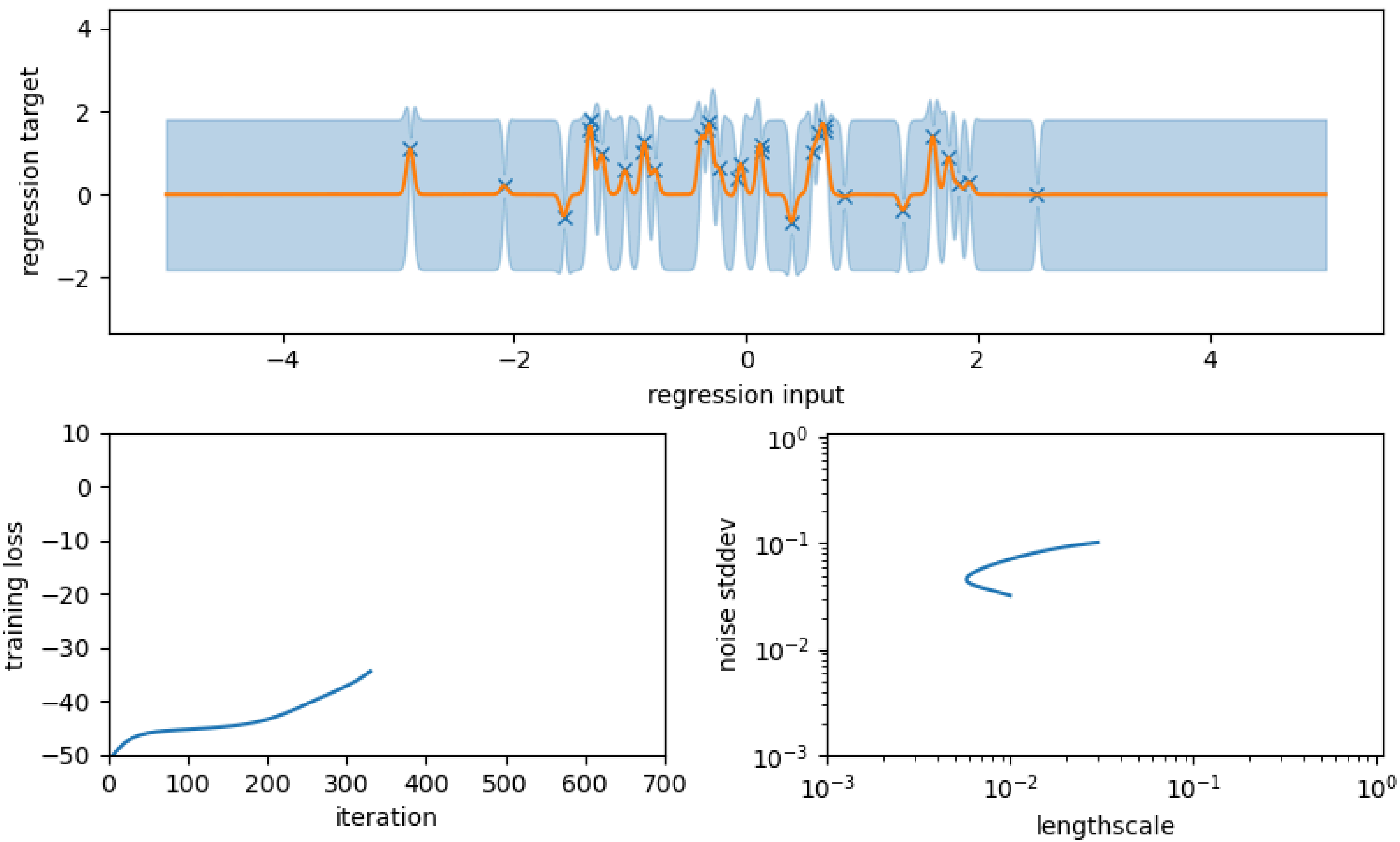

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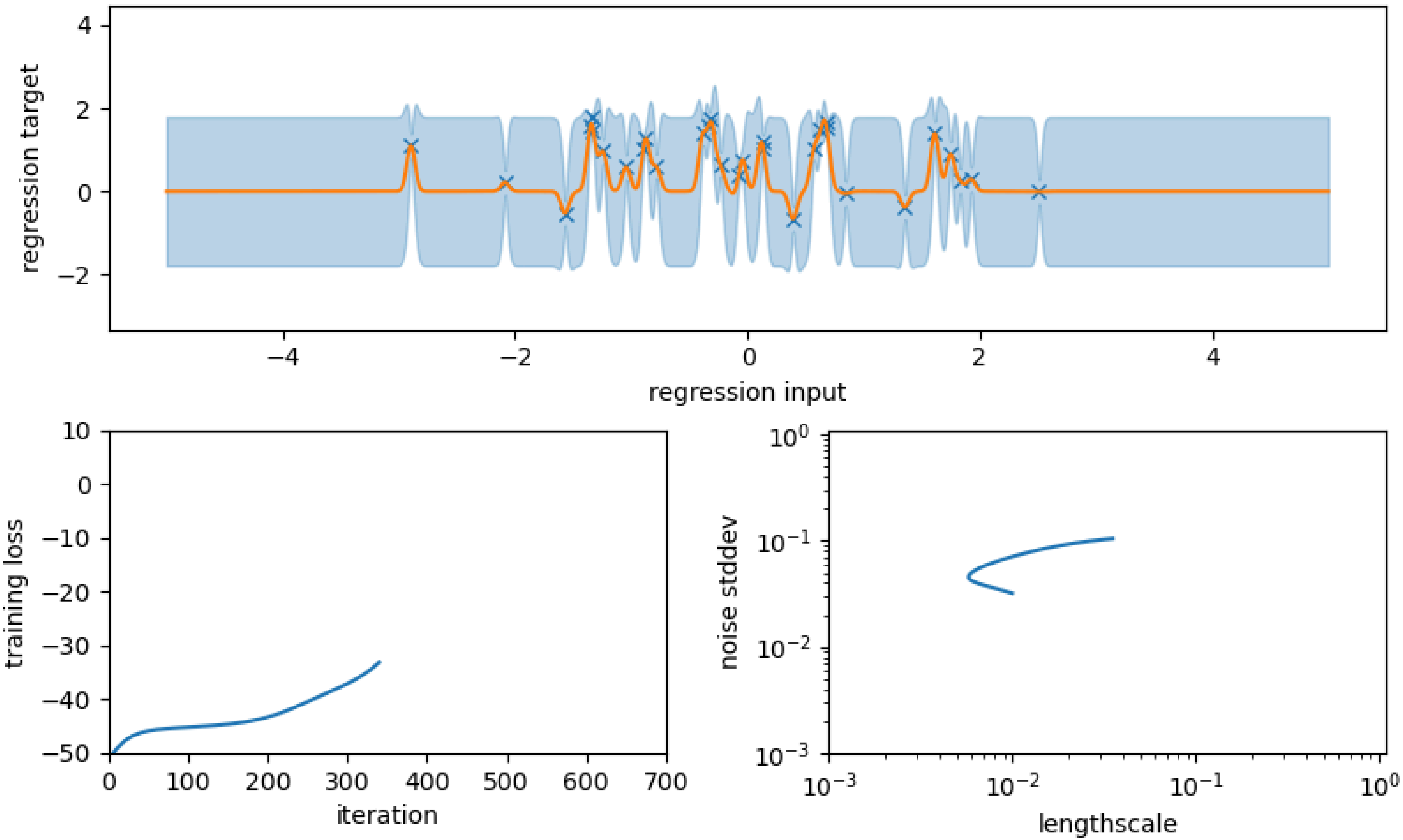

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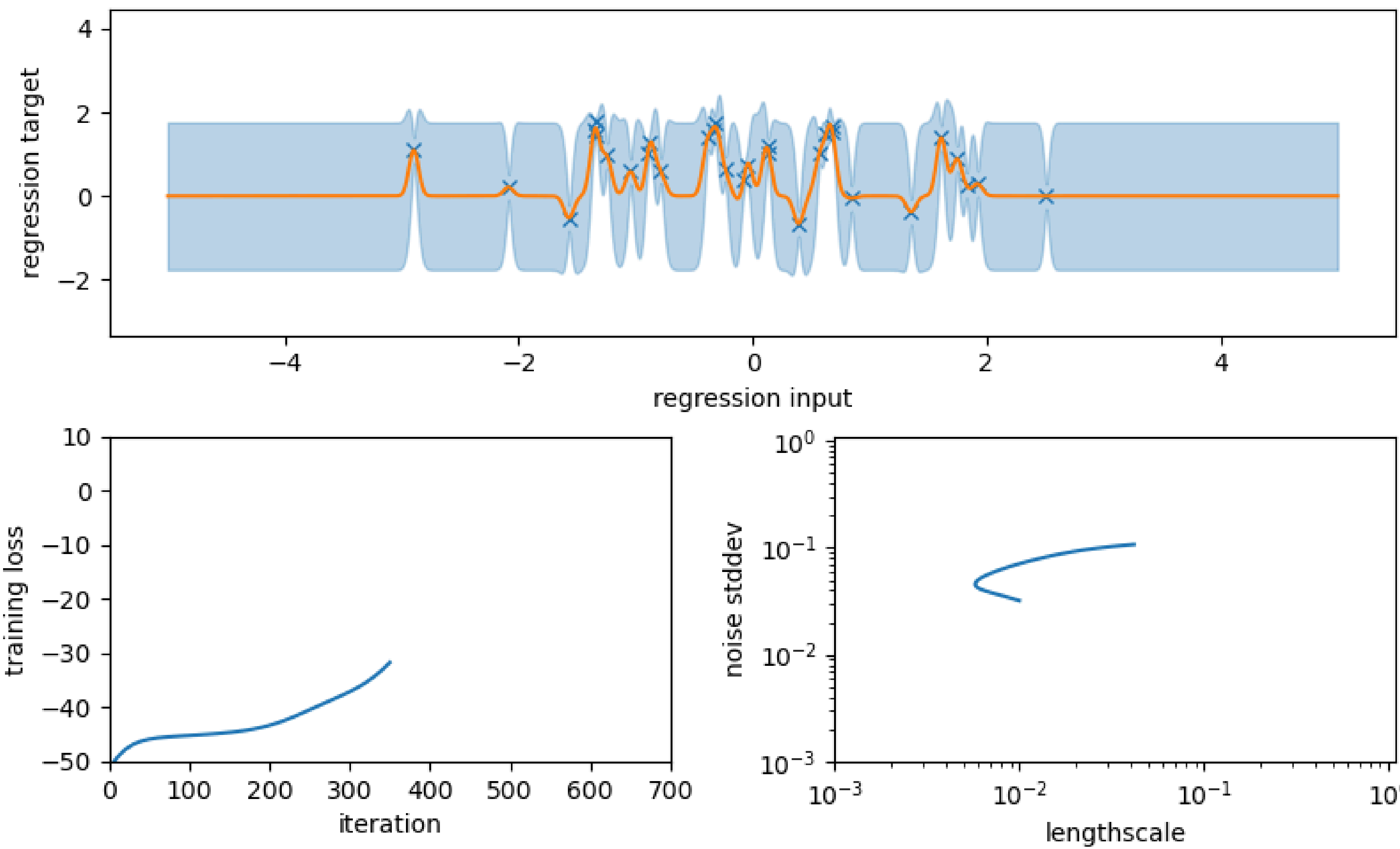

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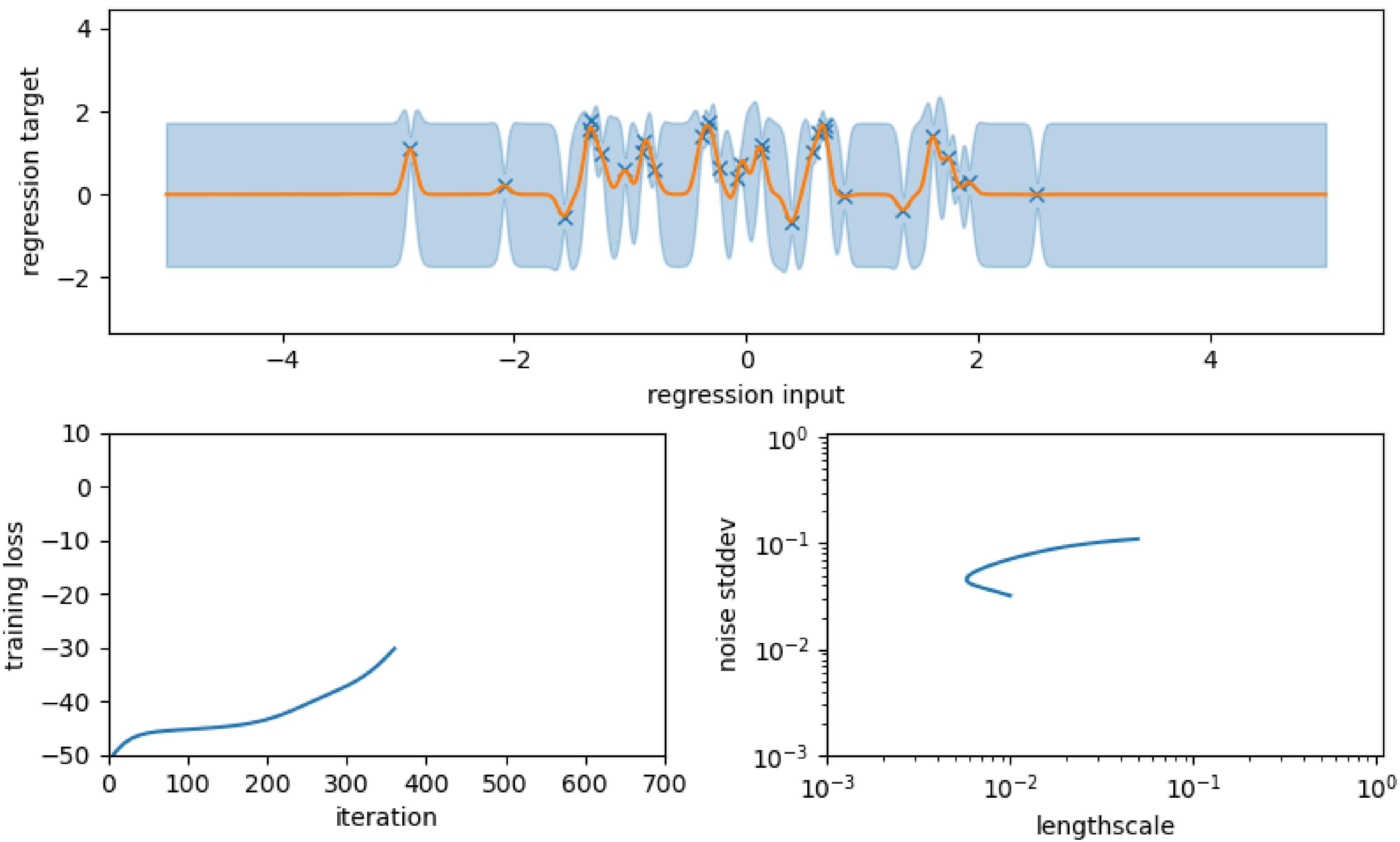

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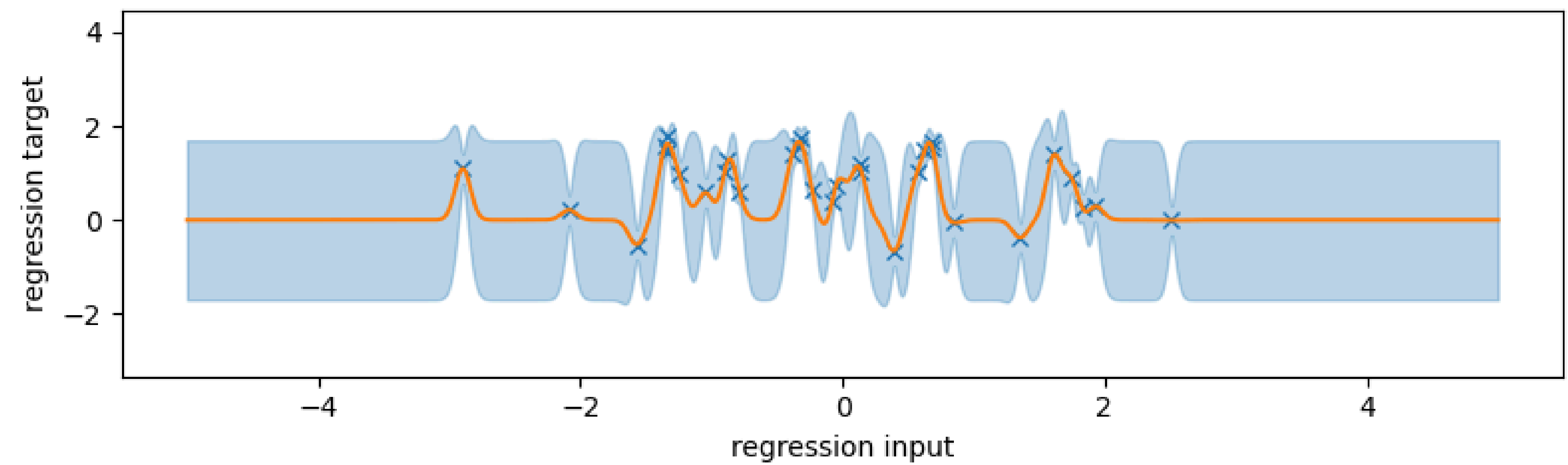


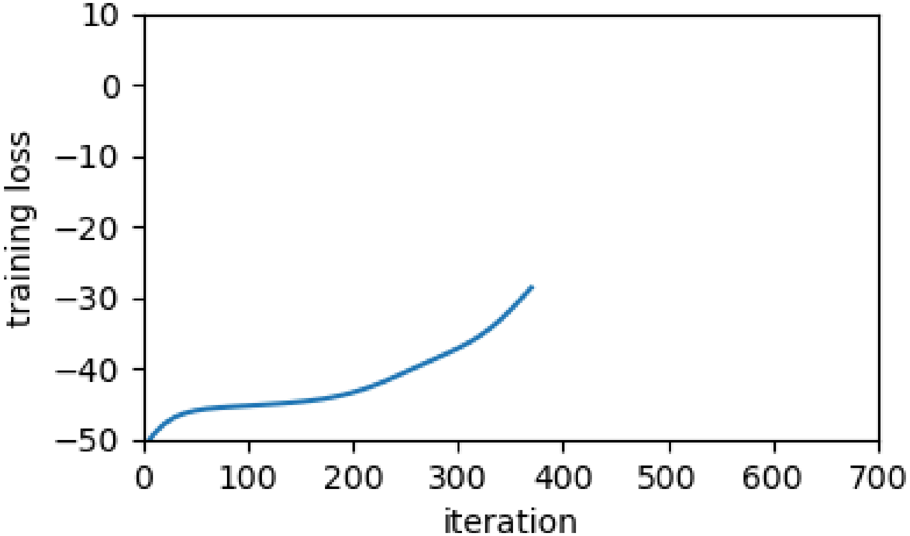


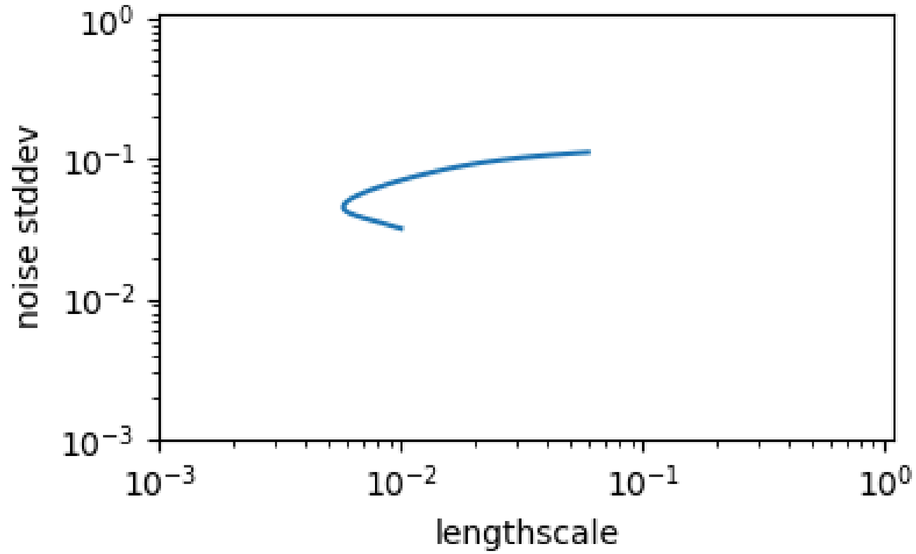

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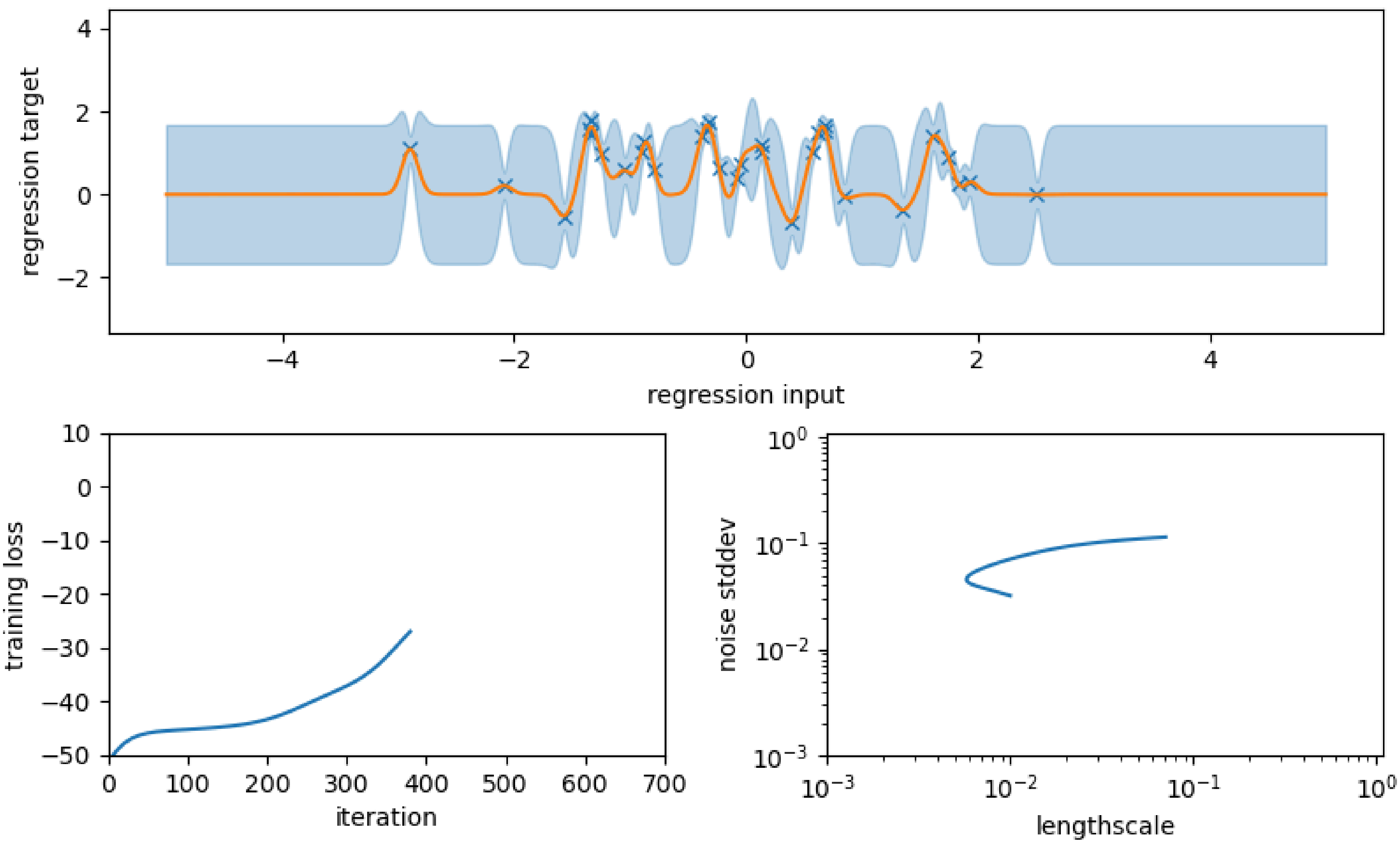

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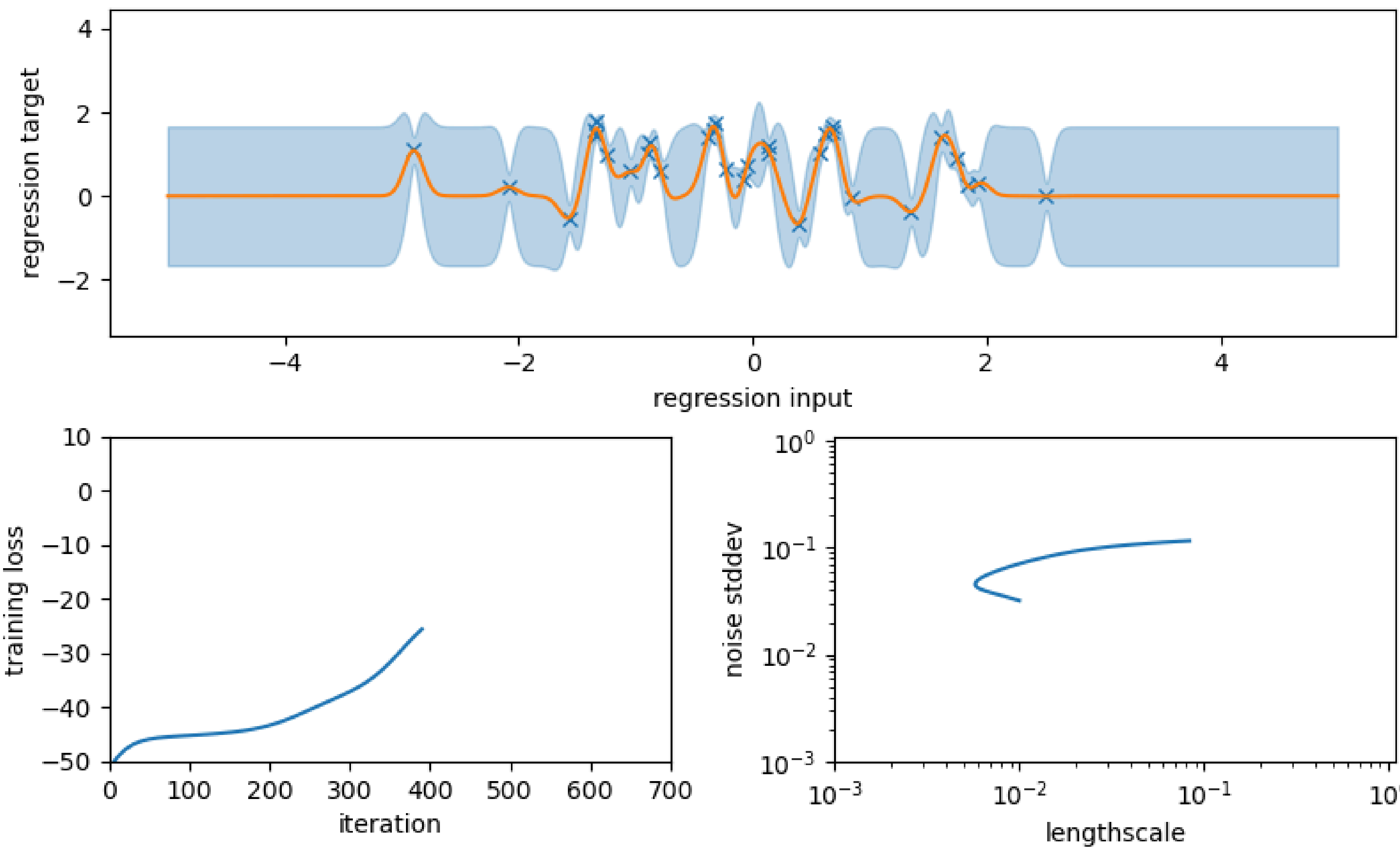

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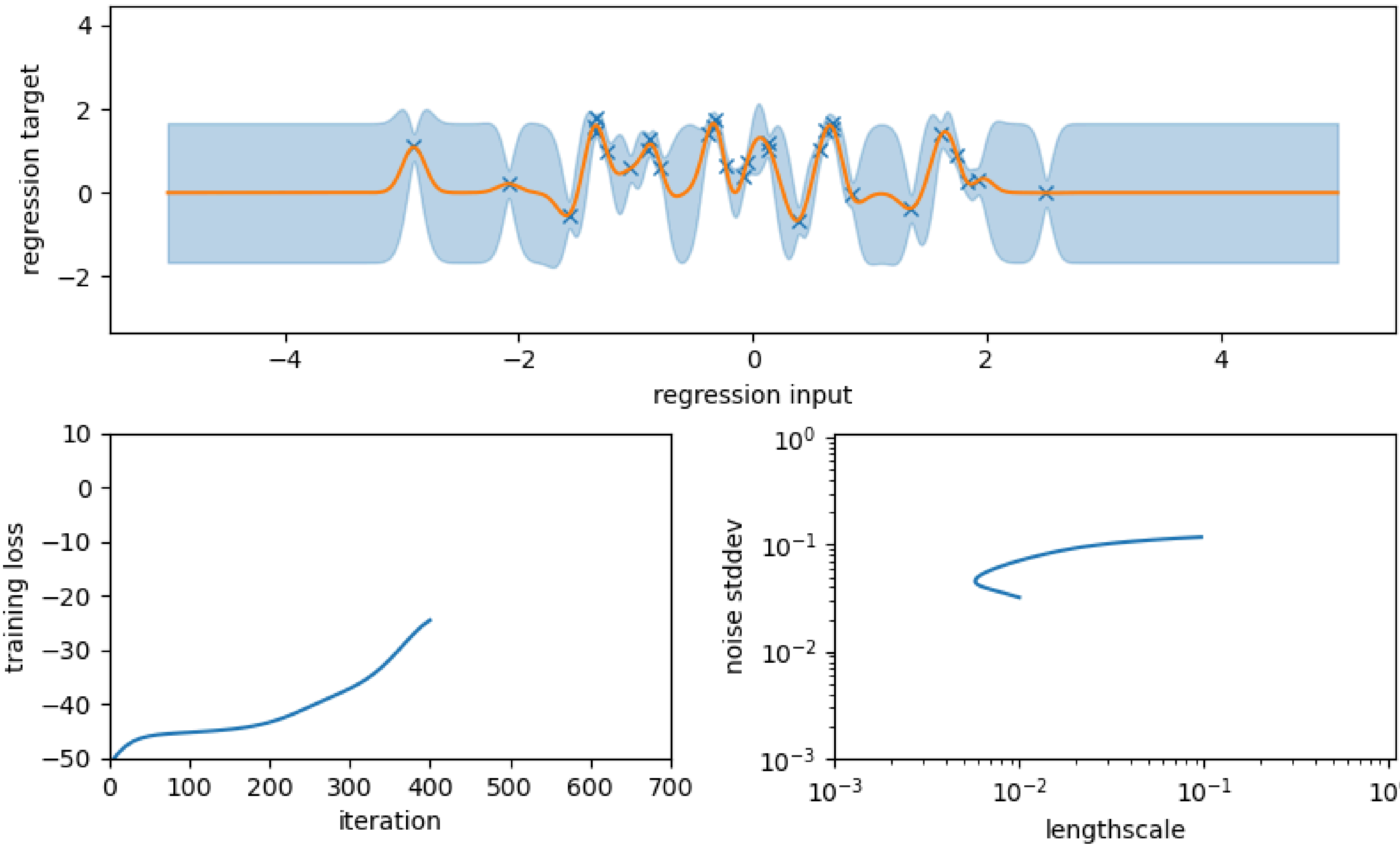

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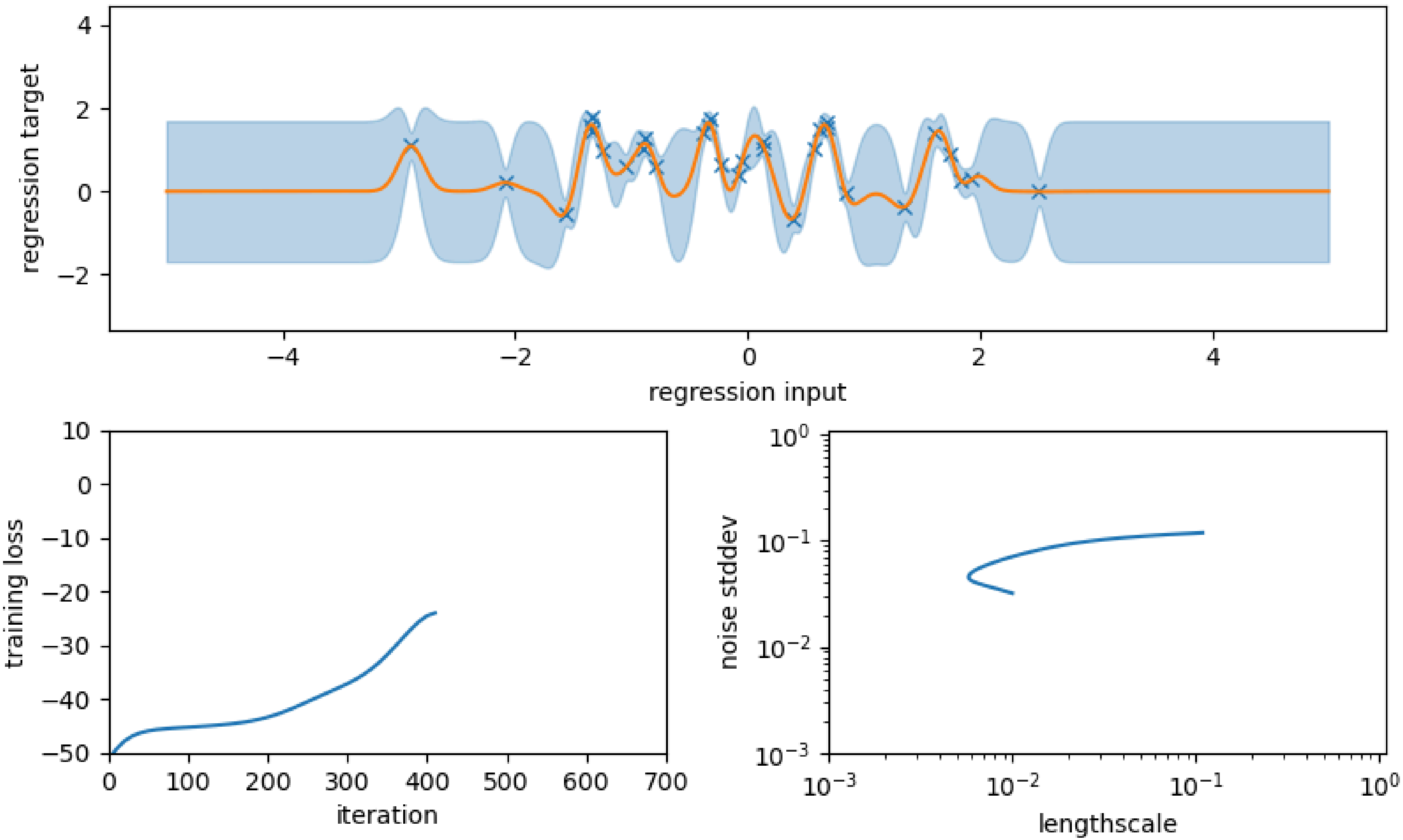

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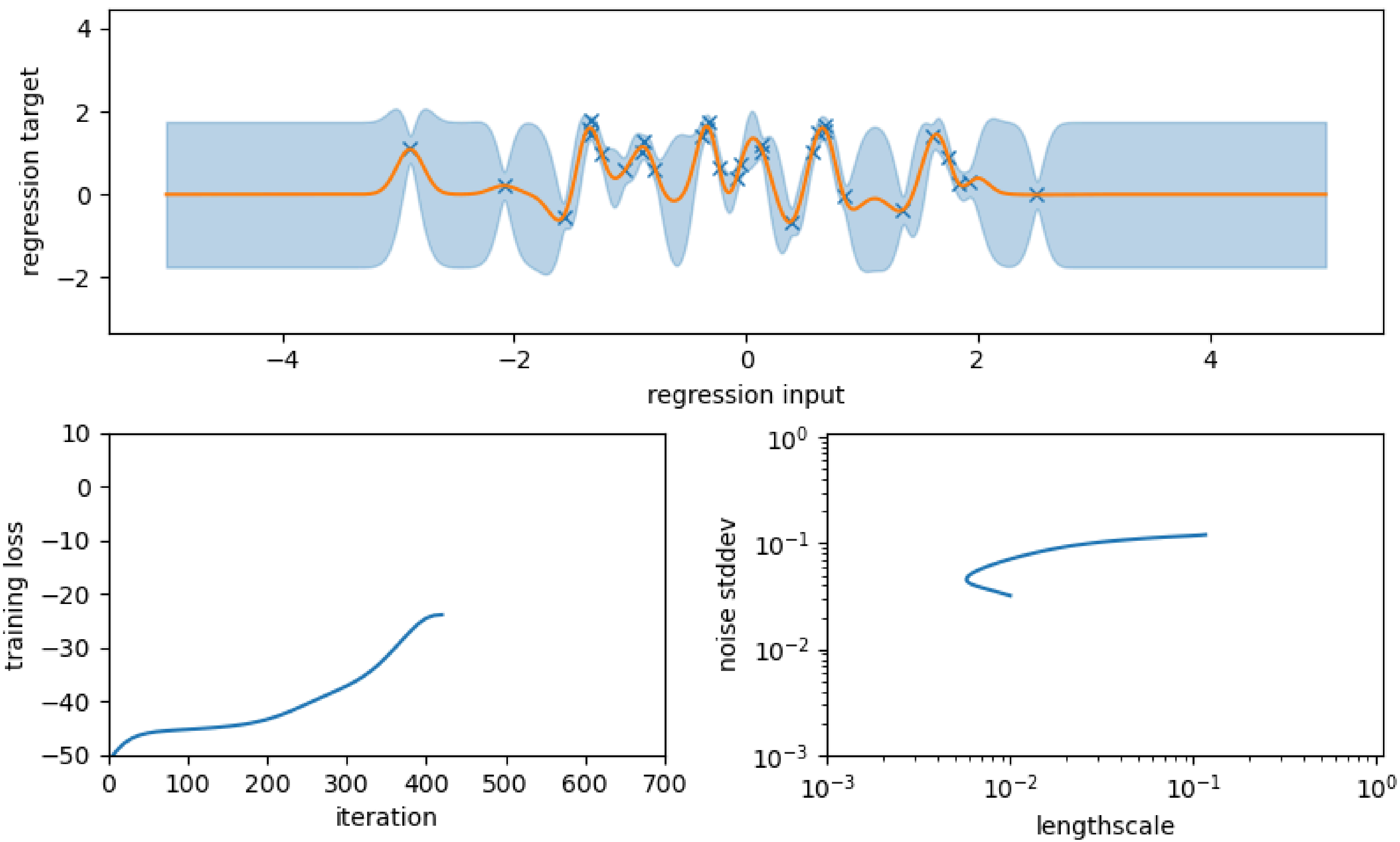

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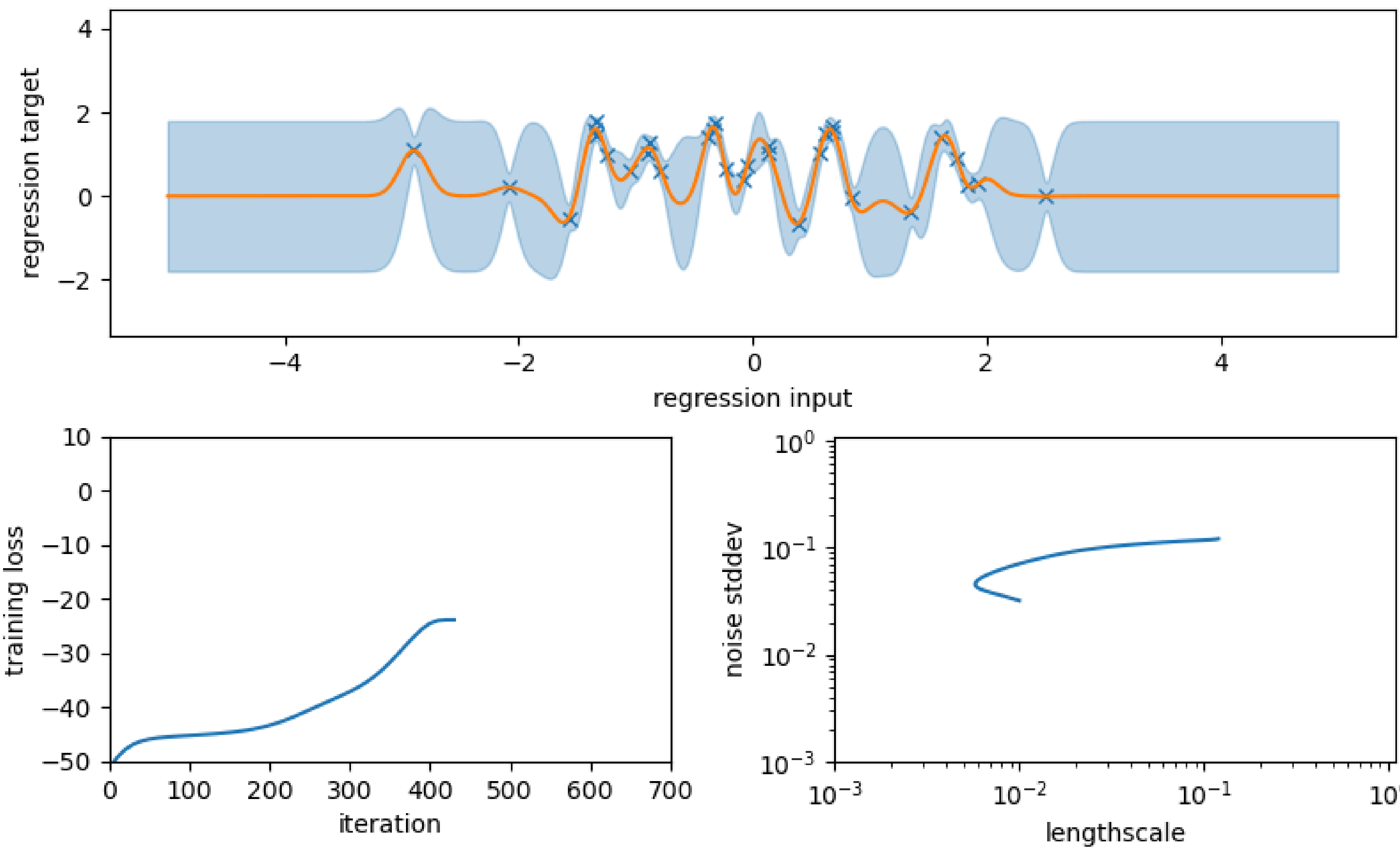

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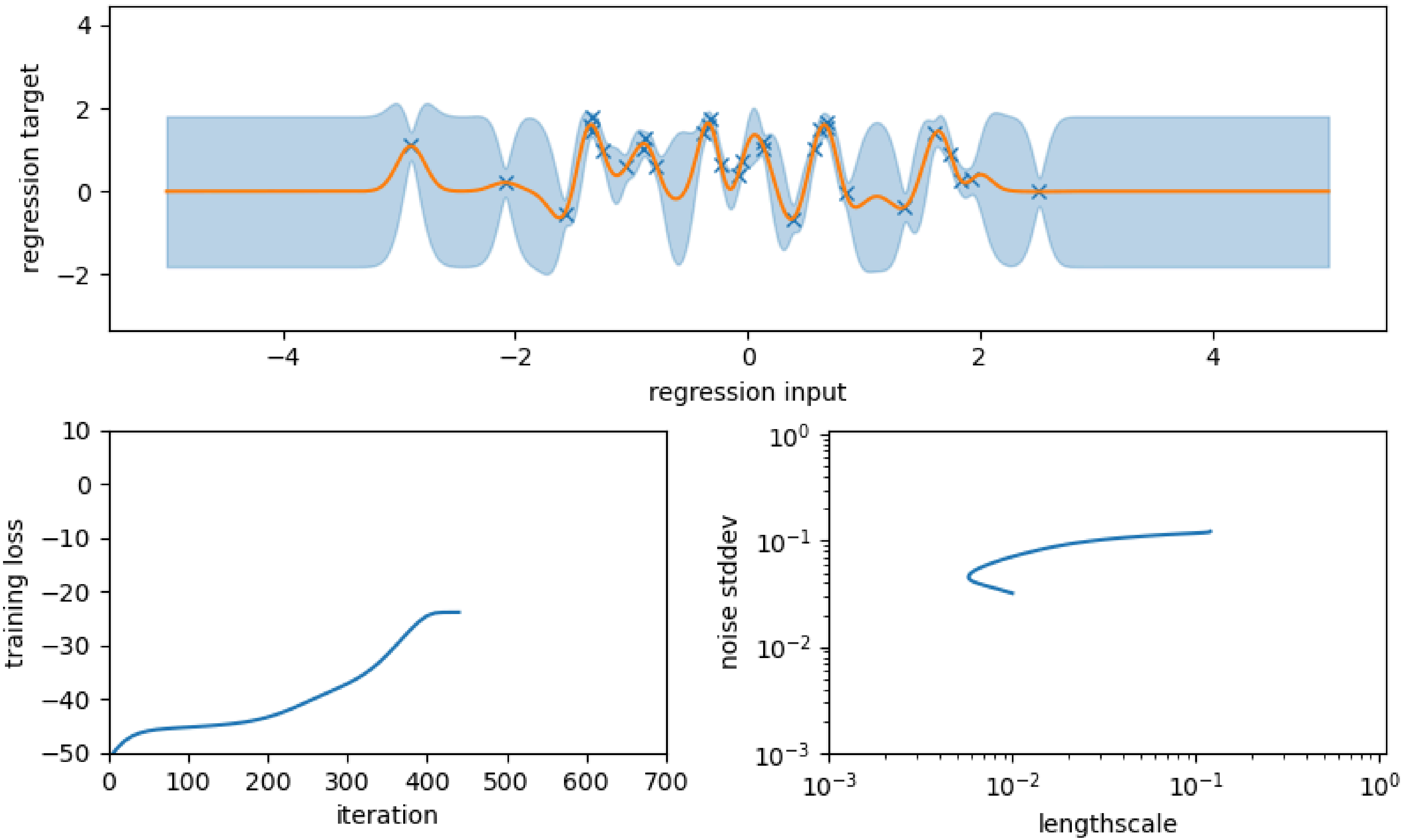

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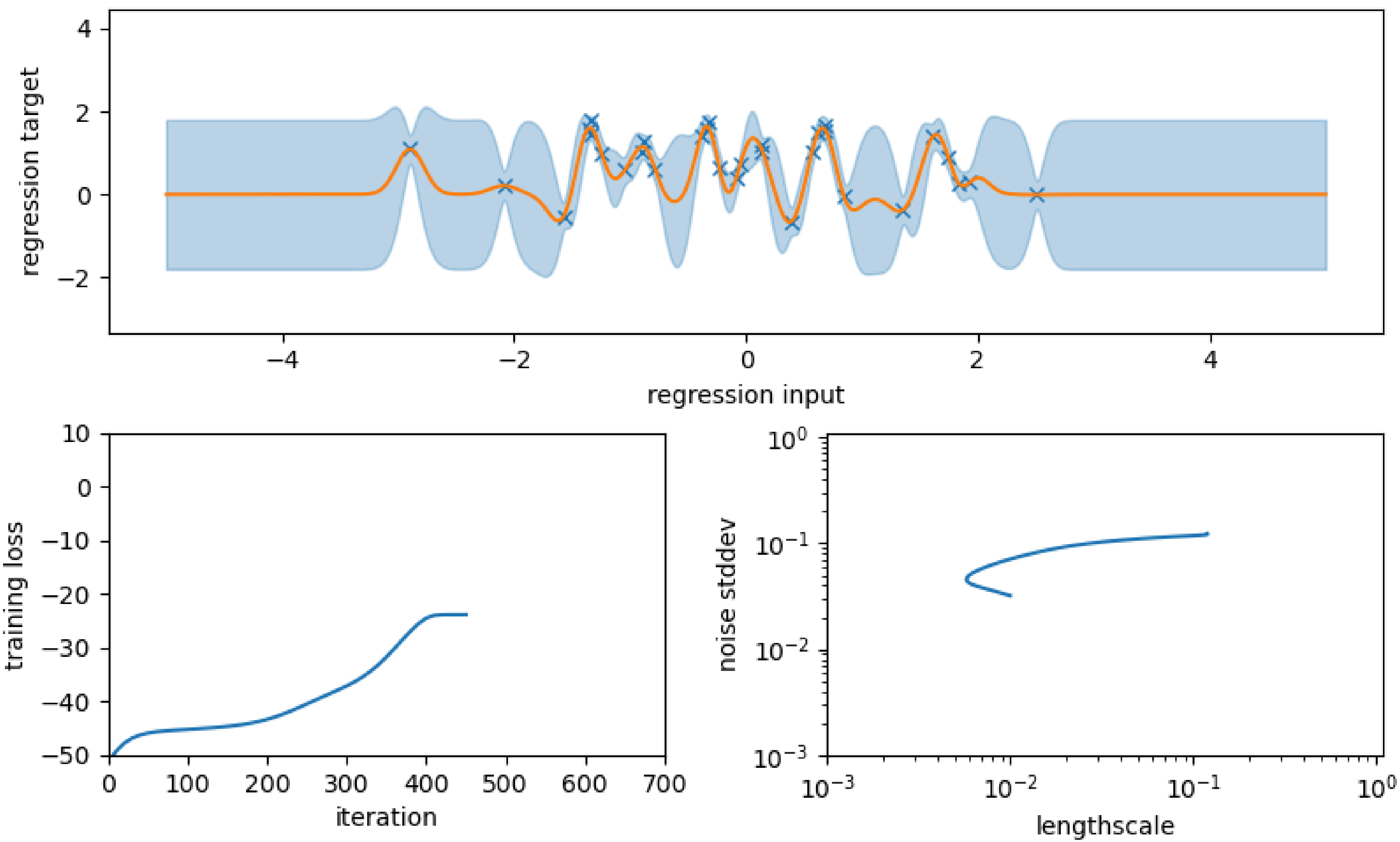

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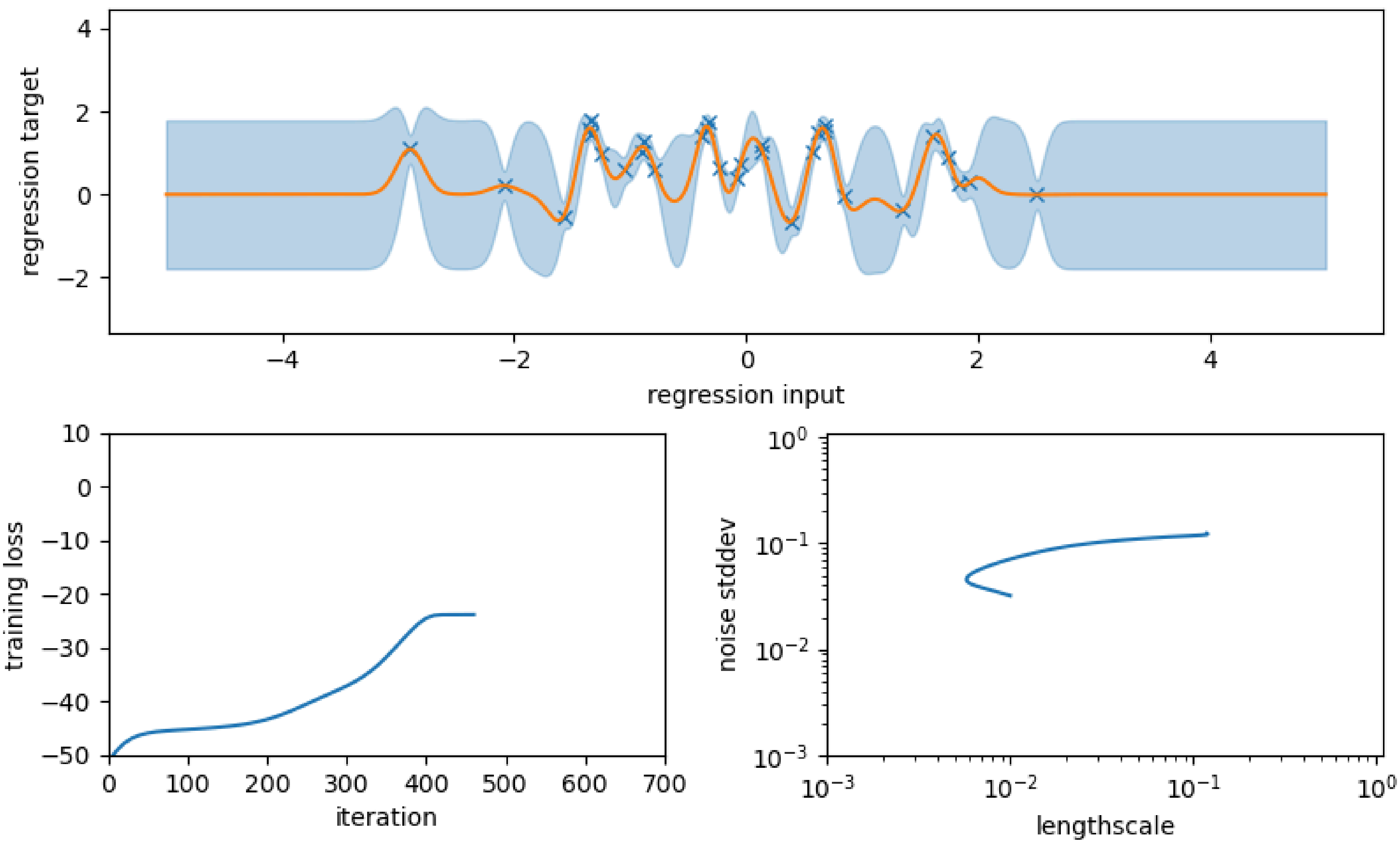

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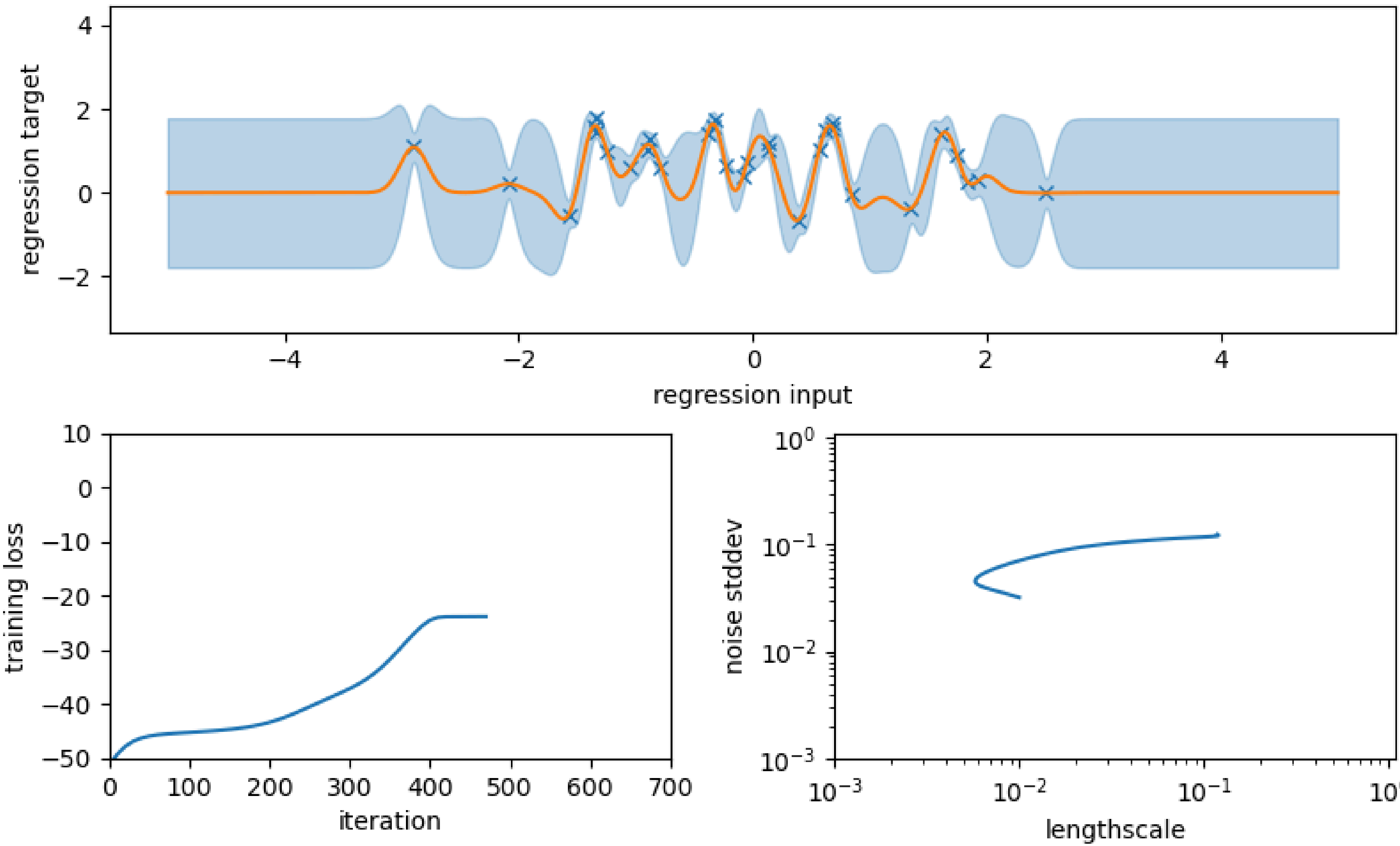

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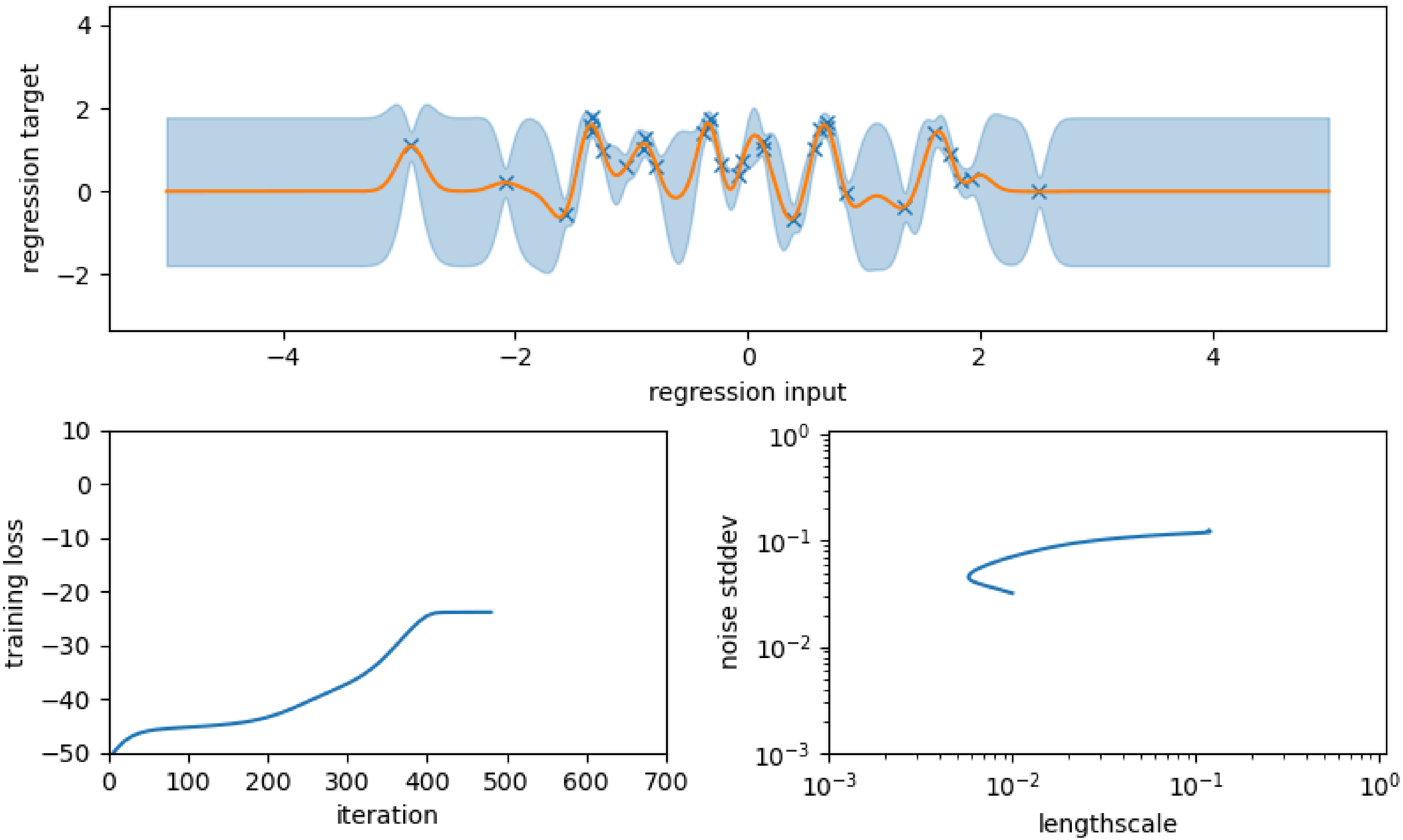

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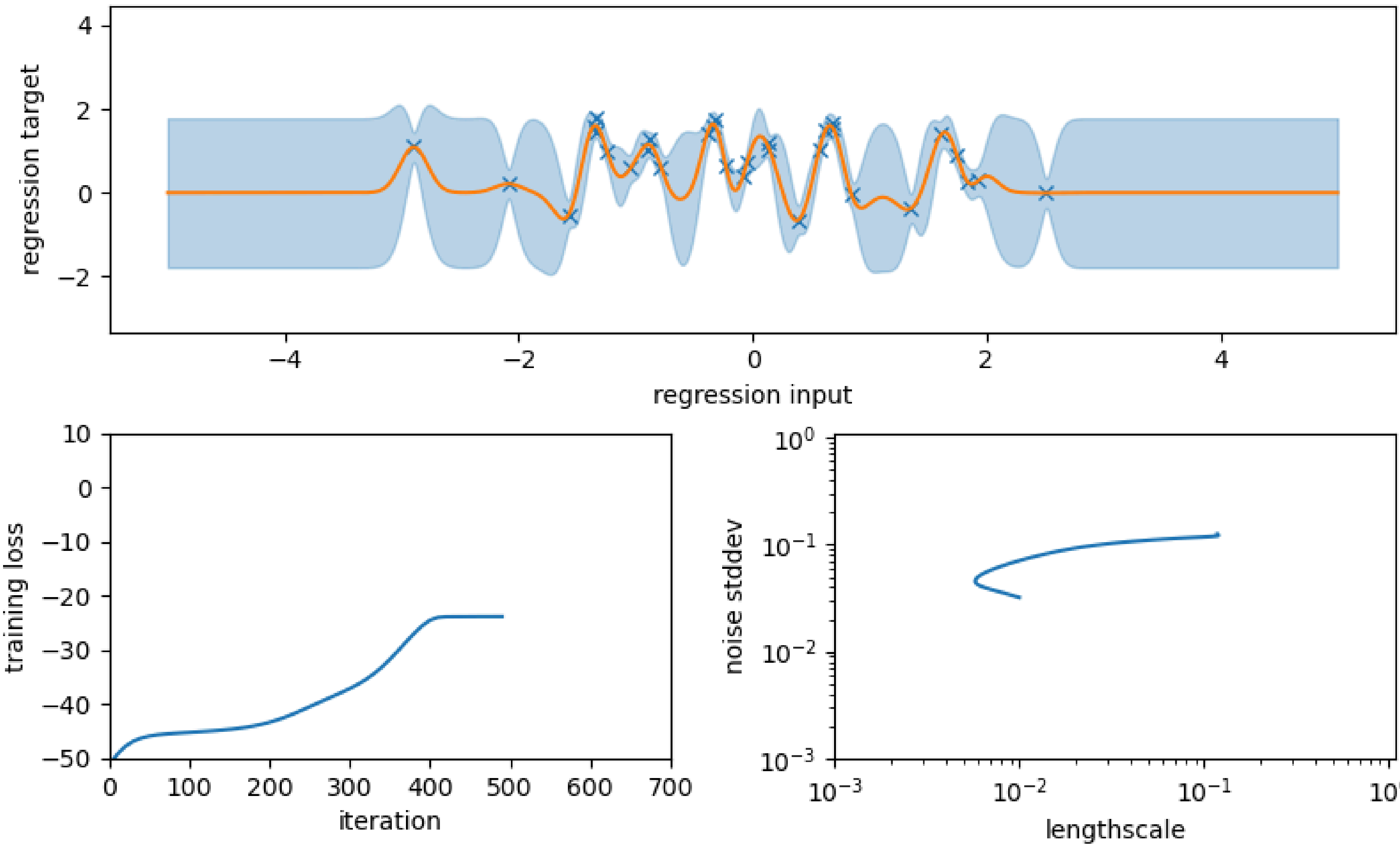

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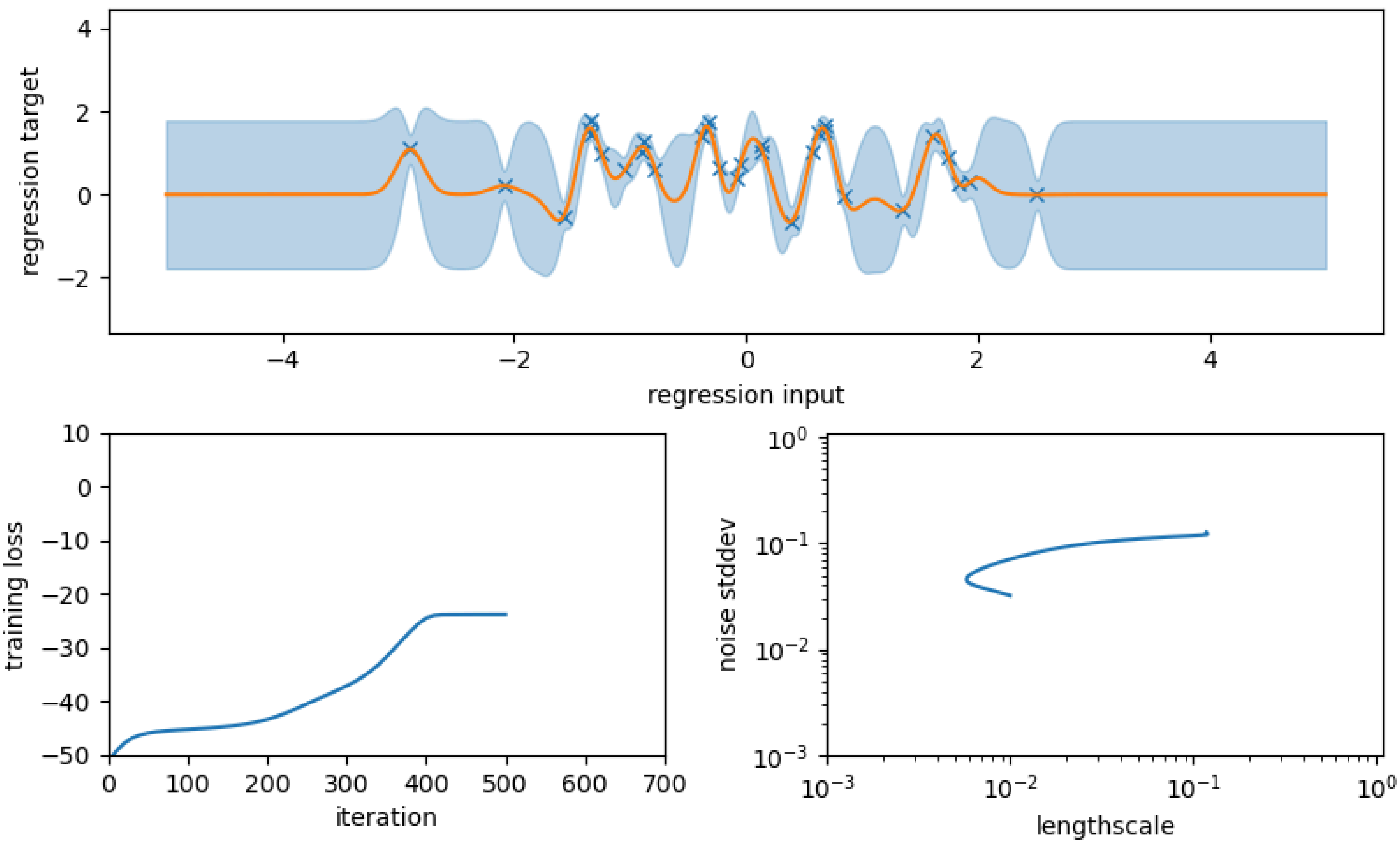

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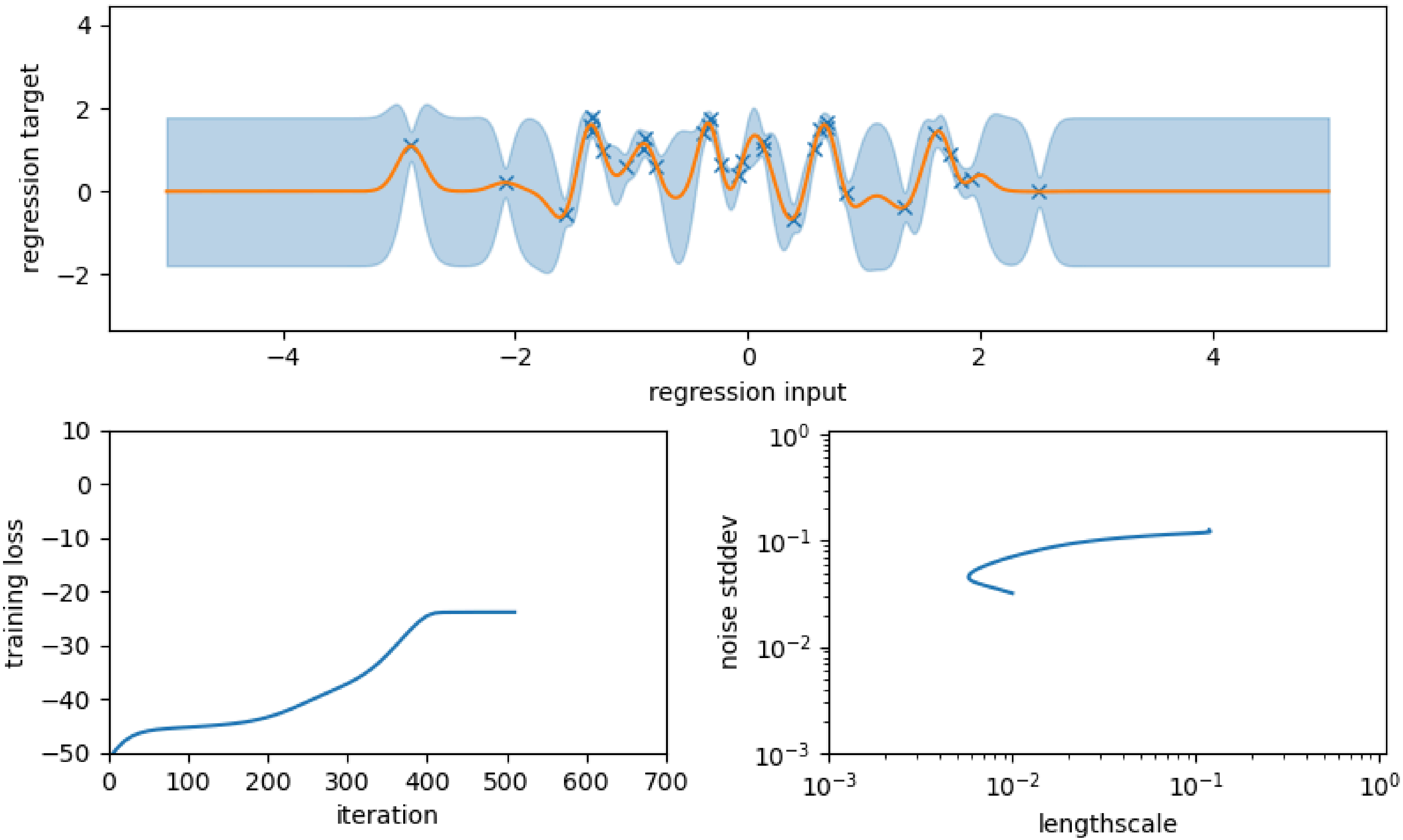

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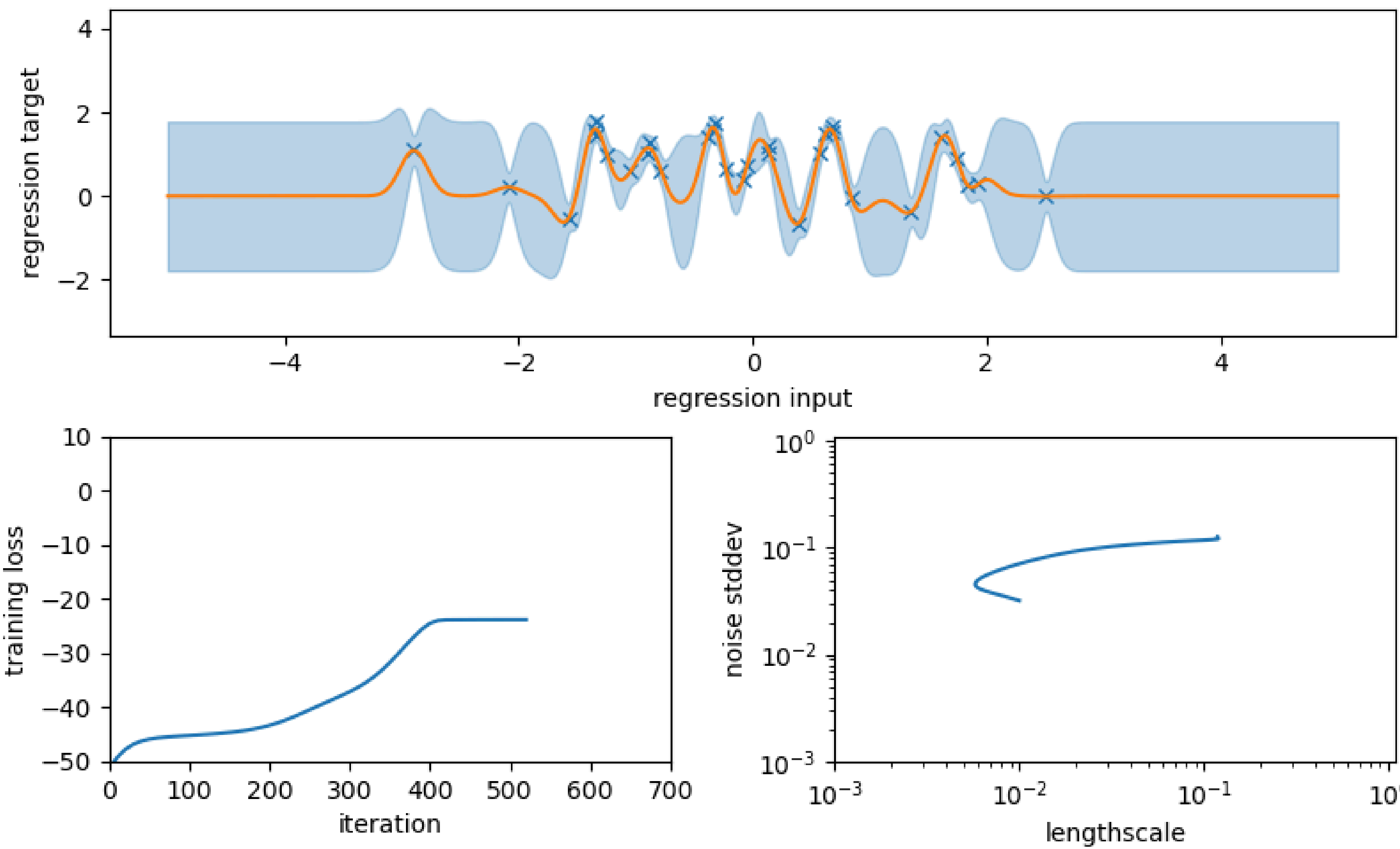

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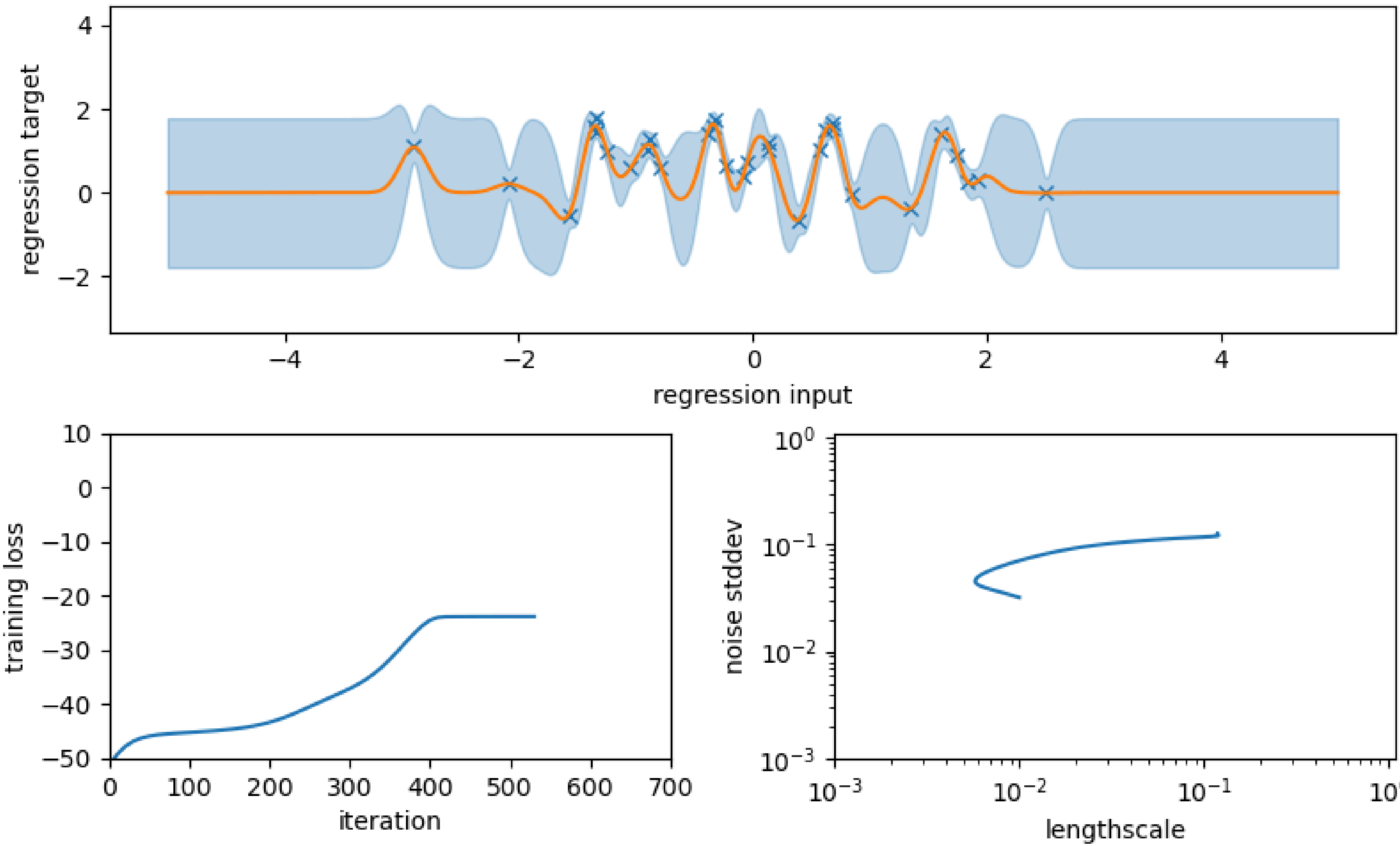

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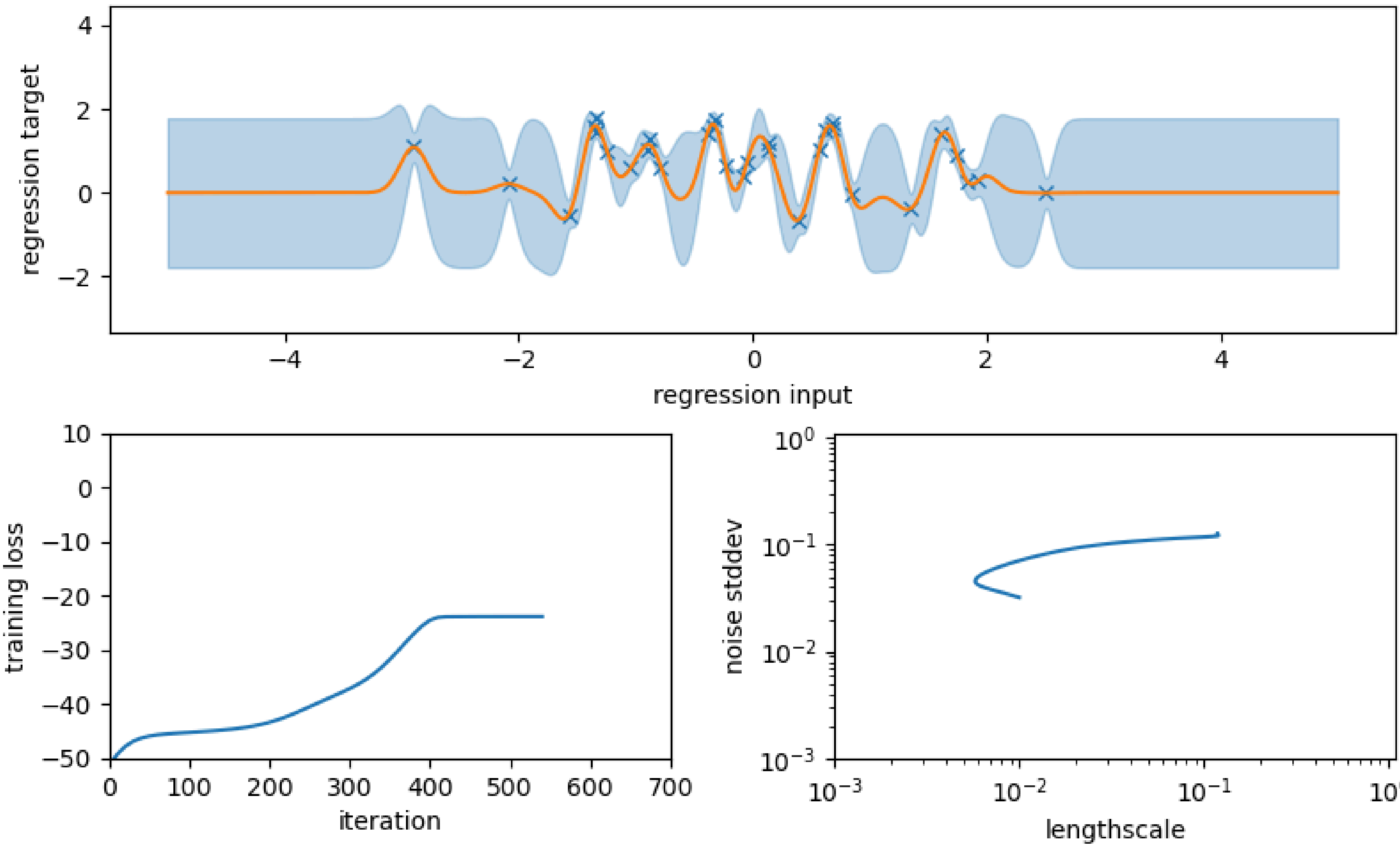

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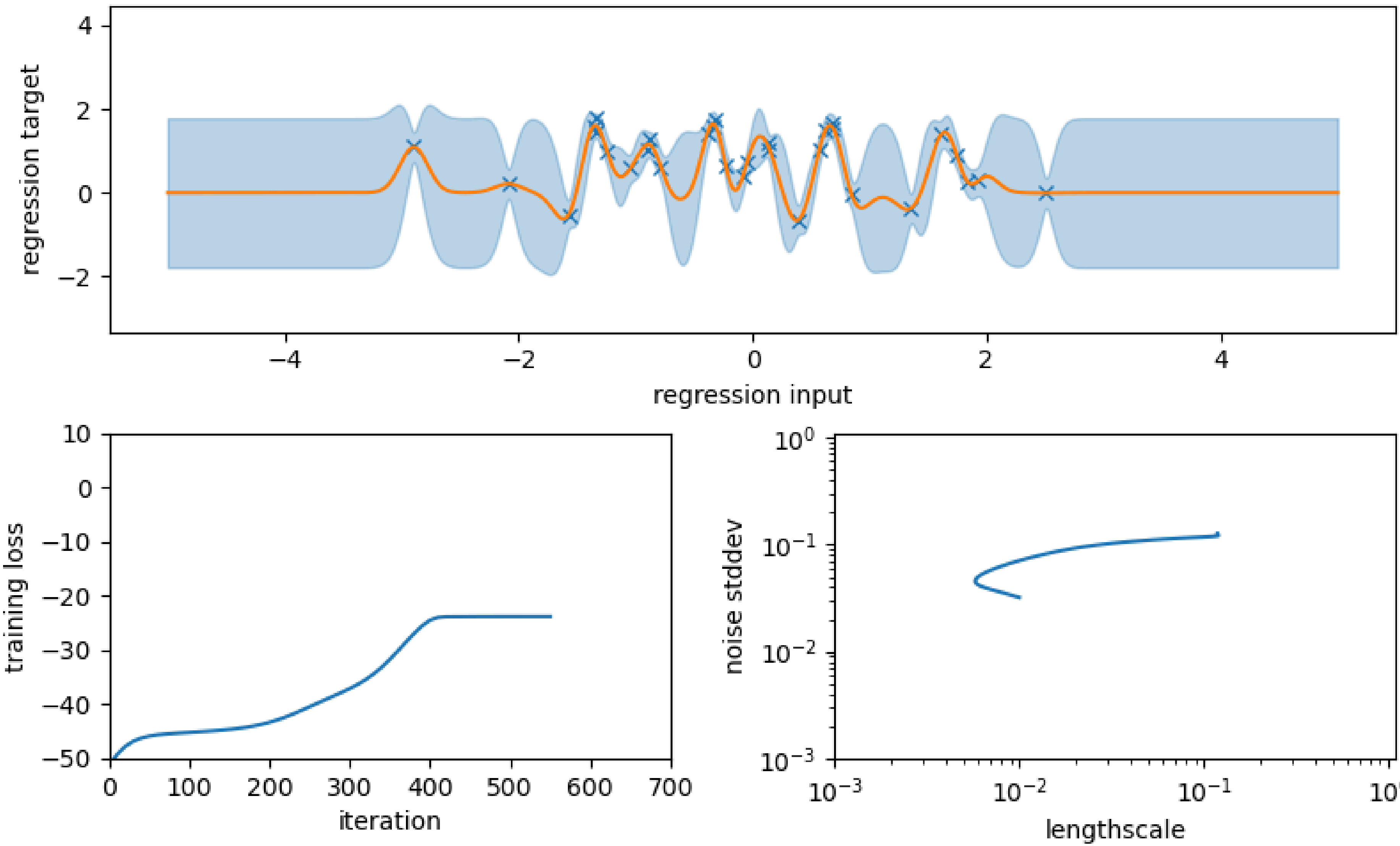

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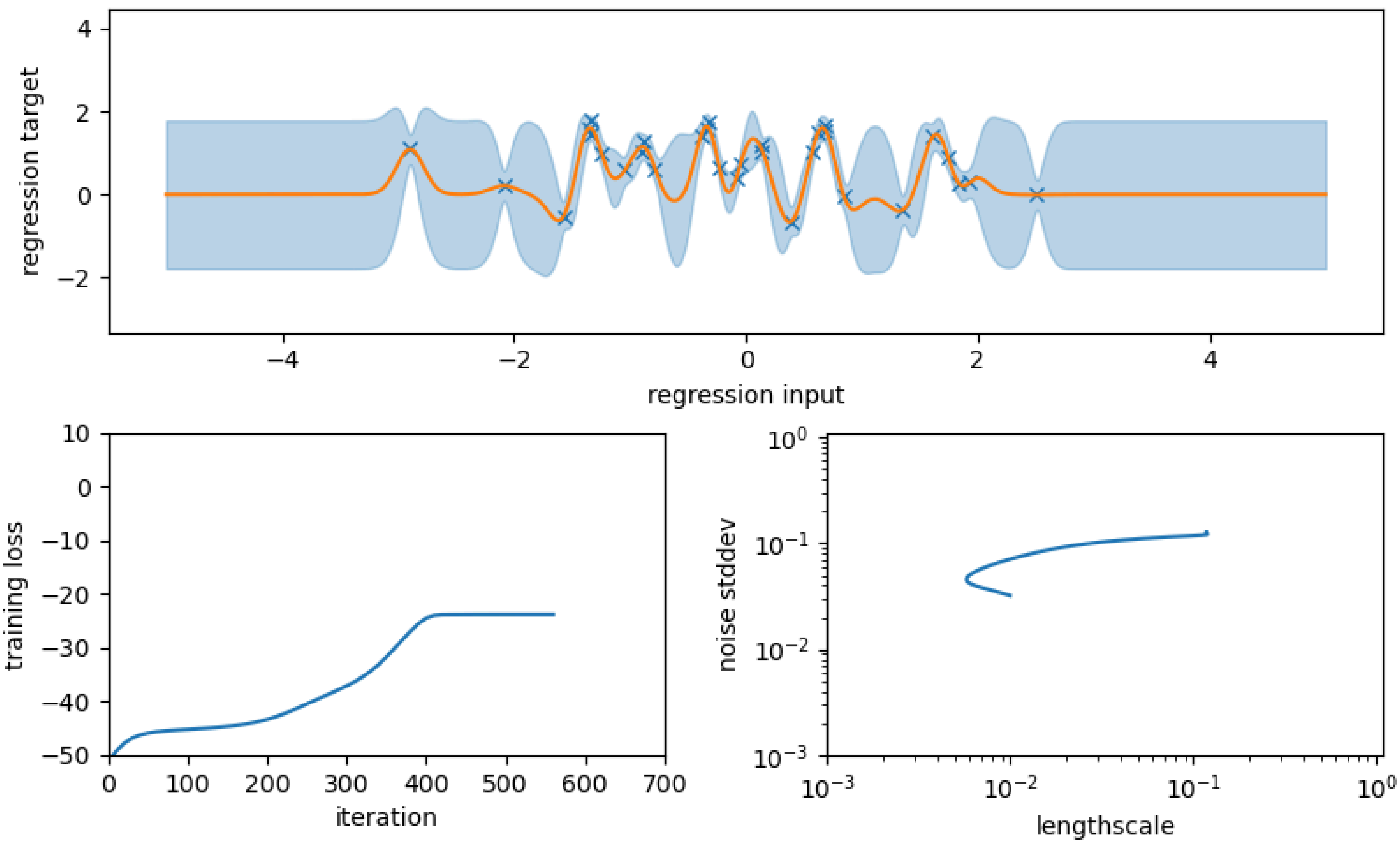

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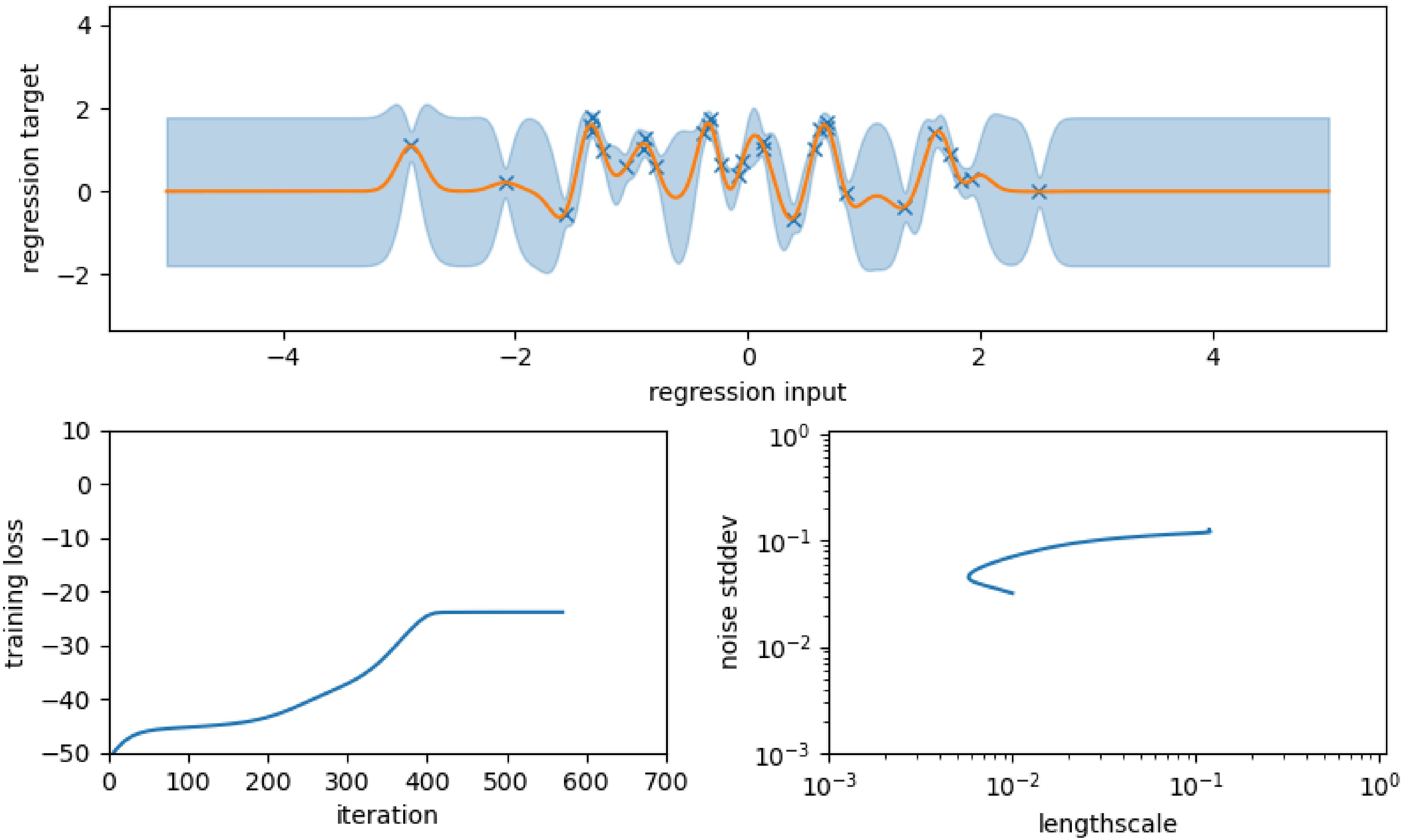

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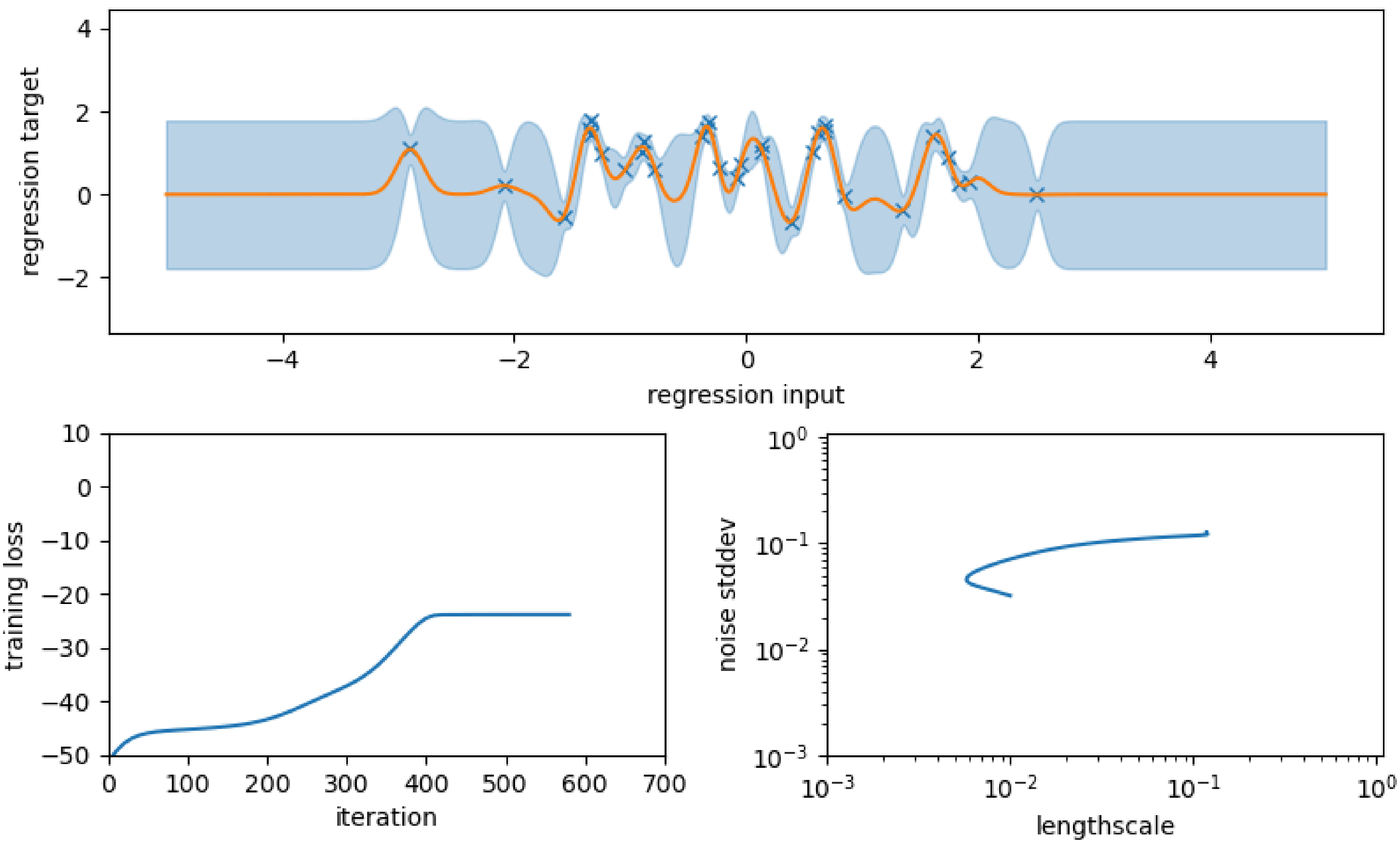

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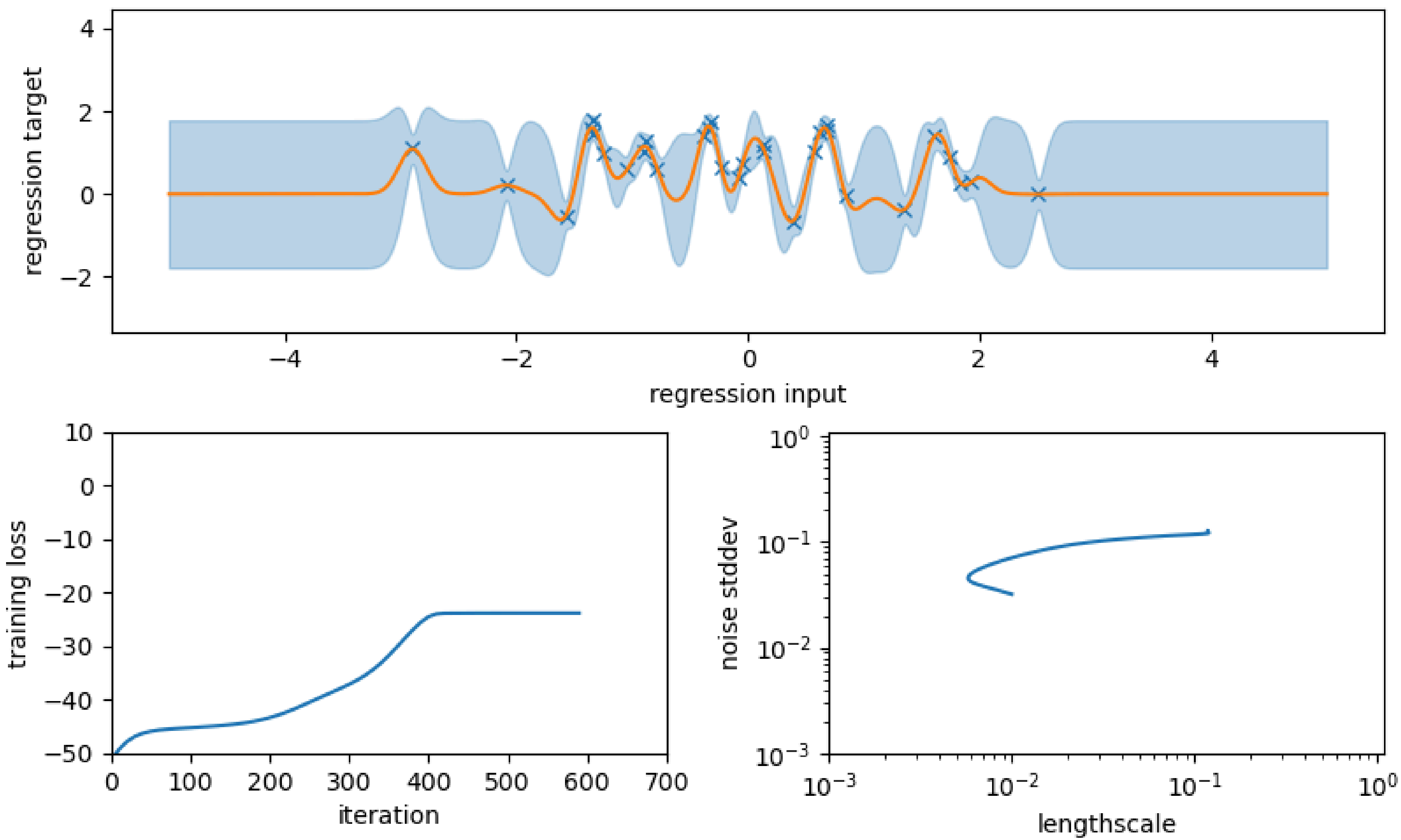

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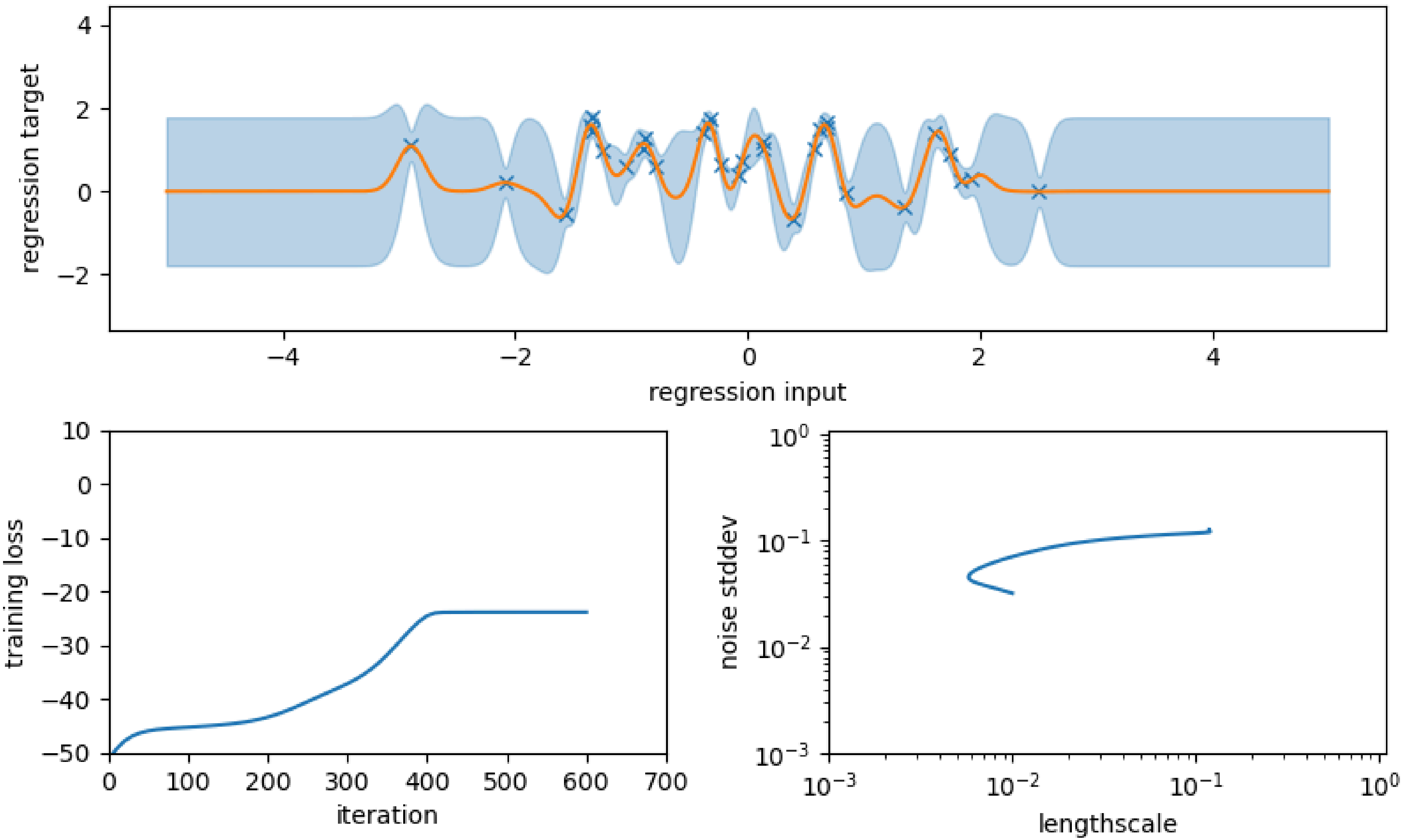

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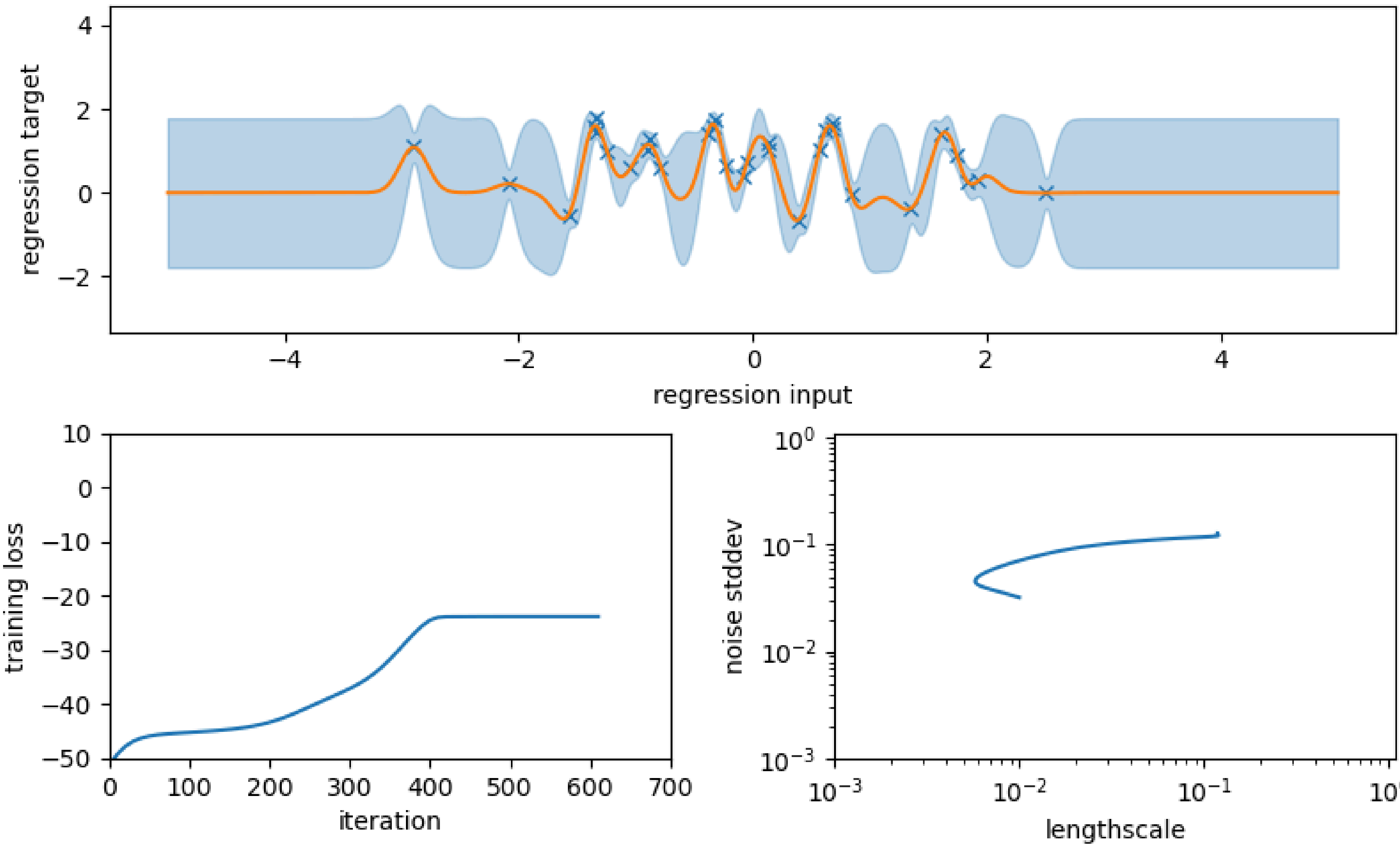

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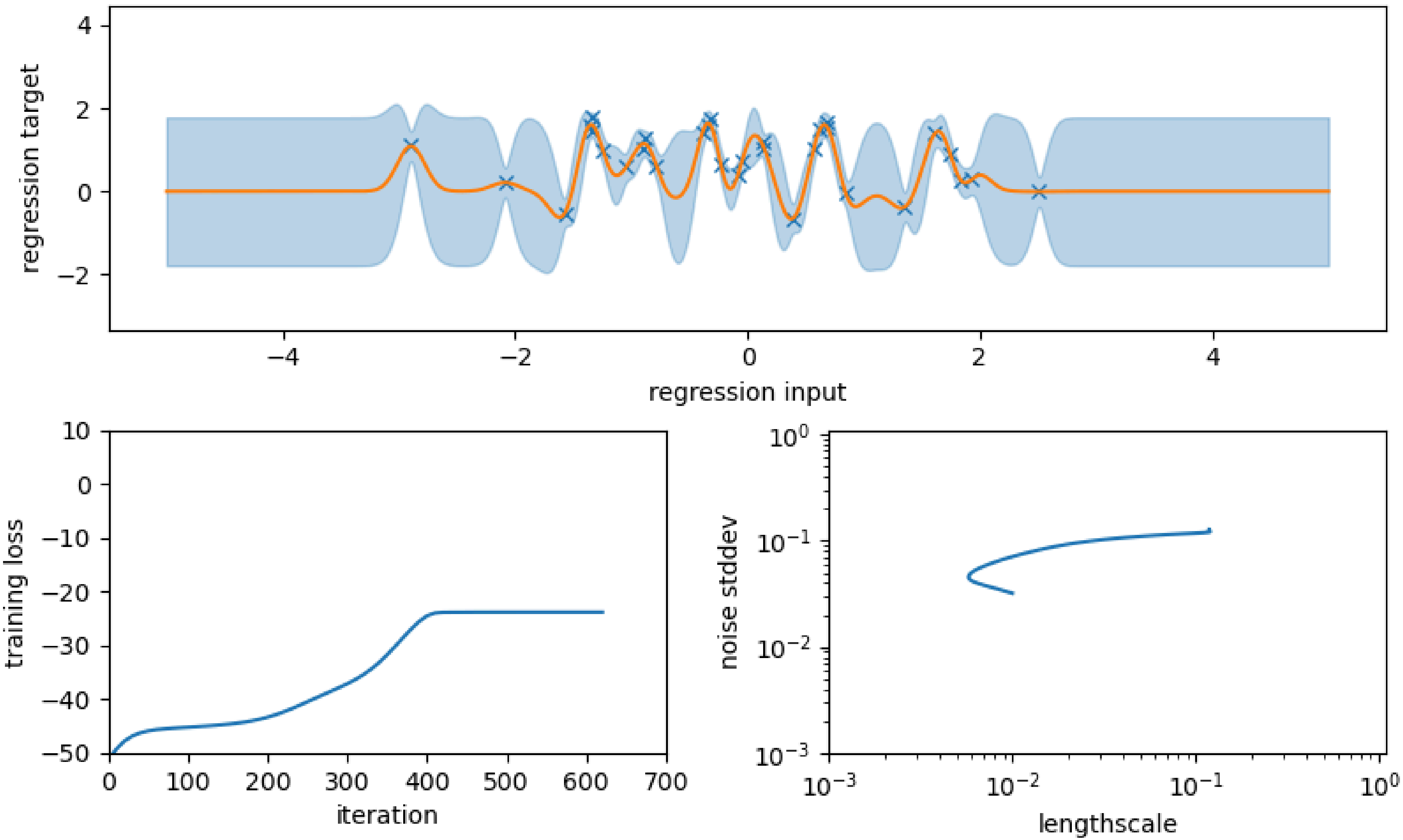

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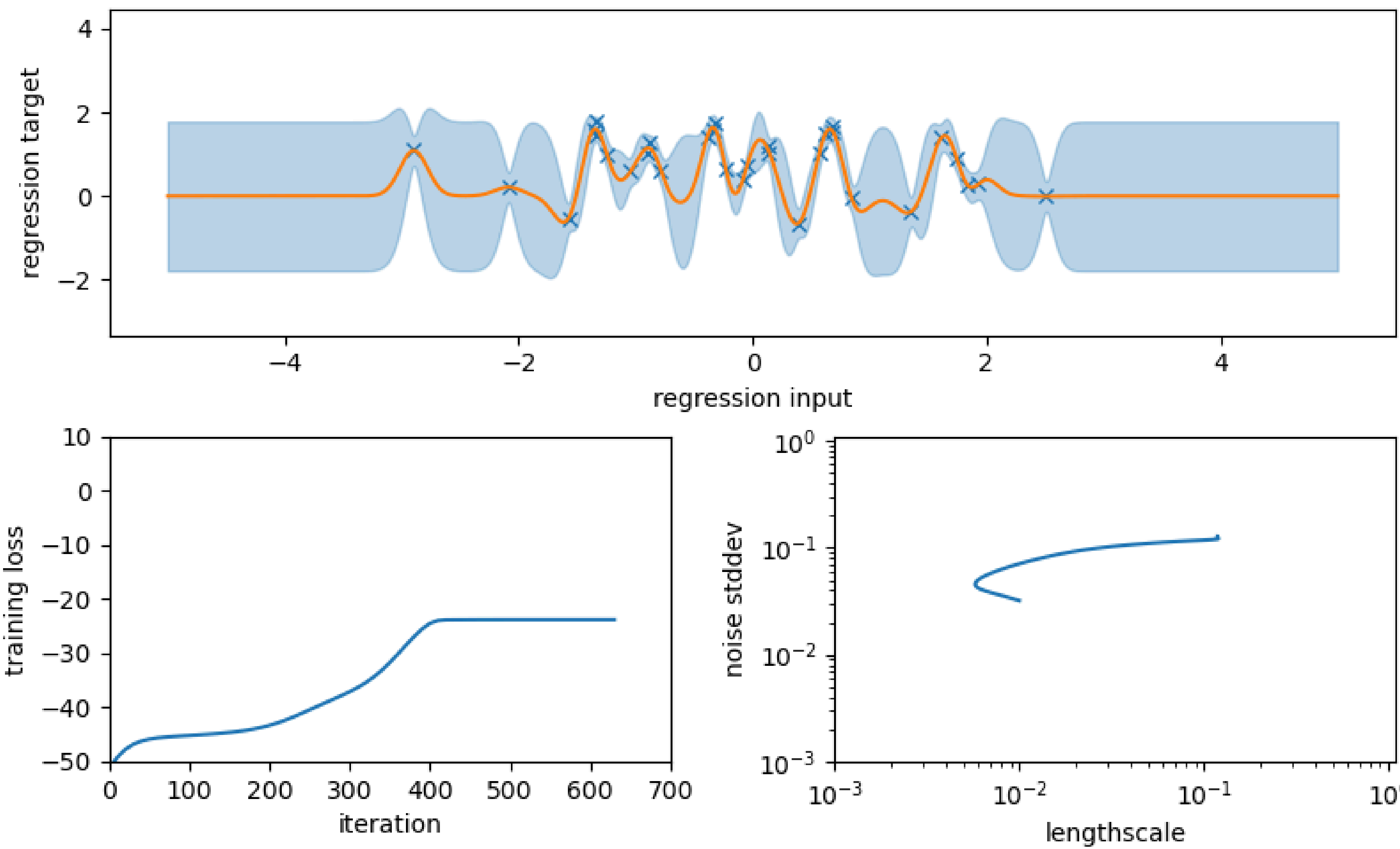

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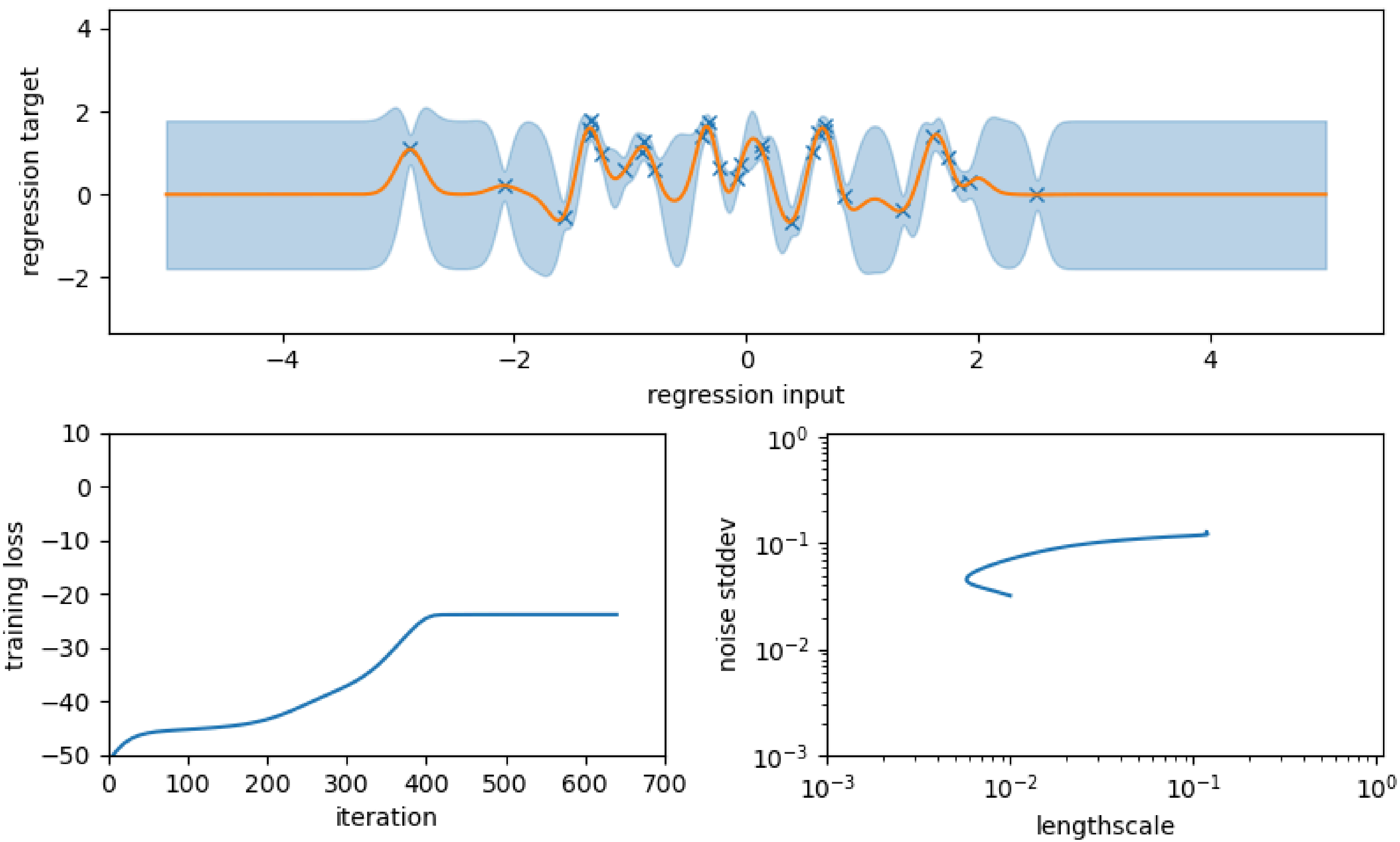

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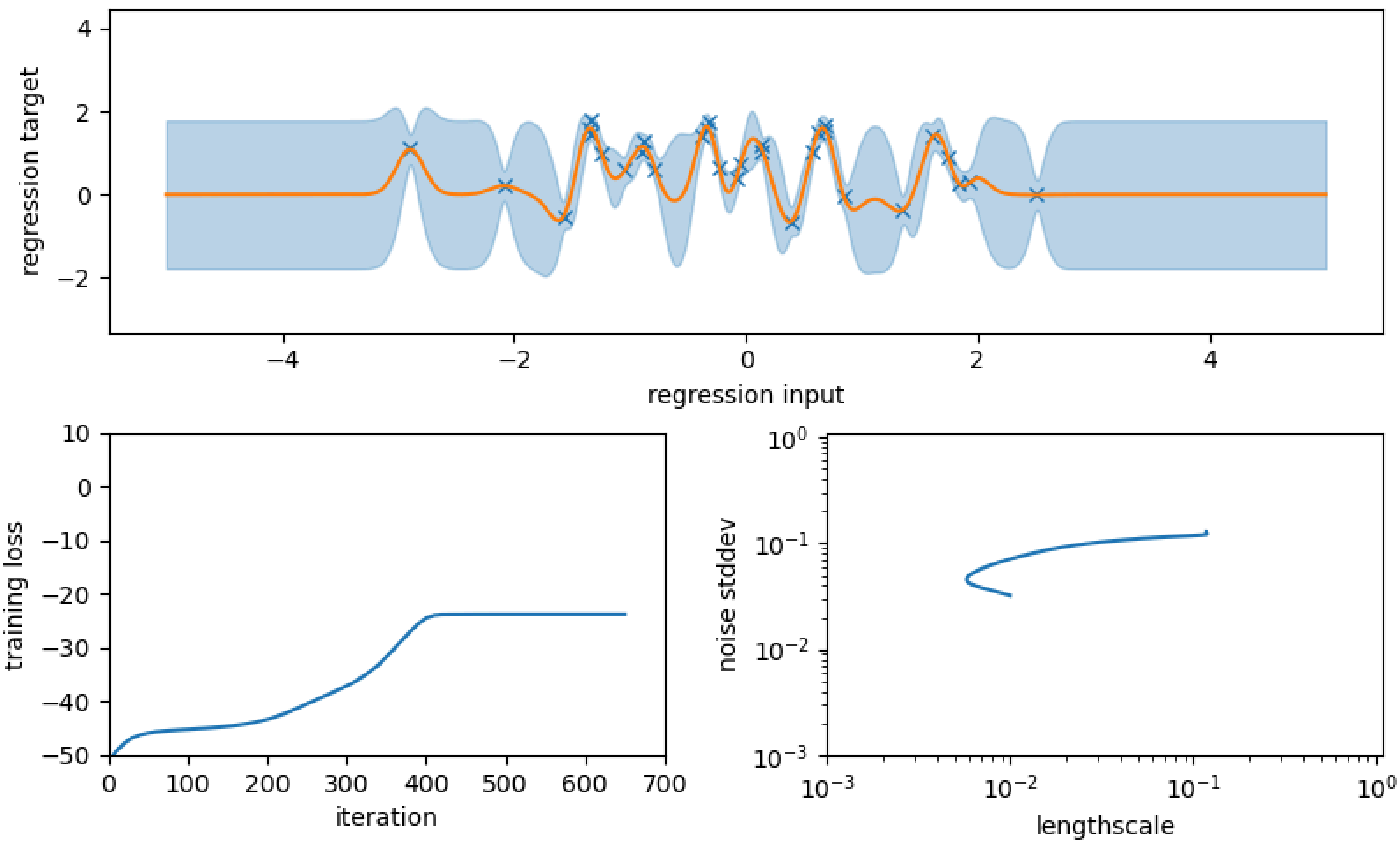

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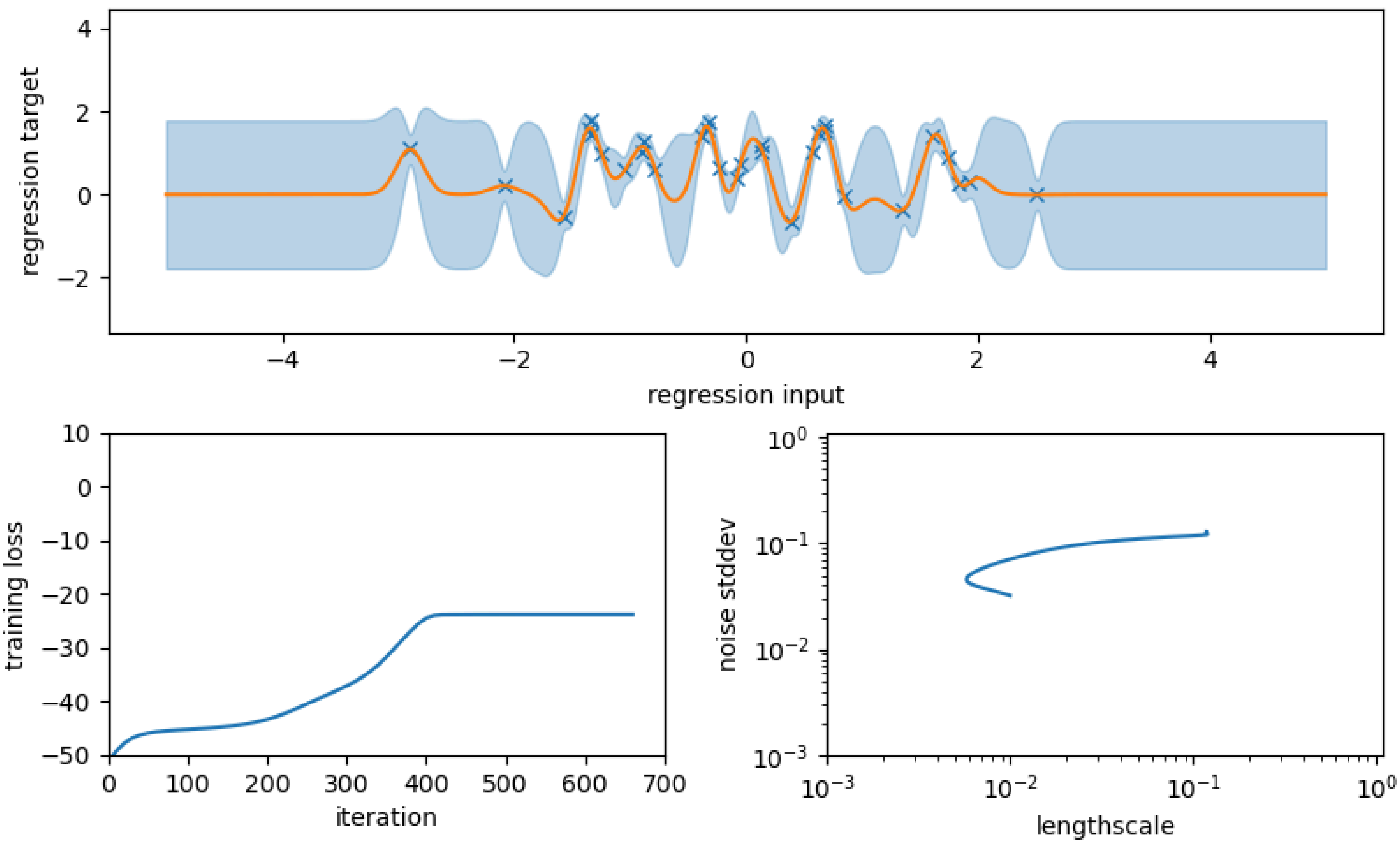

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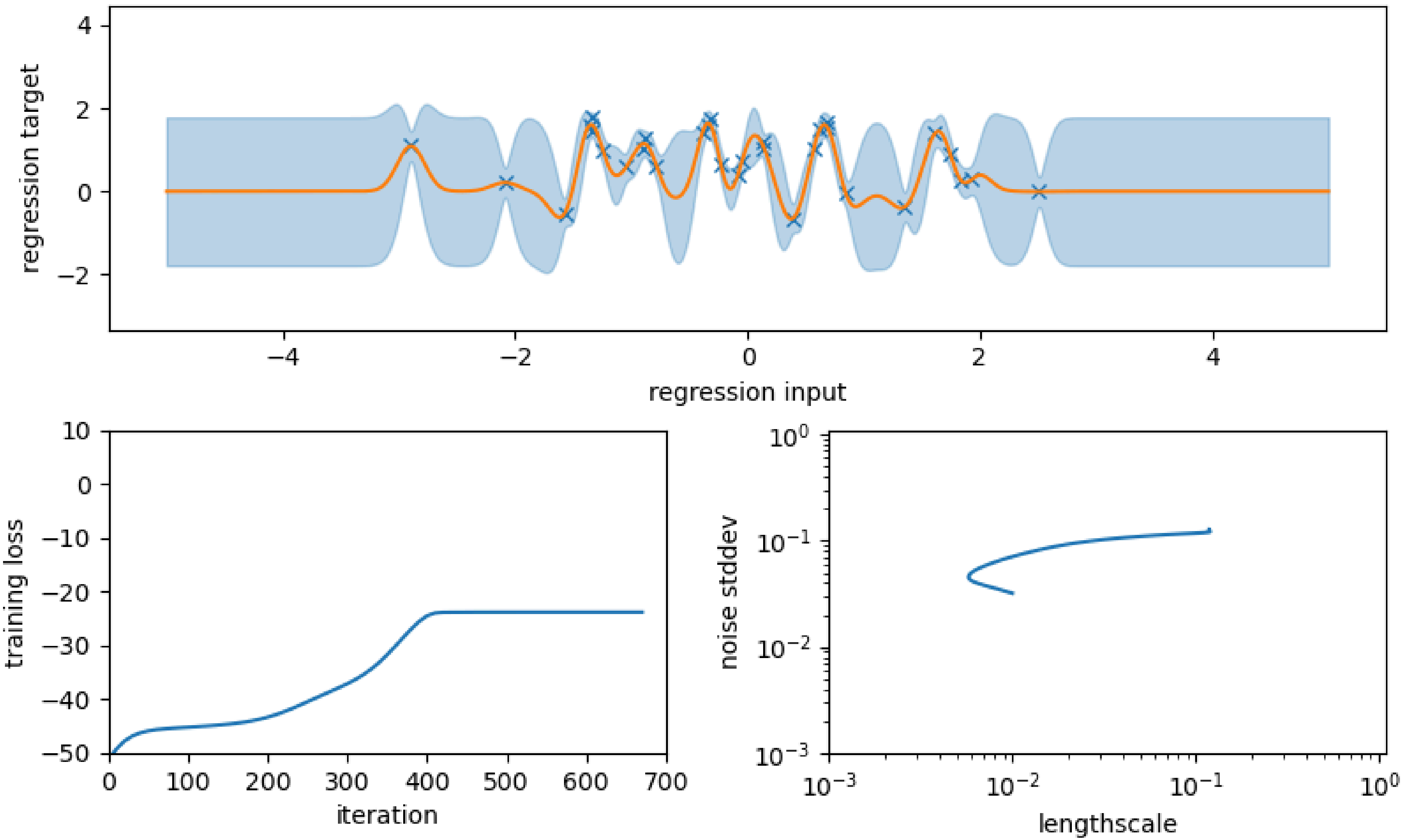

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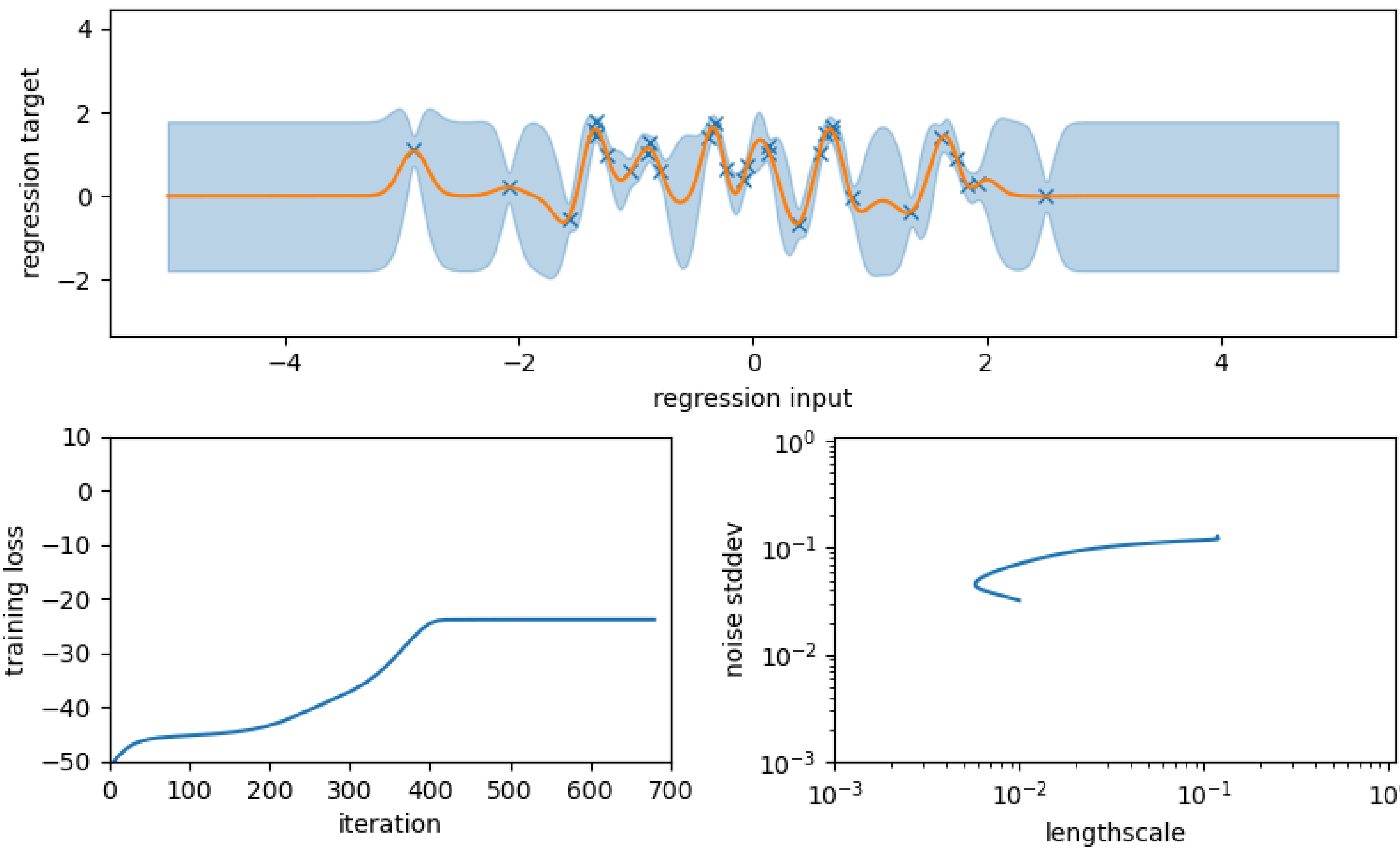

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- Interestingly, form of predictor is still single-layer NN:

$$f(x) = \sum_{m=0}^{N} \varphi(x;\theta,Z_m)w_m$$

$$\varphi(x;\theta,Z_m) = k_\theta(x,X_m) \qquad\qquad \boldsymbol{w} = \left(K(X,X)+\sigma^2 I\right)^{-1}\boldsymbol{y}$$

# Where are we in our goals?

Predictor is a *single layer* neural network:

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⚠ **Our model grows, but by memorising *all* data!**

1. What is wrong with minimising losses.
2. Bayesian Model Selection?
2. **The Bayesian answer to model size: Nonparametrics.**
3. A principle for selecting size

# Why use Nonparametric models?

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Classic arguments:

1.
2.
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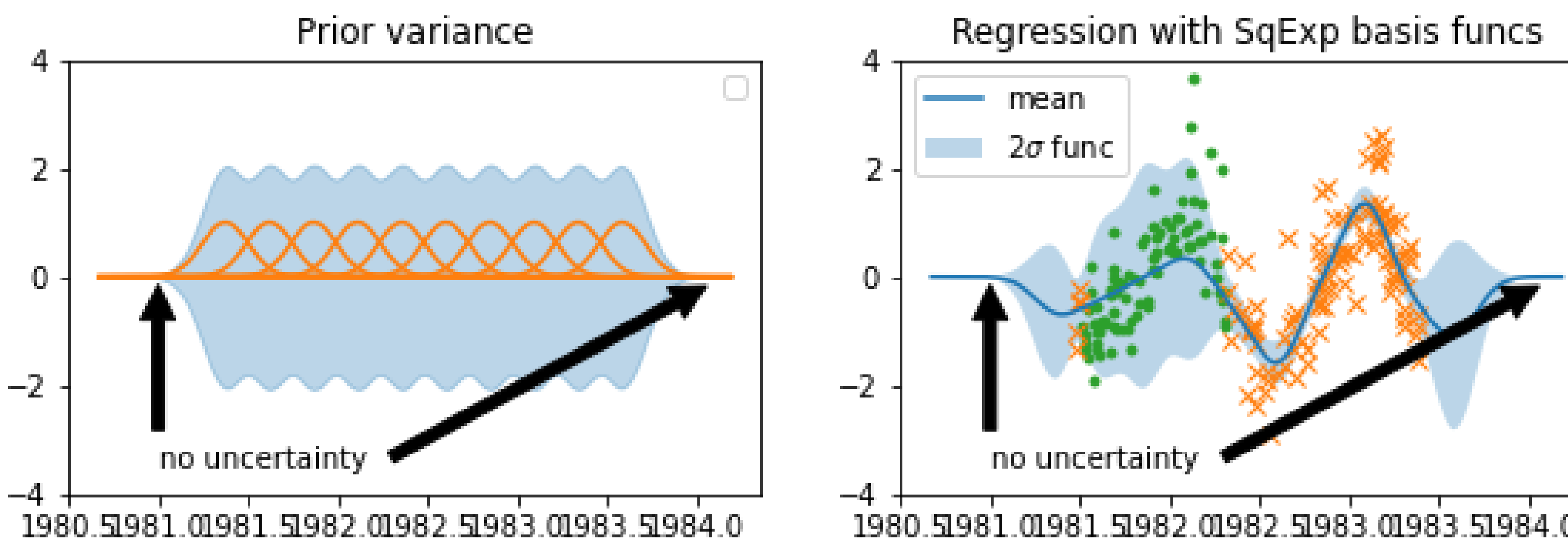

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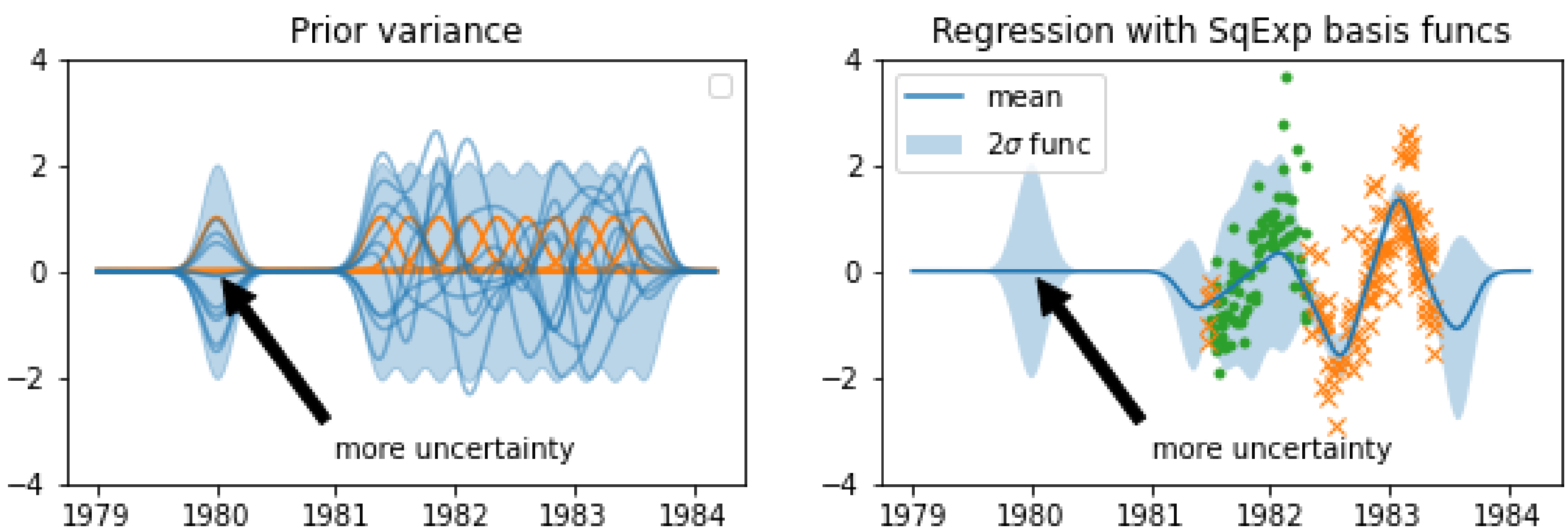

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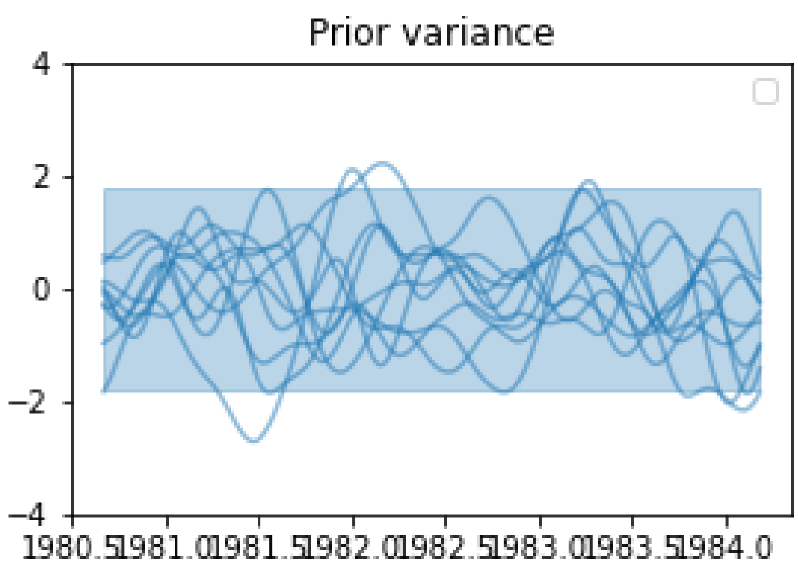


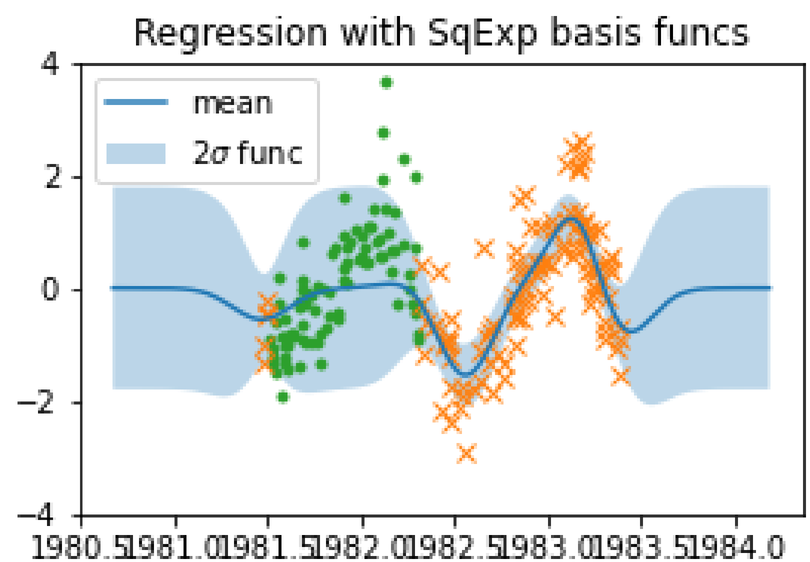

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Or are we stuck with memorising all the data?

We could do model selection over the model size...

$$p(W, \theta, M|\mathcal{D}) = \frac{p(\mathcal{D}|W, \theta, M)p(W|\theta, M)}{p(\mathcal{D}|\theta, M)} \frac{p(\mathcal{D}|\theta, M)p(\theta)}{p(\mathcal{D})}$$

$$\theta^*, M^* = \underset{\theta, M}{\operatorname{argmax}} \log p(\mathcal{D}|\theta, M)$$

# Bayesian Model Selection of Model Size is BAD

1.
2.

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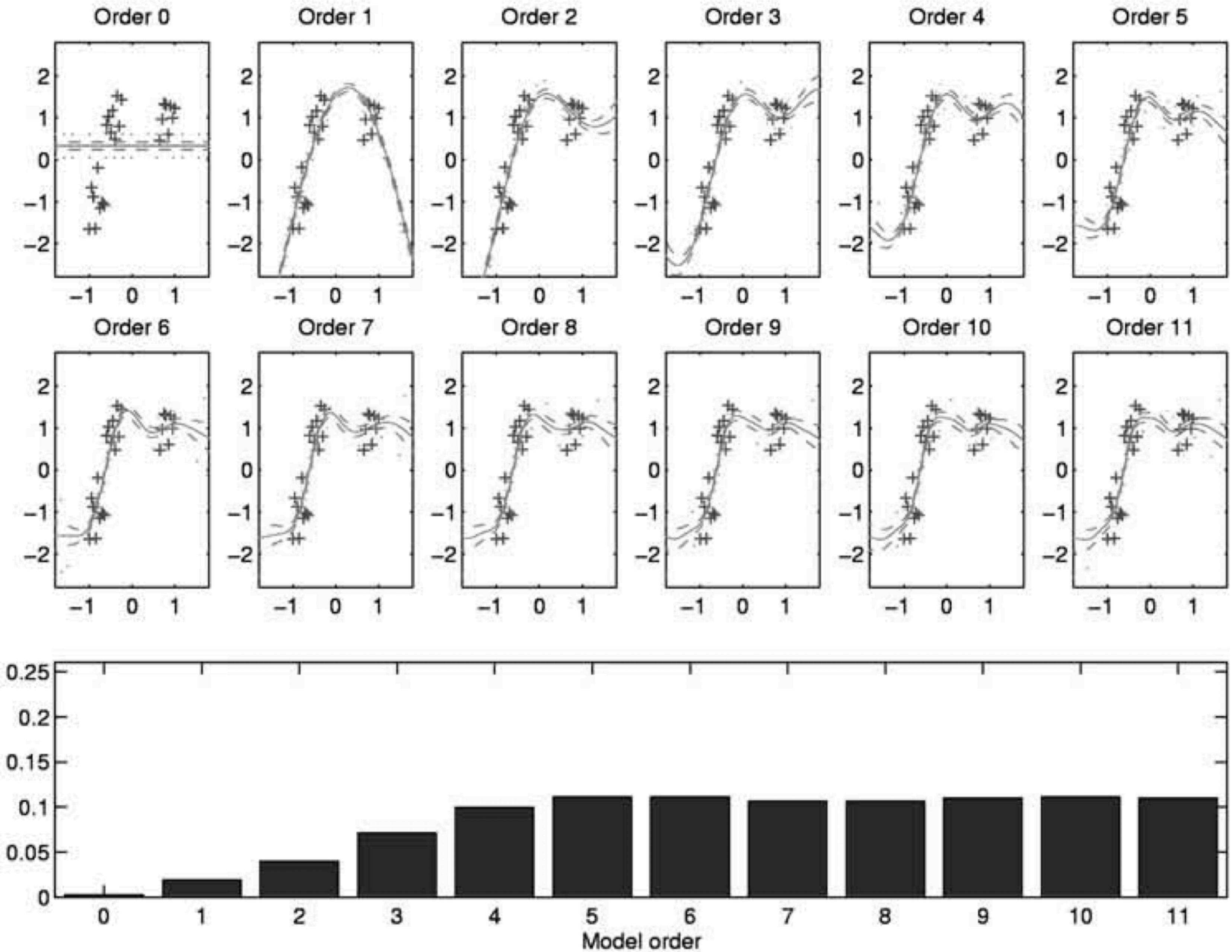
See *Occam's Razor* (Rasmussen & Ghahramani, 2000). One of my favourite papers.



Order 0
Order 1
Order 2
Order 3
Order 4
Order 5
Order 6
Order 7
Order 8
Order 9
Order 10
Order 11
Model order

### Bayes selects a nonparametric model!

- Bayes itself is pushing us to use "large" nonparametric models!
- Cannot rely on Bayes to choose a "small" model!

1. What is wrong with minimising losses?
2. Bayesian Model Selection
2. Model Selection over Model Size? Or Nonparametrics?
3. **A Principle for Selecting Model Size**

**What principle can determine a compressed model size,
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**Define a nonparametric model,
then approximate it with $M < N$ basis funcs.**

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Step 1: Introduce family of approximate predictors

$$q(f(x)) = \mathcal{N}\left( f(x); \sum_{m=1}^{M} \varphi(x; Z_m, \theta) w_m, \ldots \right)$$

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**ELBO is a *unified objective* for all our questions!**

- Optimising w.r.t. $\boldsymbol{w}$, $Z$: finds weights (min KL)
- Optimising w.r.t. $\theta$: finds hyperparameters (max $\log p(\mathcal{D}|\theta)$)
- Select M large enough, that more gives diminishing returns!

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More basis functions is always better:

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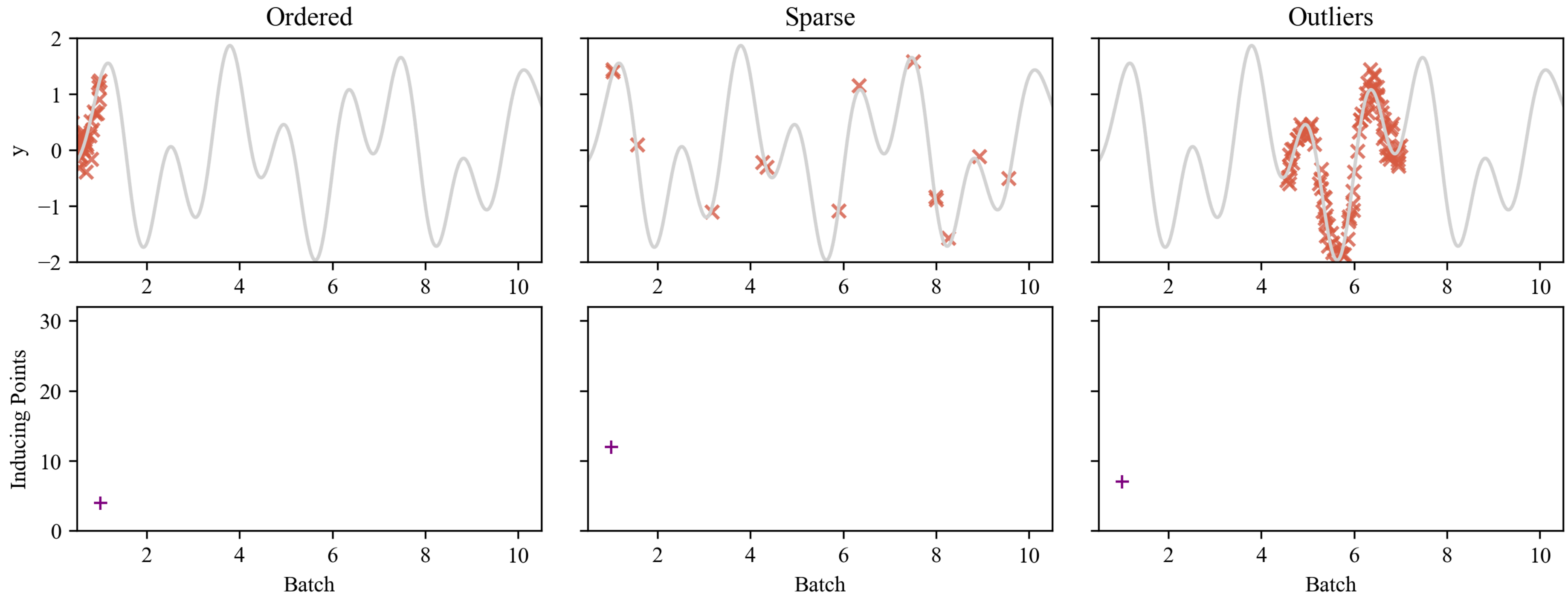
**Simple Rule, Interesting Adaptive Behaviour!**

# **Continual Learning** (Pescador-Barrios et al., 2024; 2025)




Growth of neurons depends on *novelty* in data.

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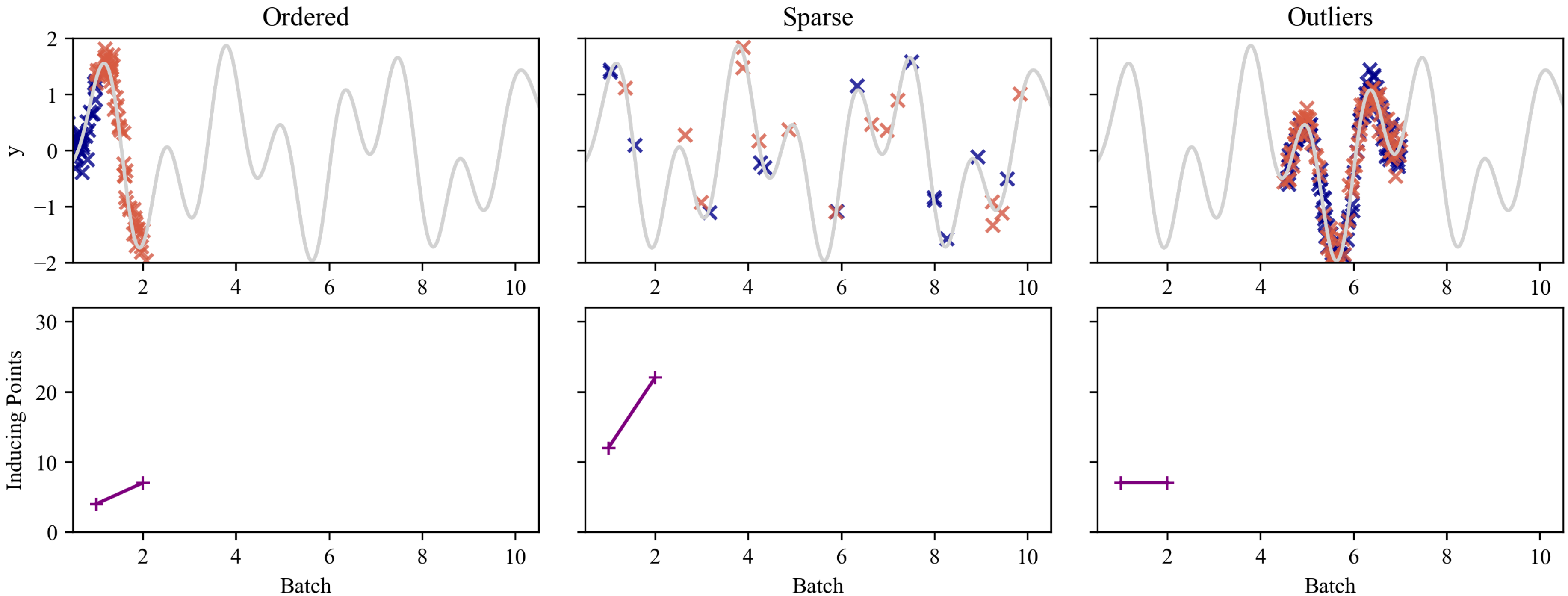


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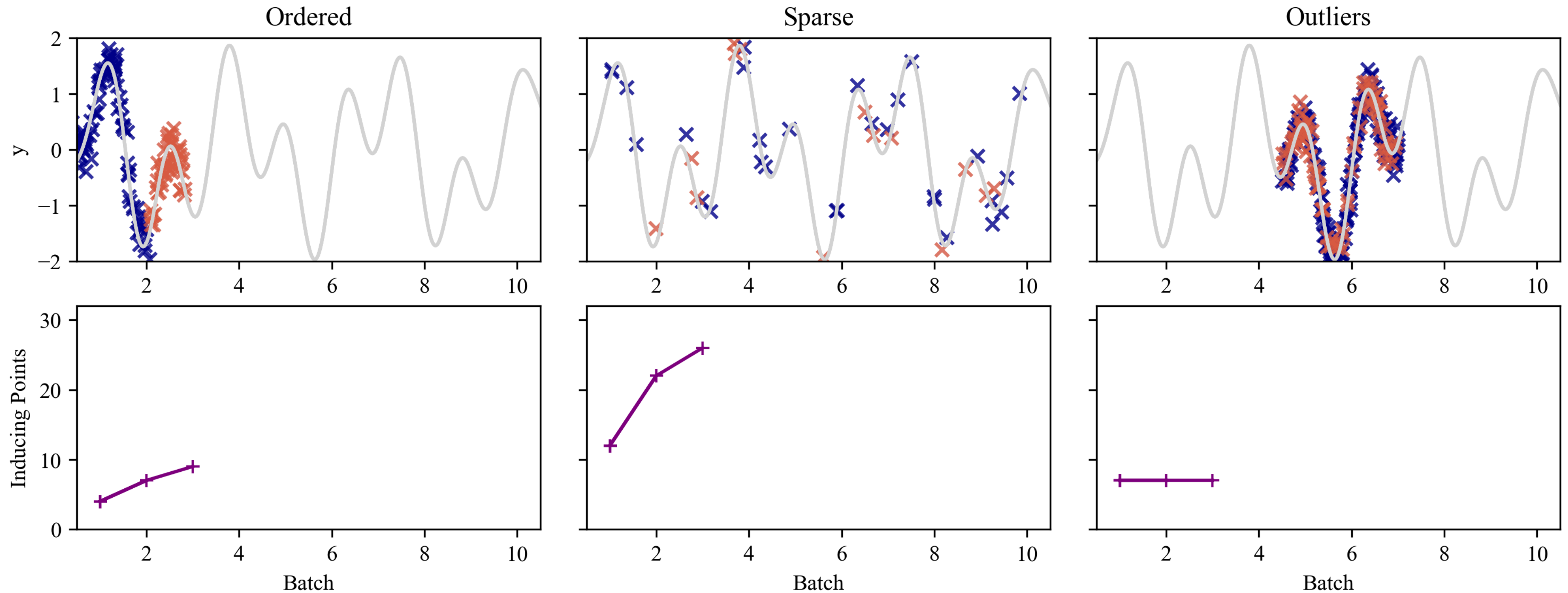


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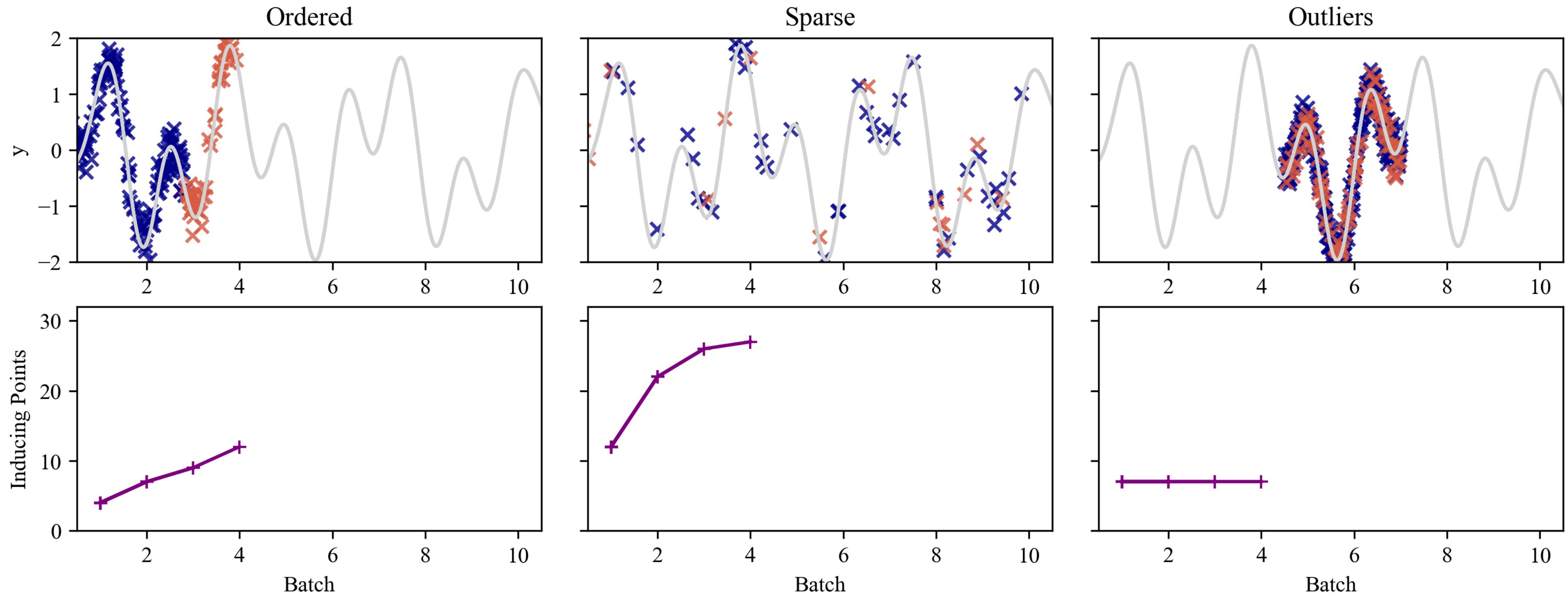


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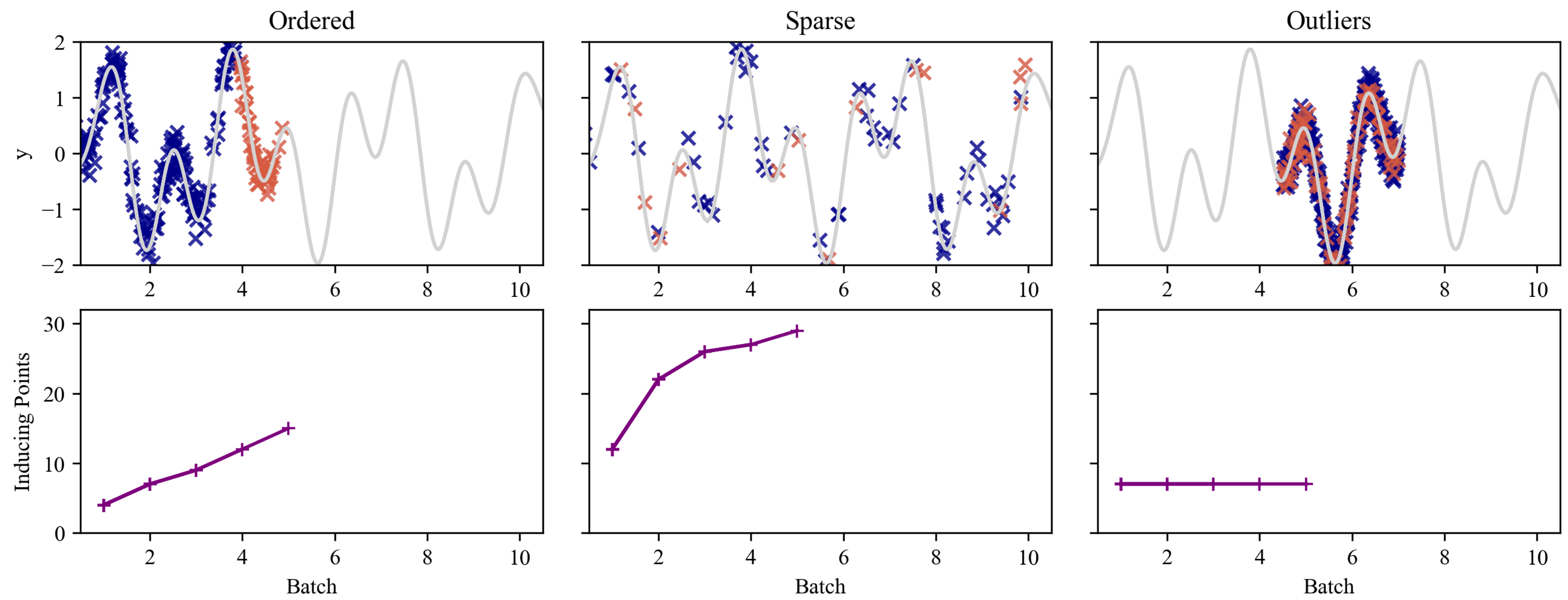


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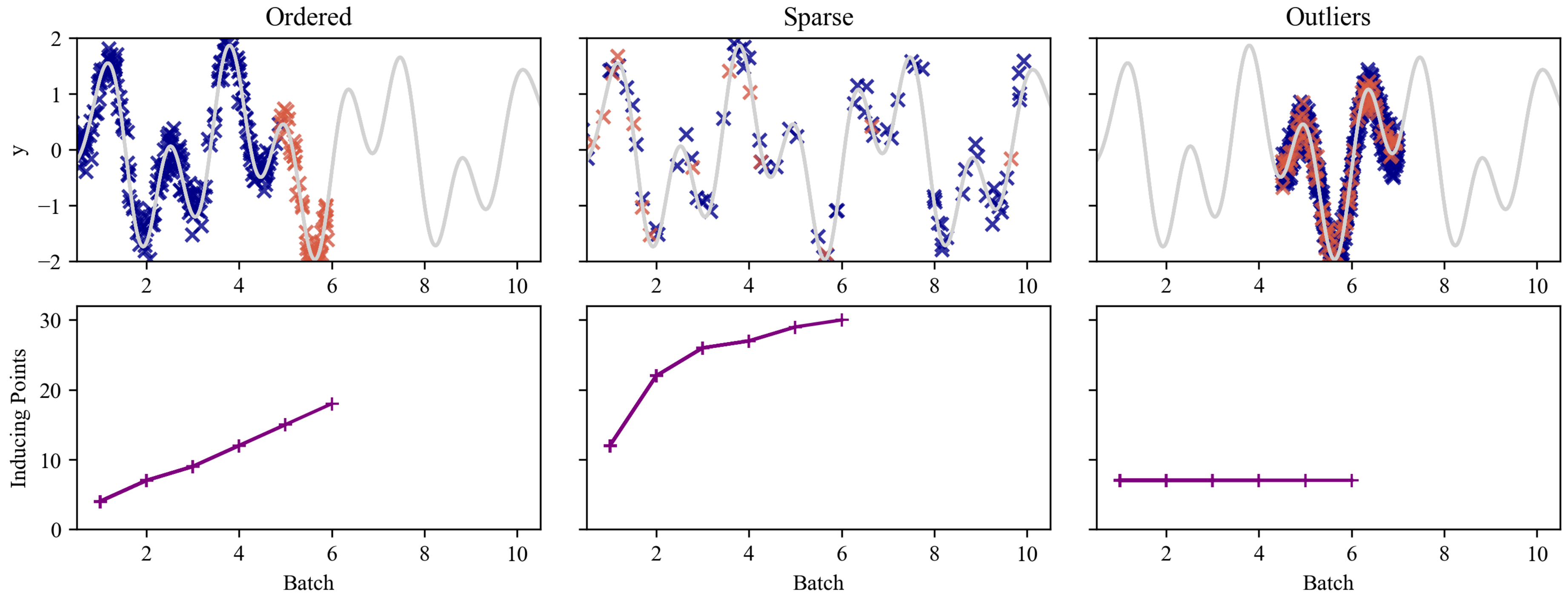


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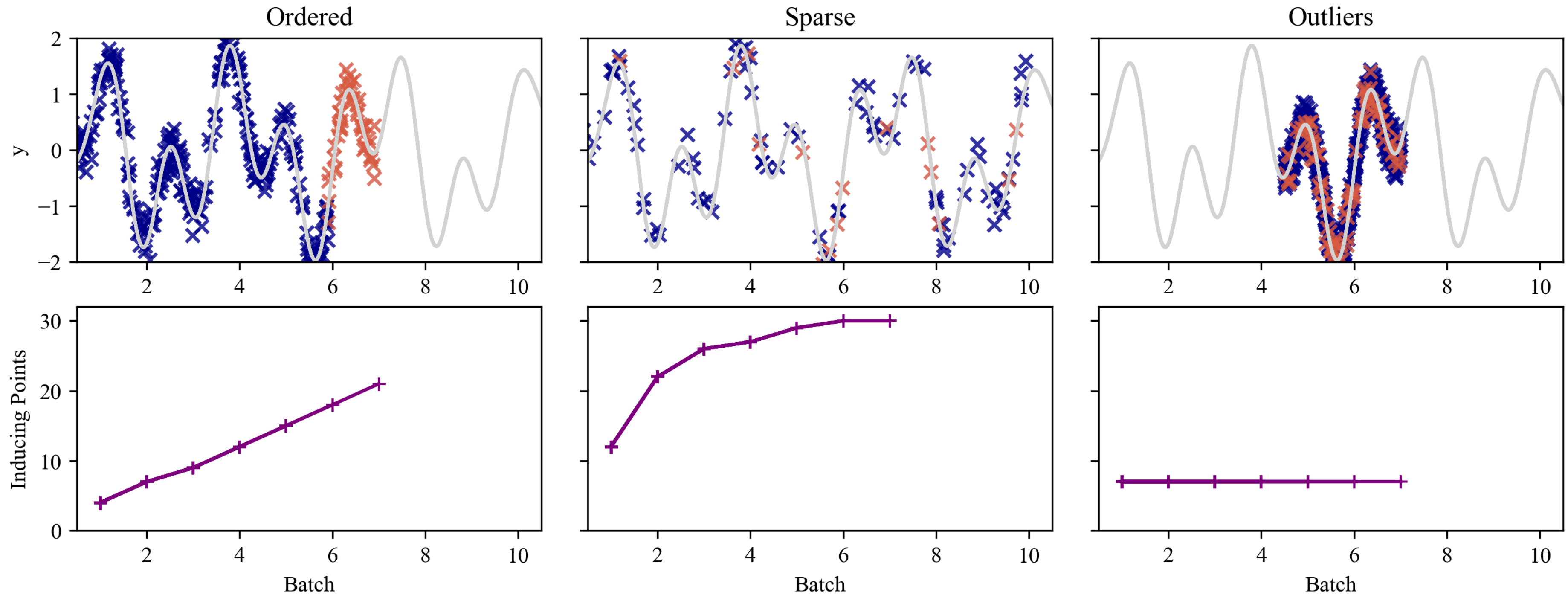


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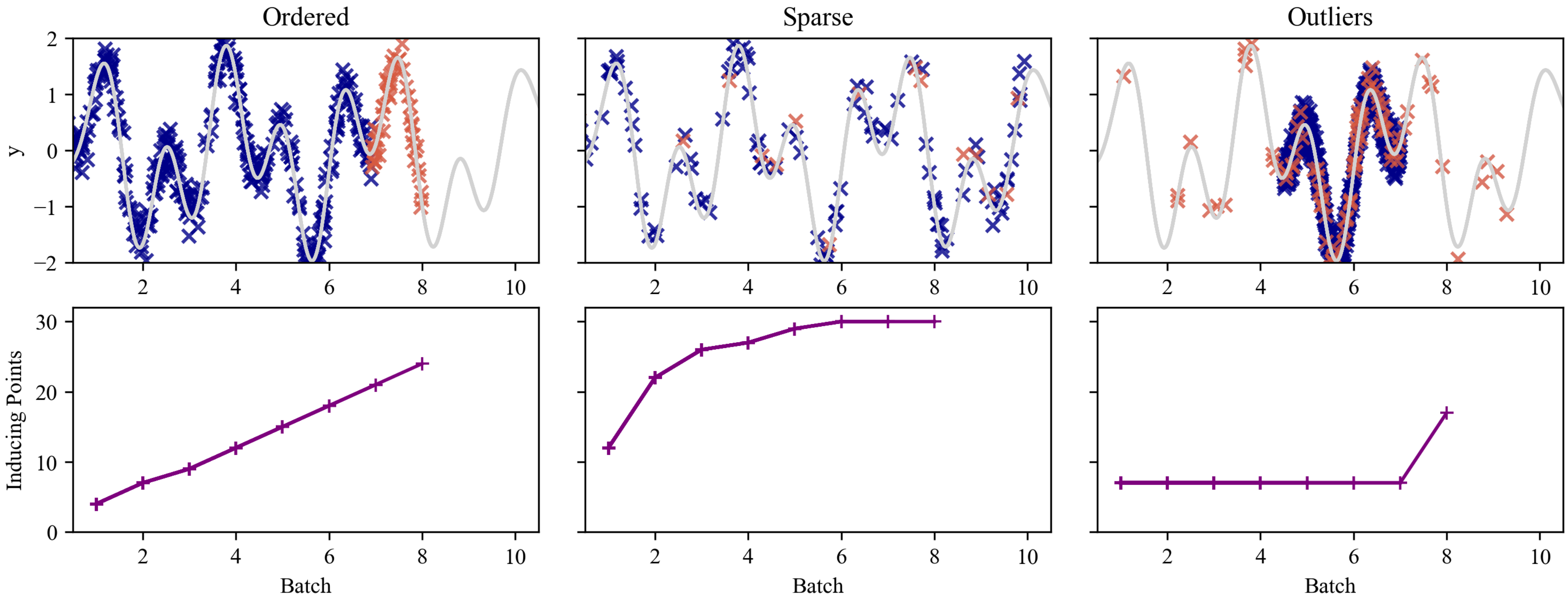


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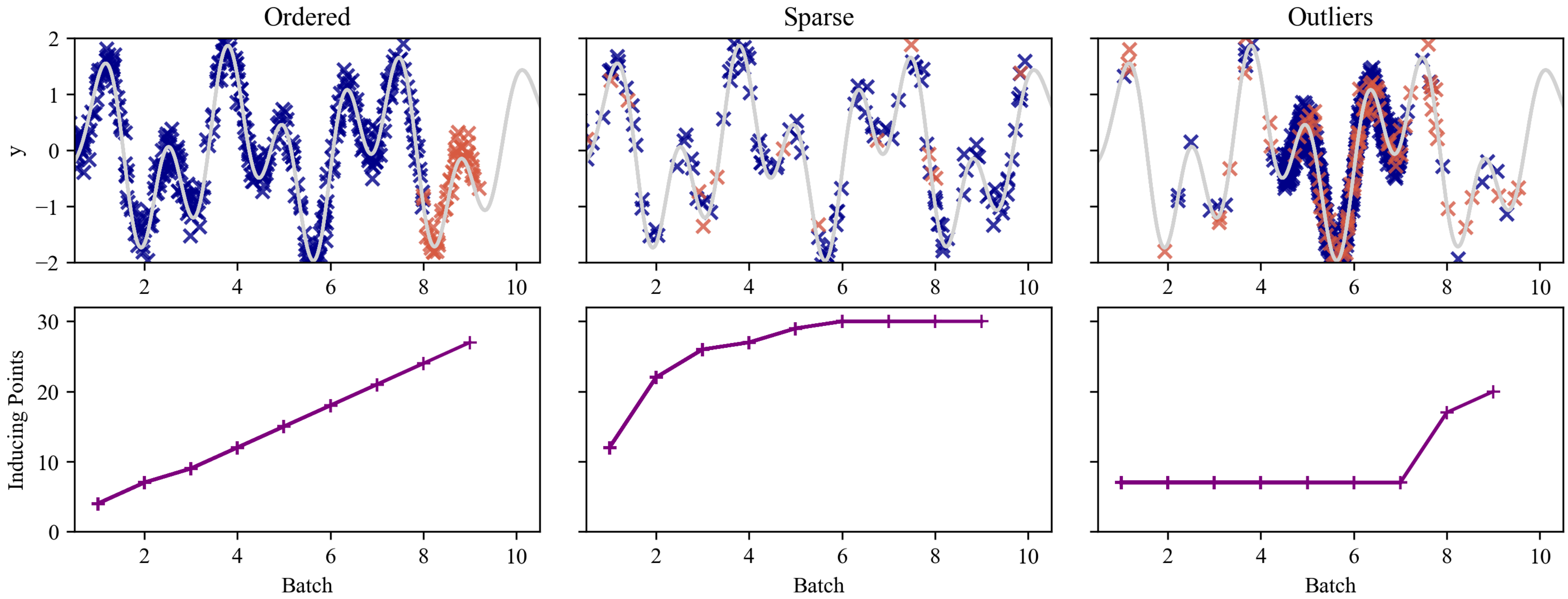


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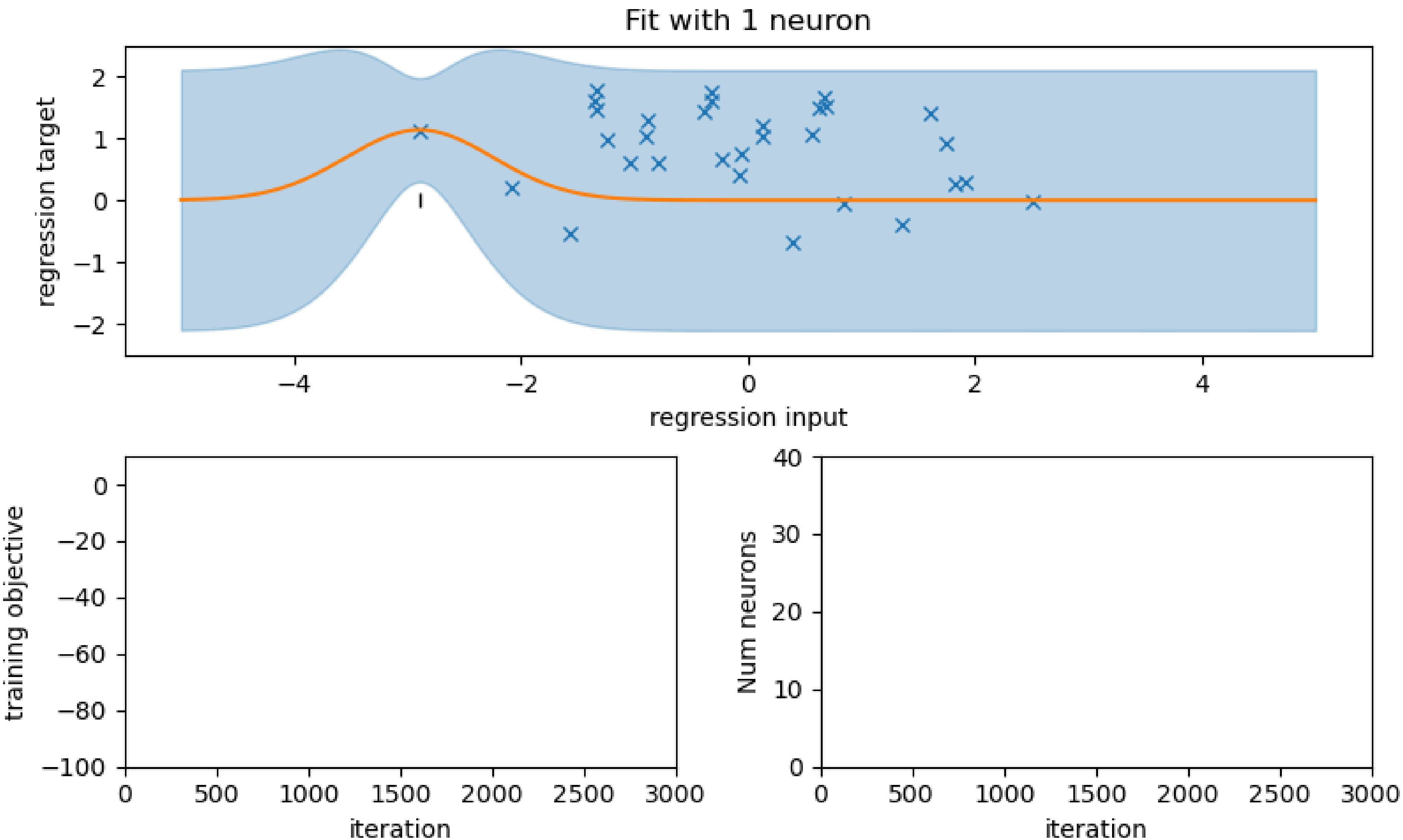

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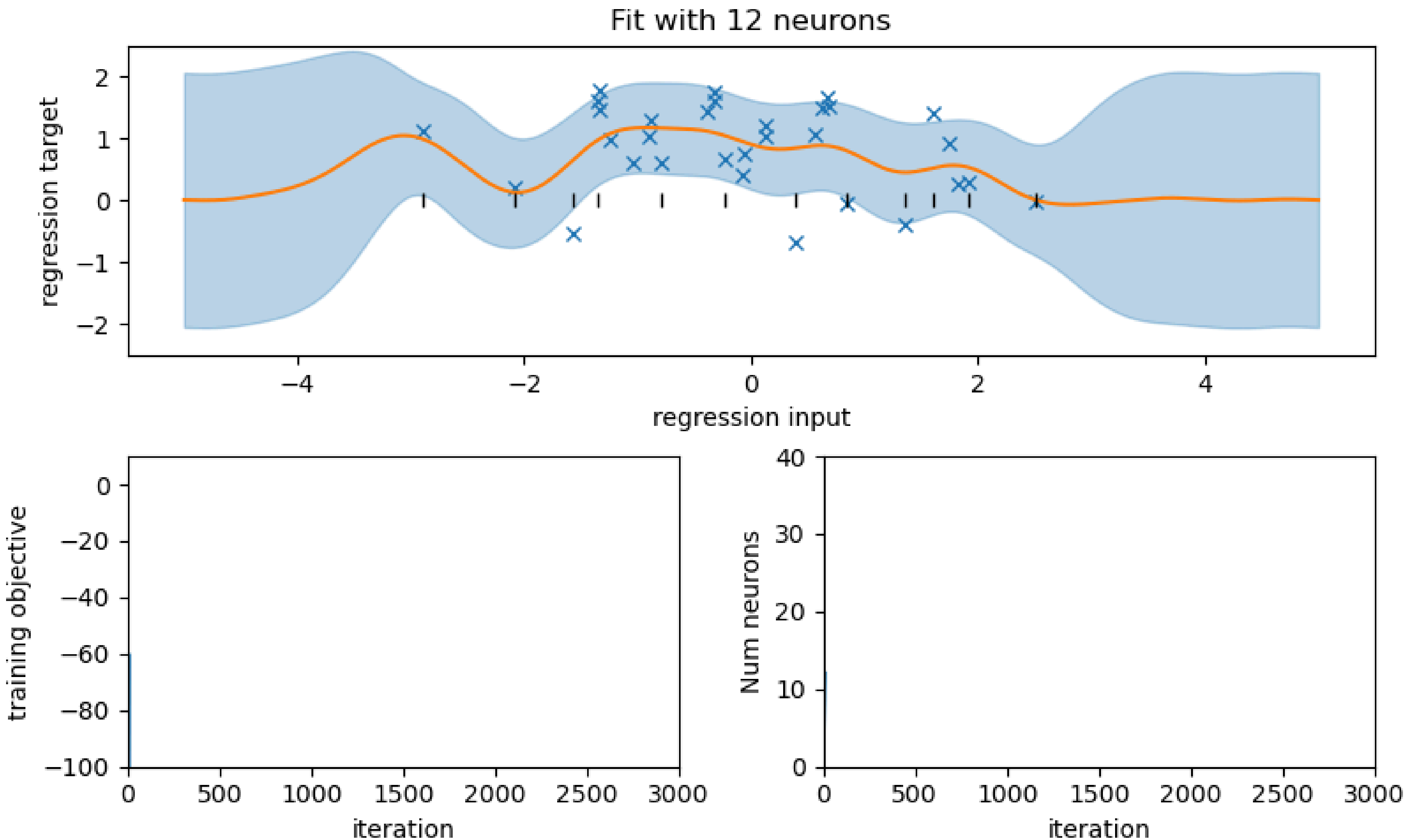

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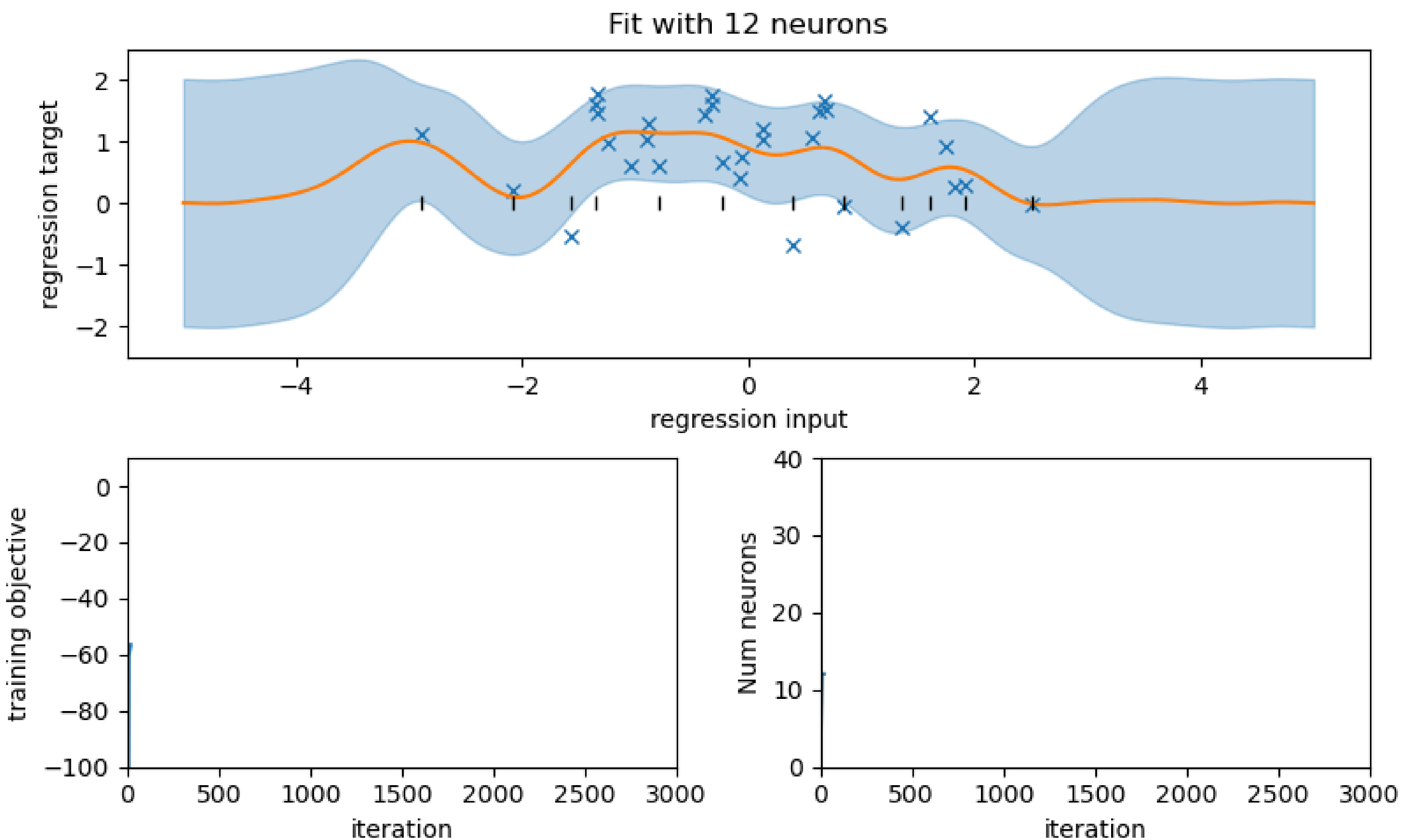

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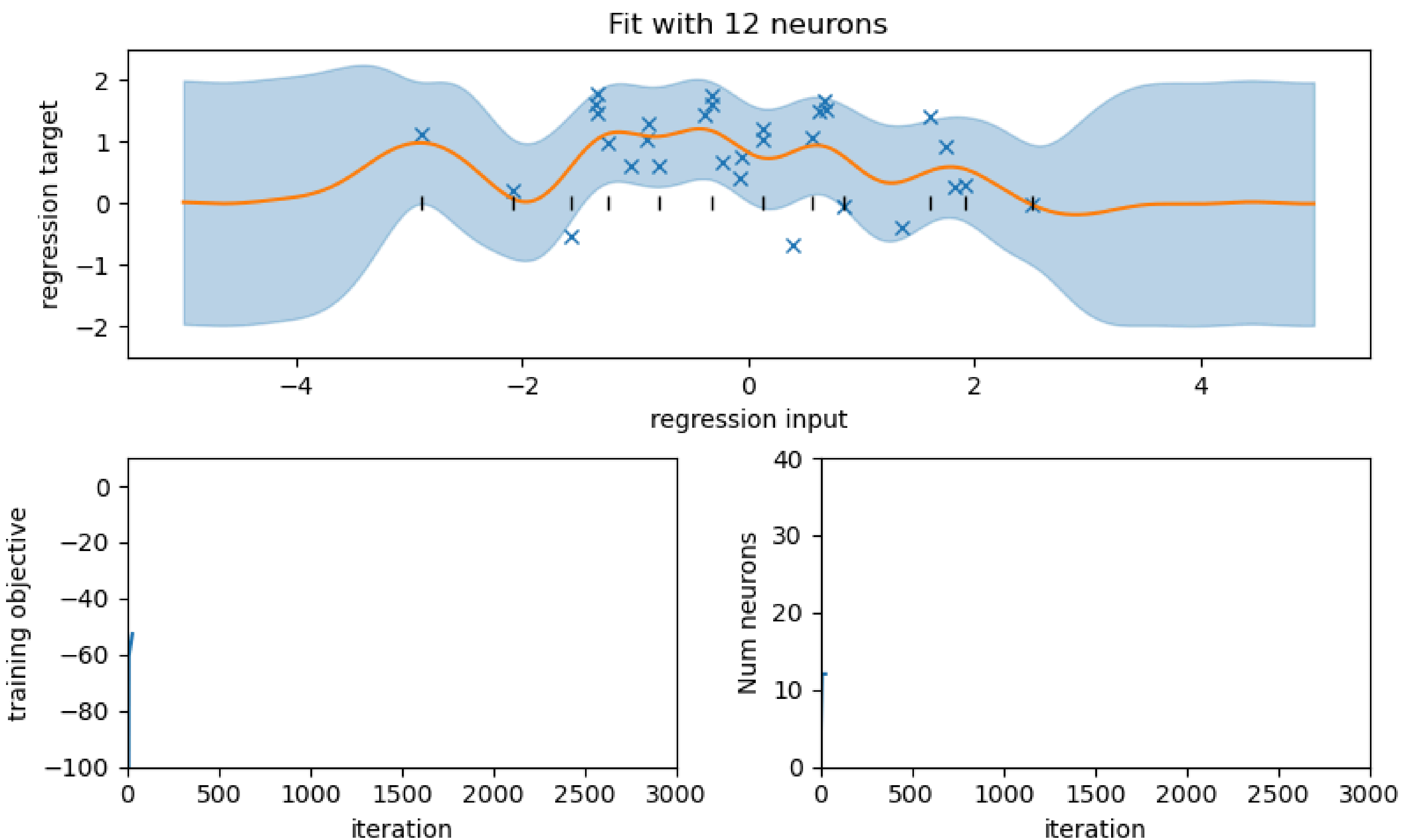

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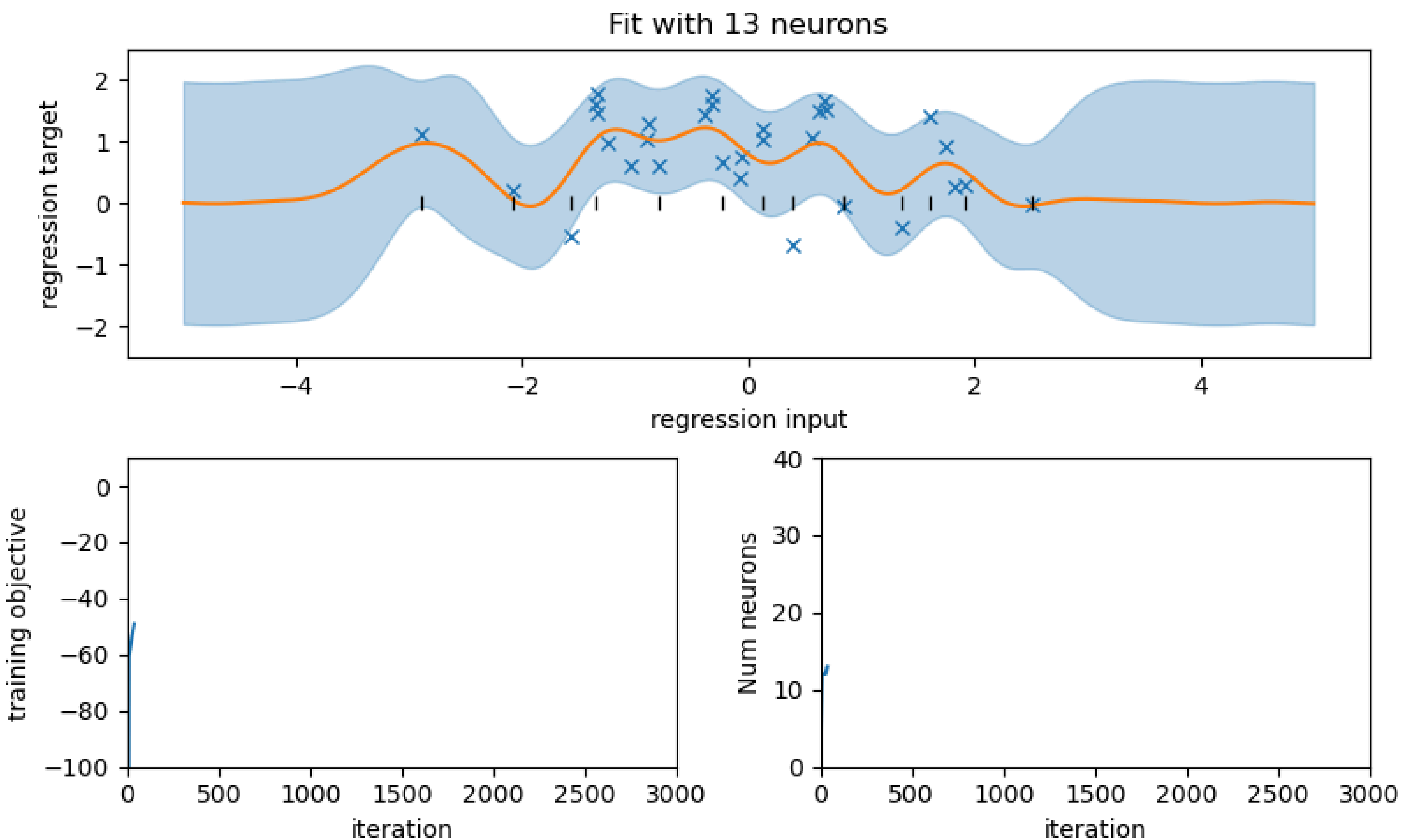

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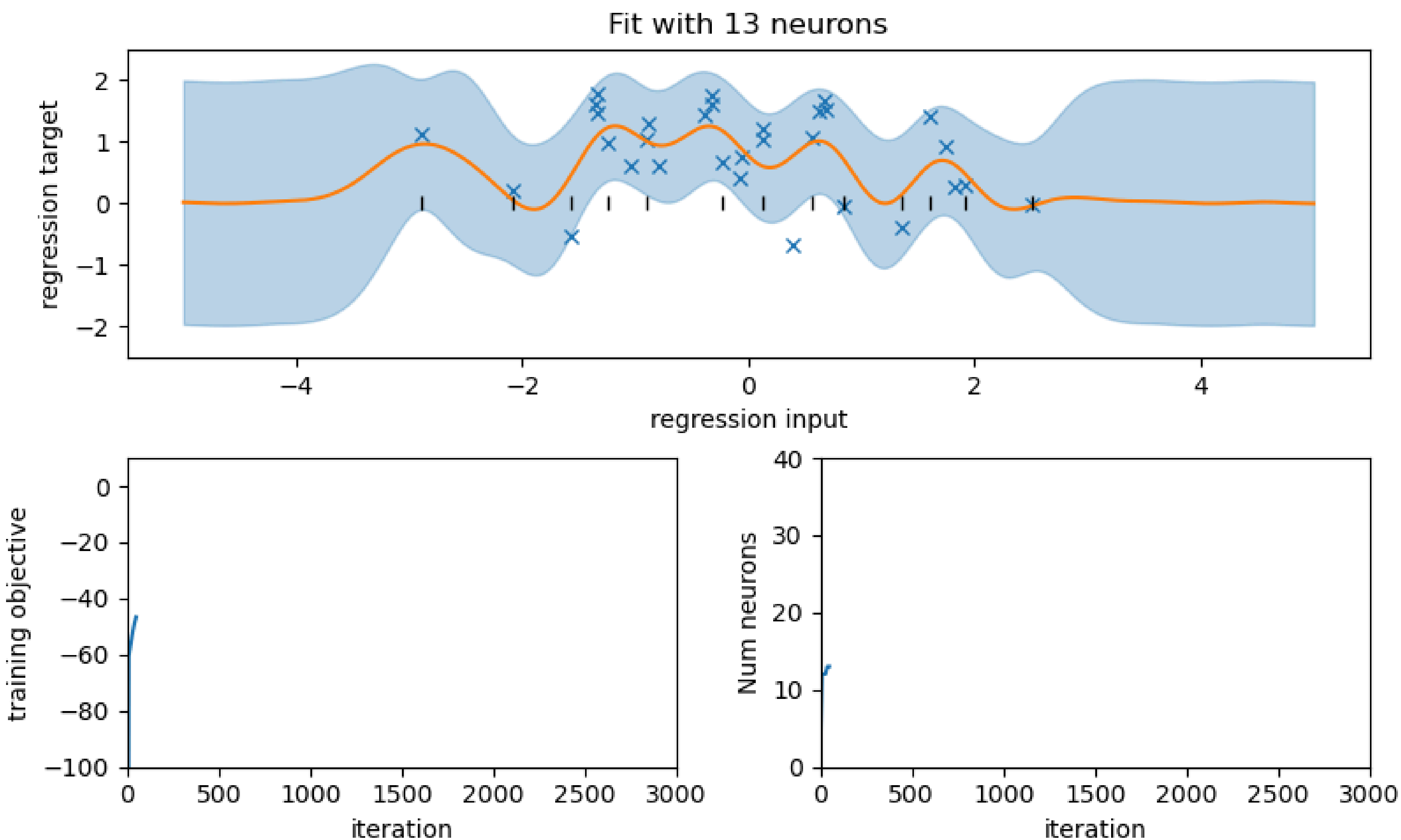

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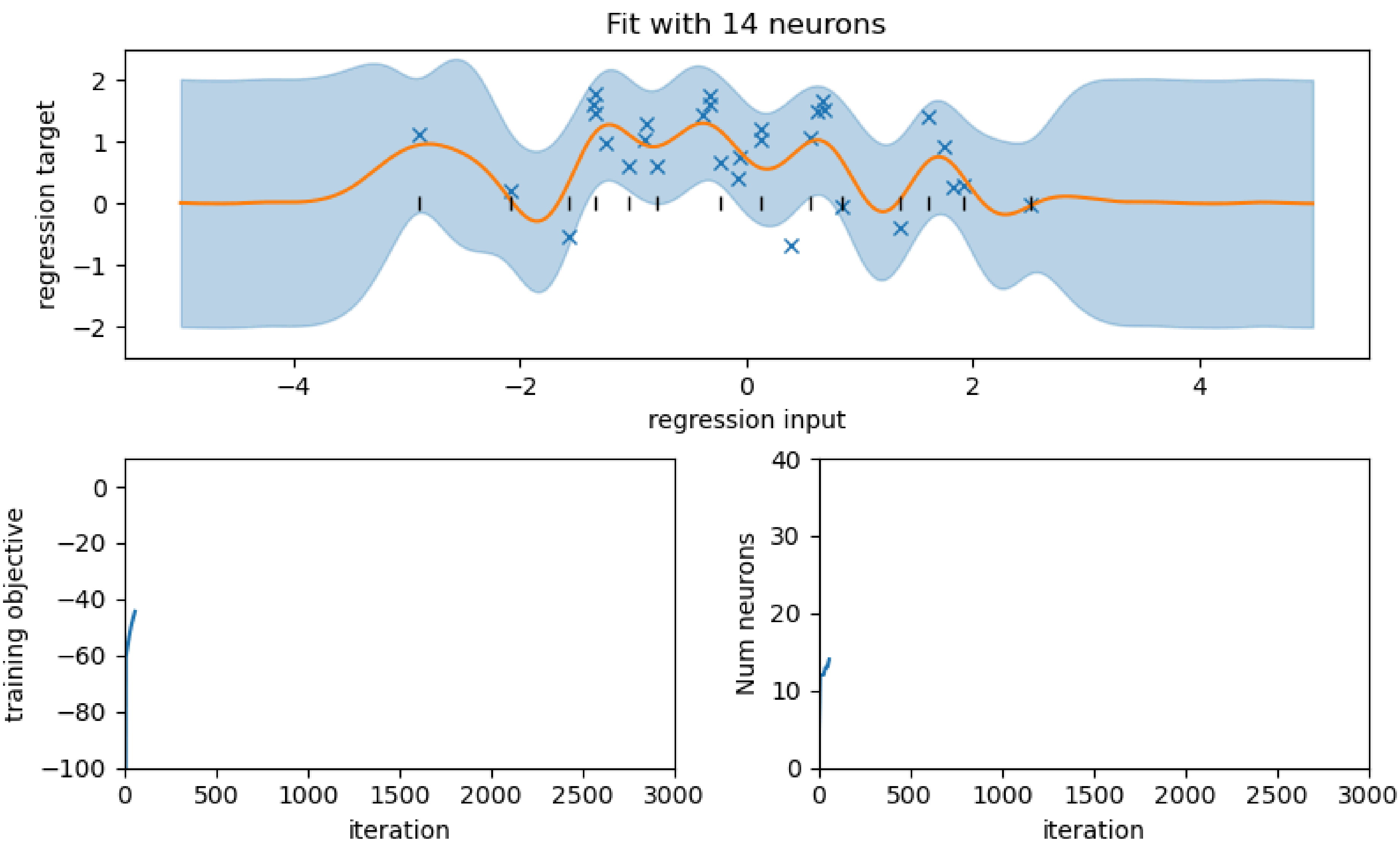

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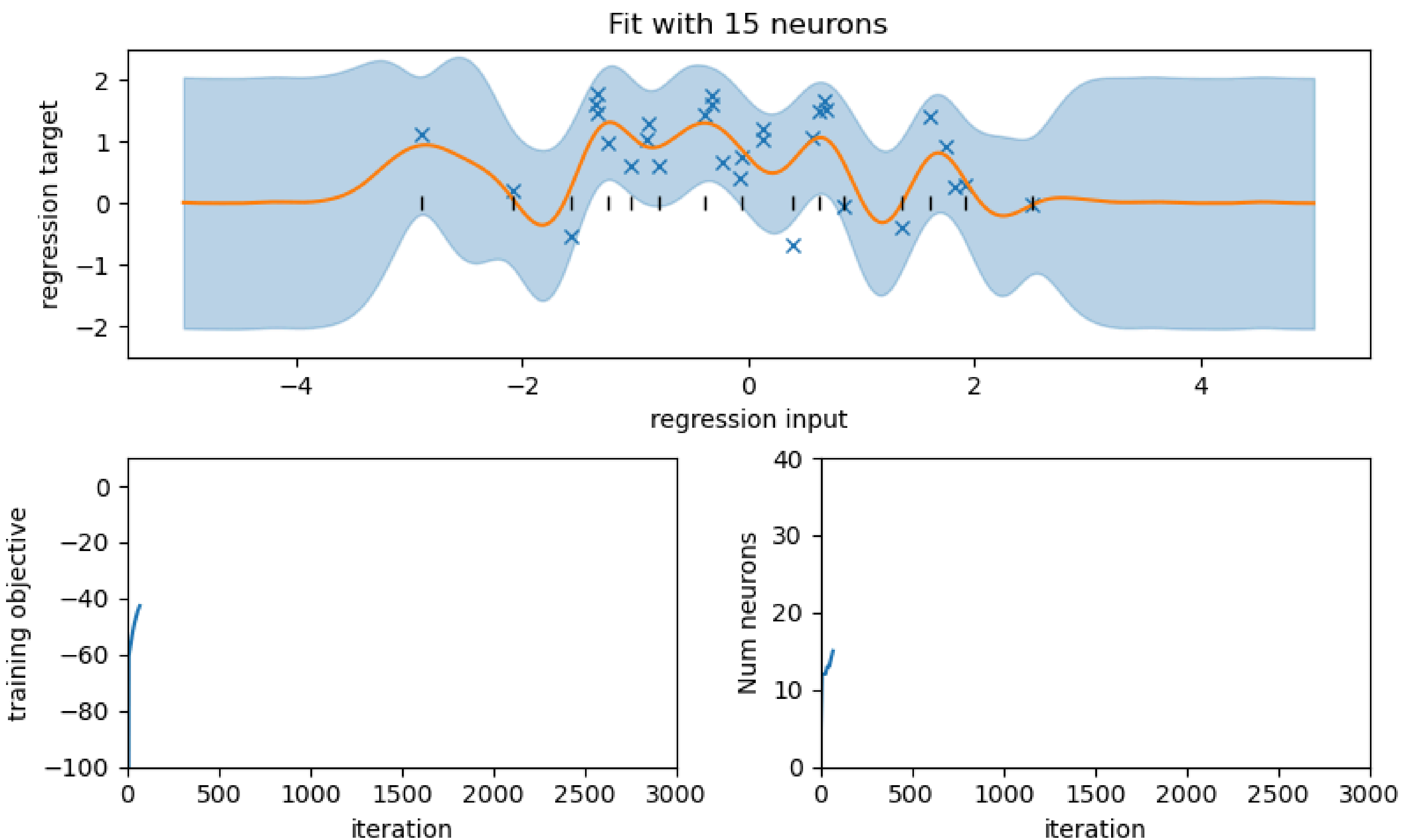

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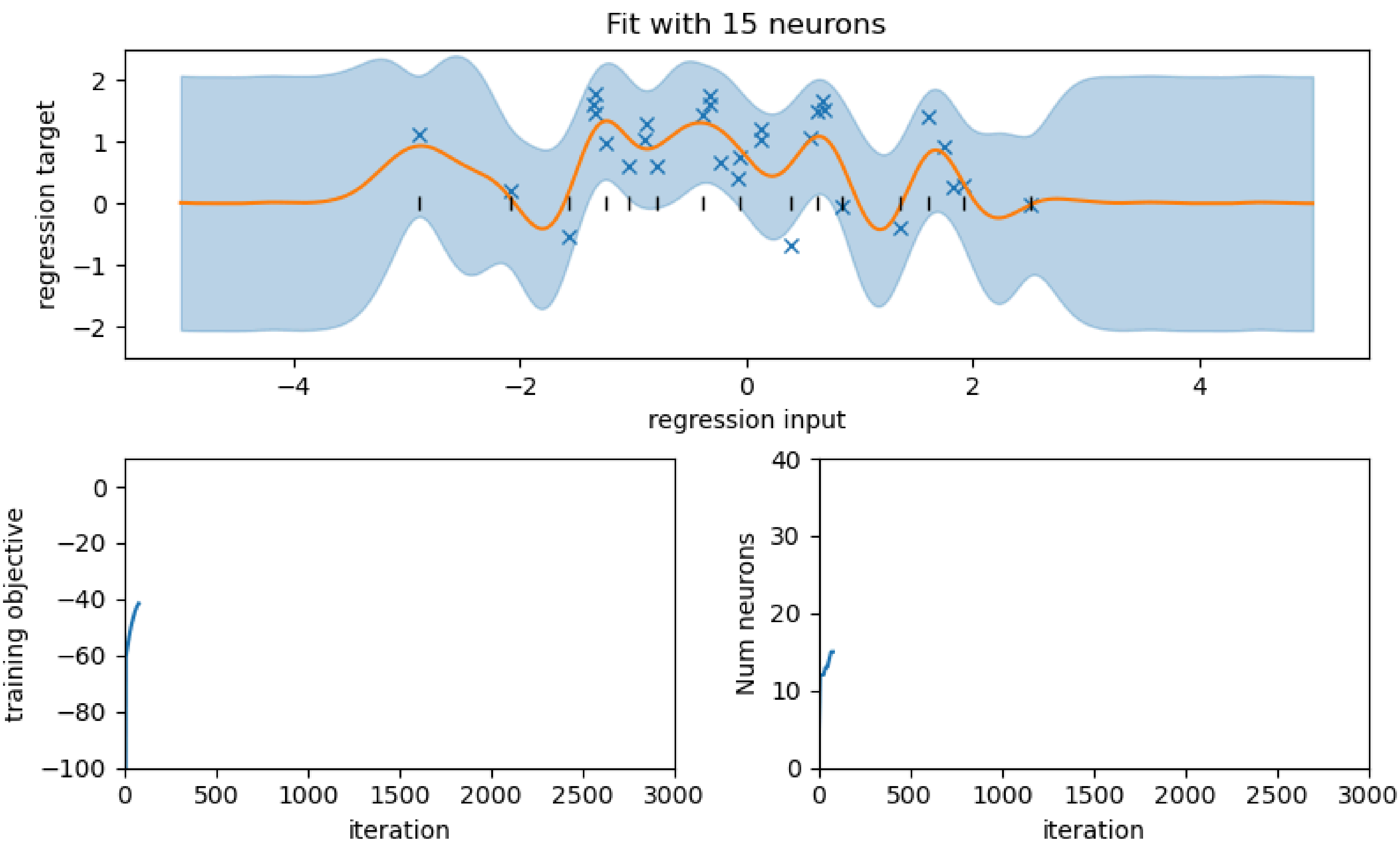

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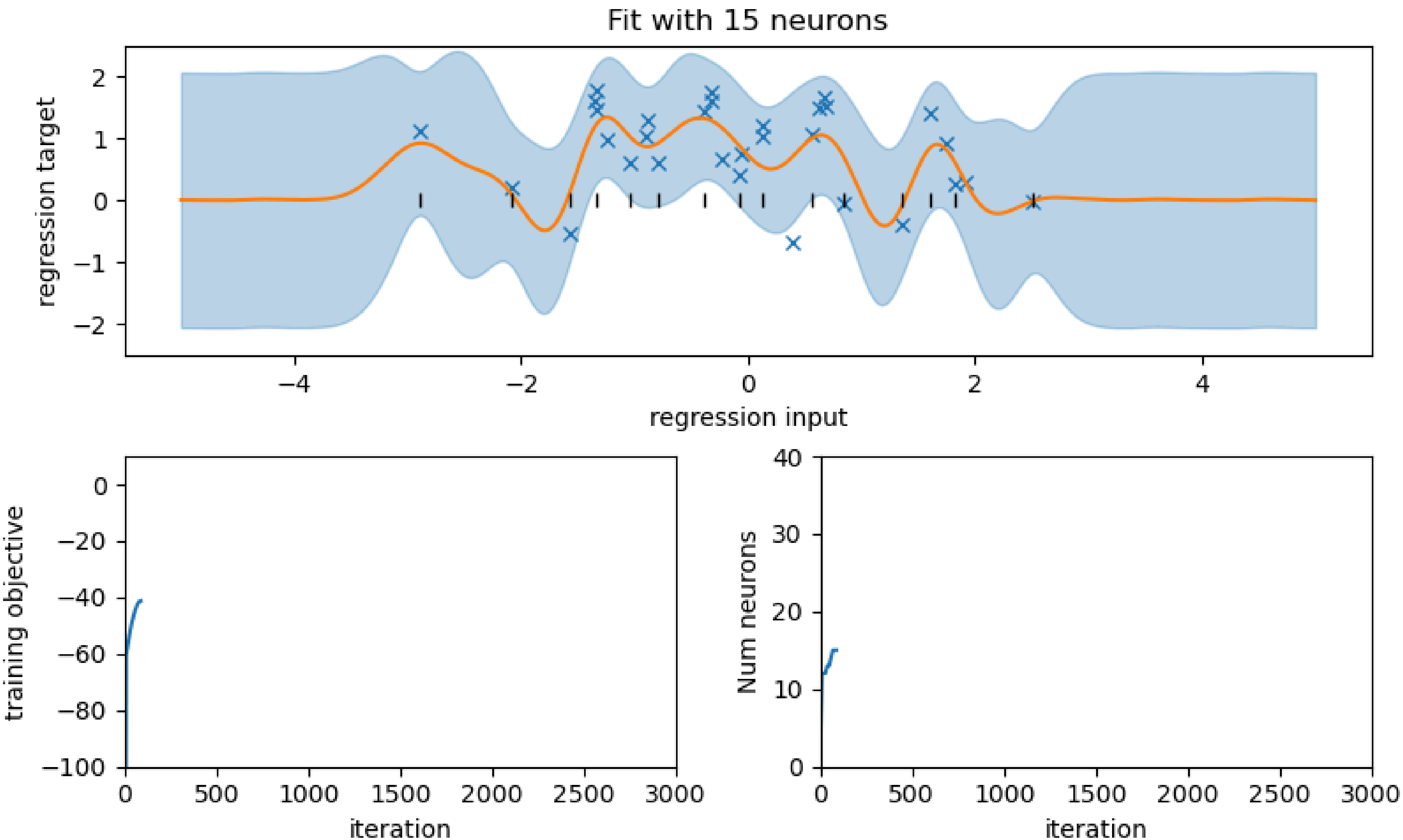

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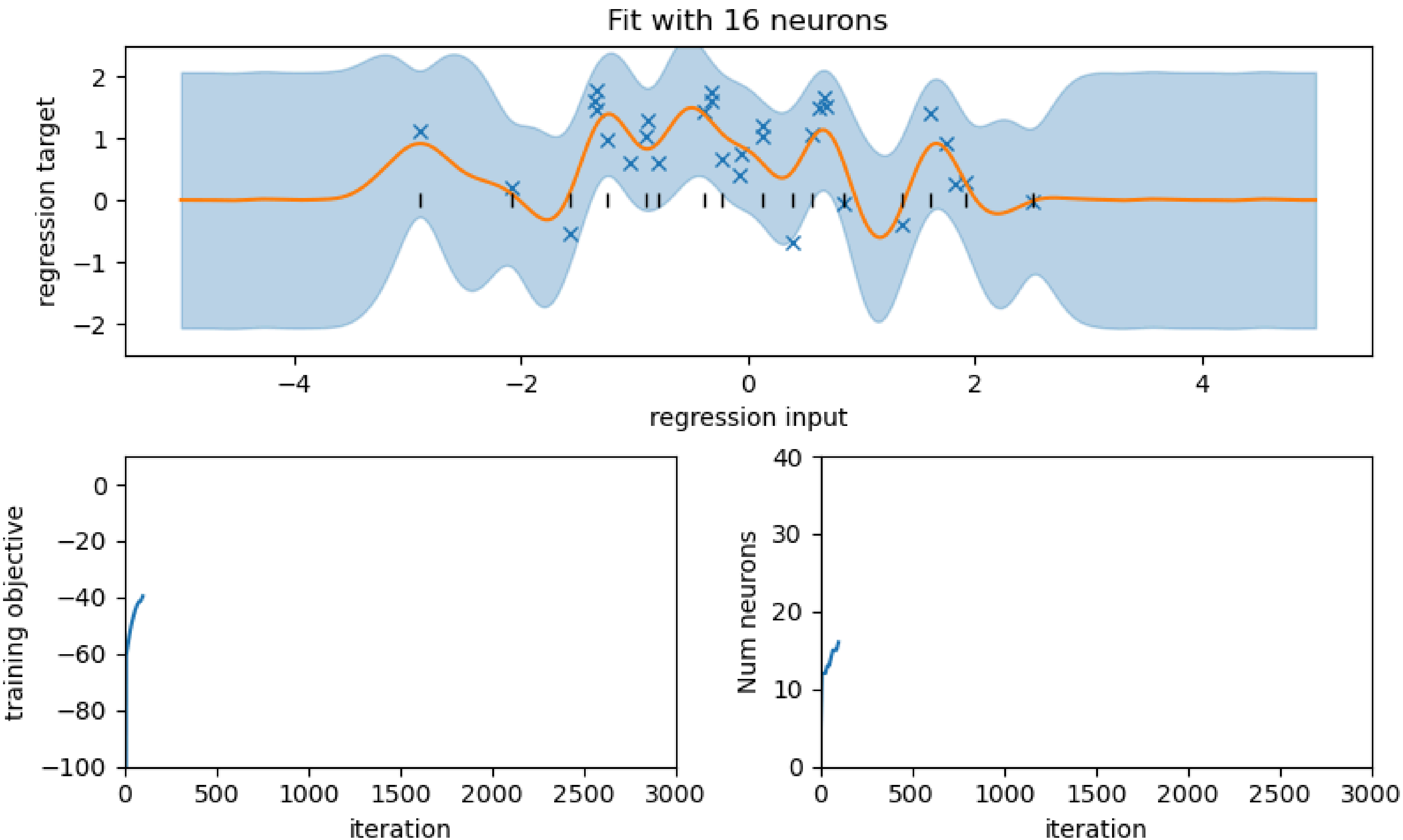

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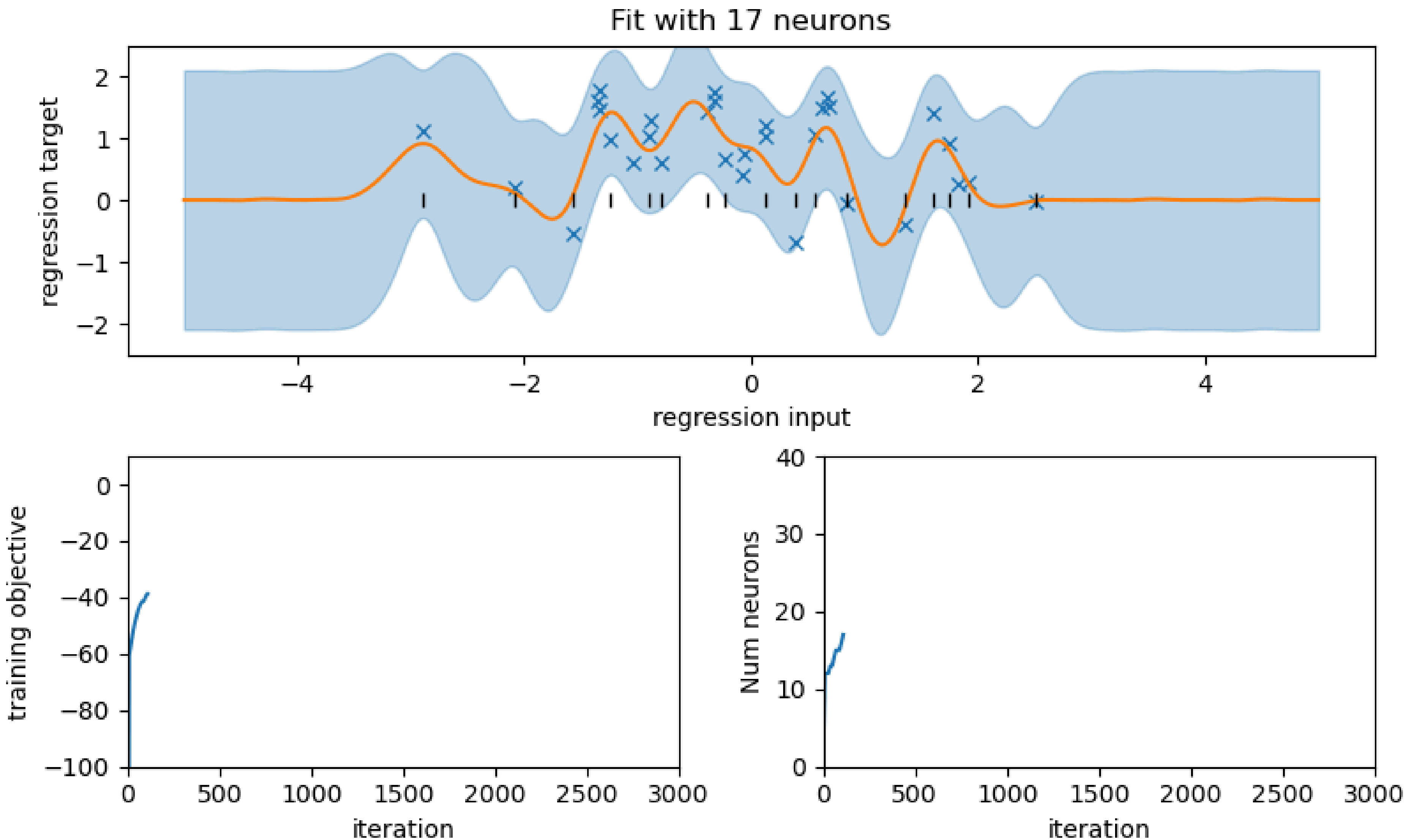

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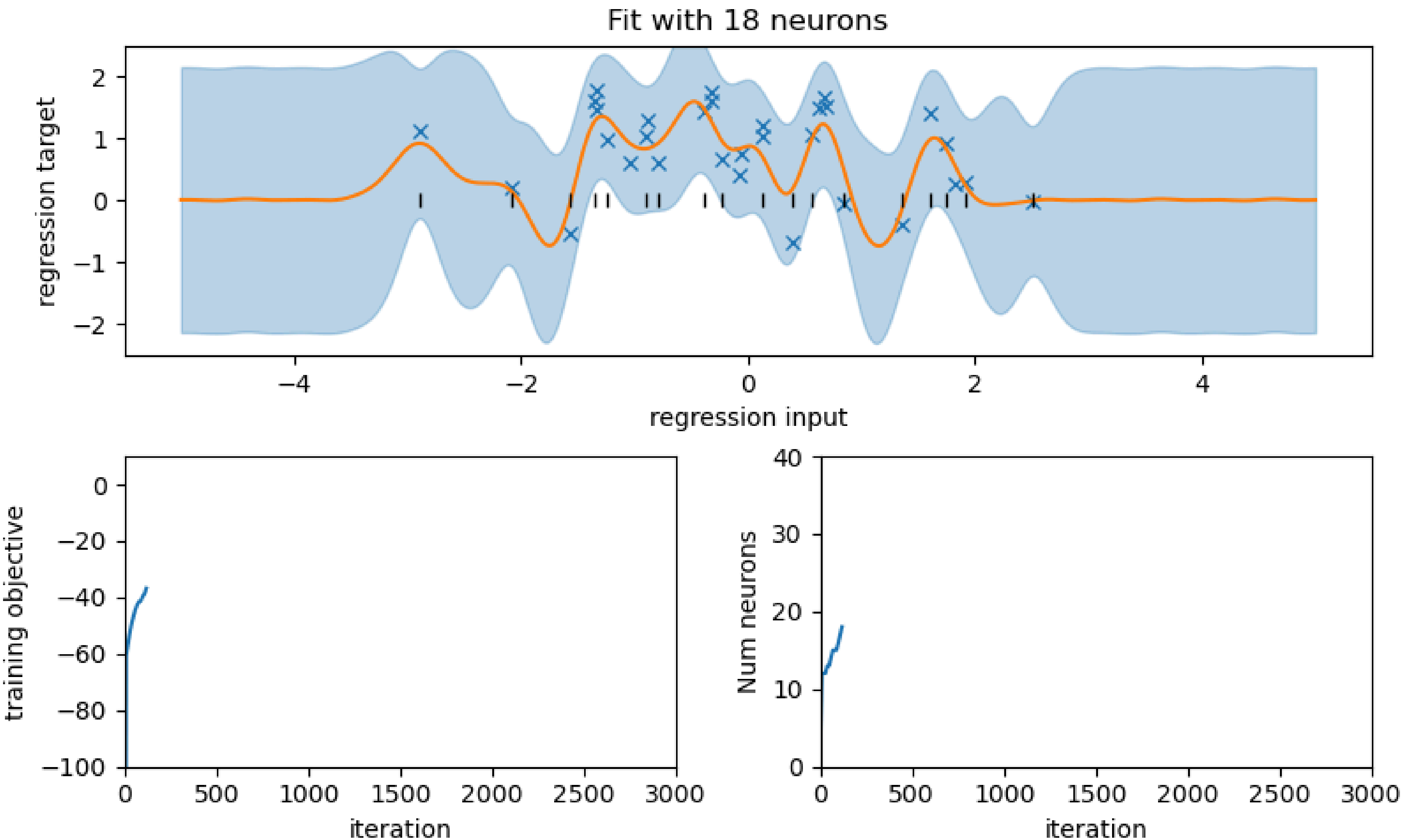

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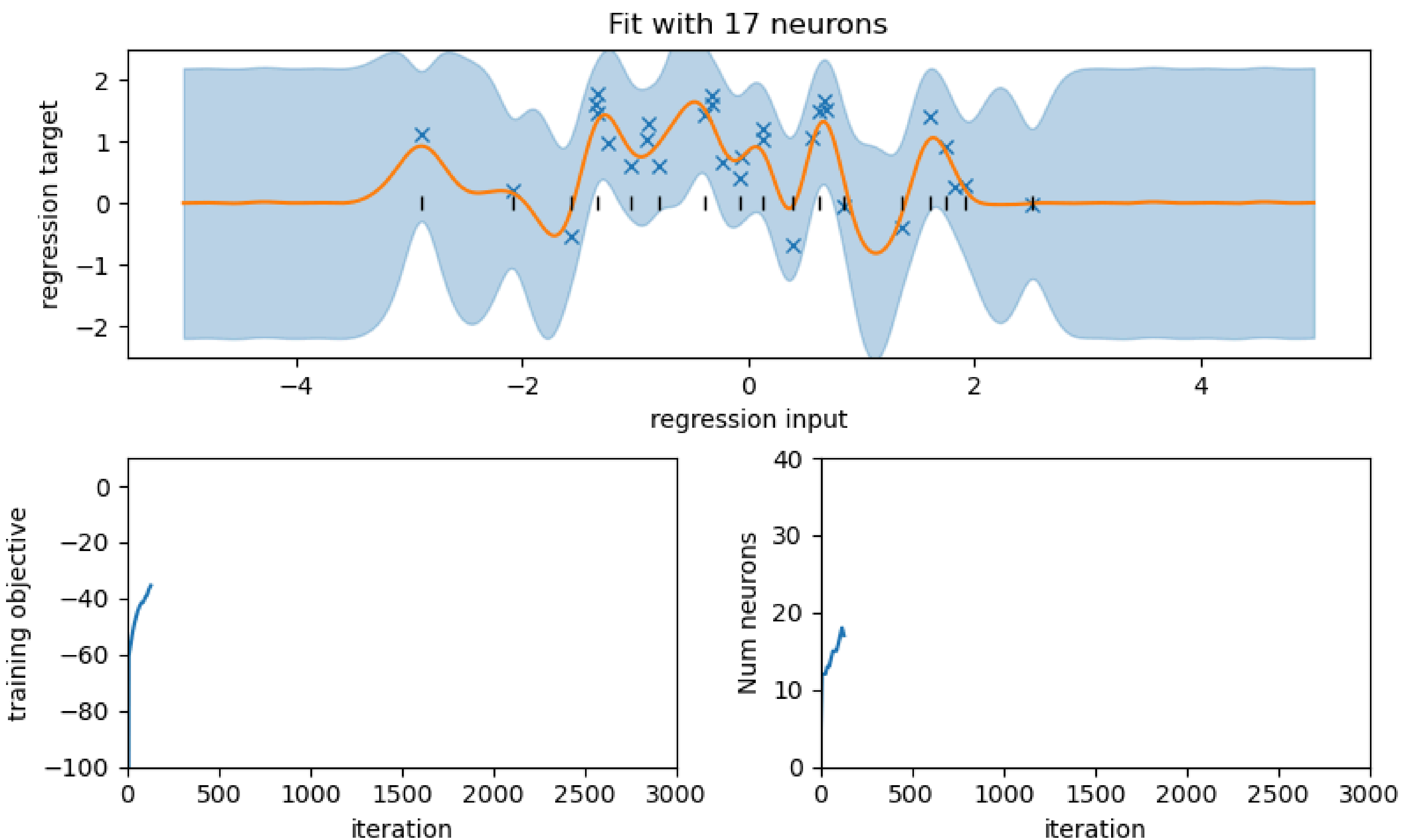

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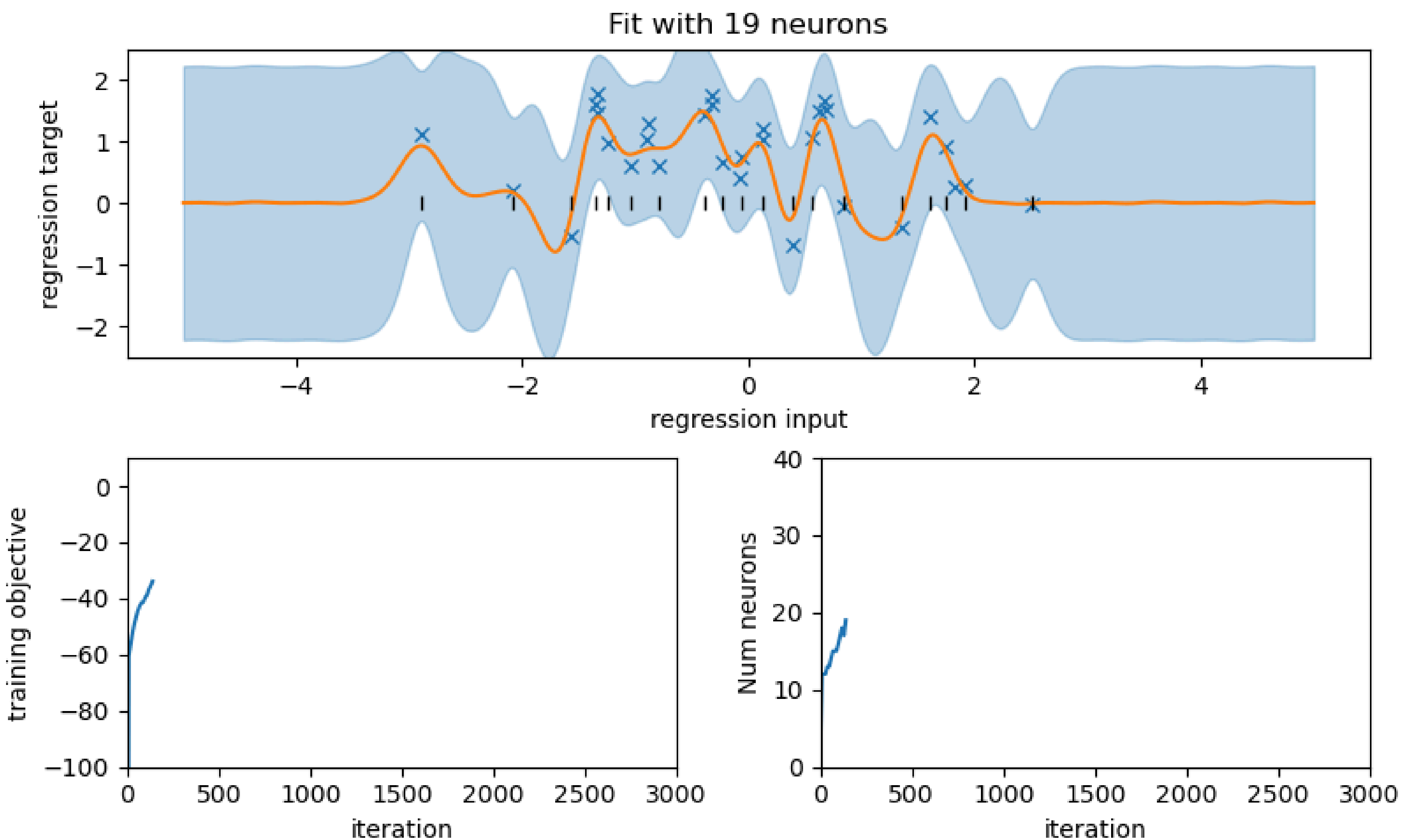

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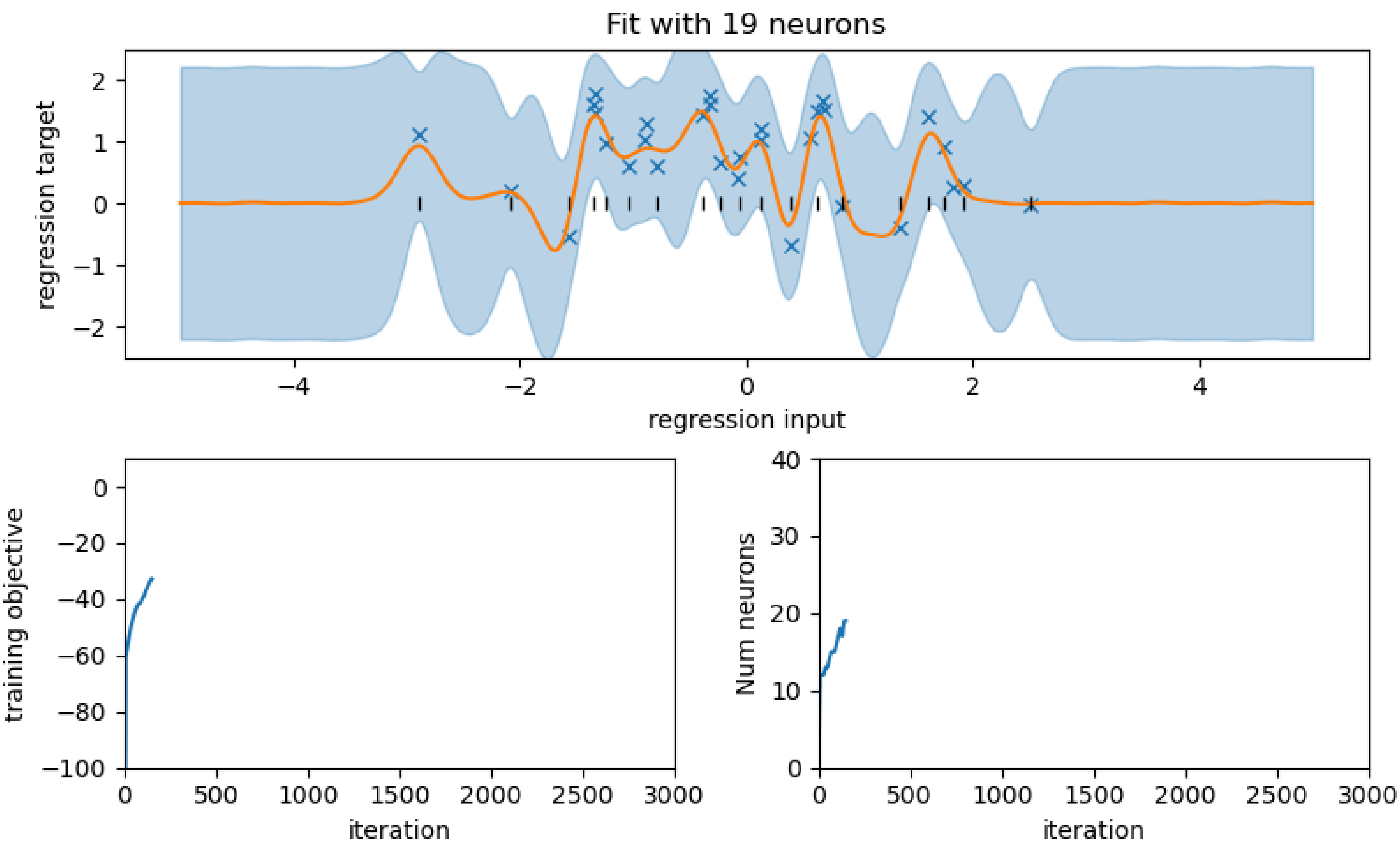

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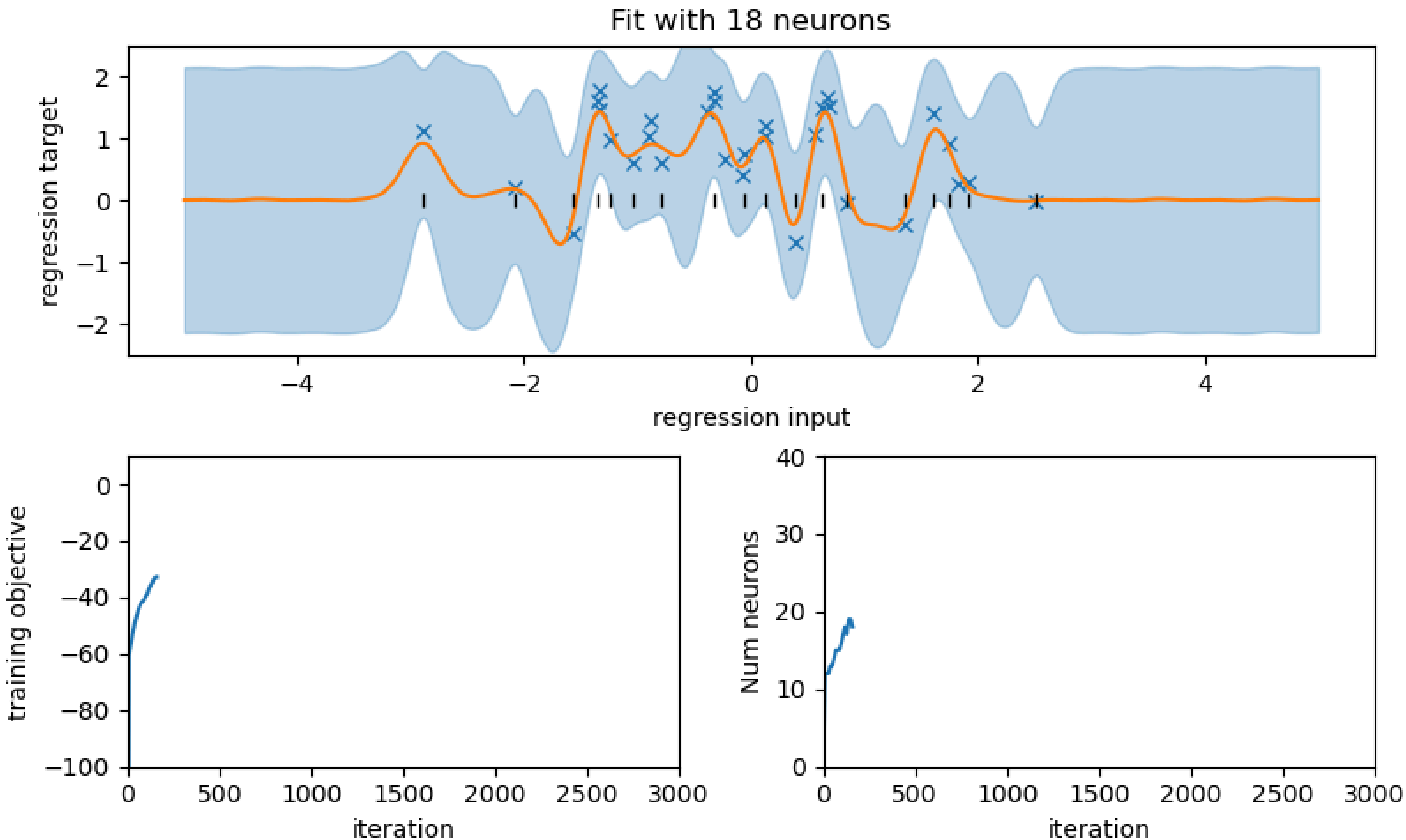

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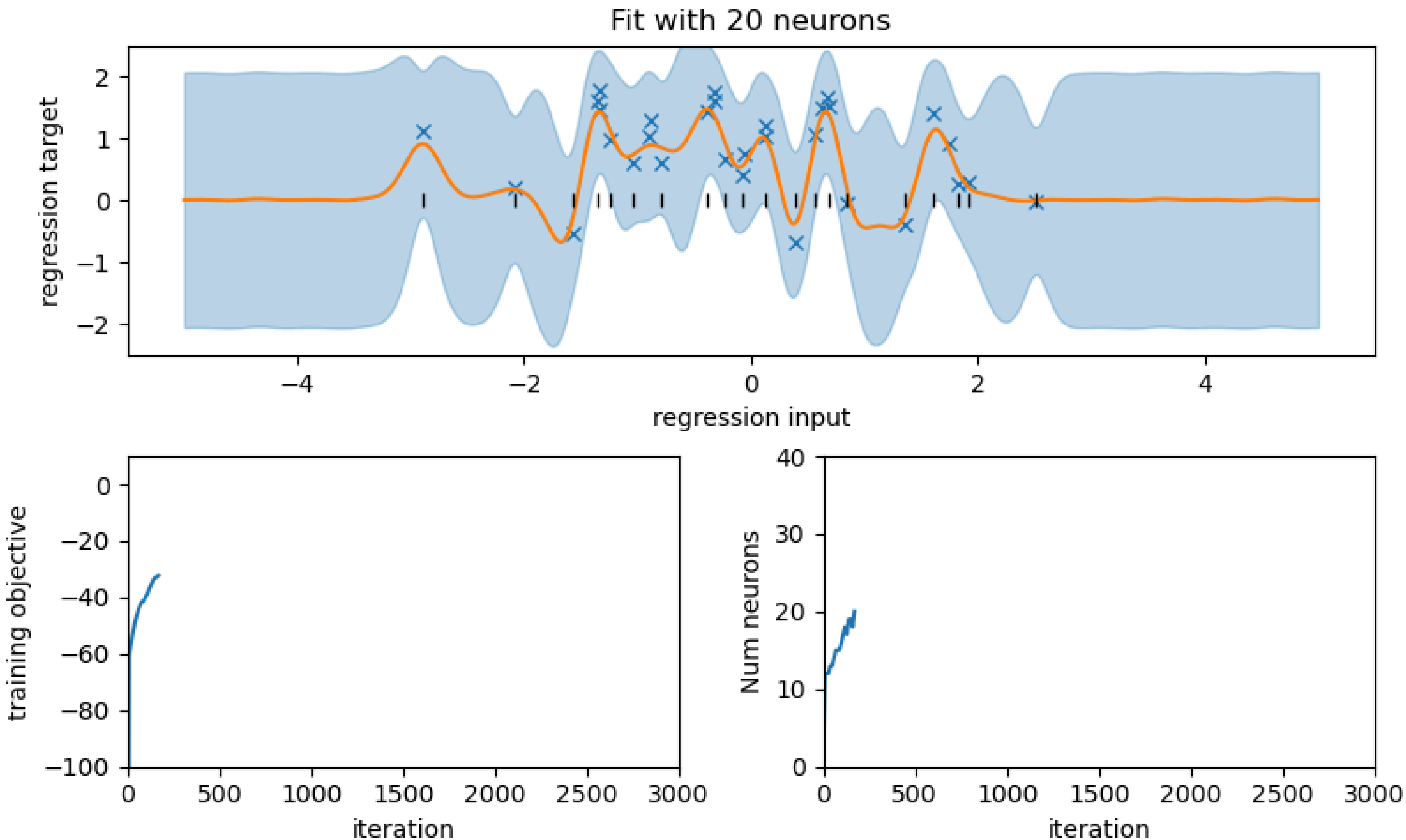

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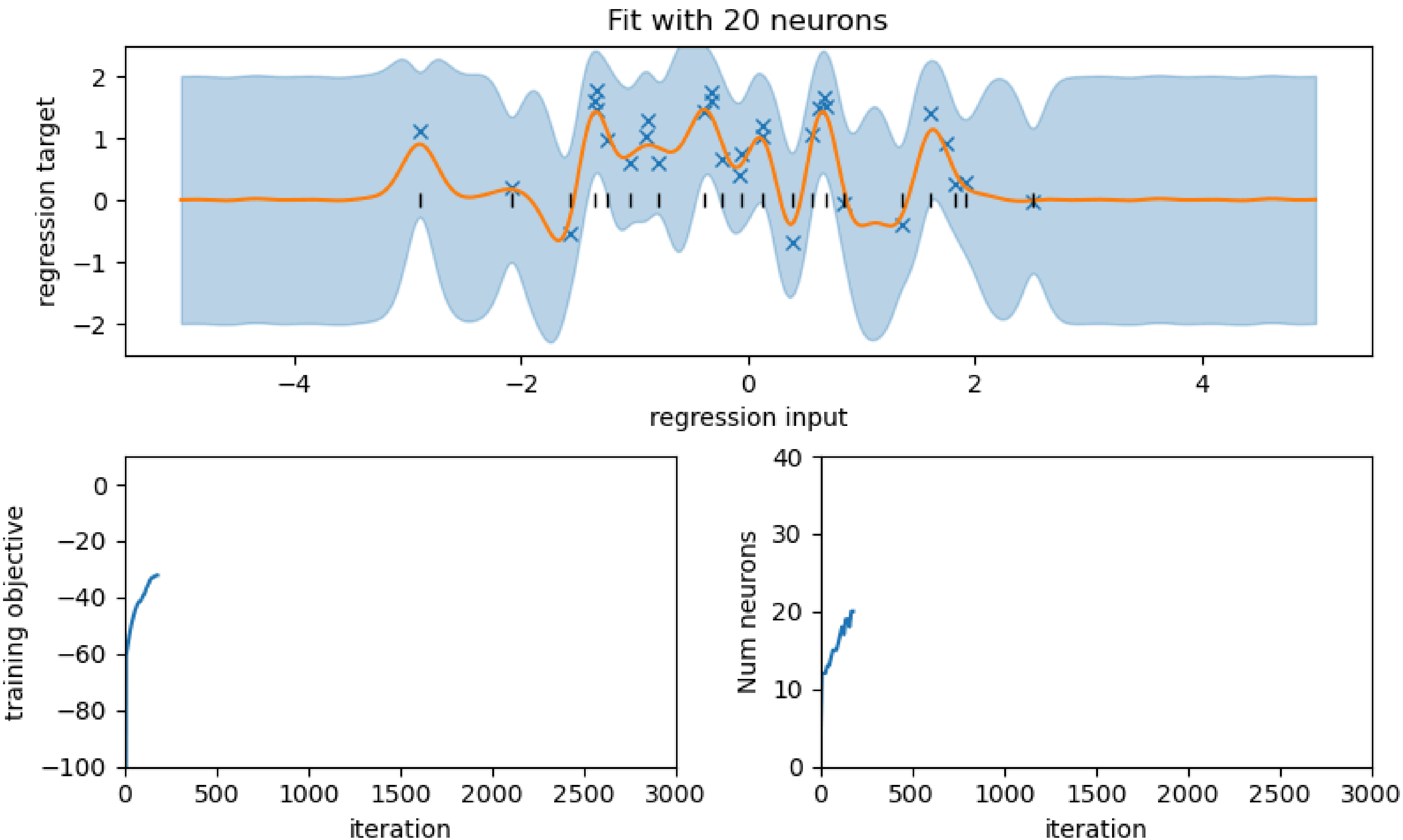

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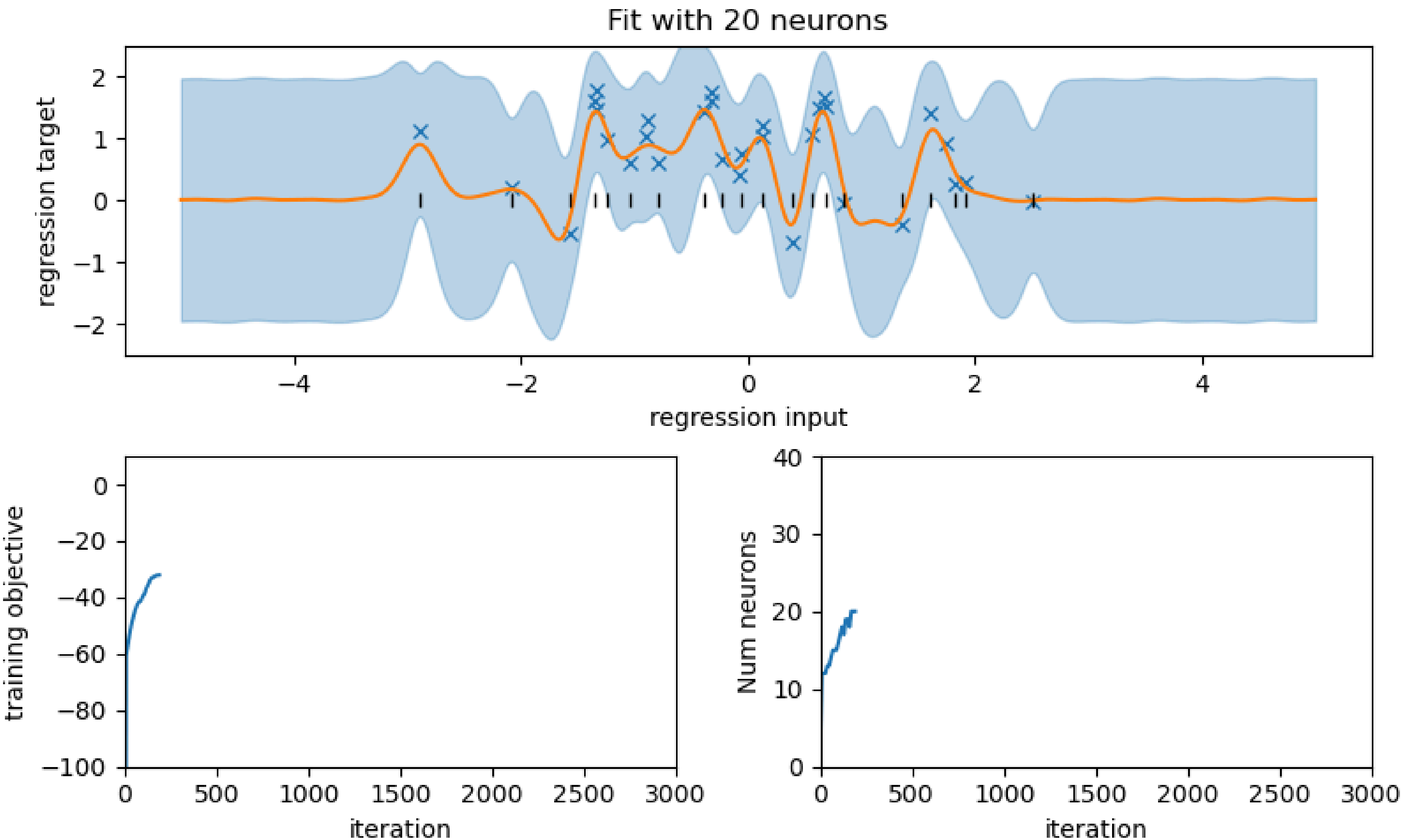

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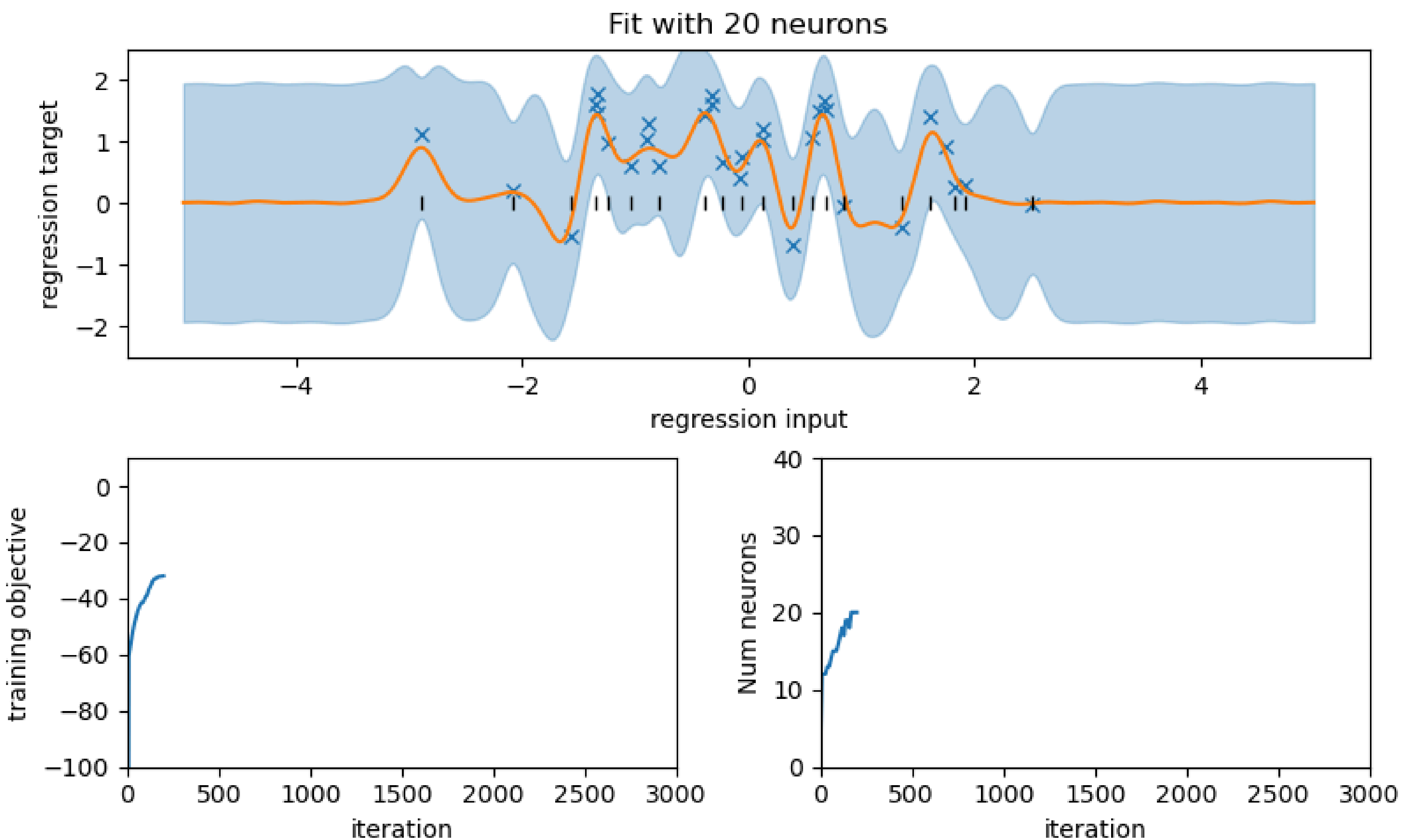

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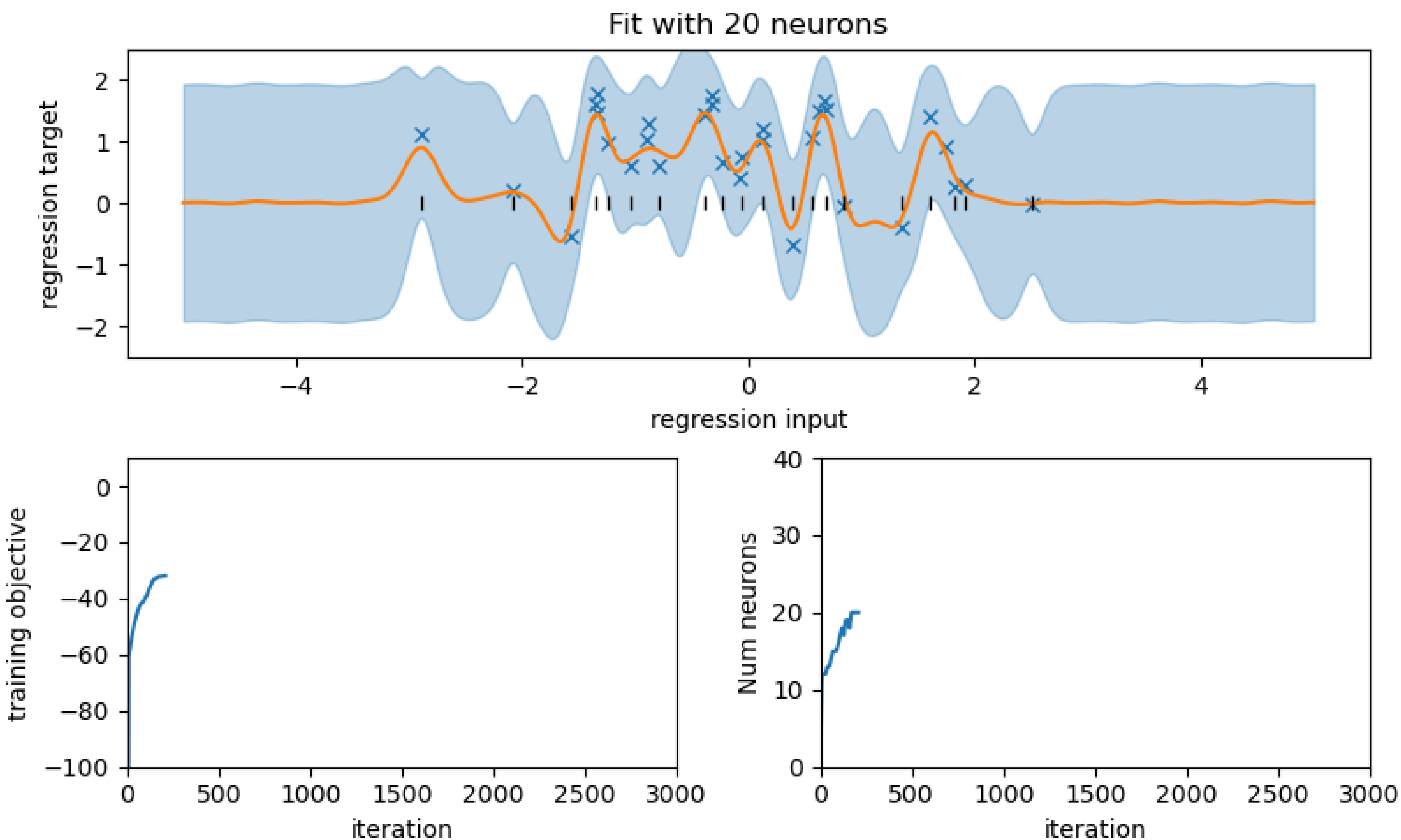

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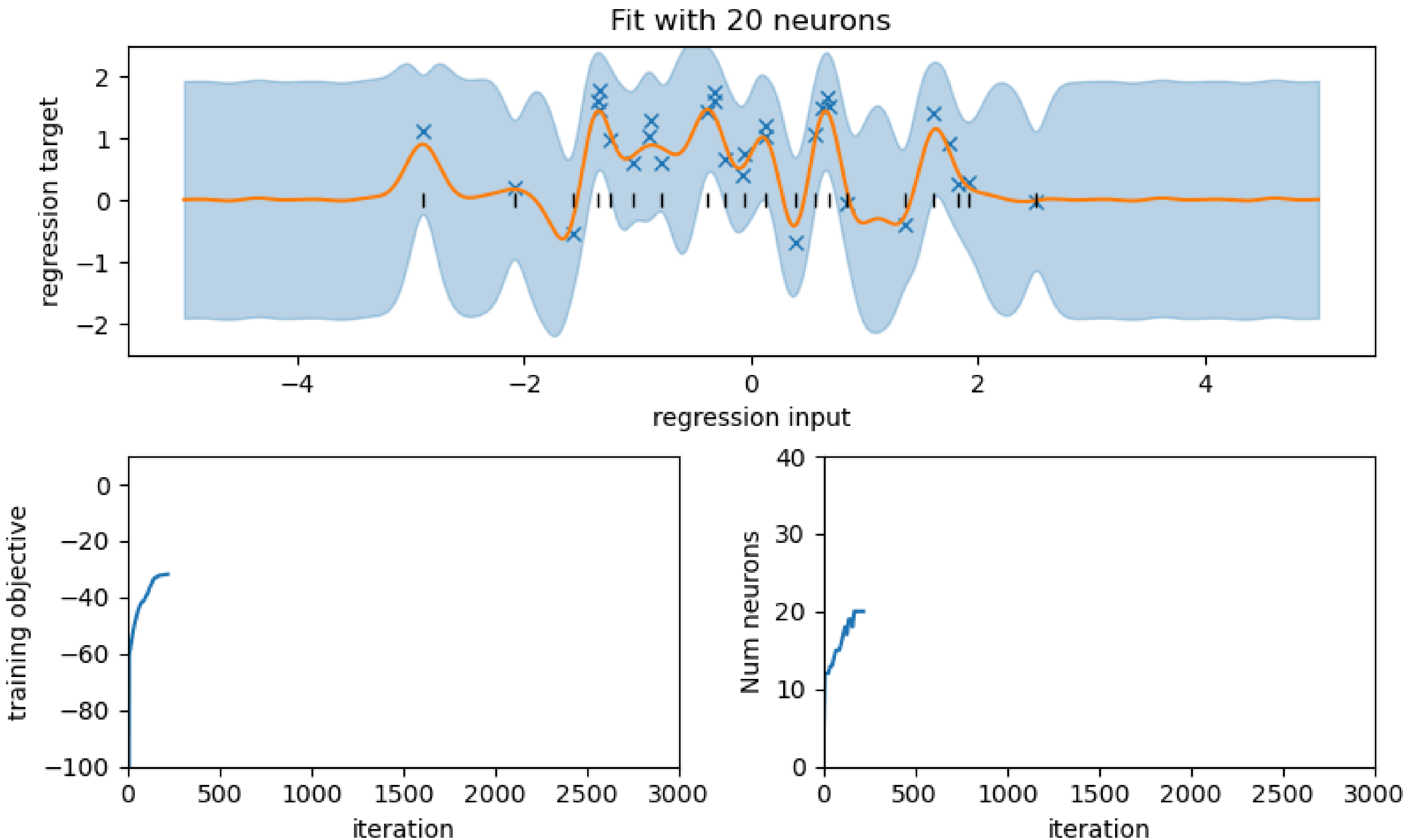

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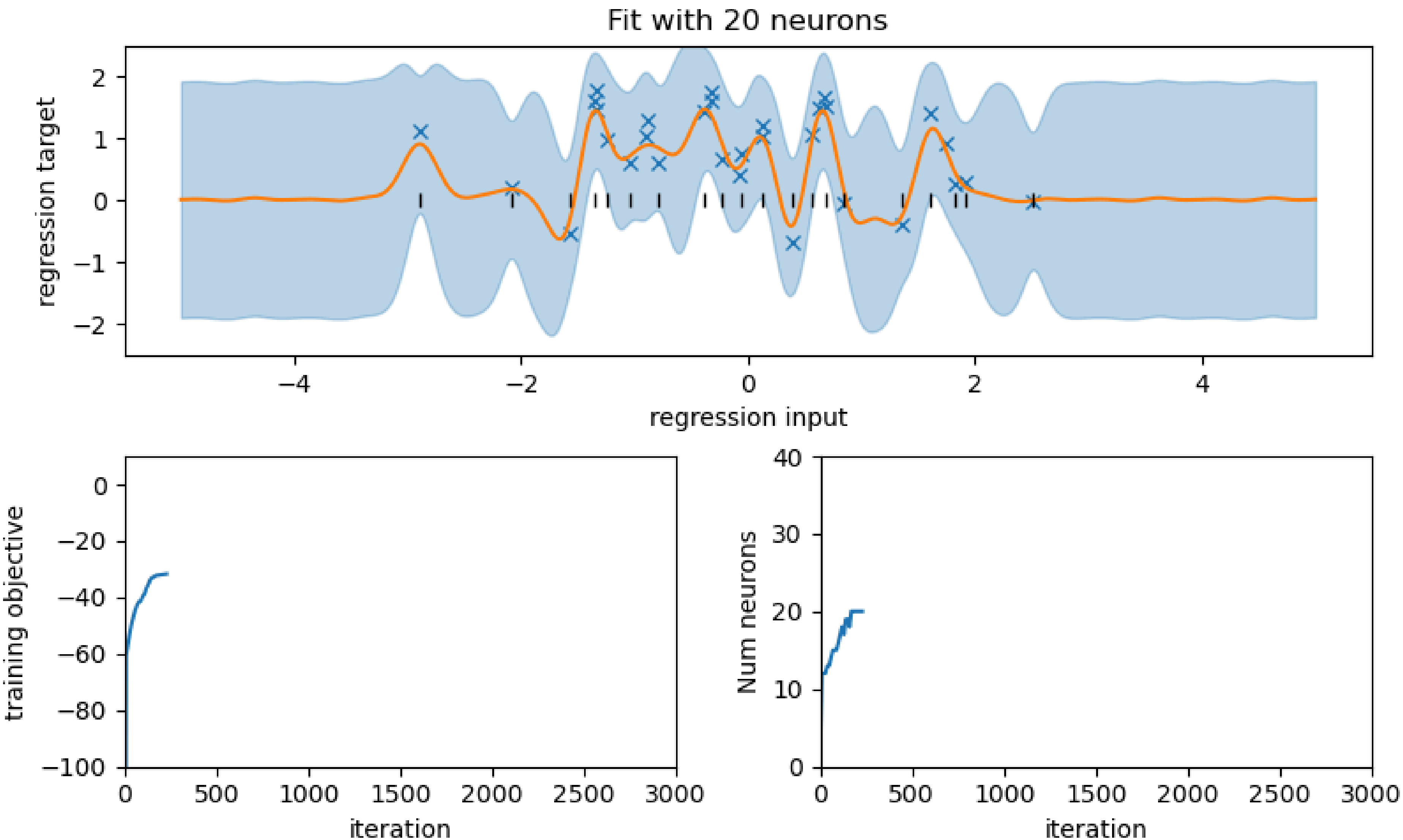

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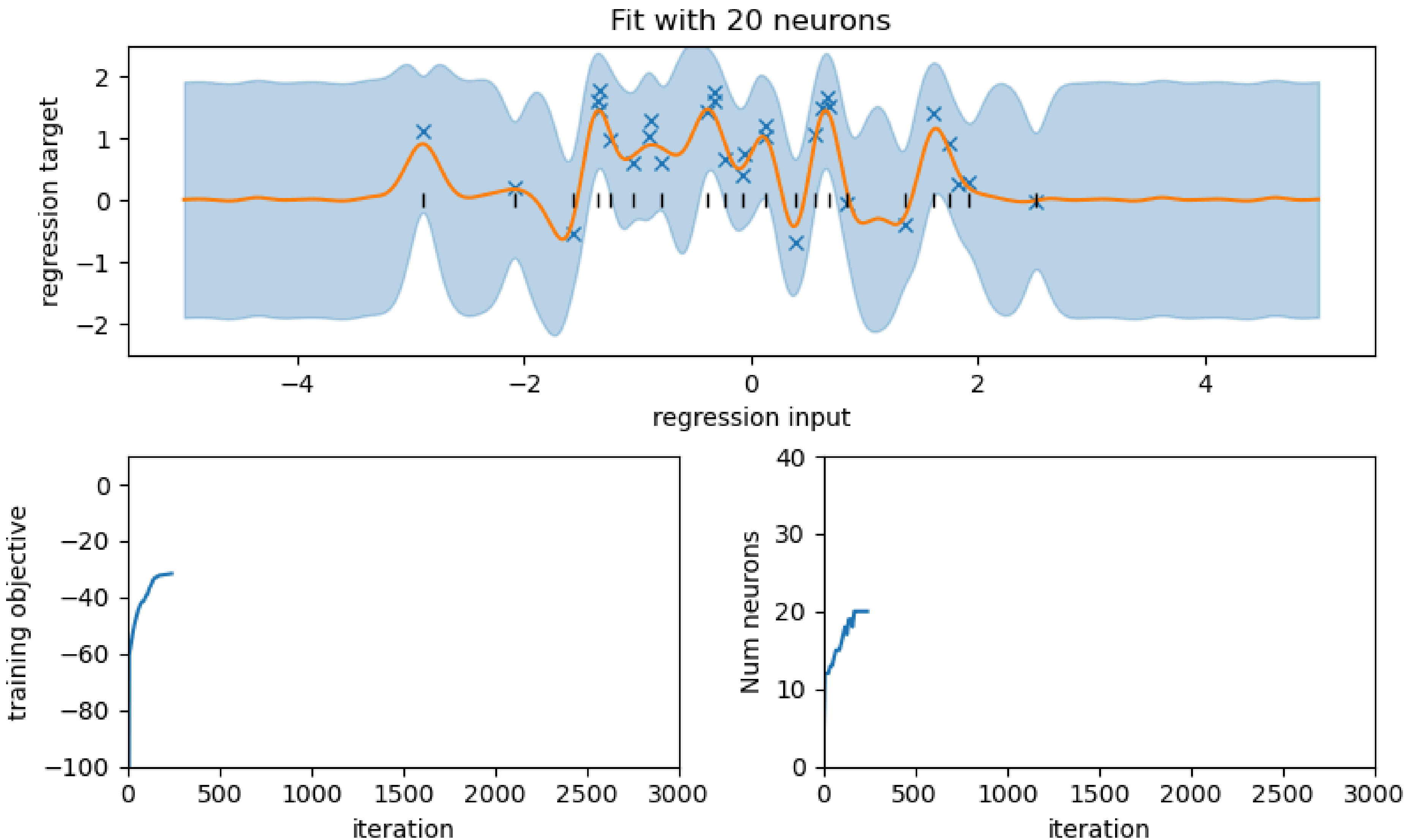

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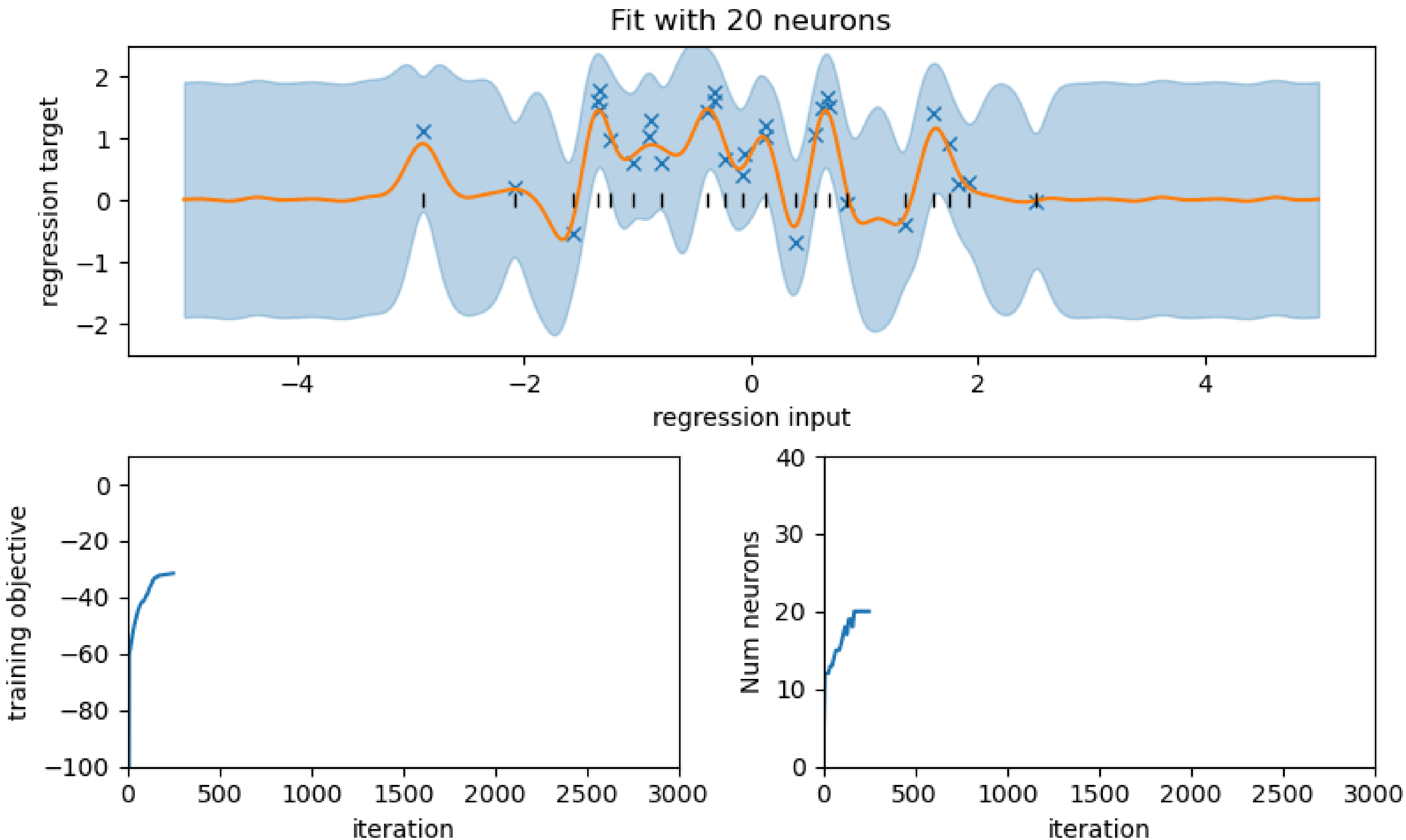

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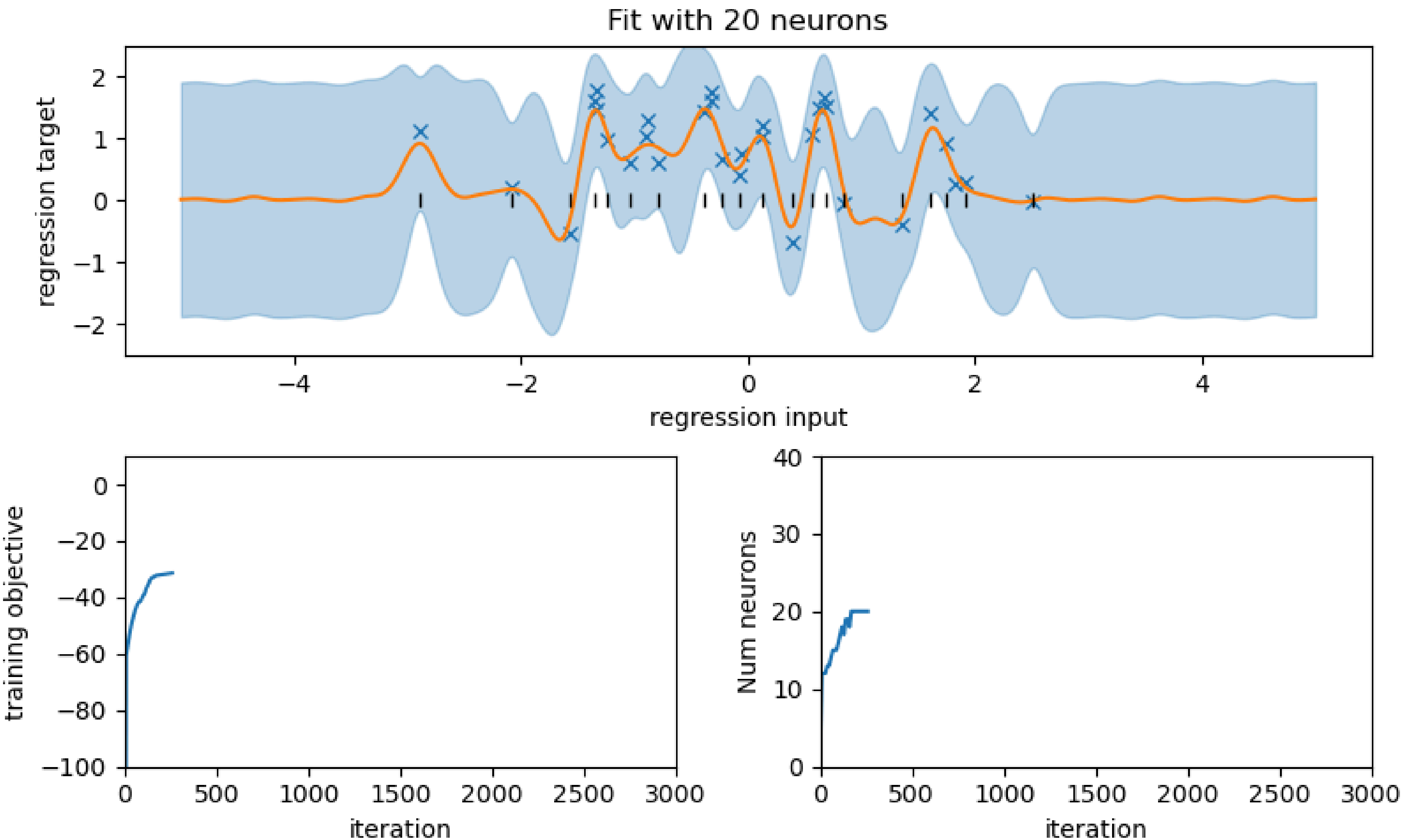

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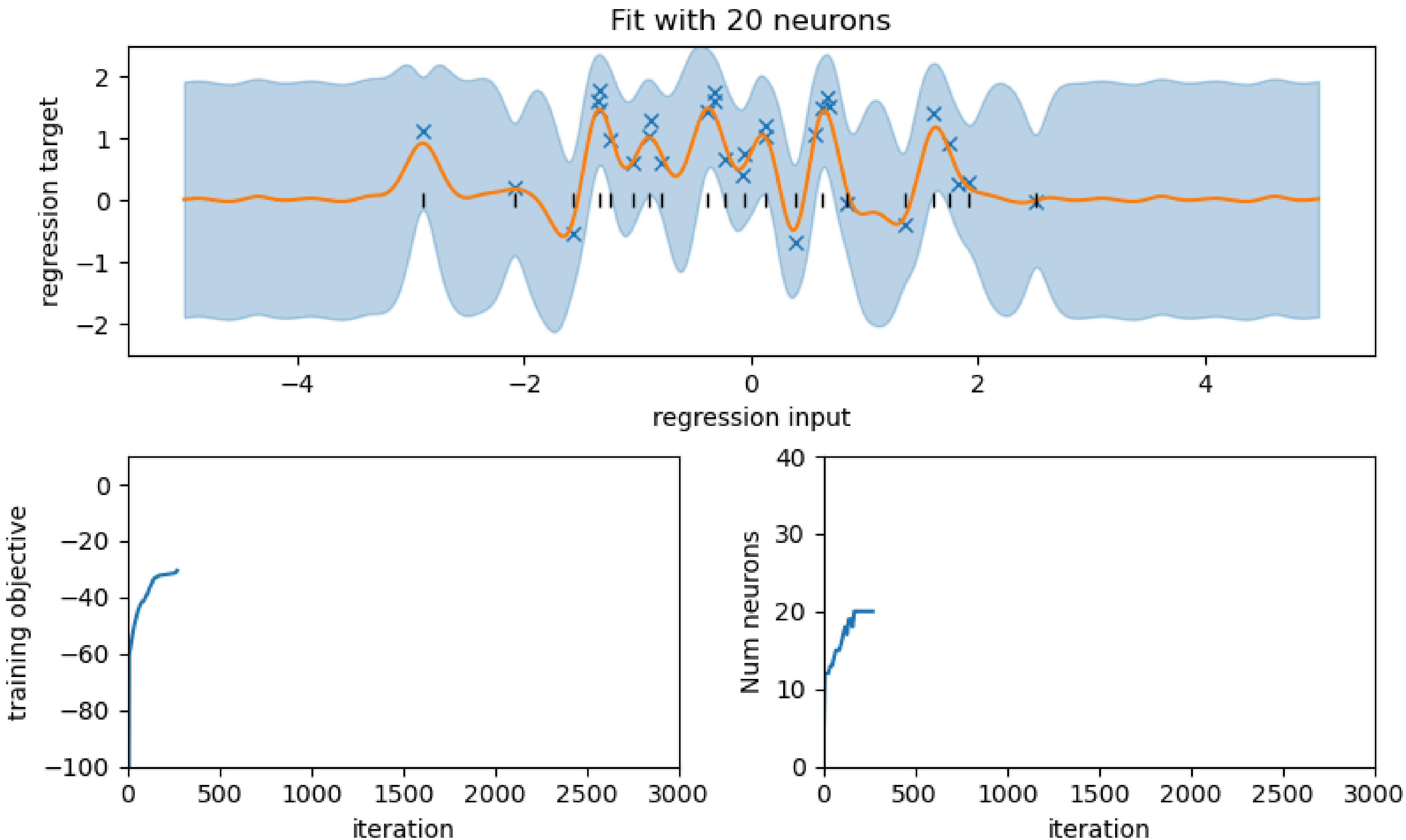

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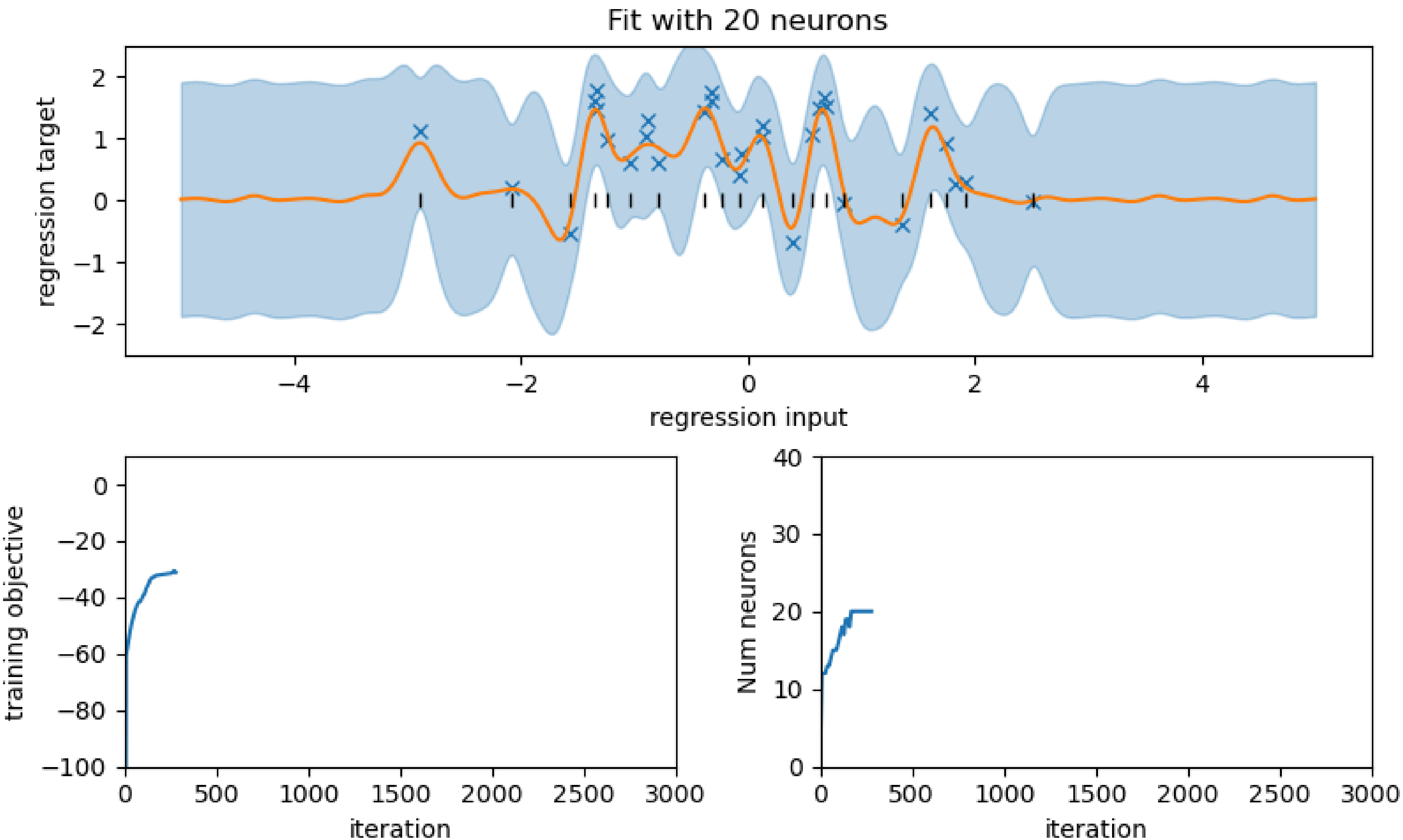

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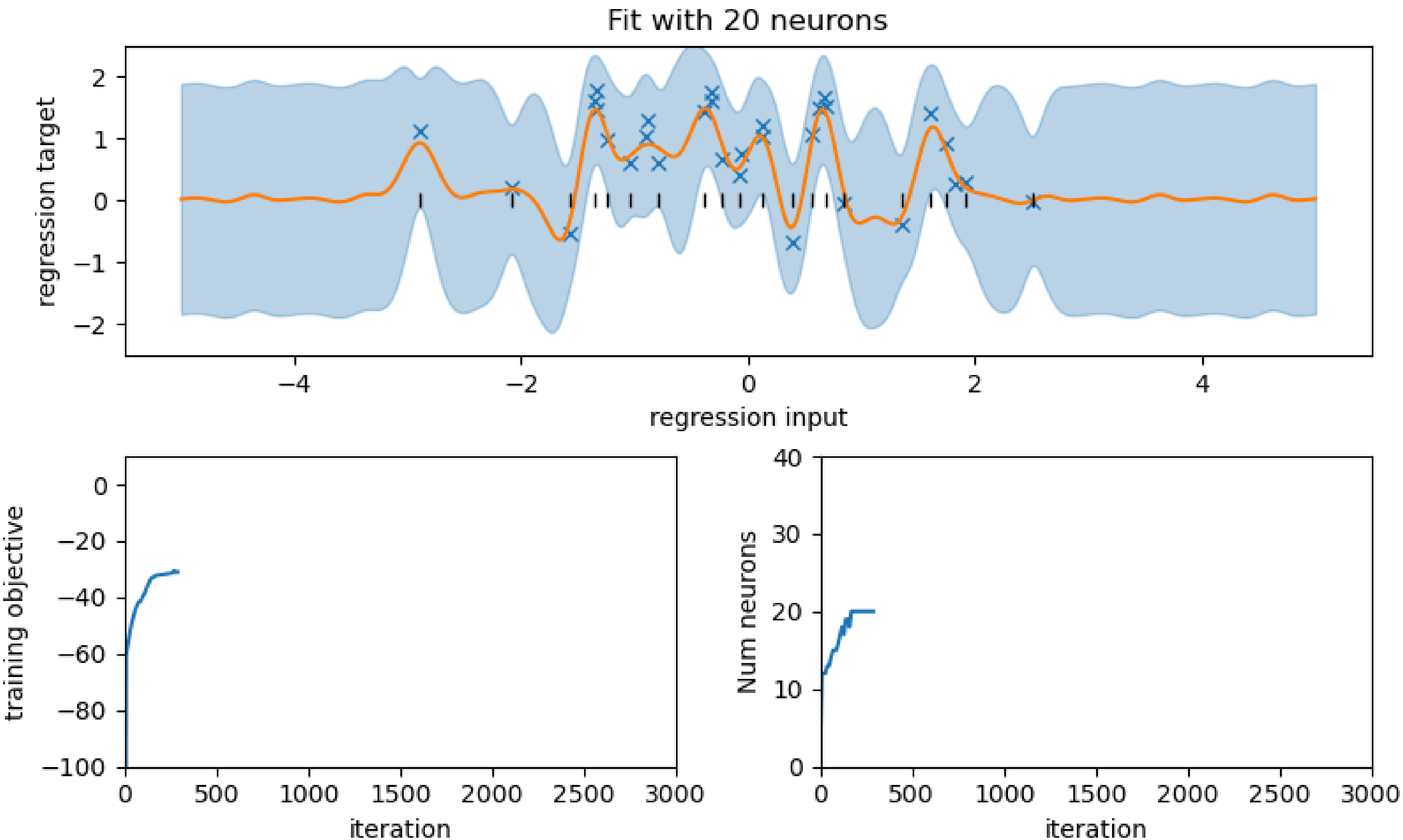

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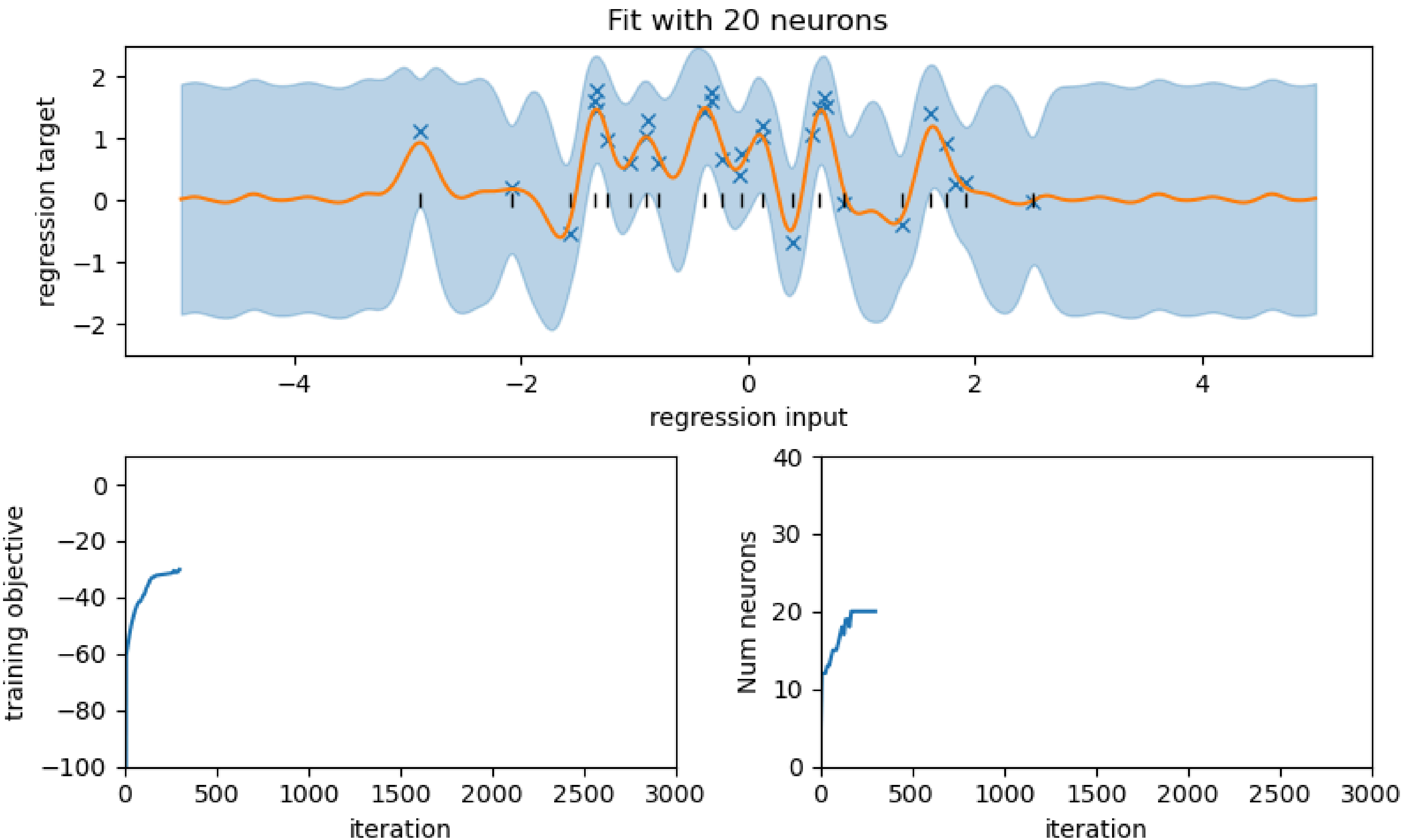

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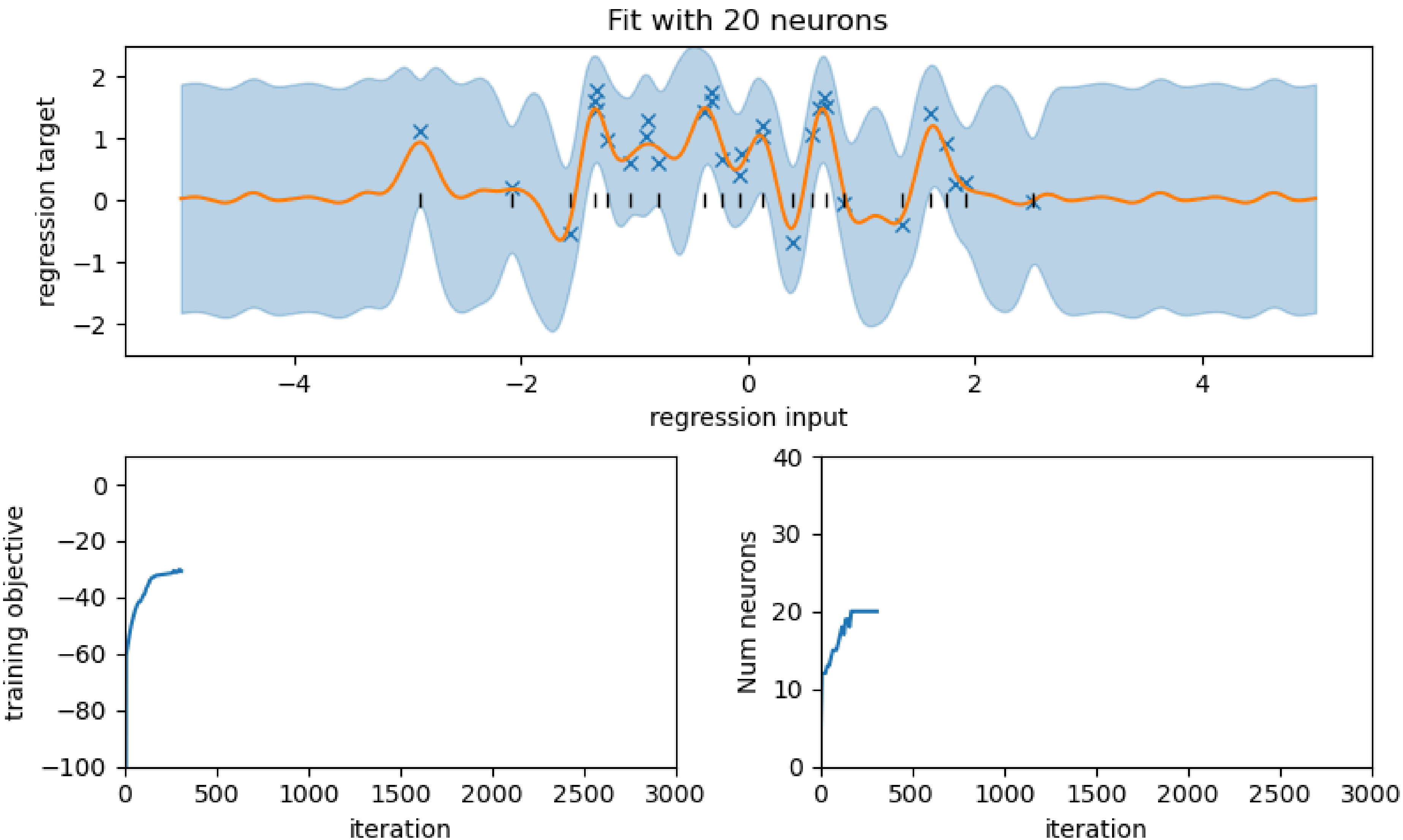

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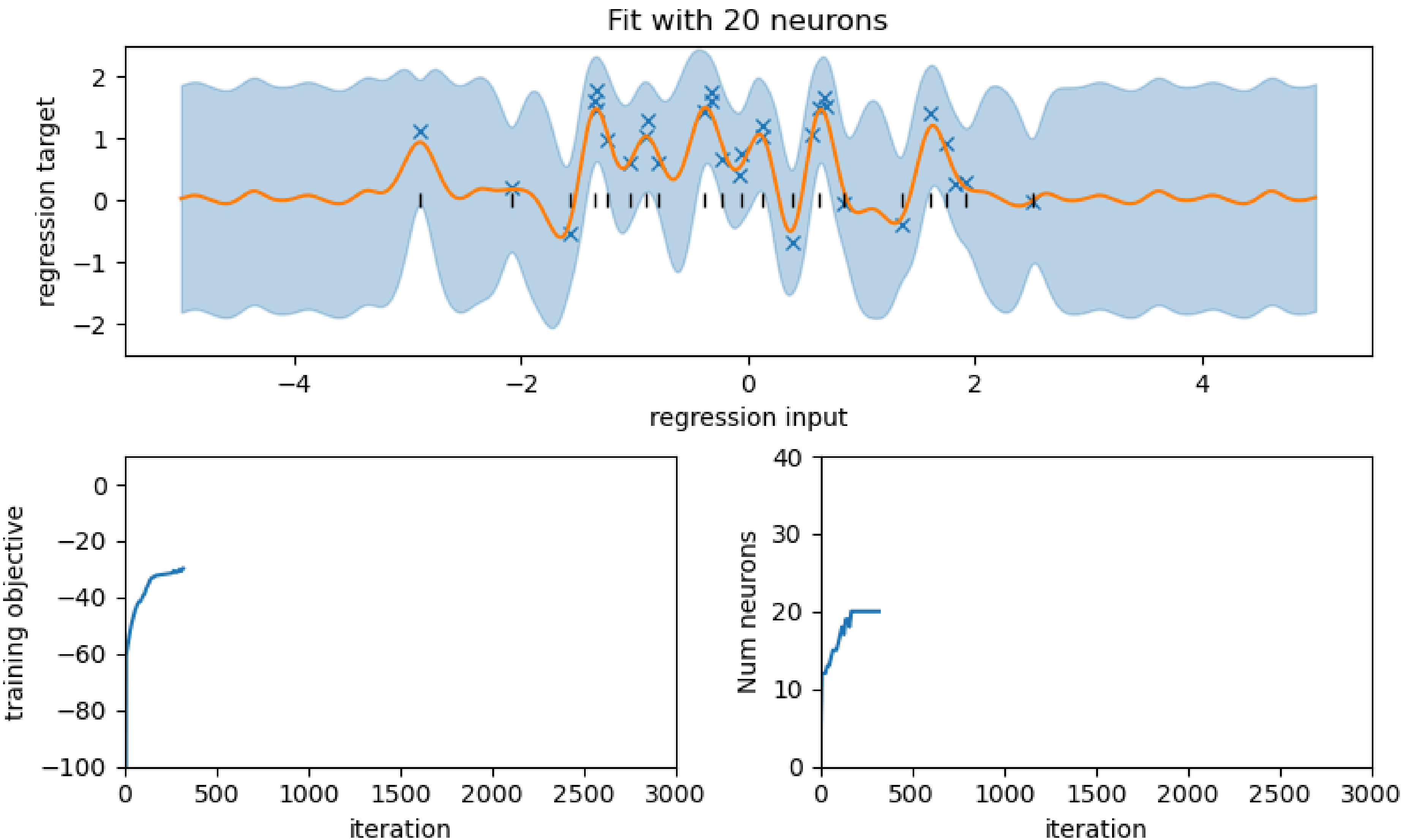

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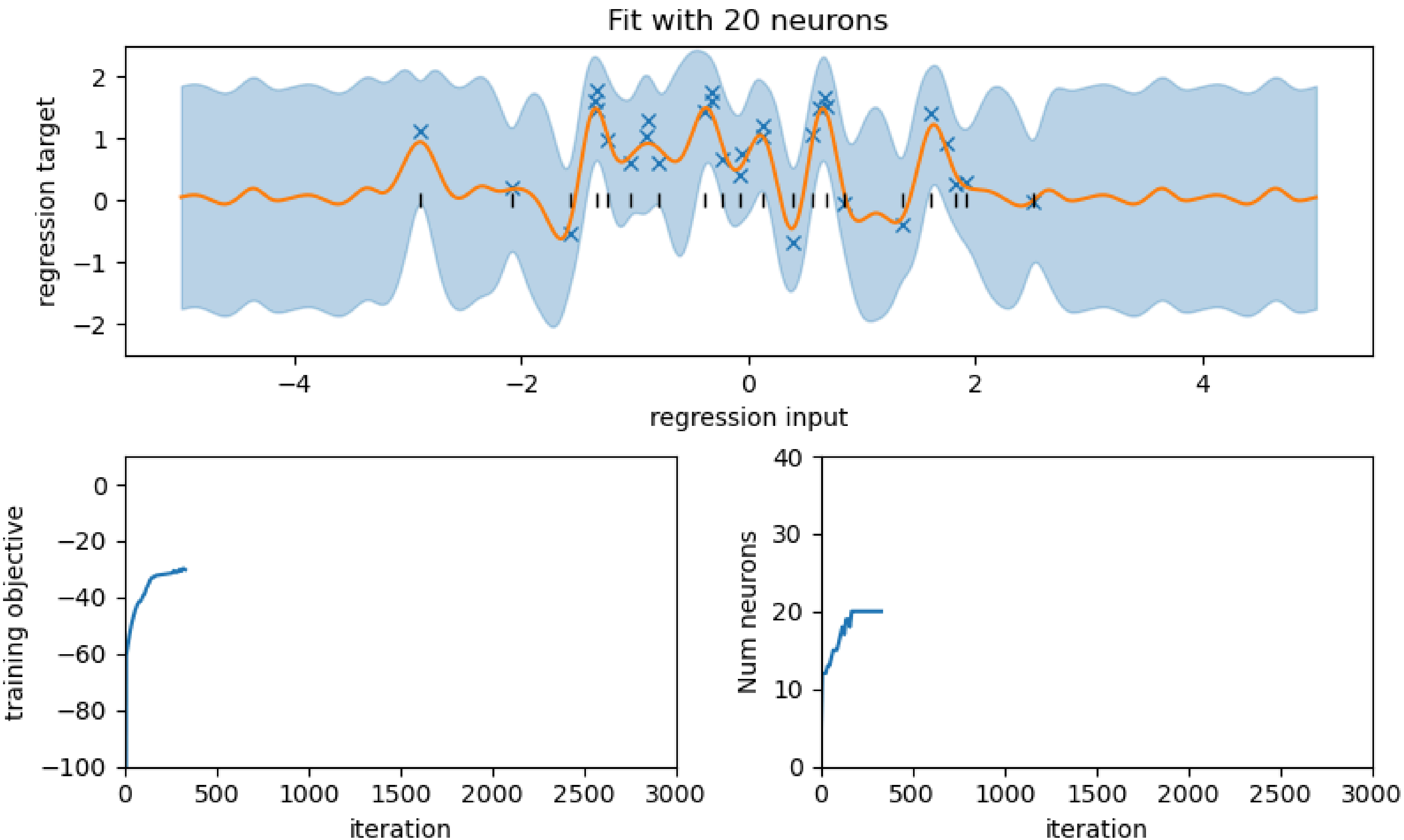

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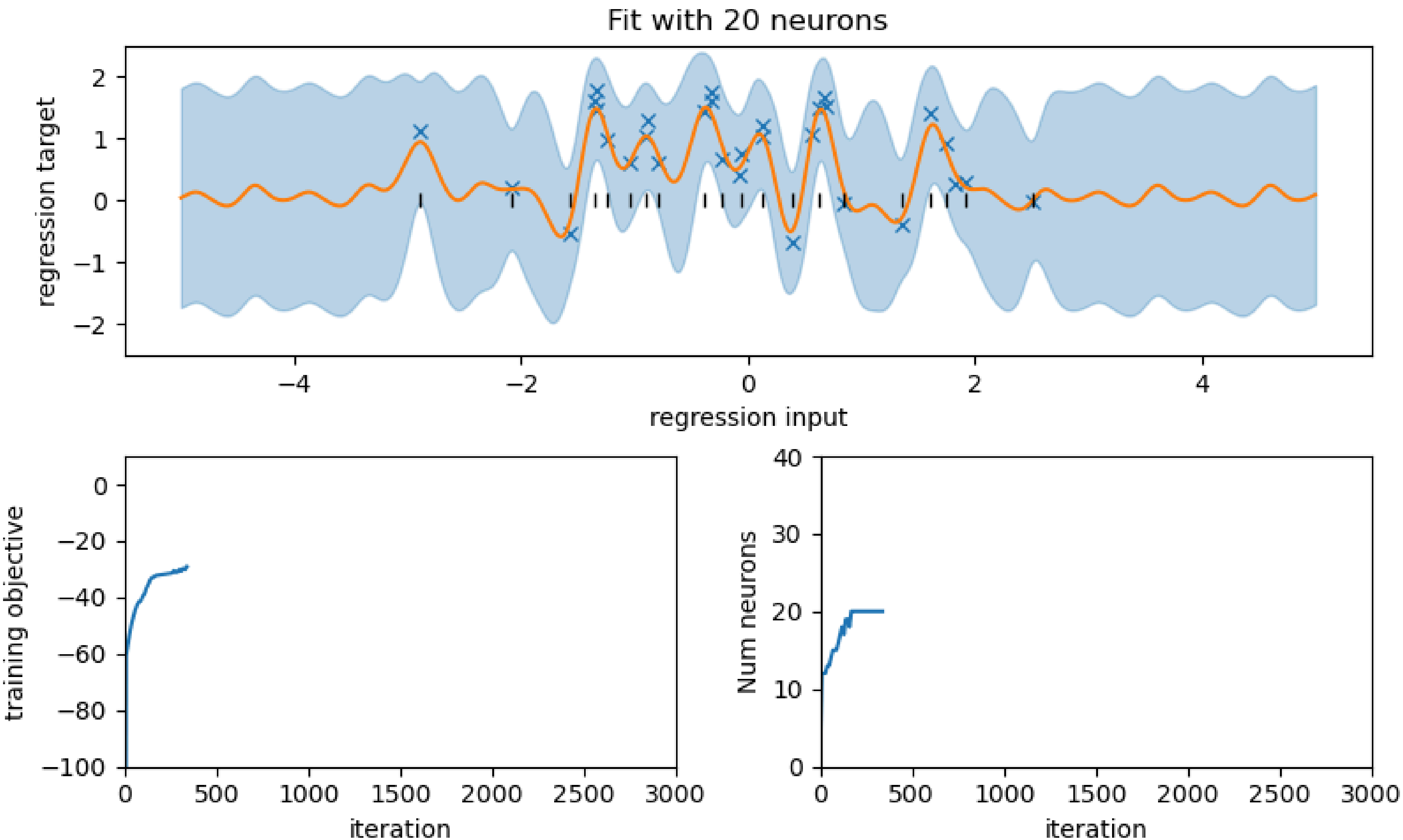

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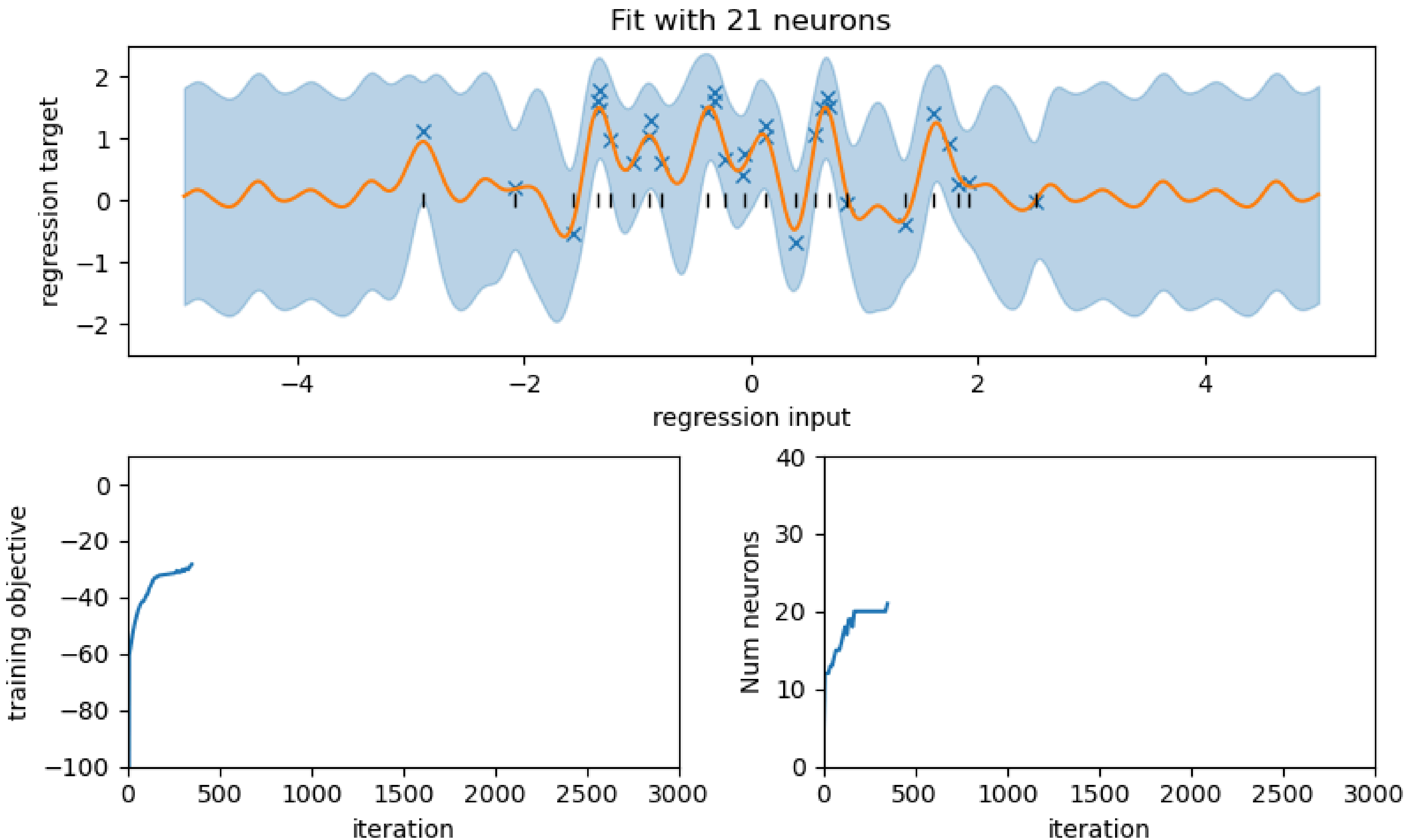

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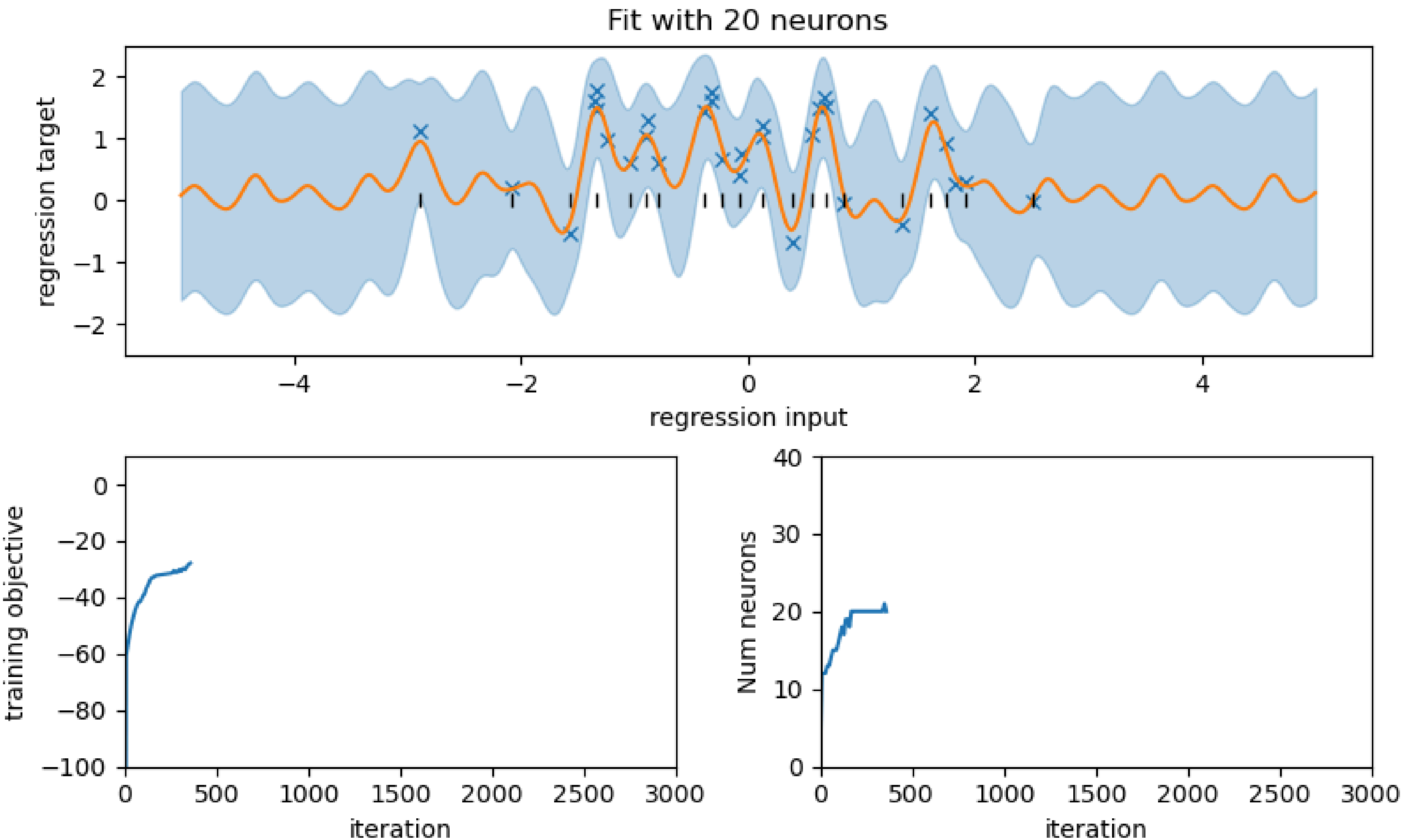

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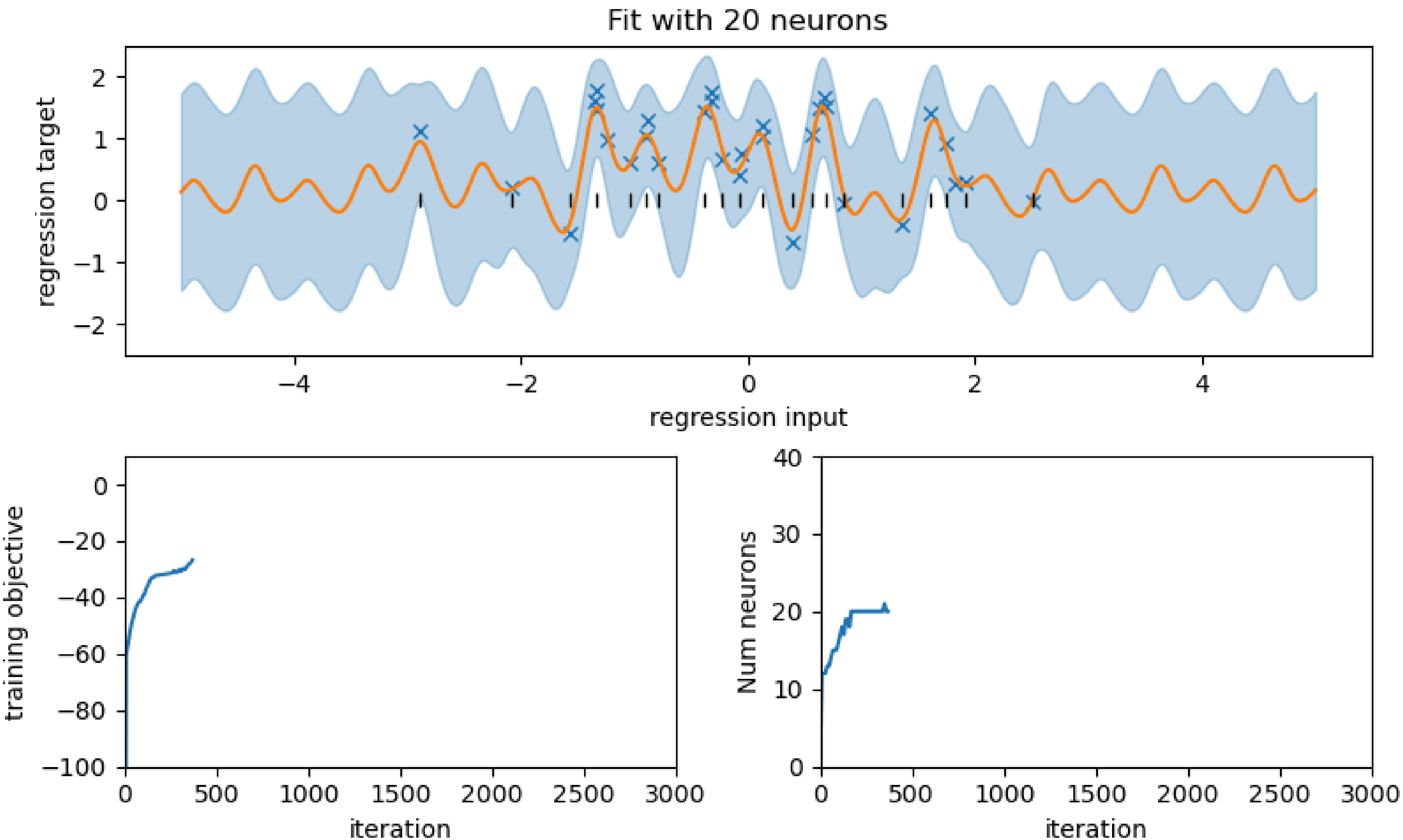

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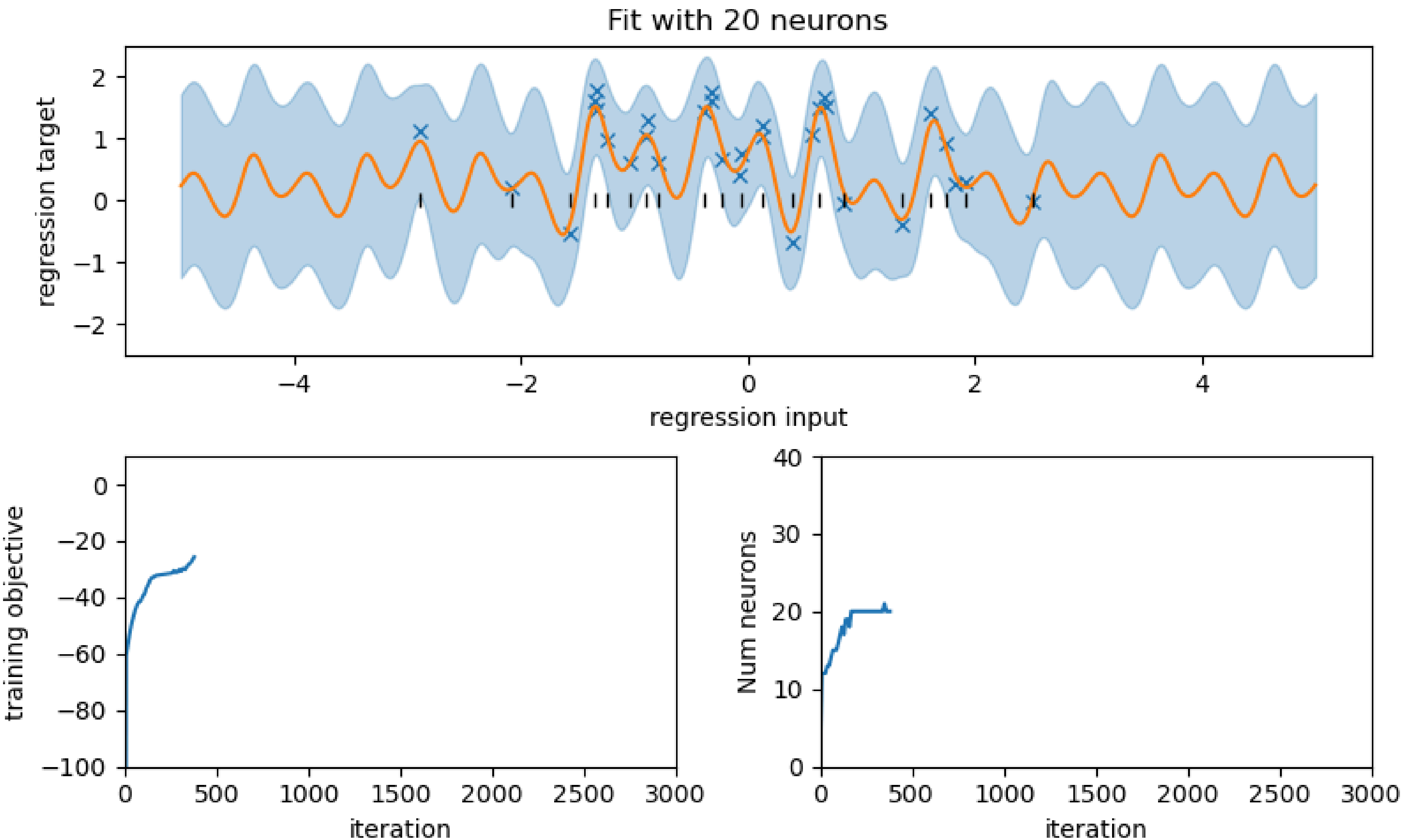

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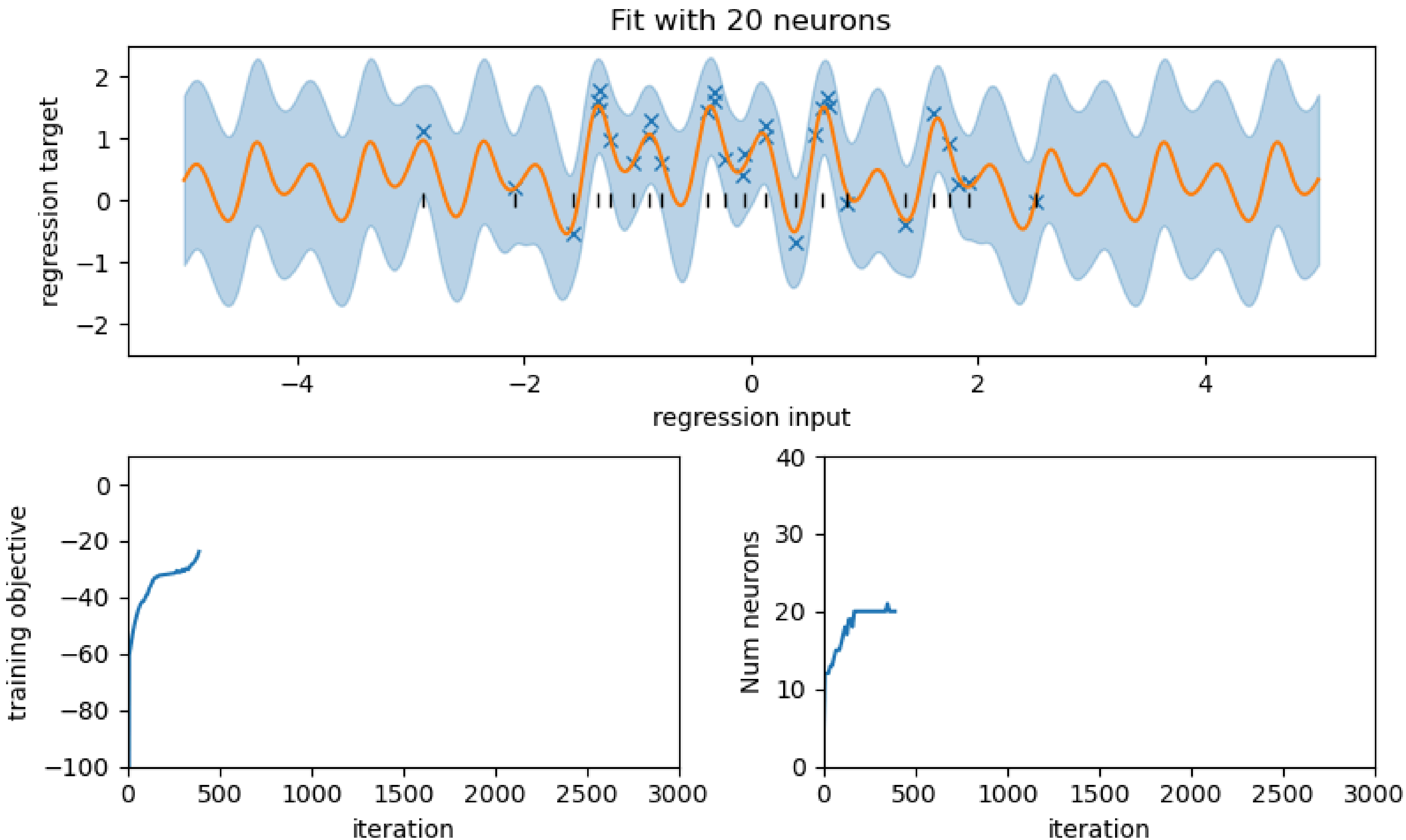

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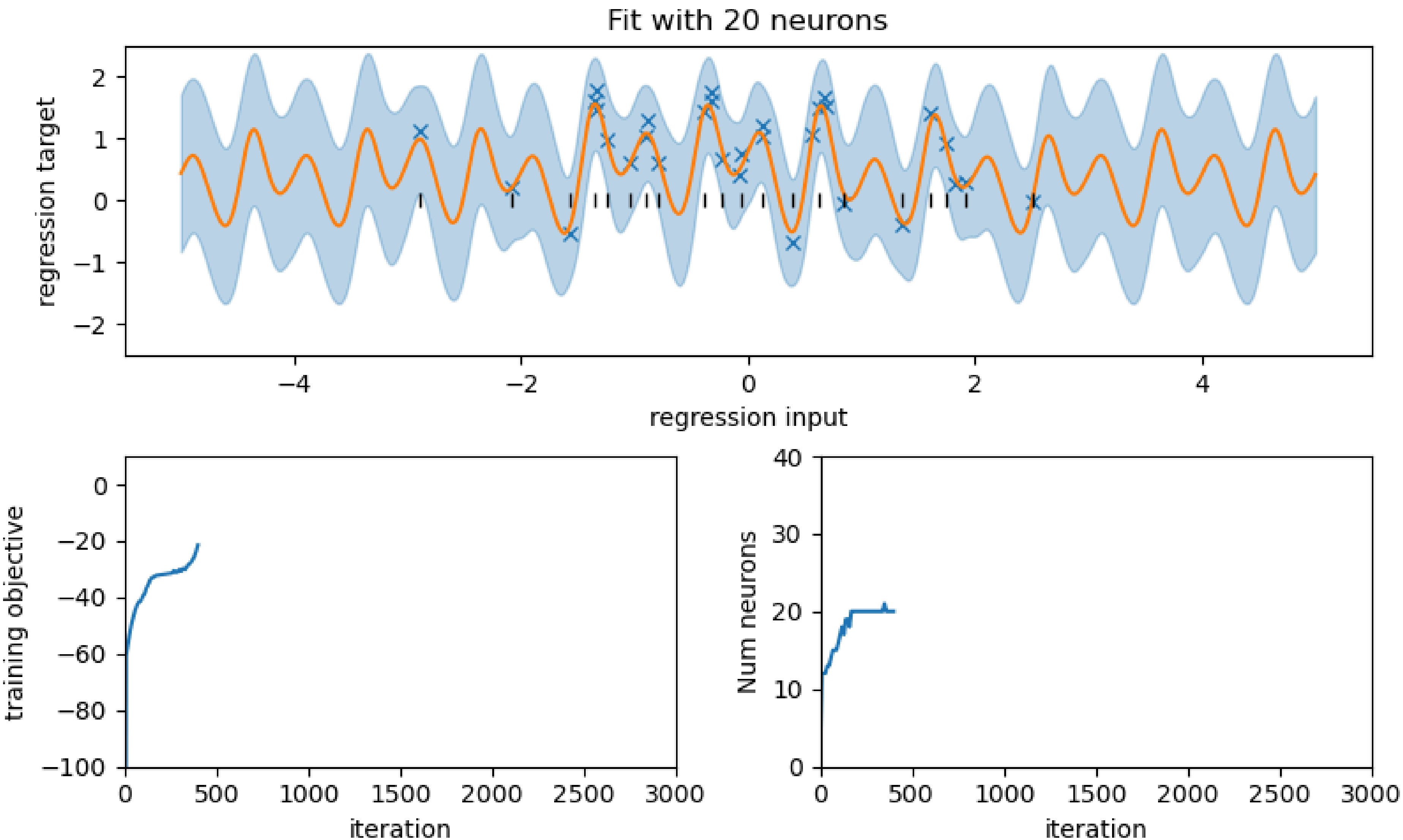

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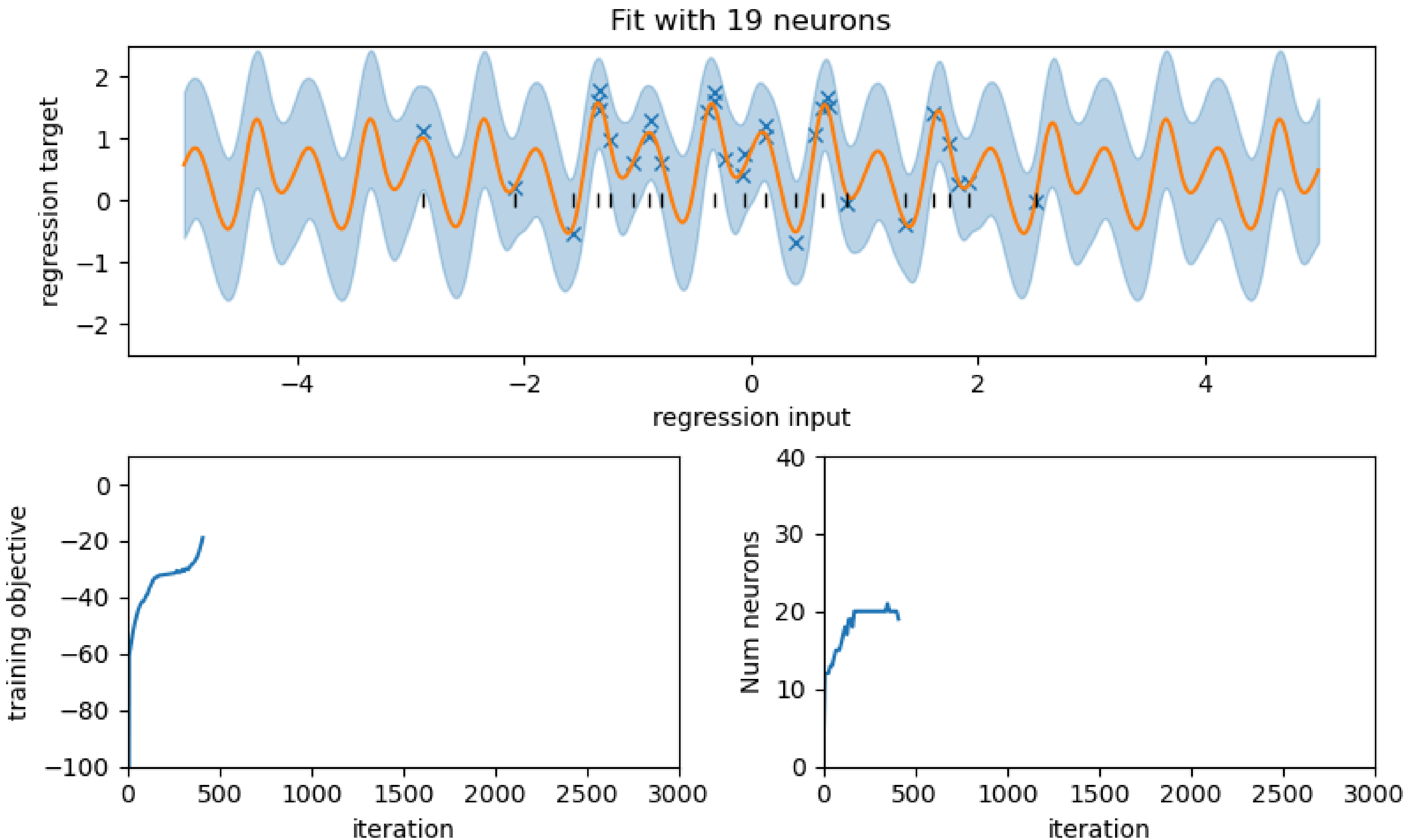

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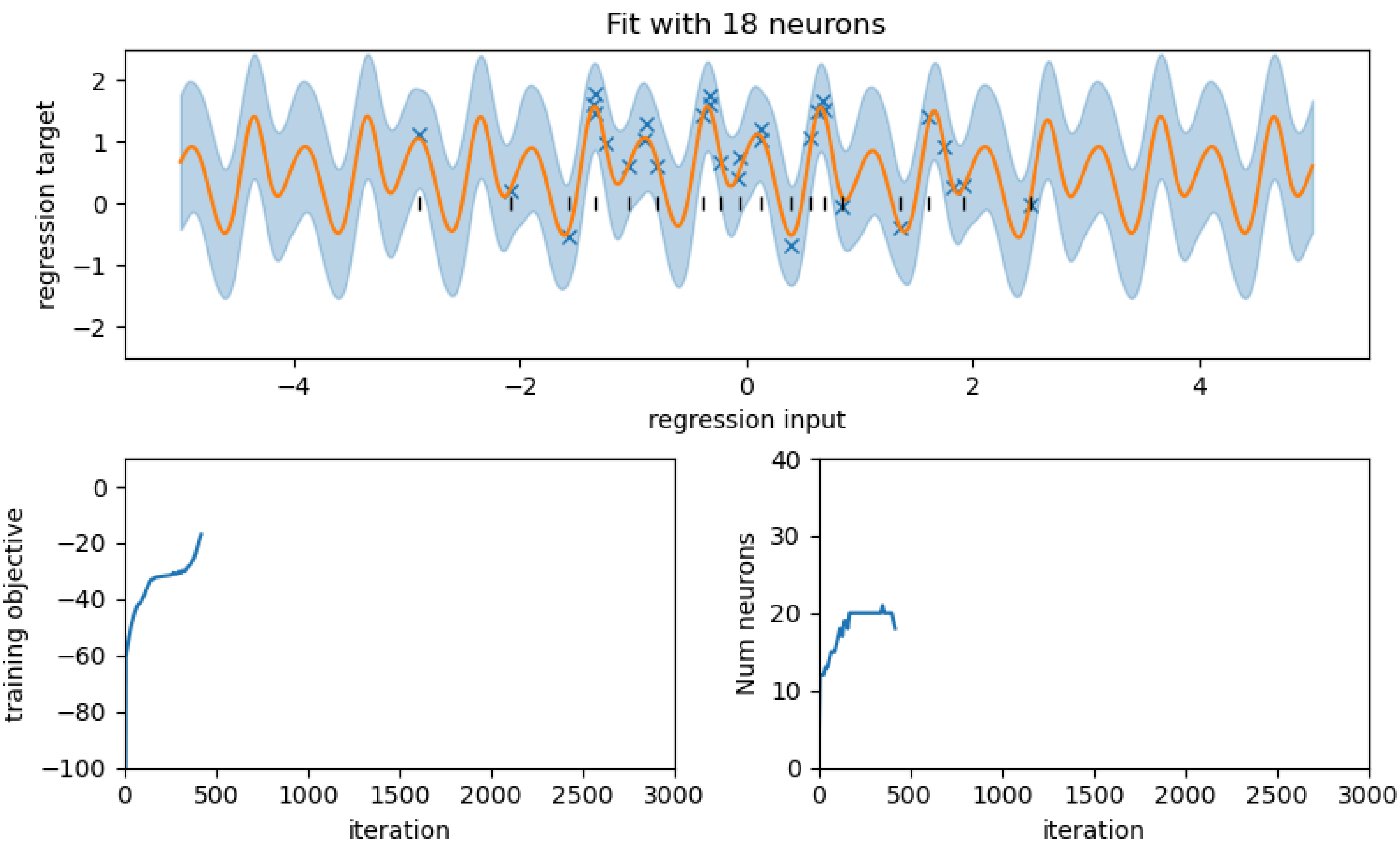

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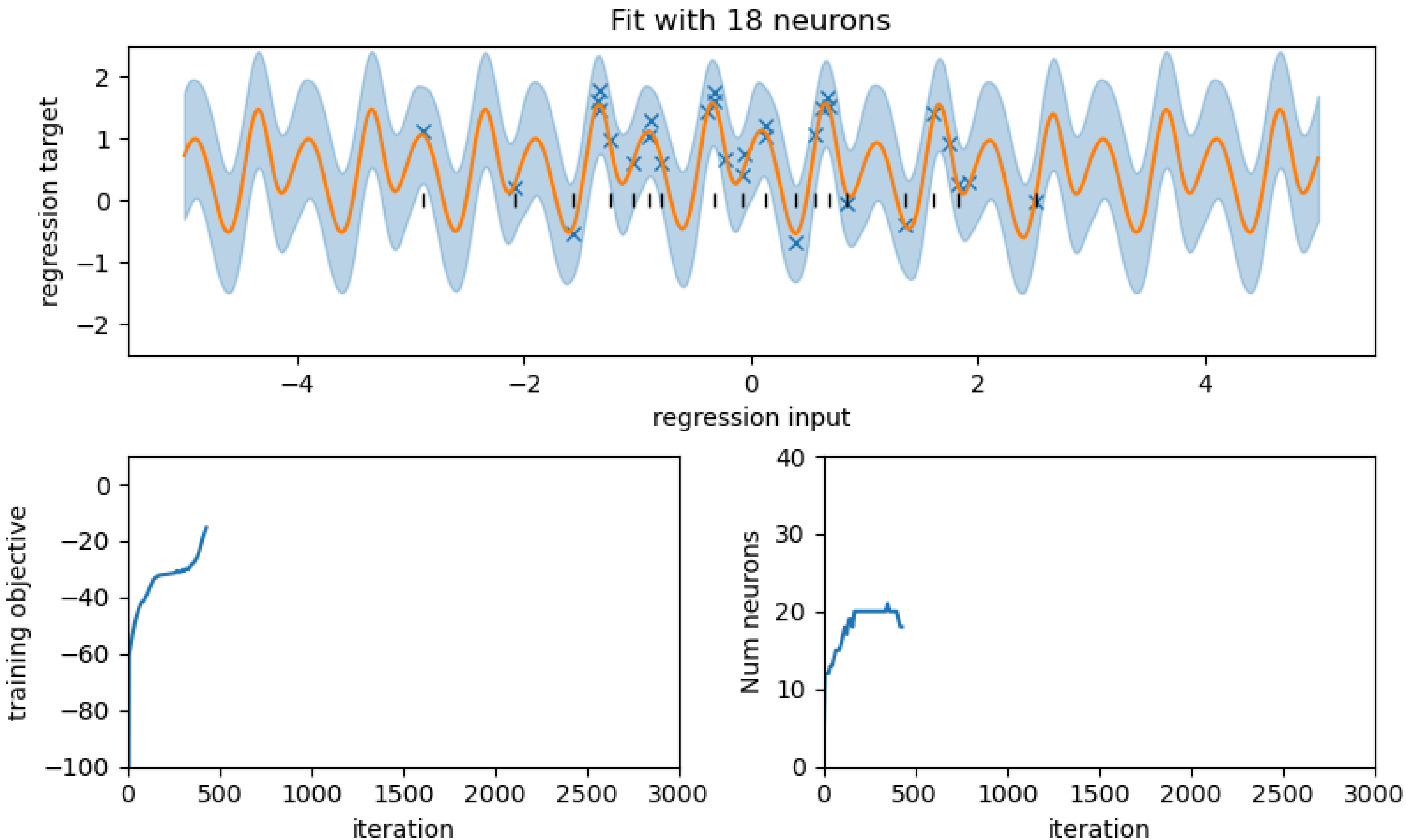

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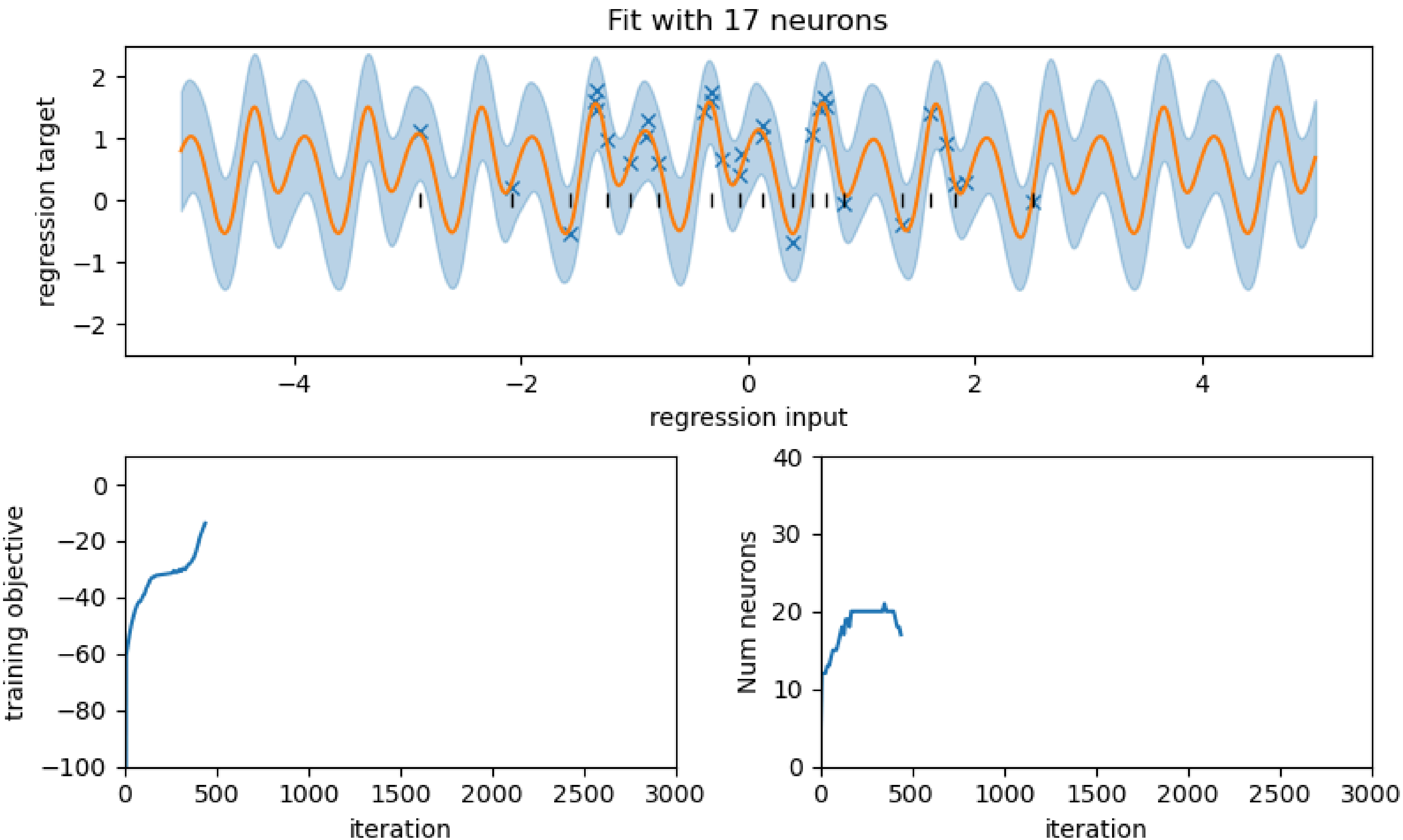

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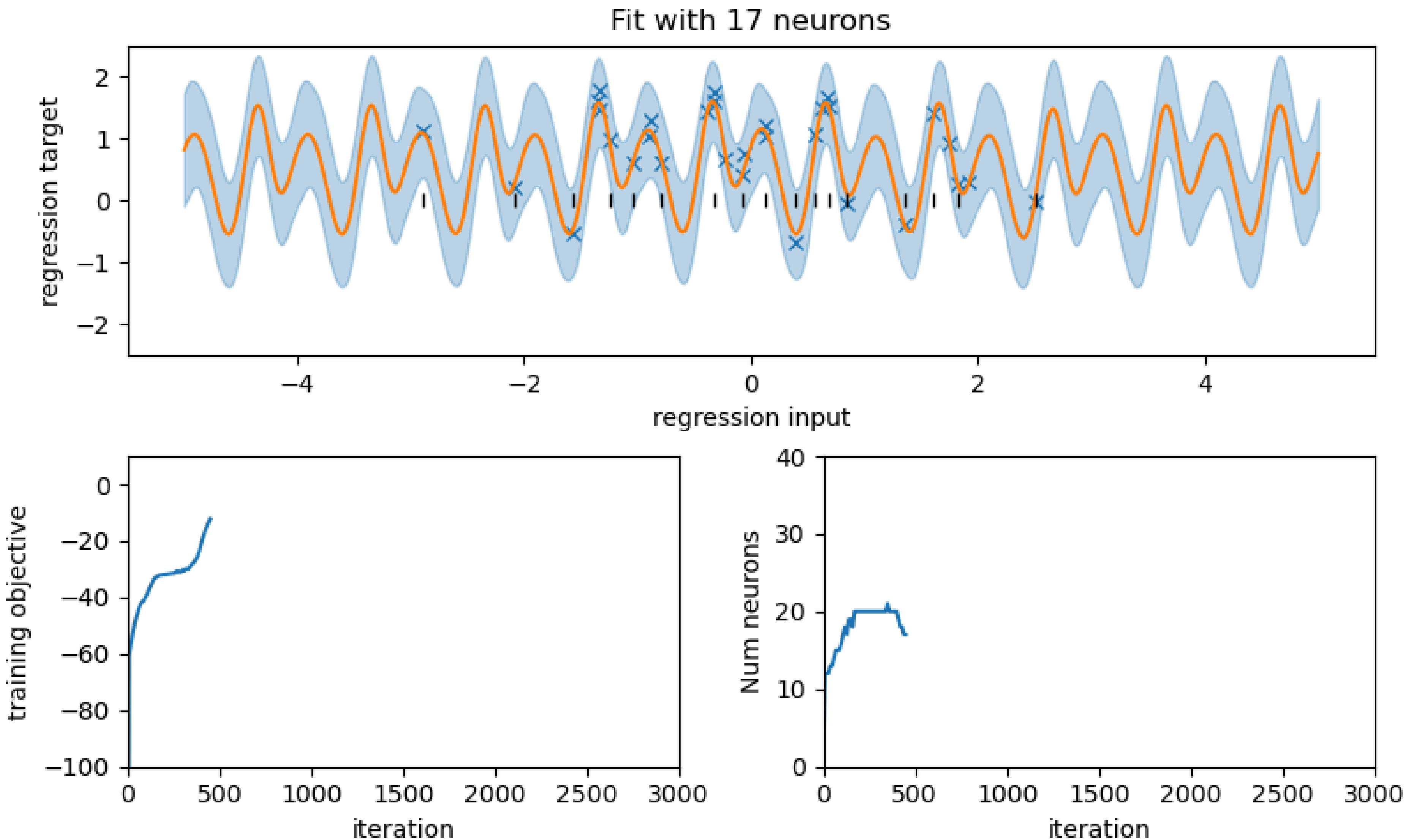

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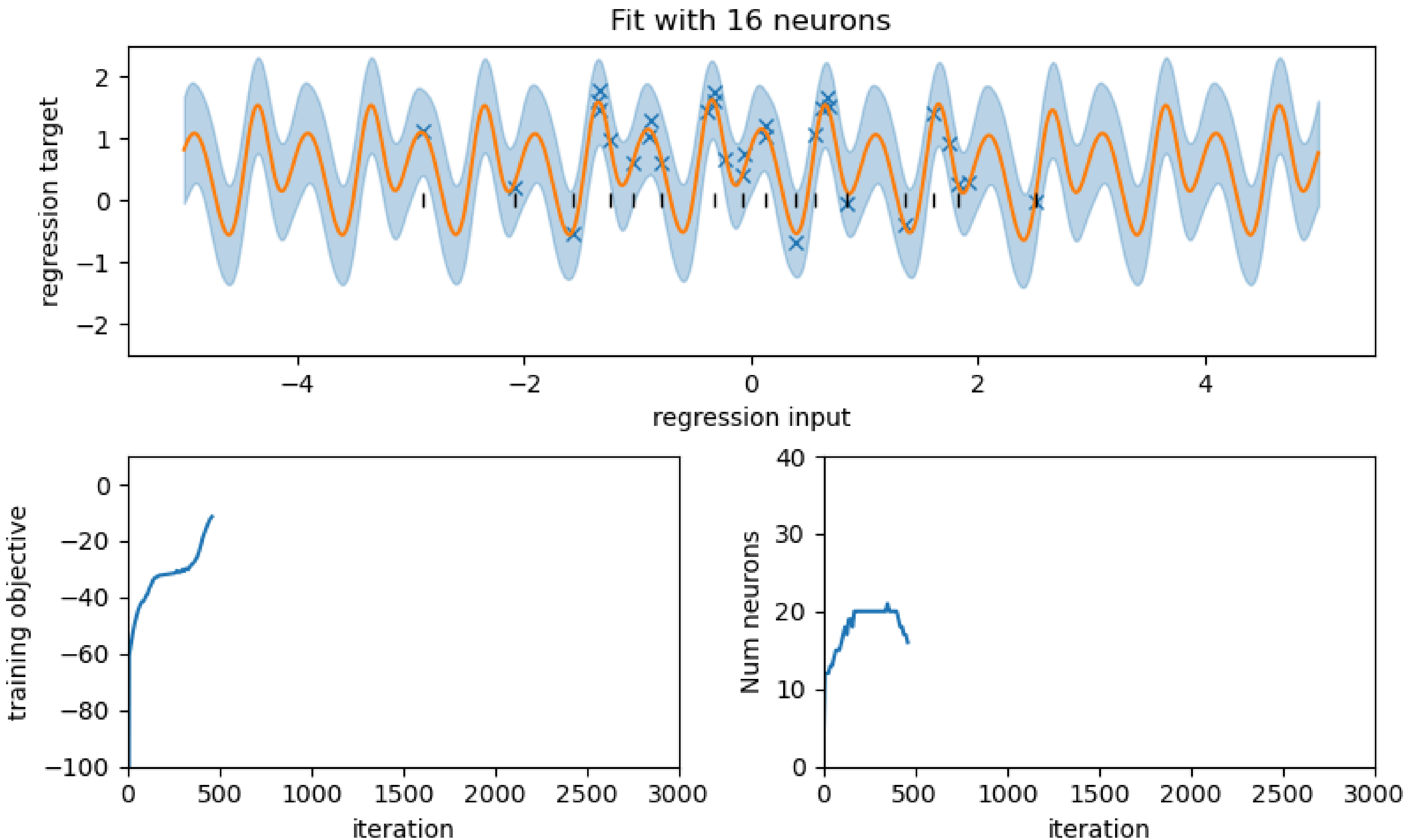

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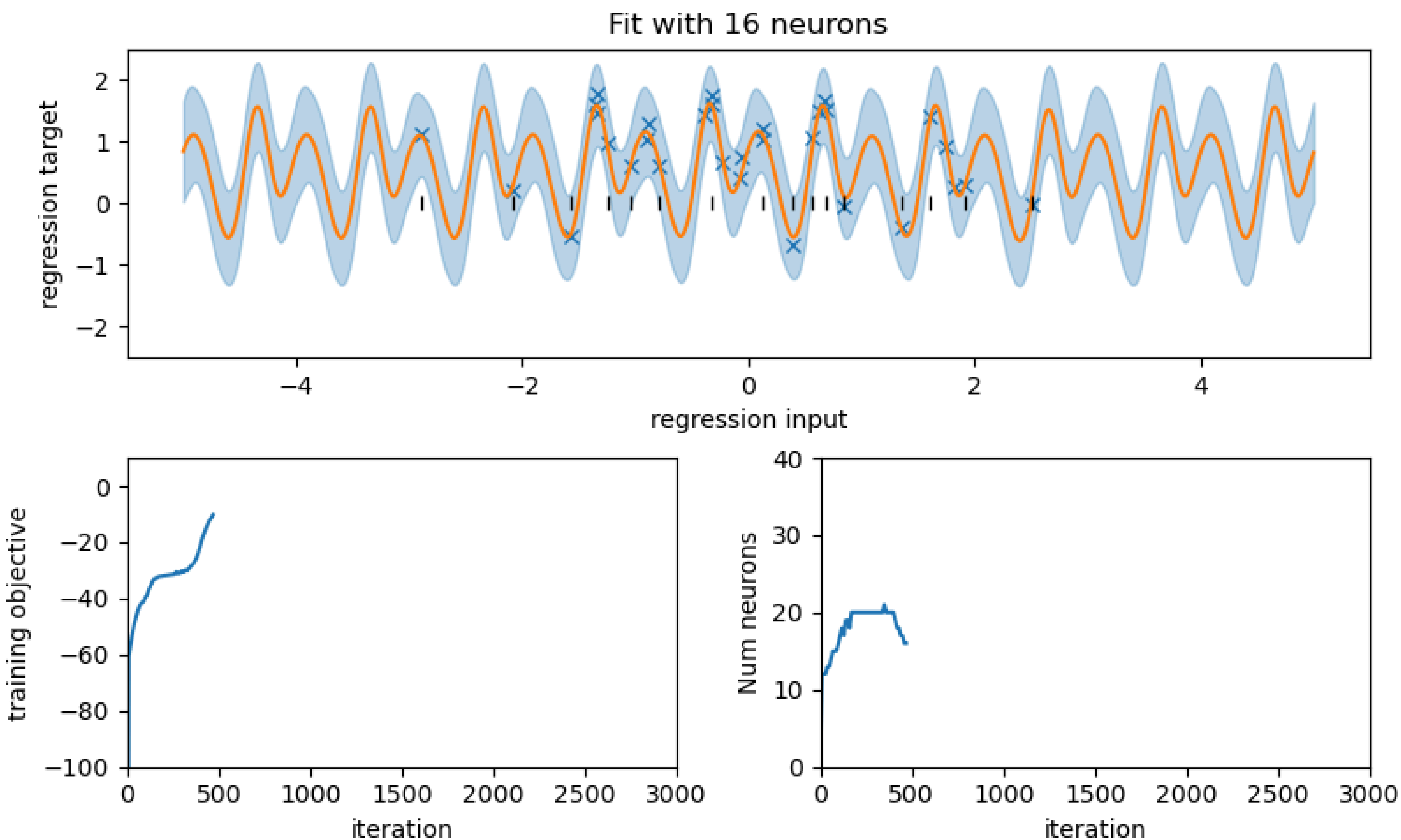

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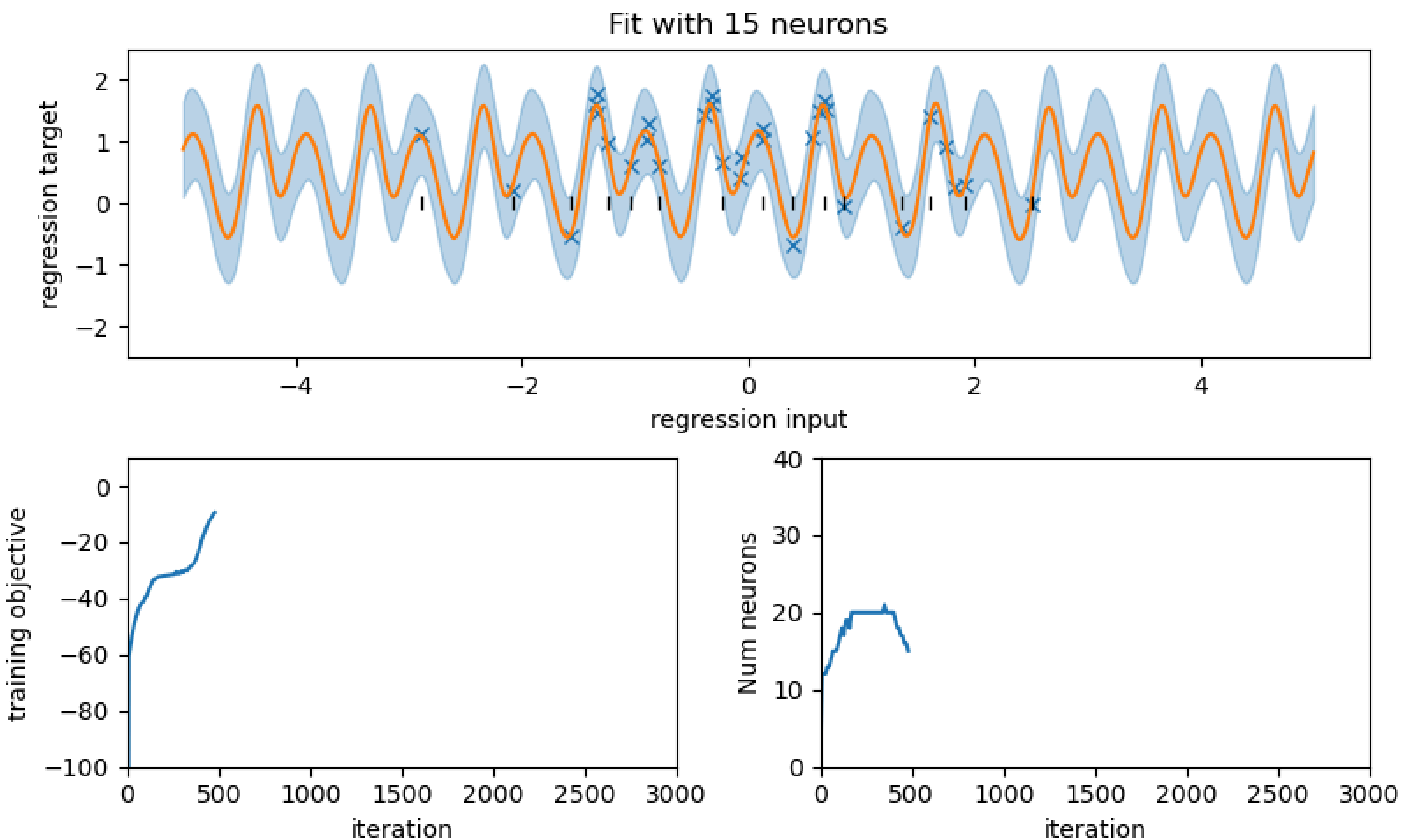

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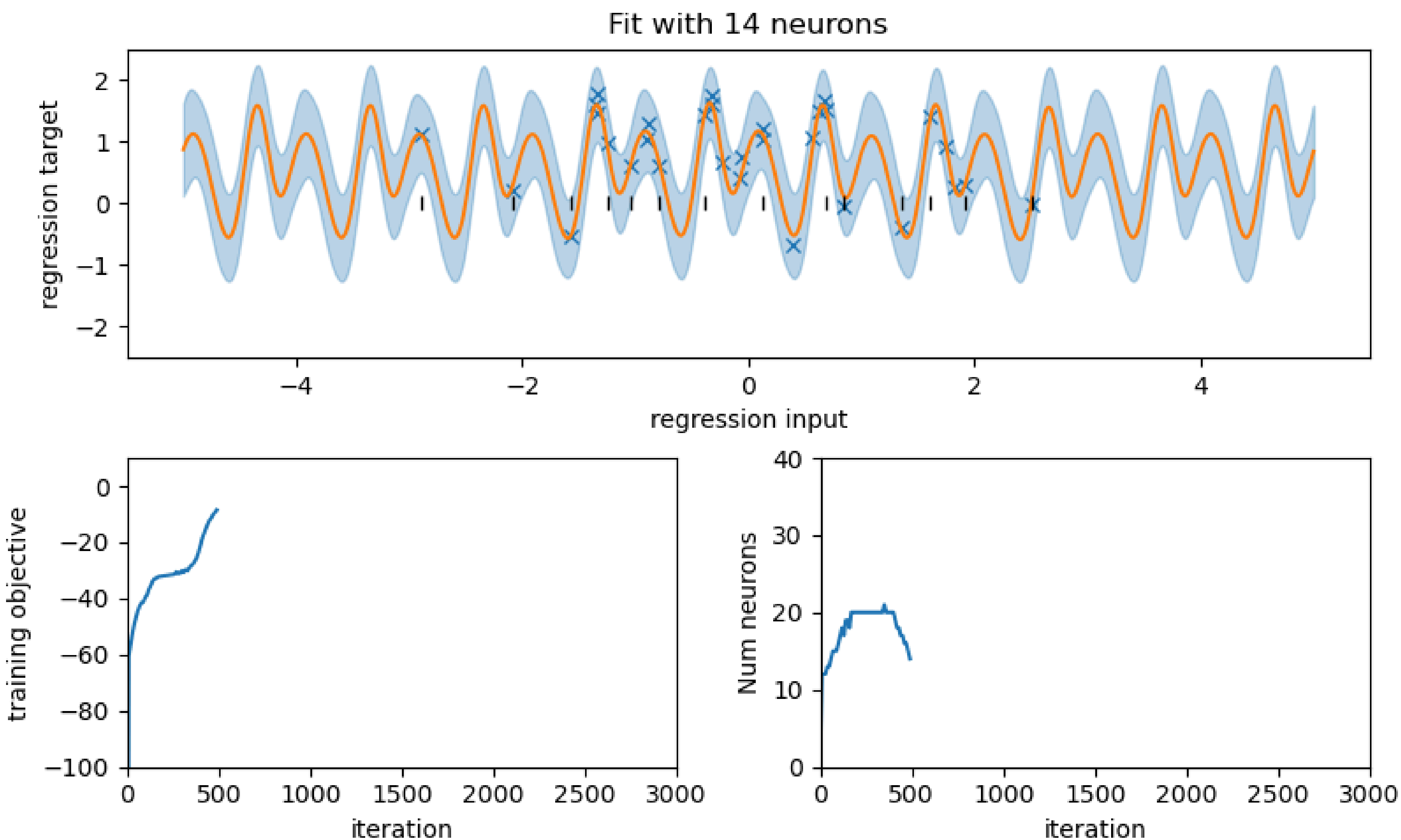

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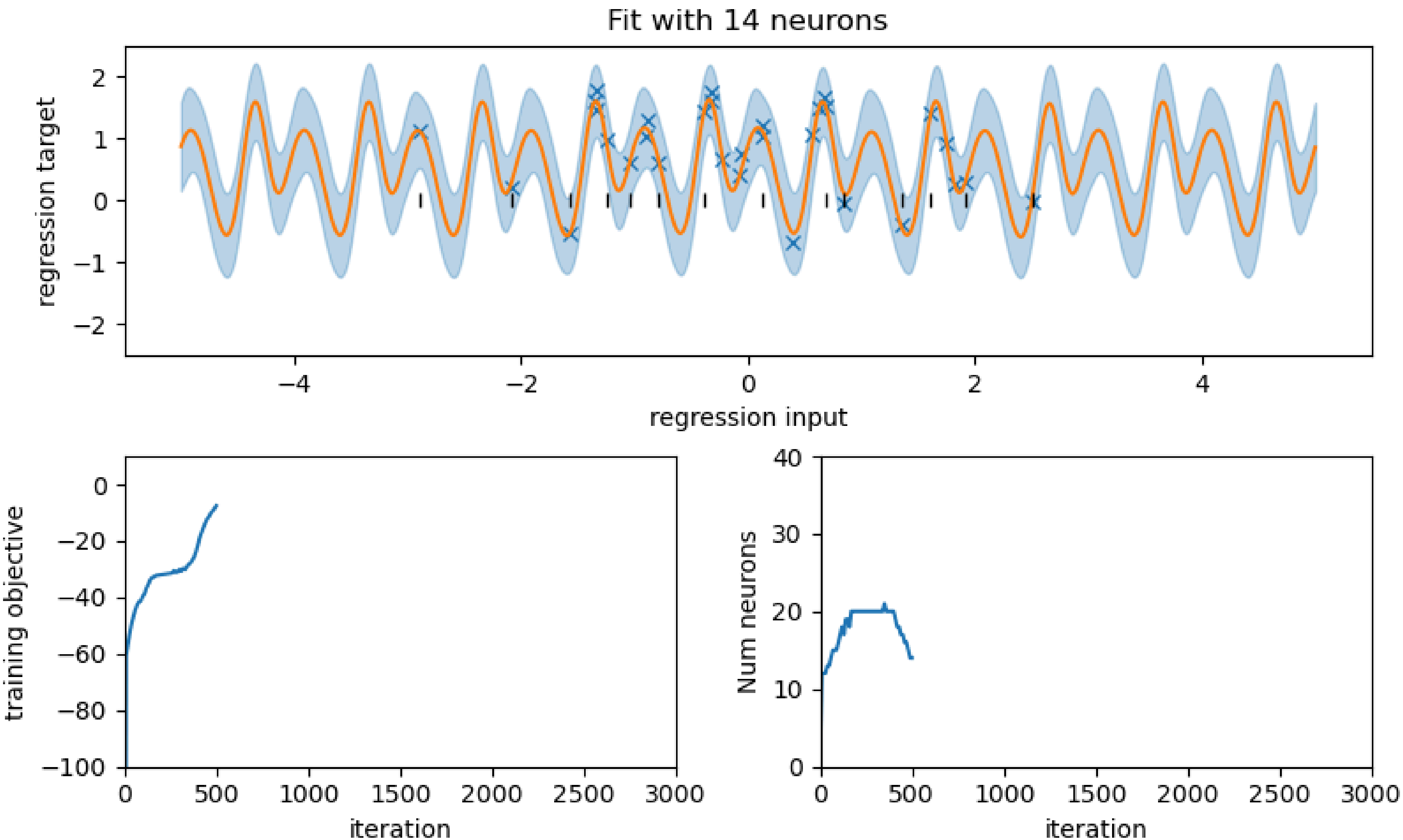

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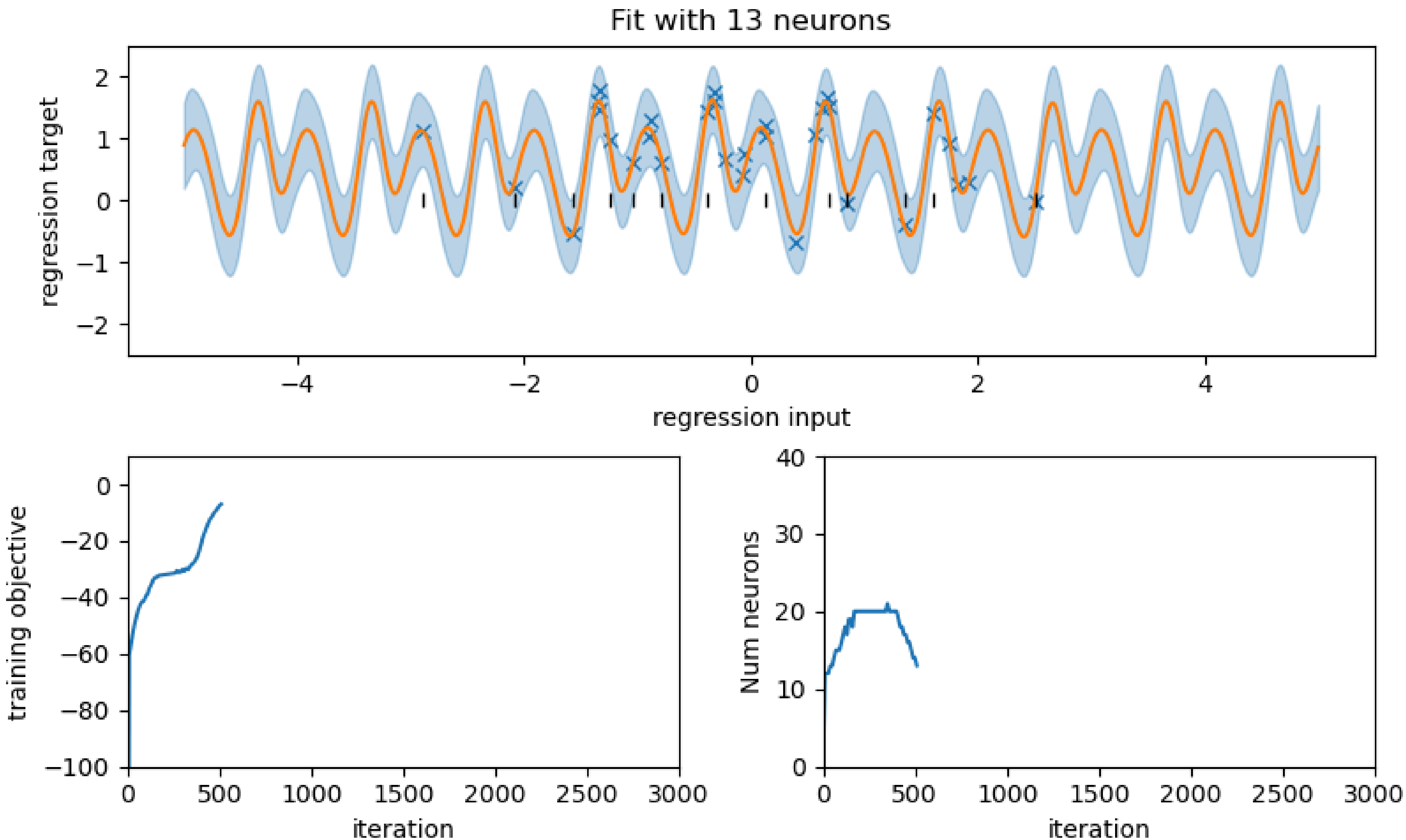

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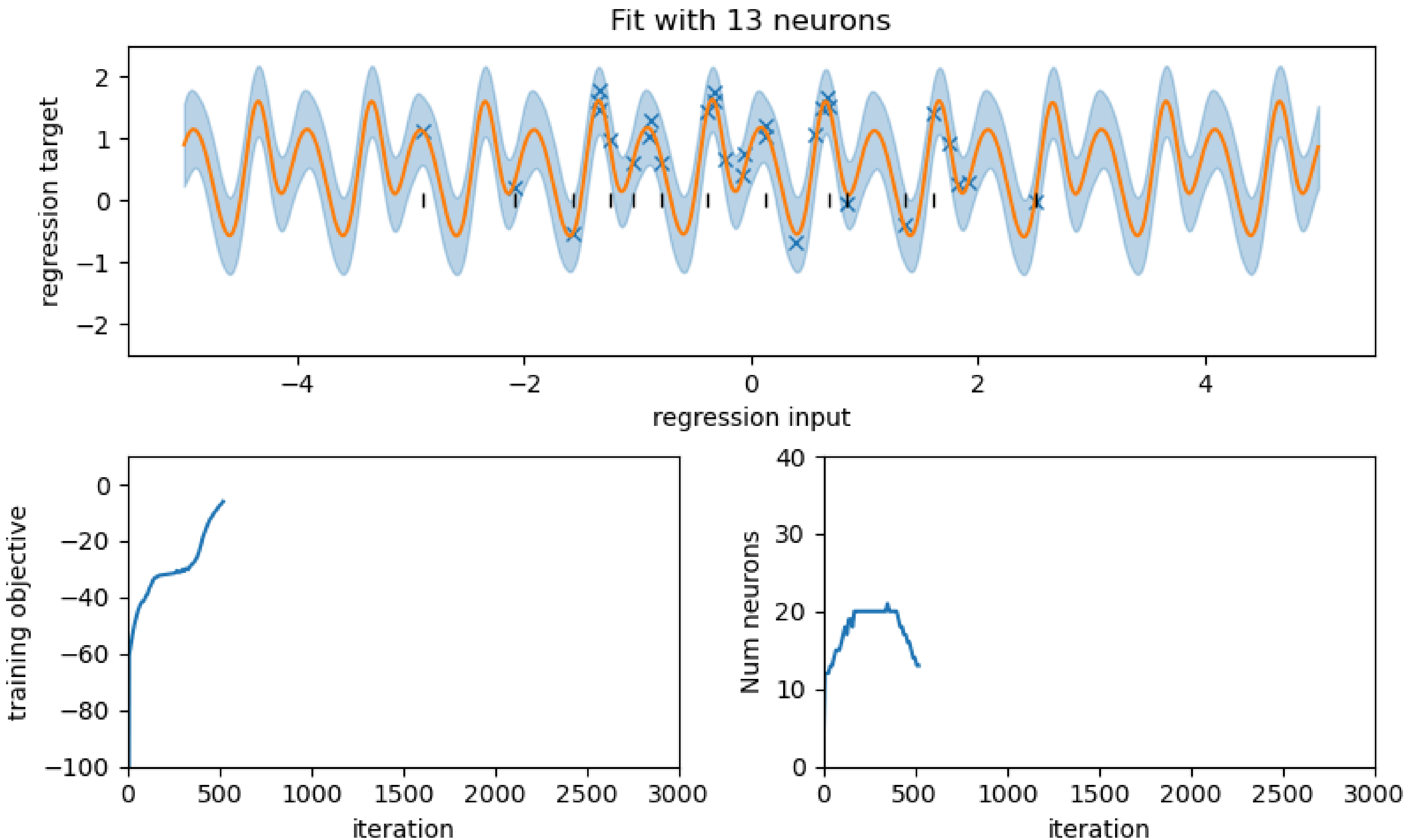

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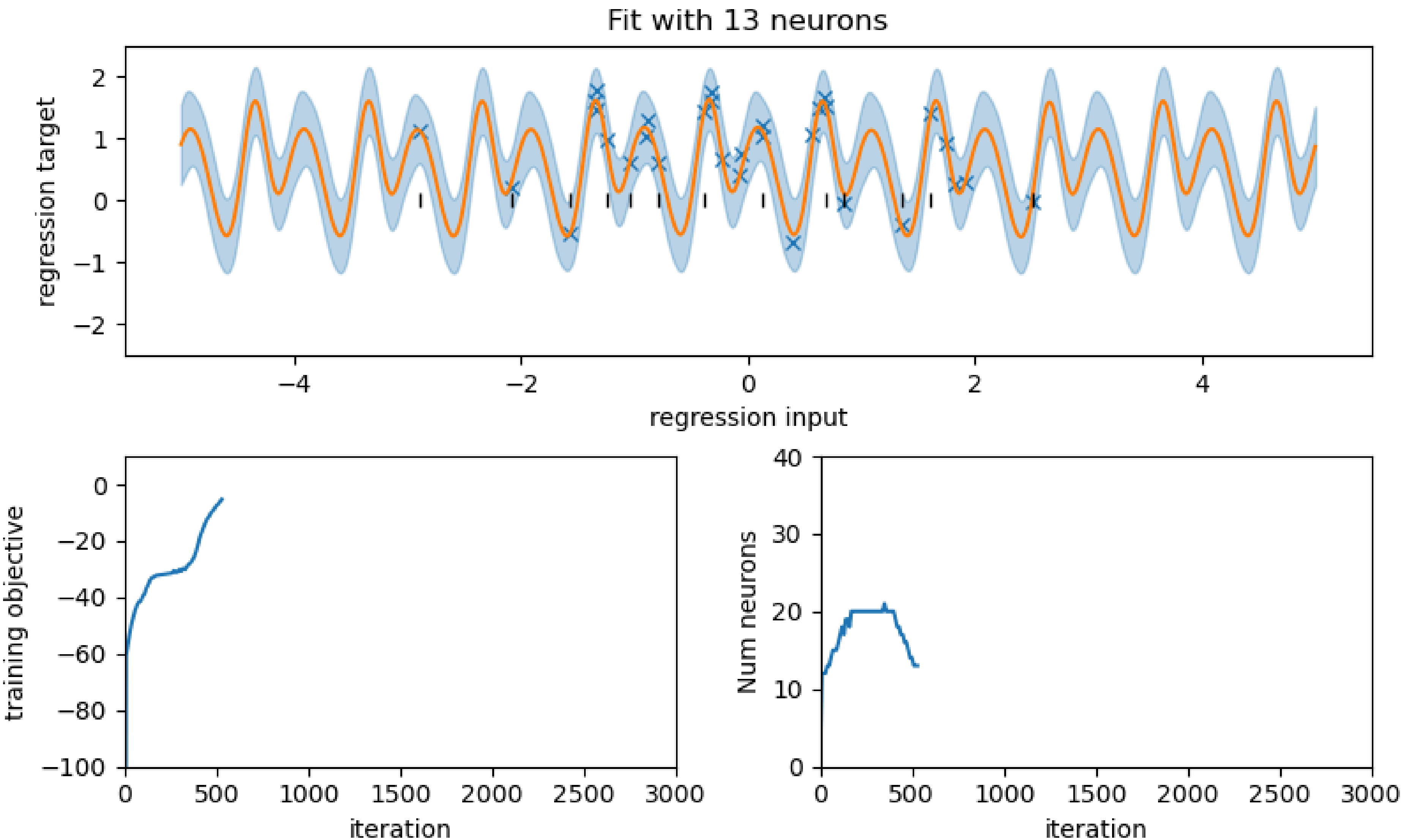

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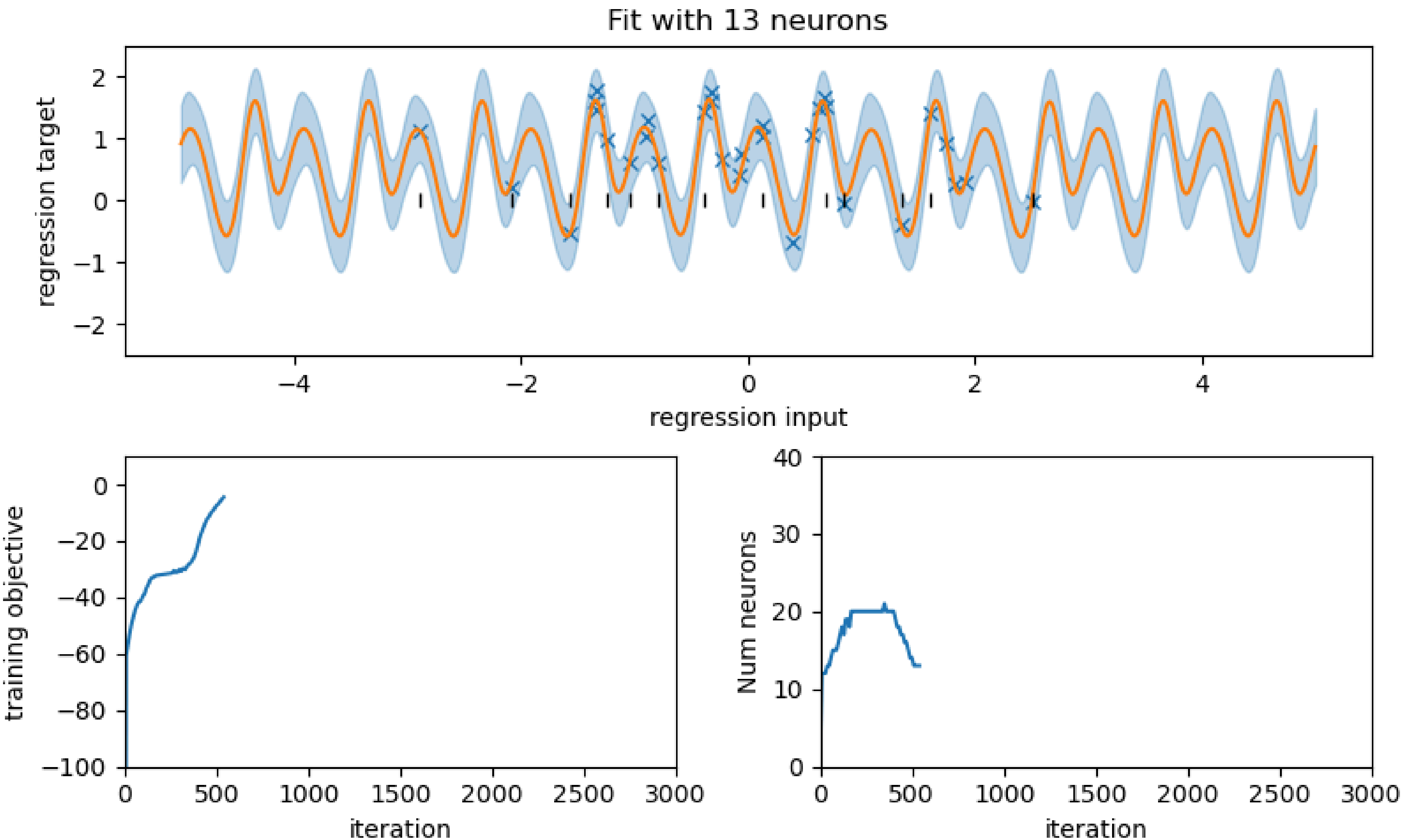

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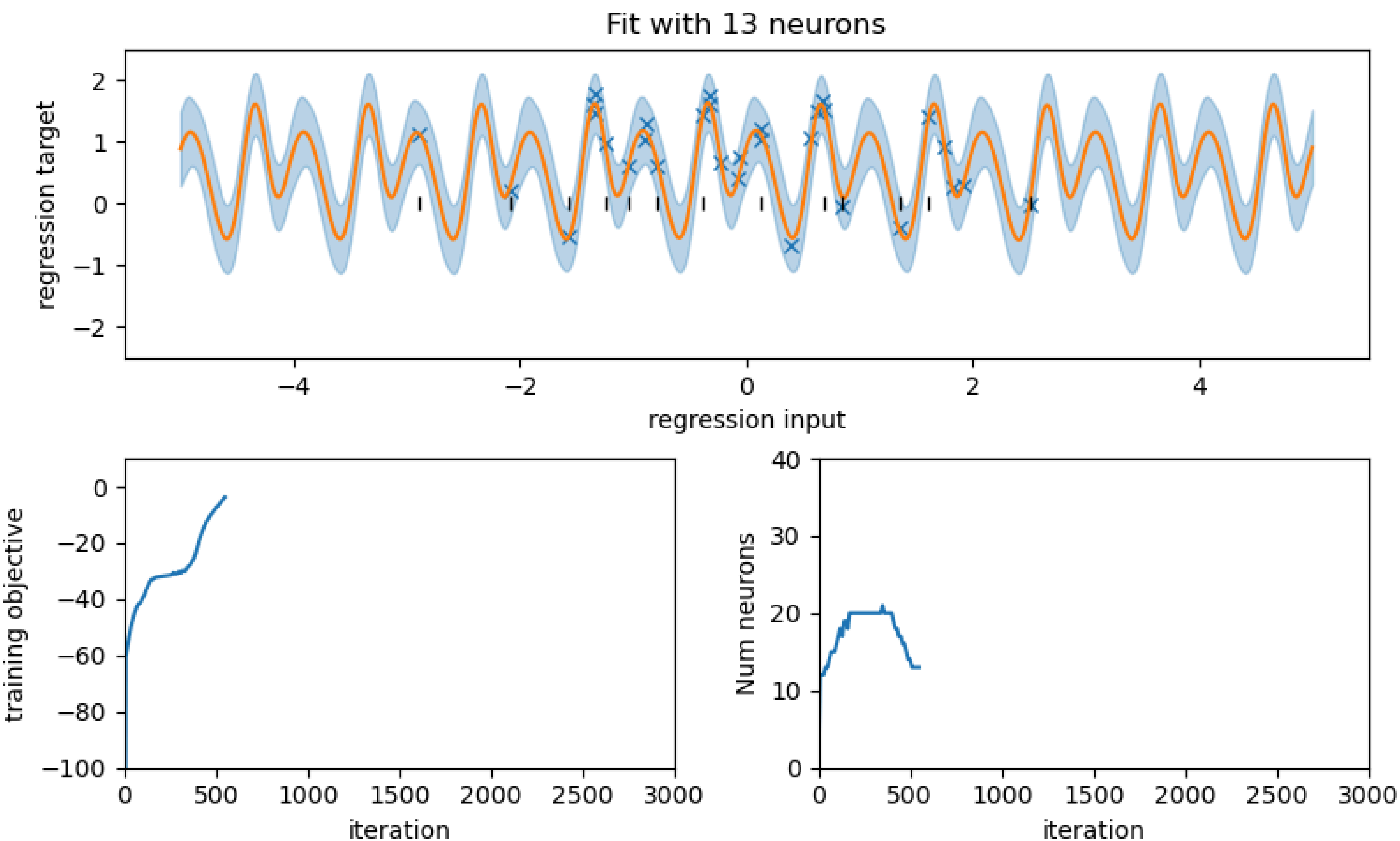

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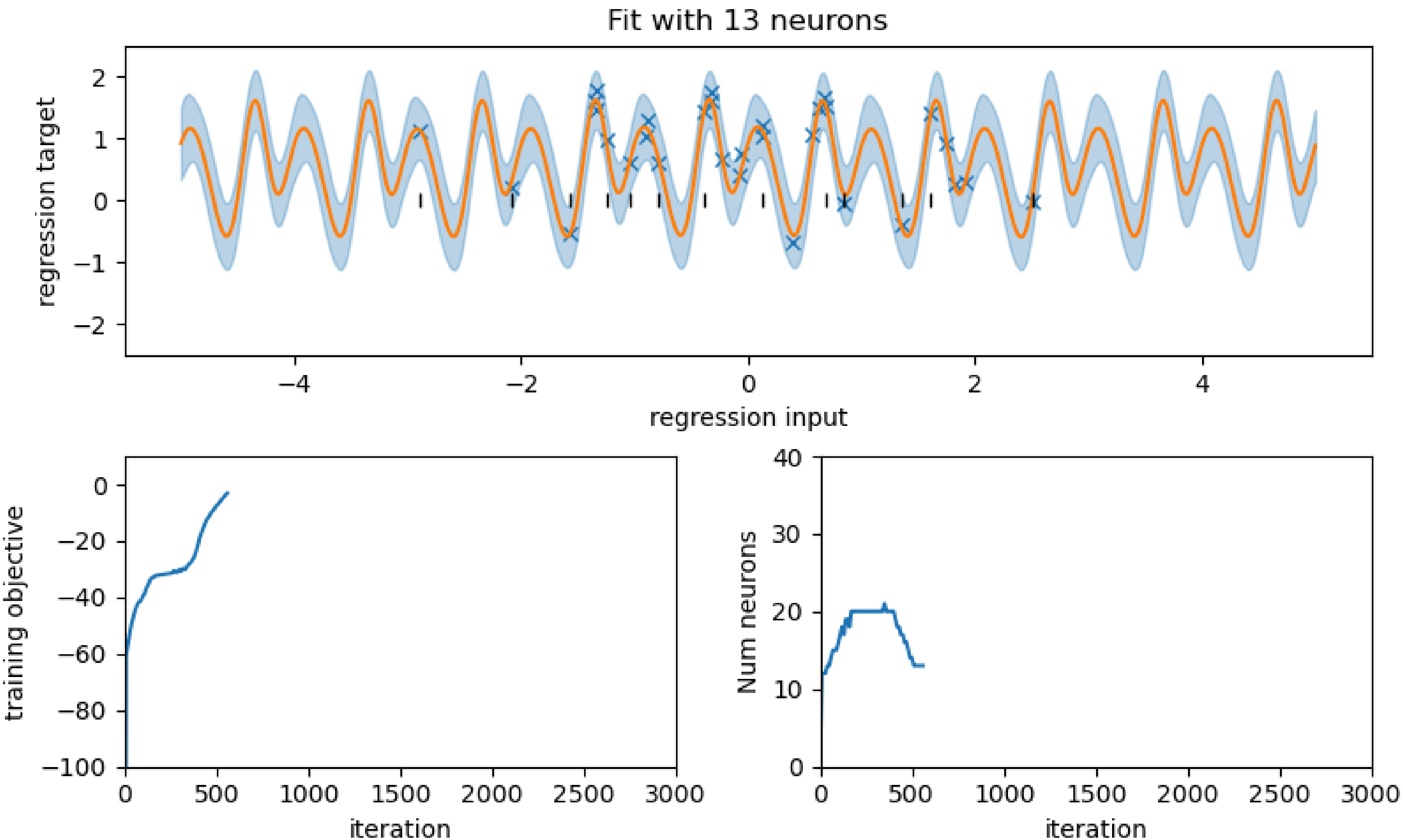

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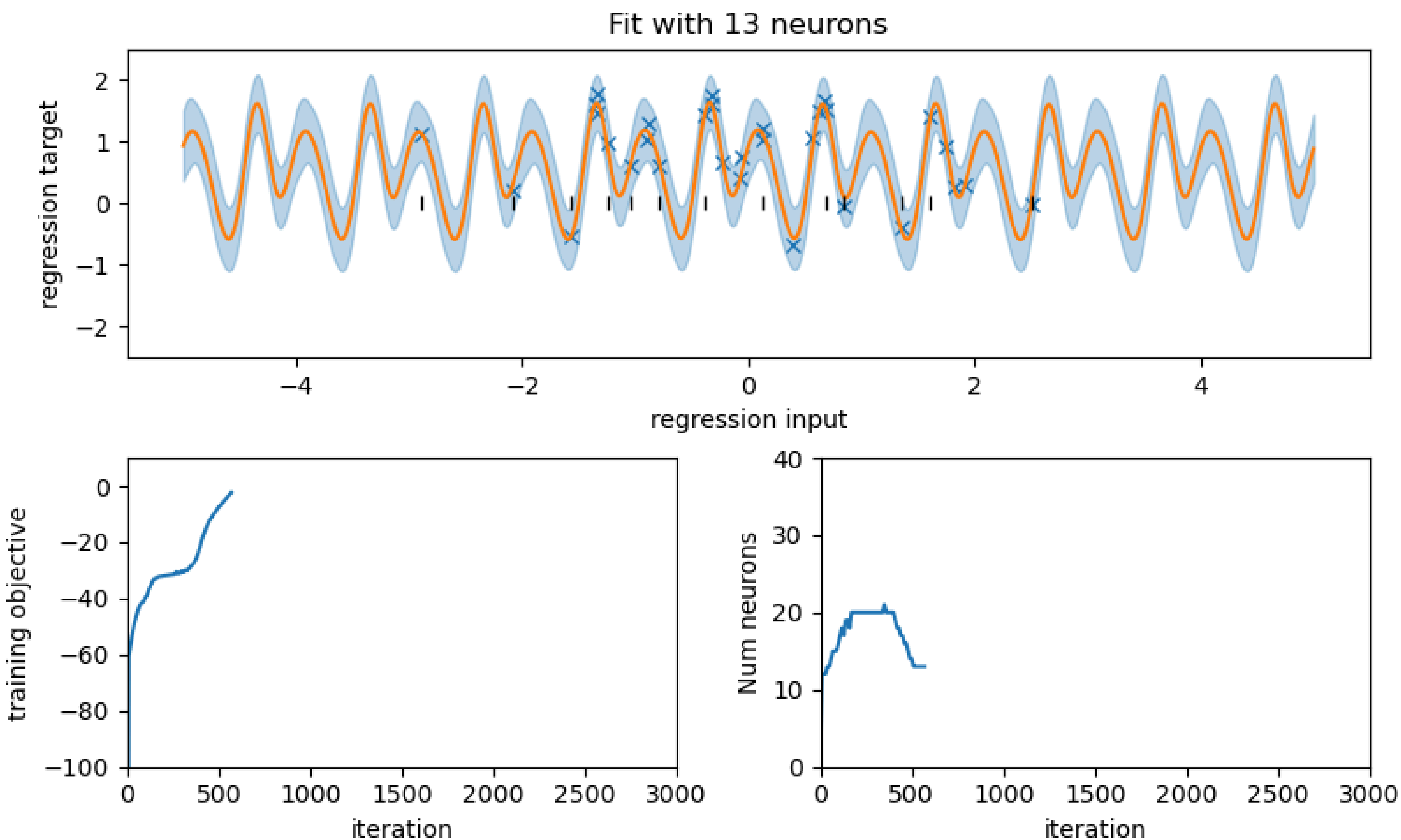

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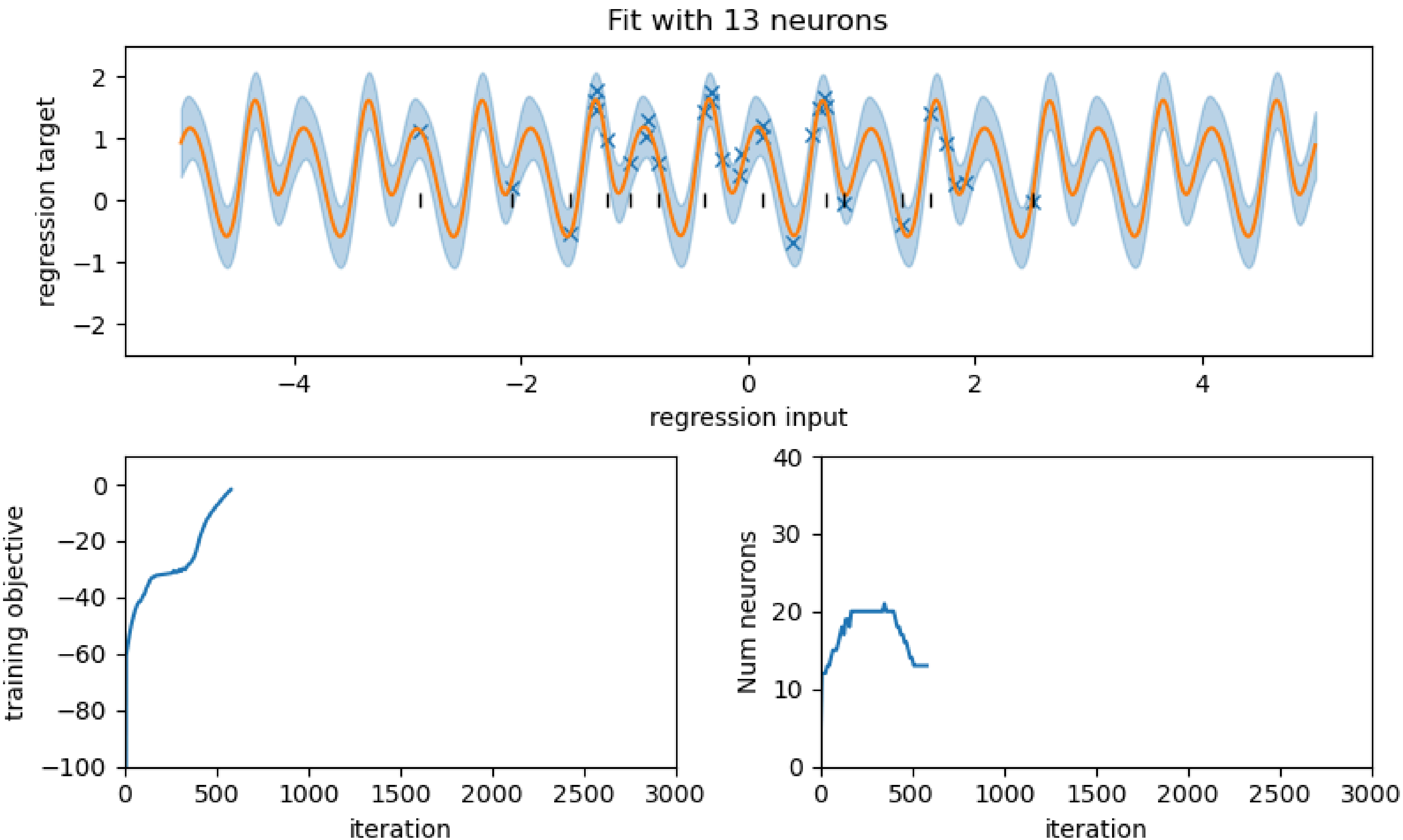

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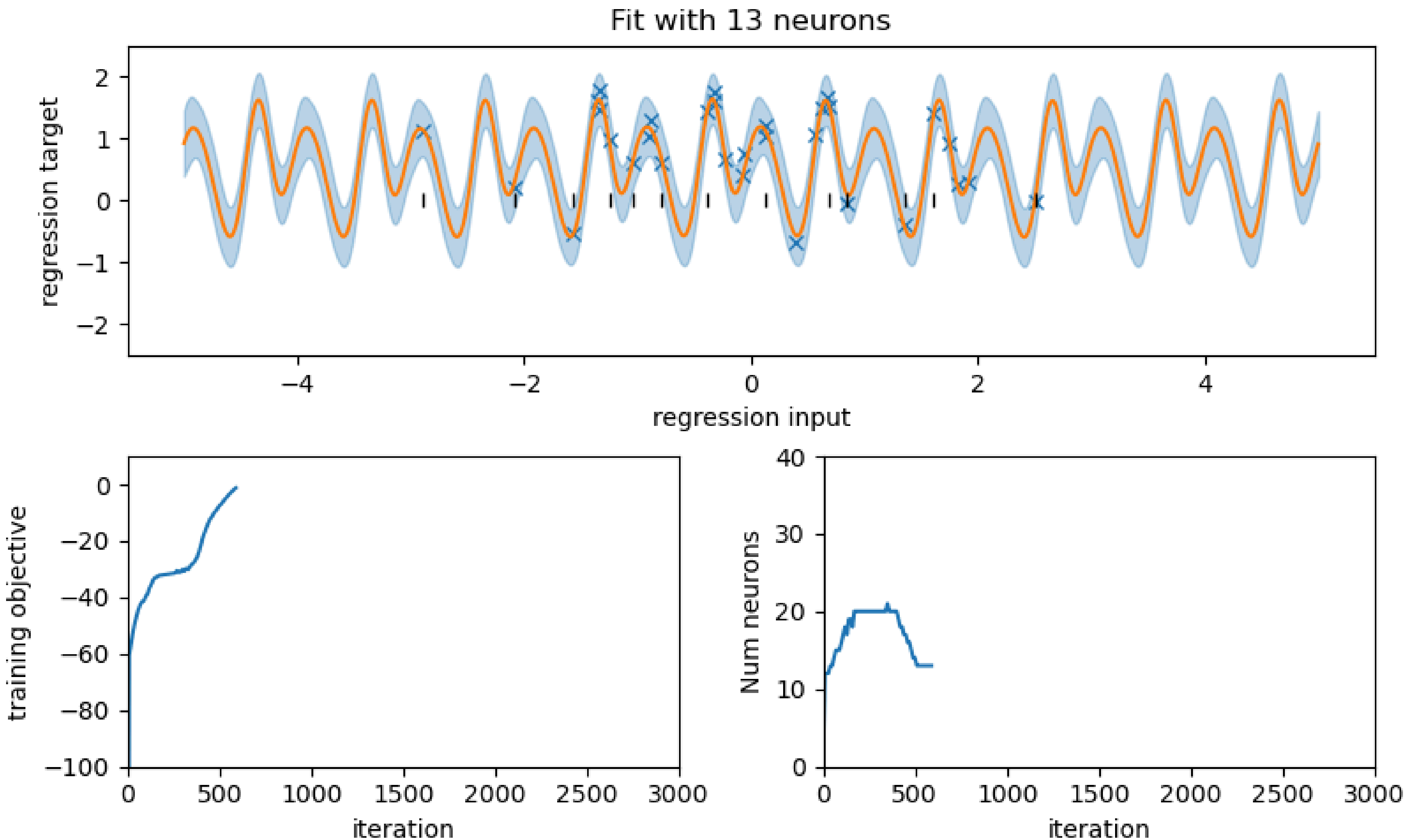

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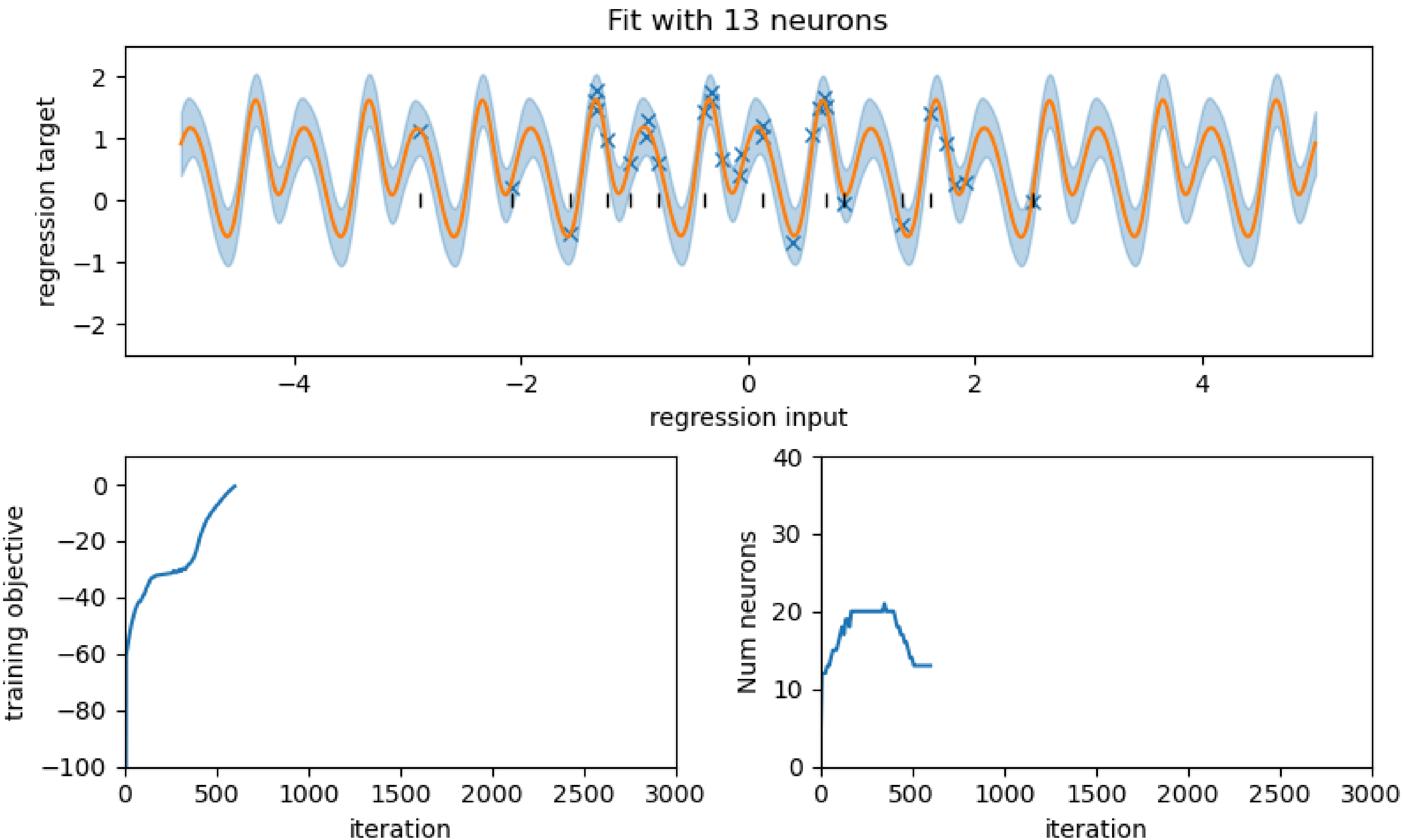

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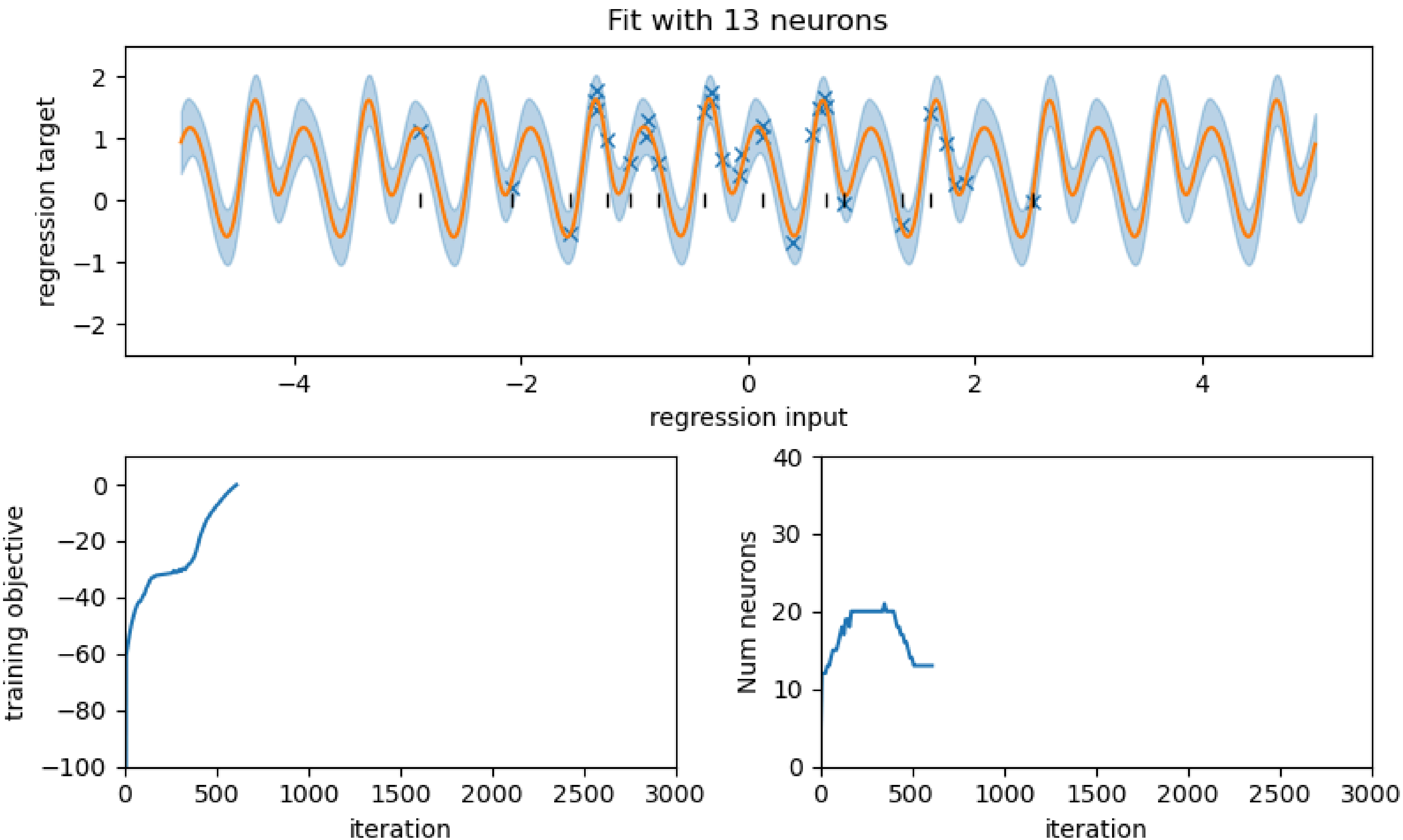

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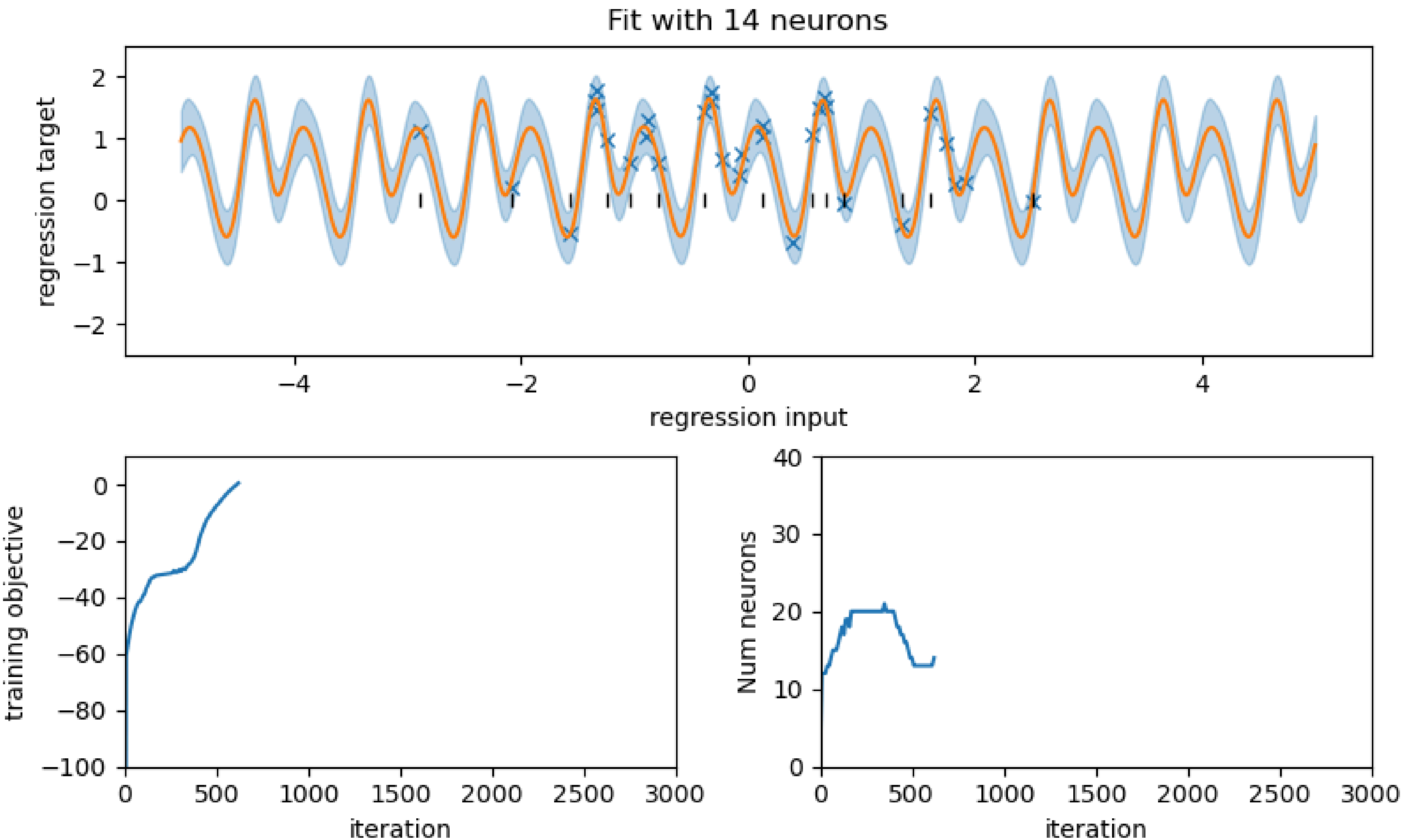

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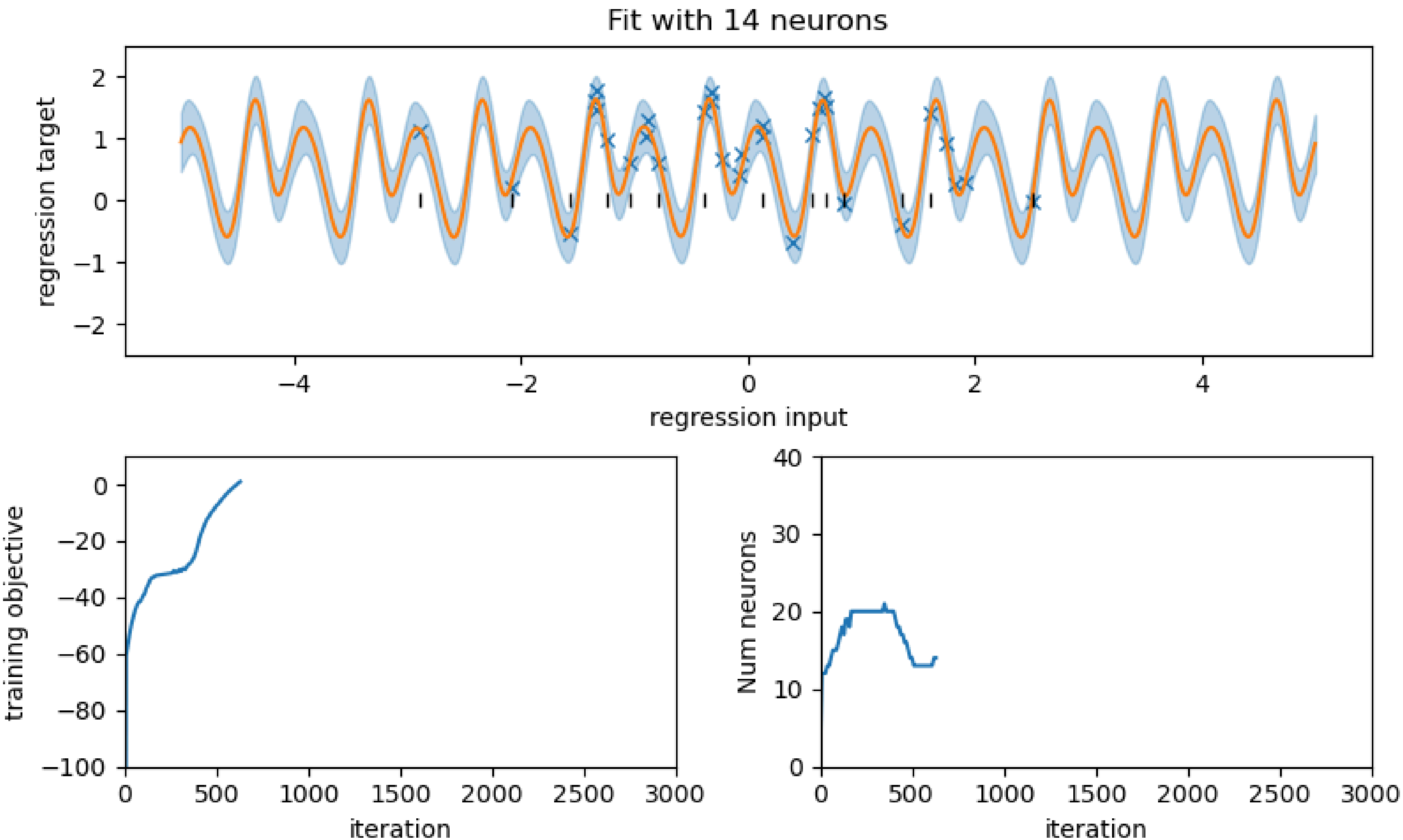

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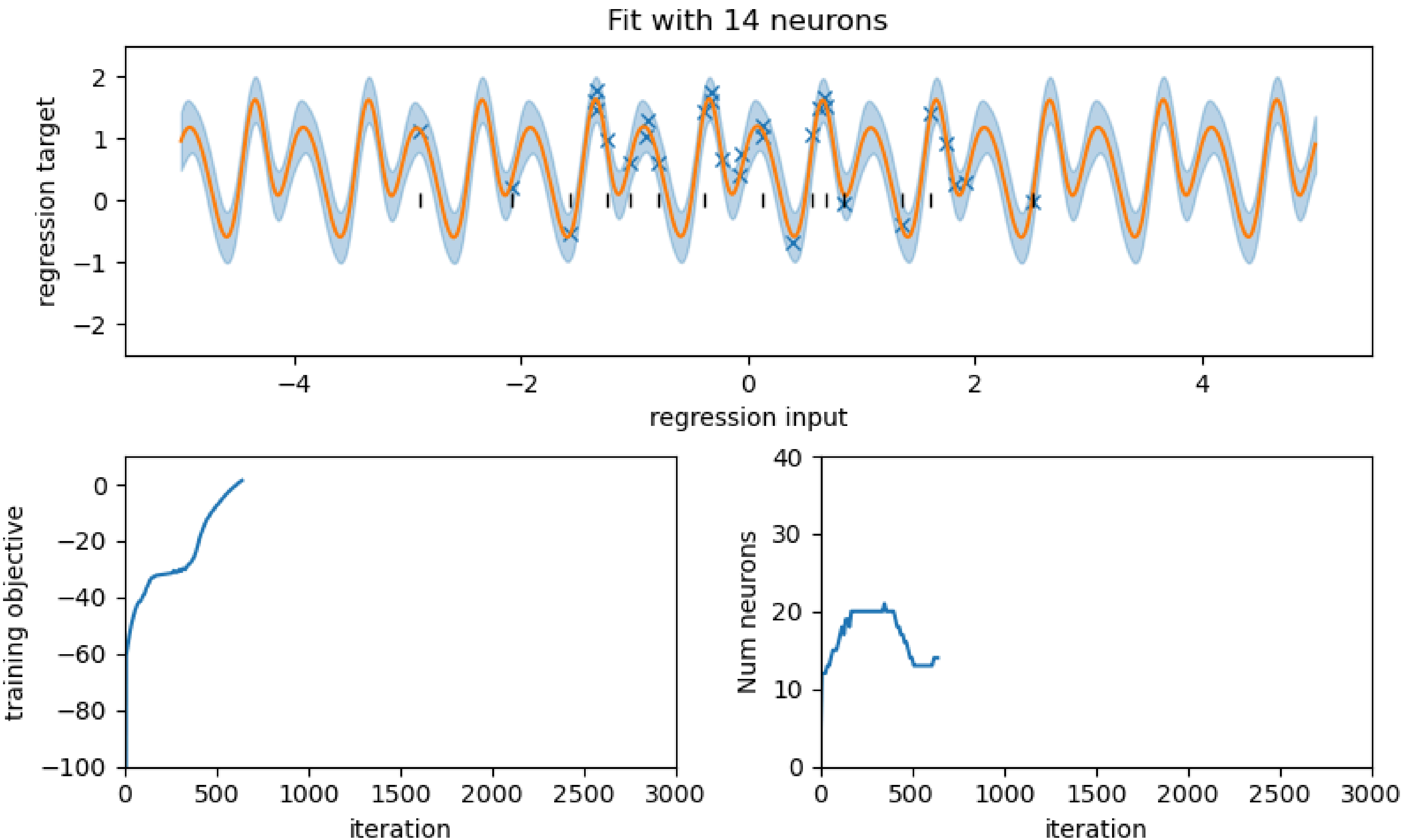

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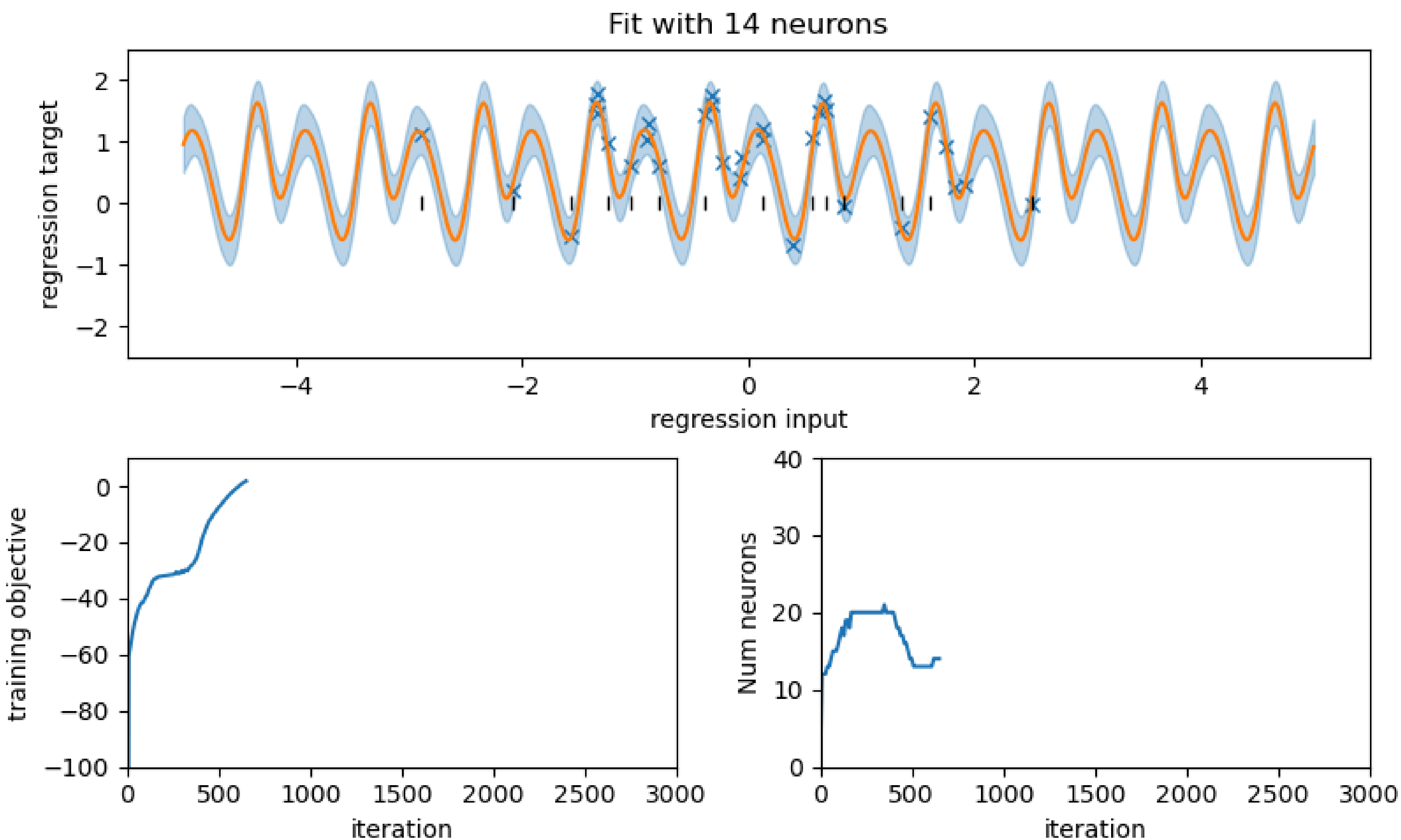

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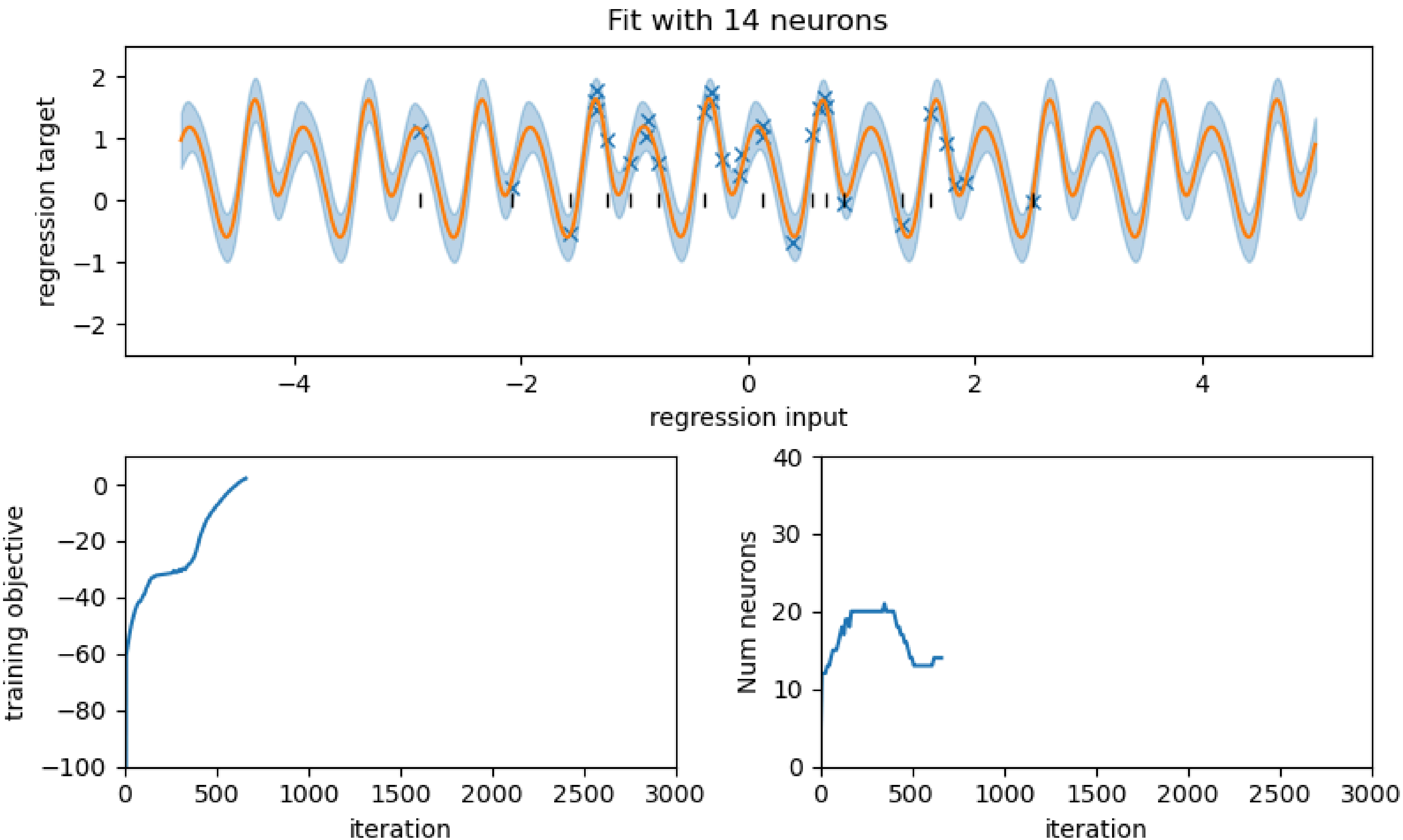

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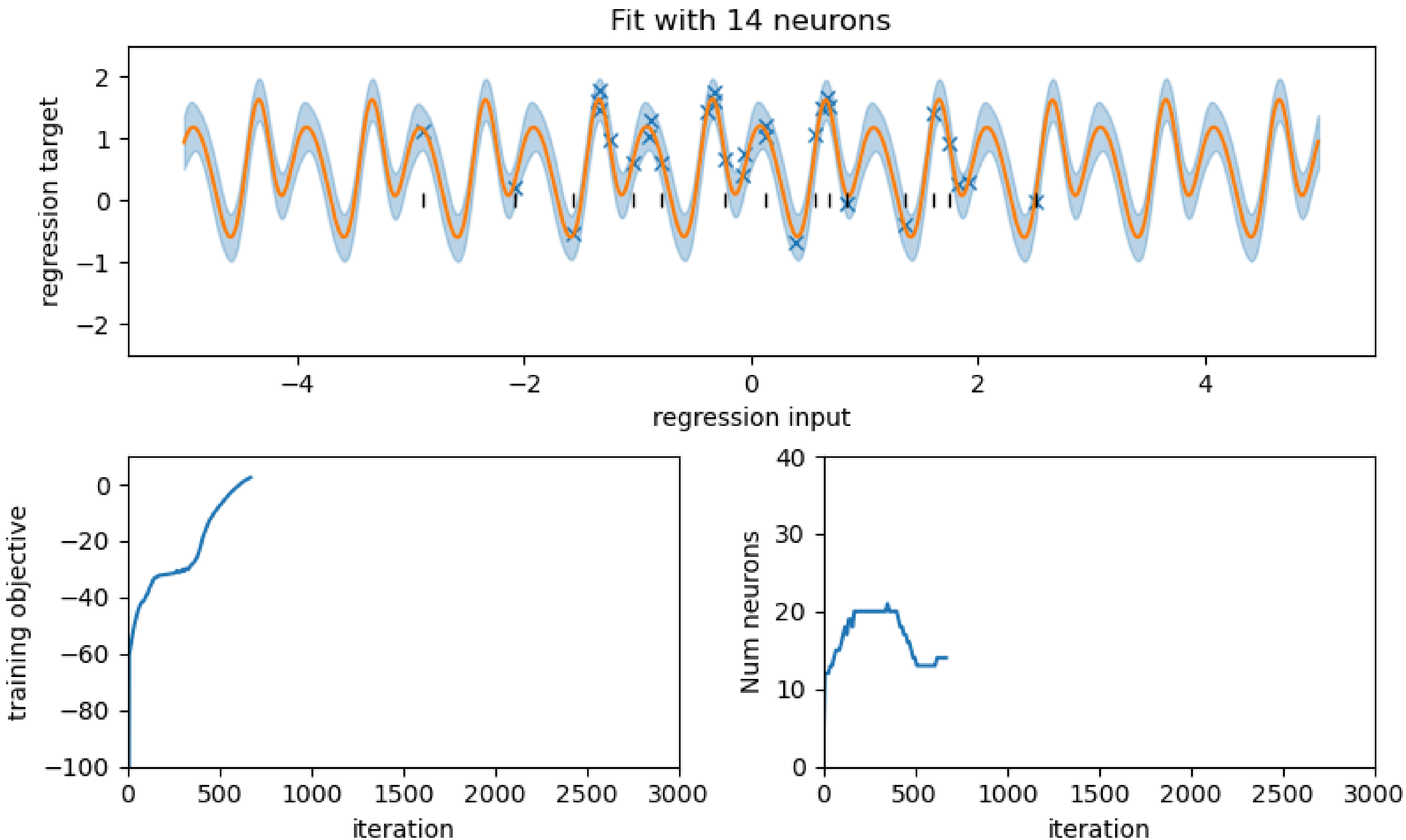

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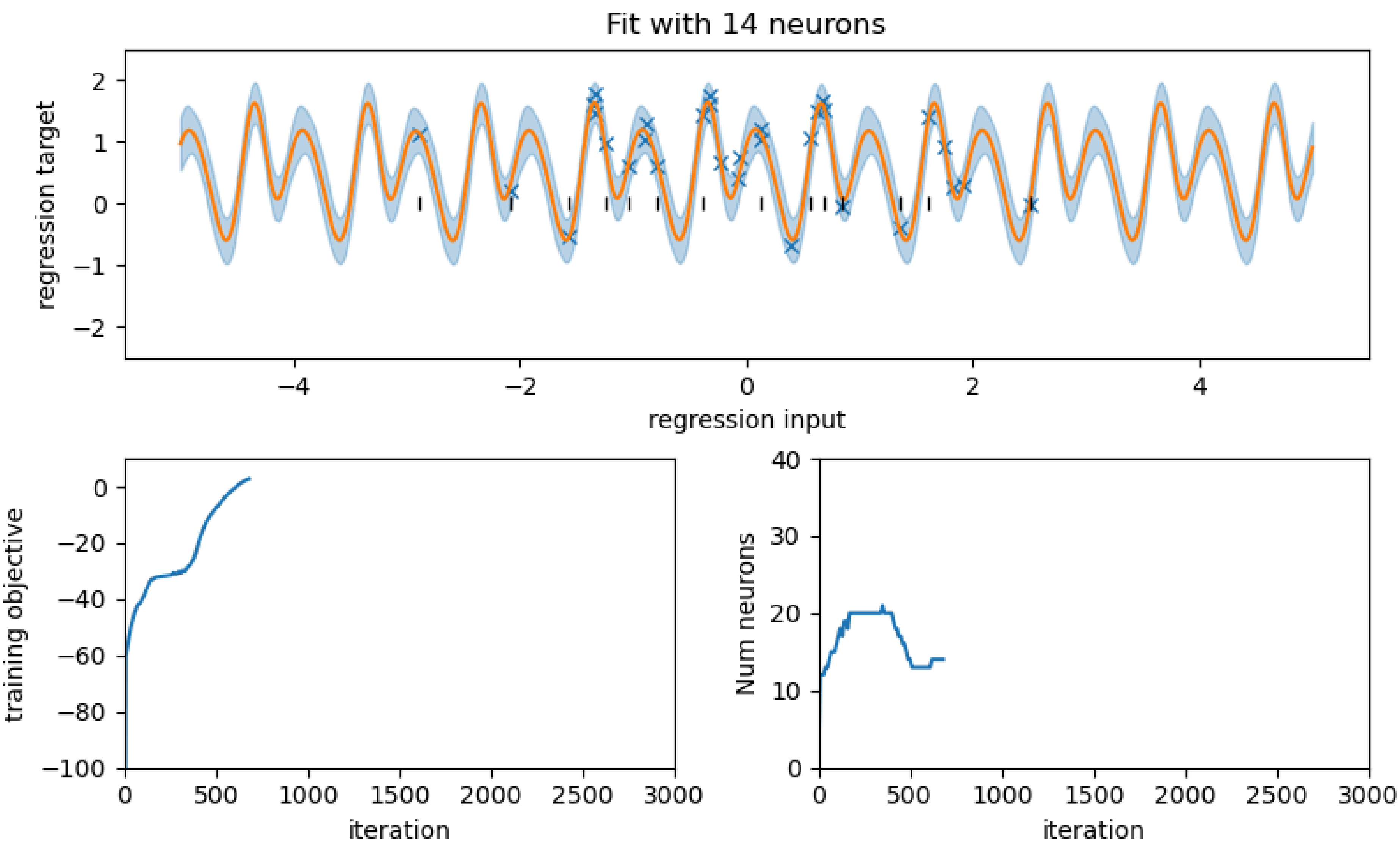

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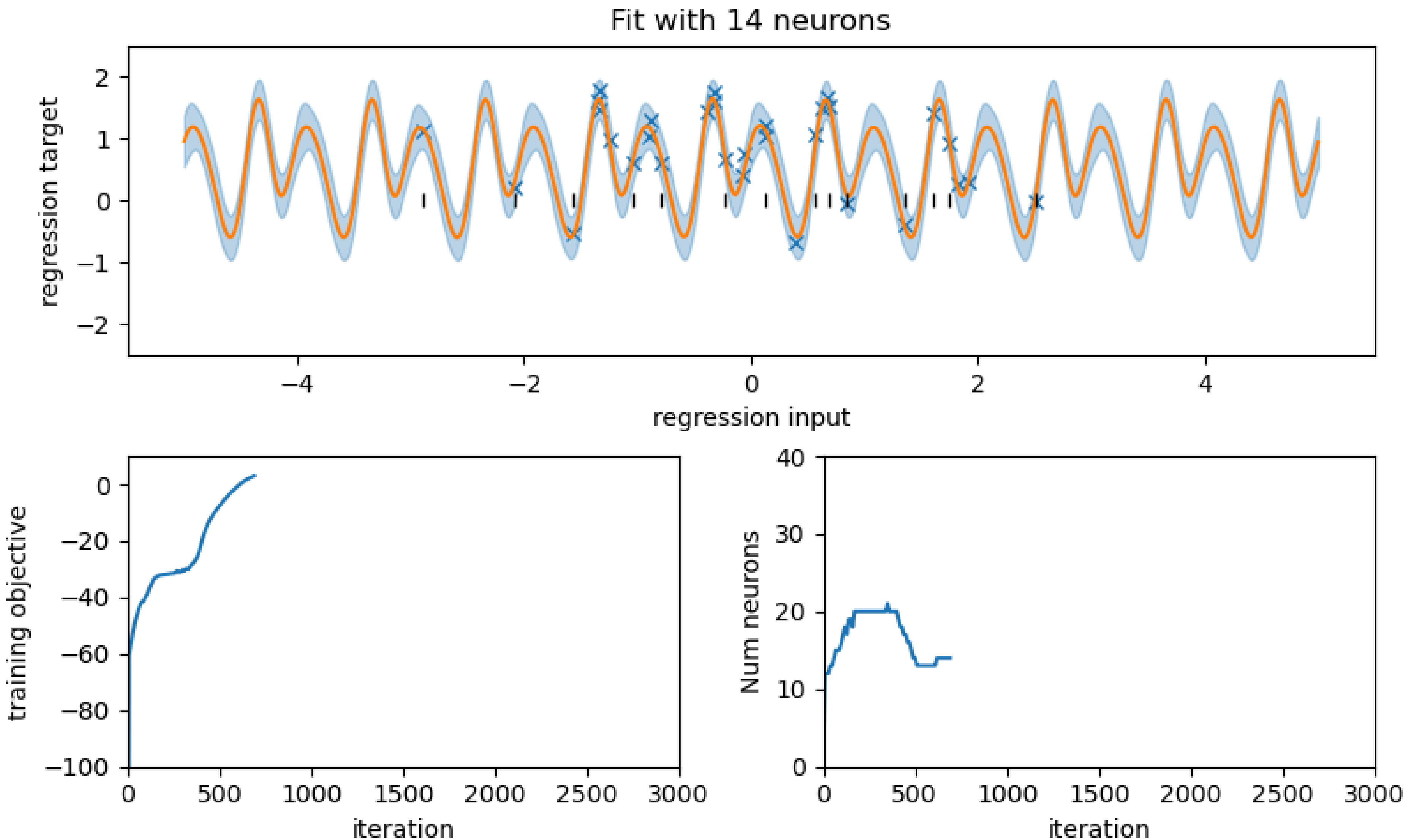

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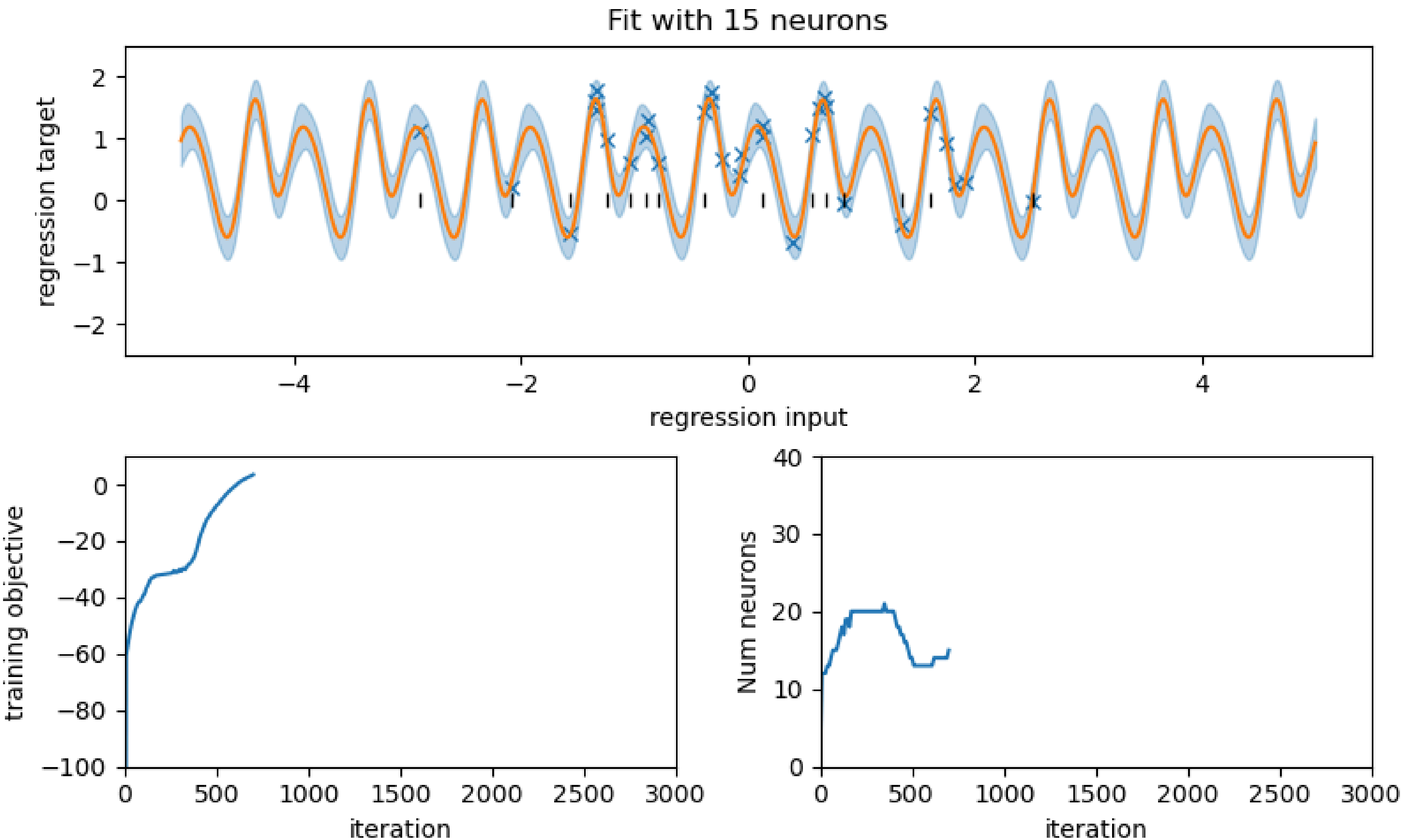

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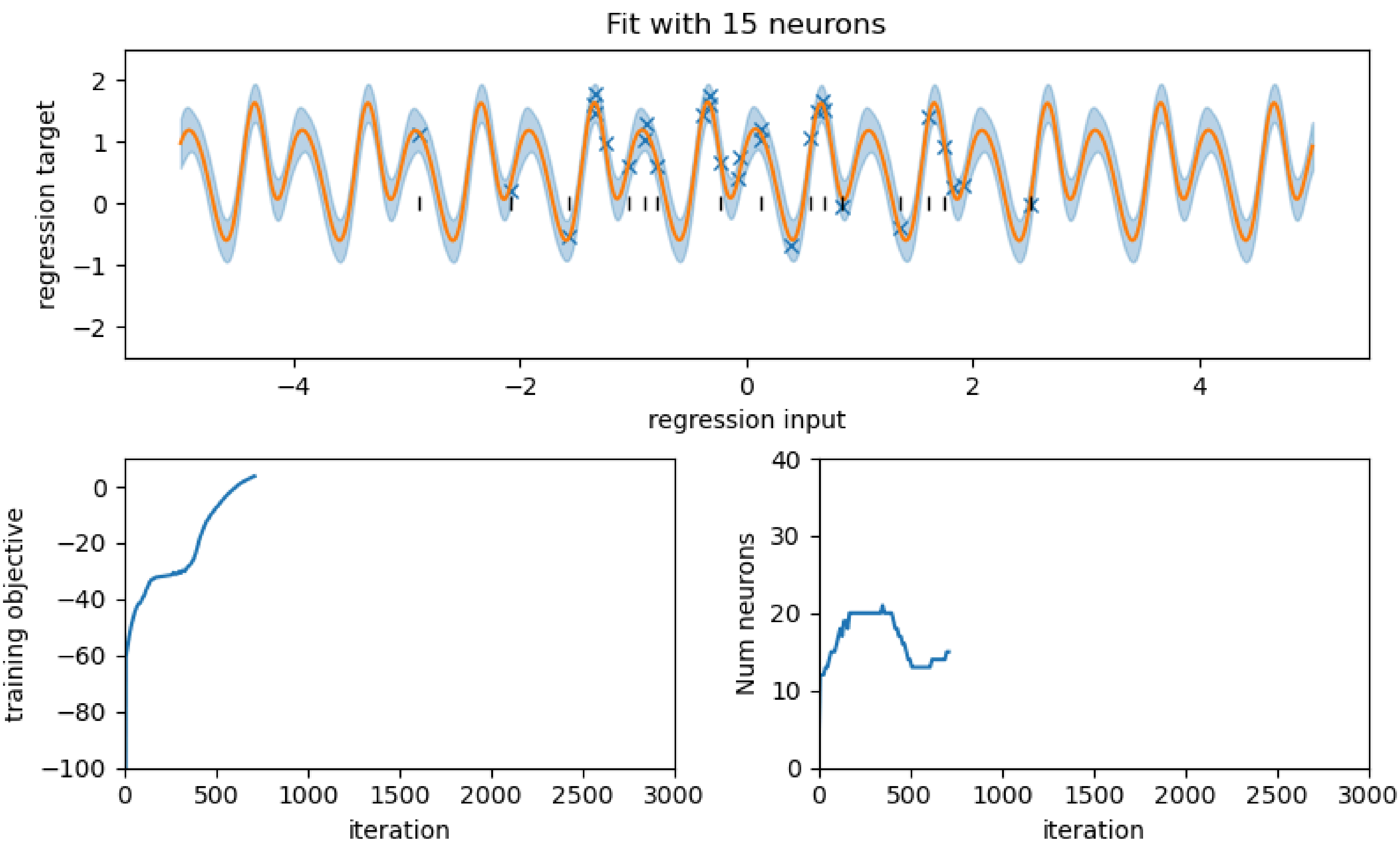

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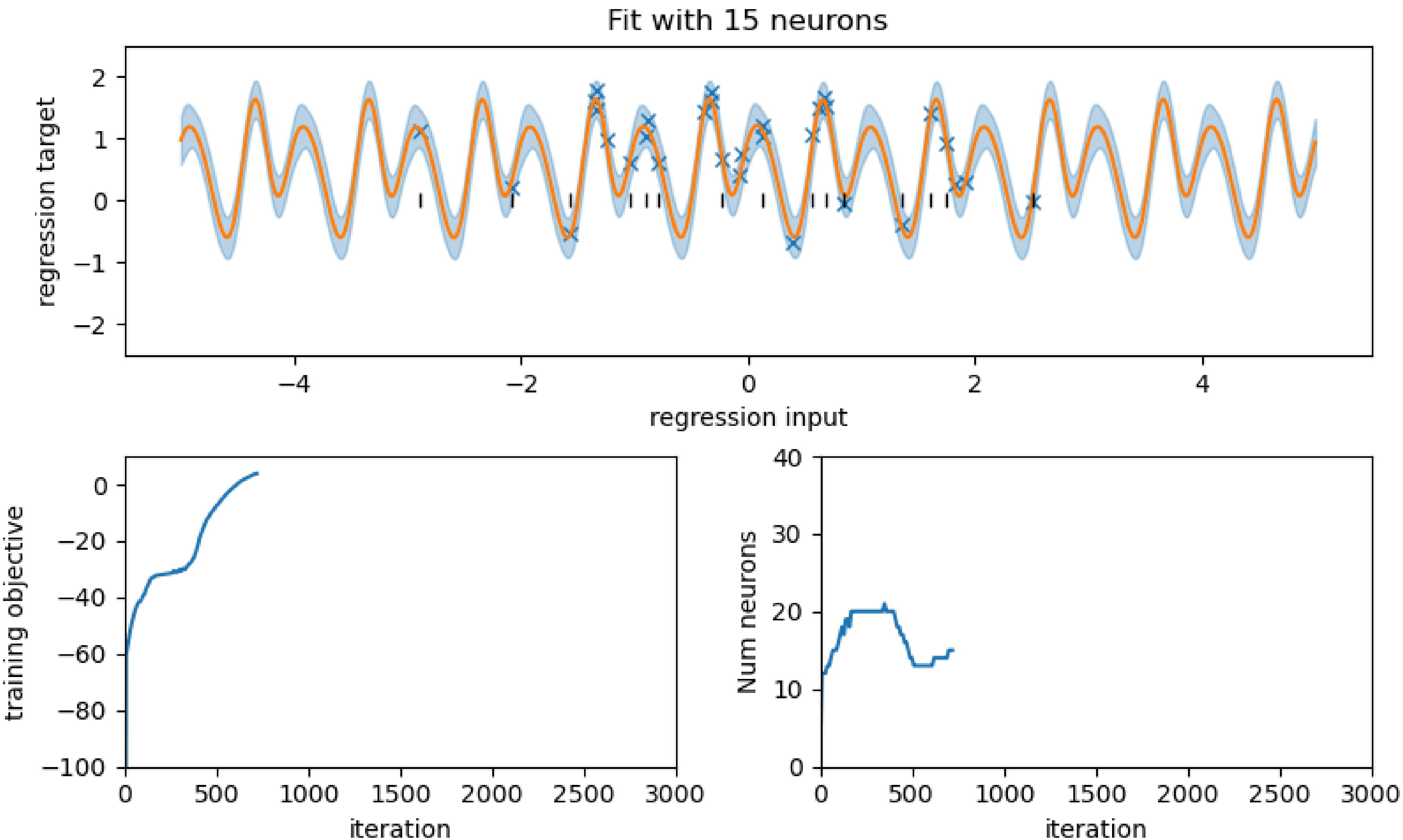

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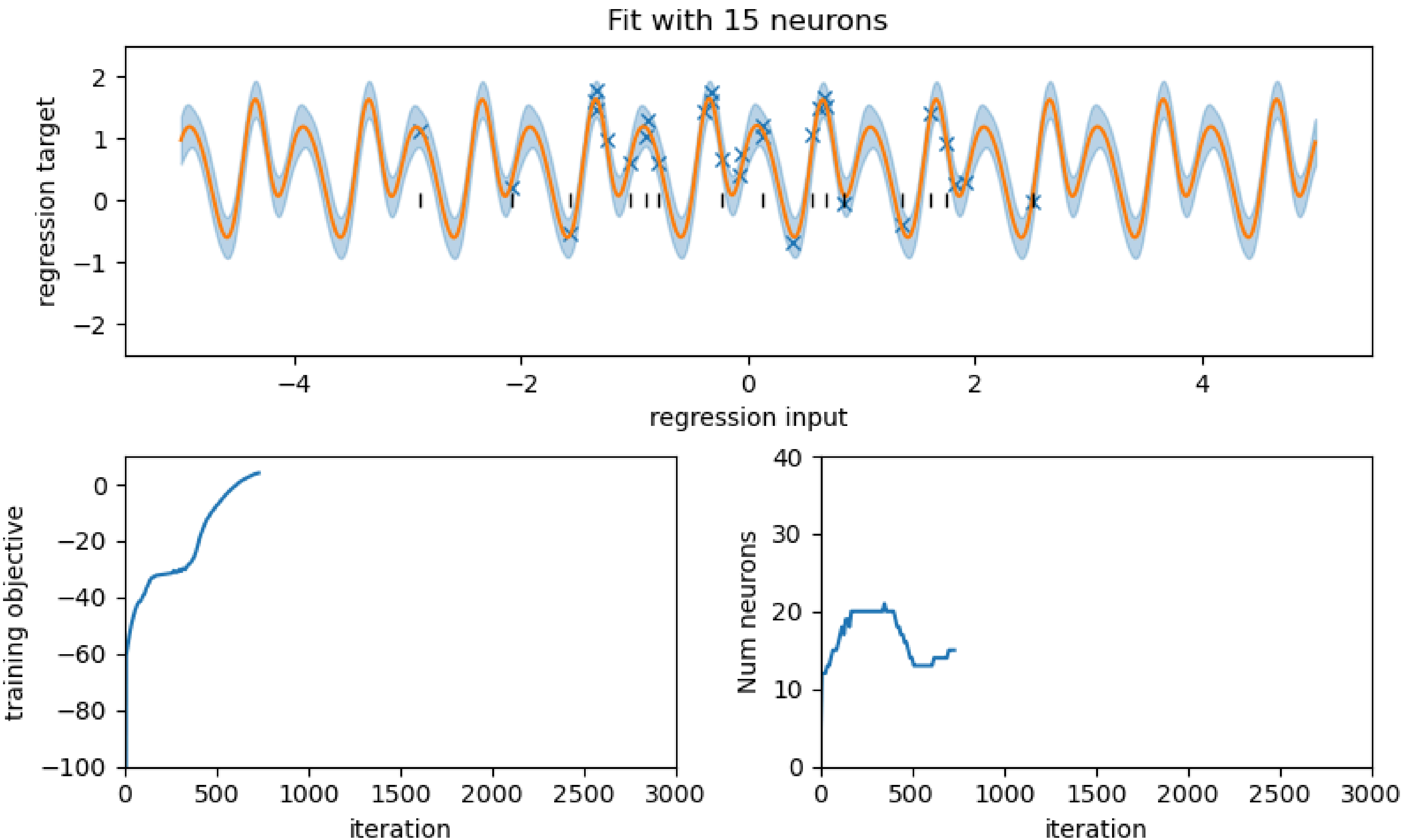

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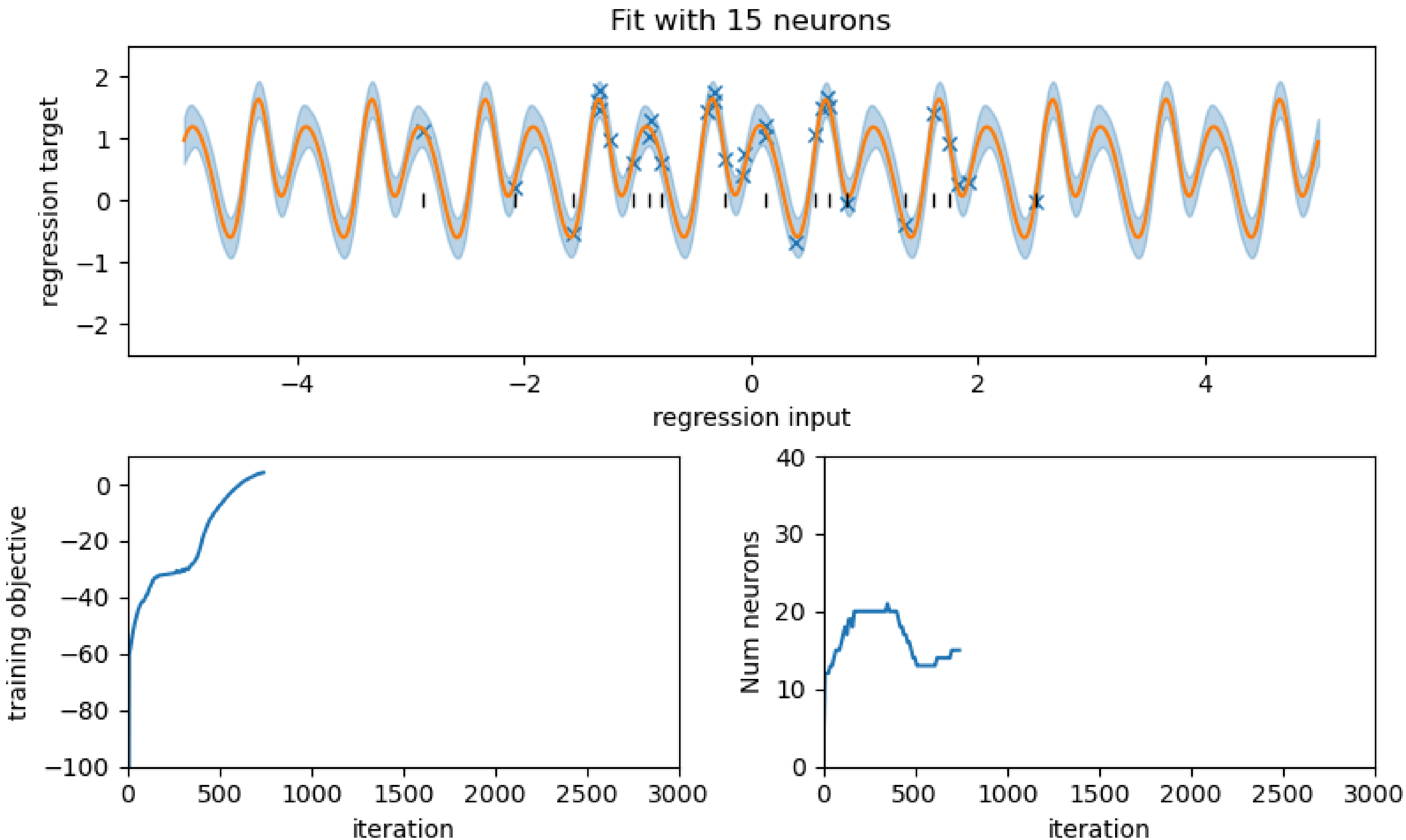

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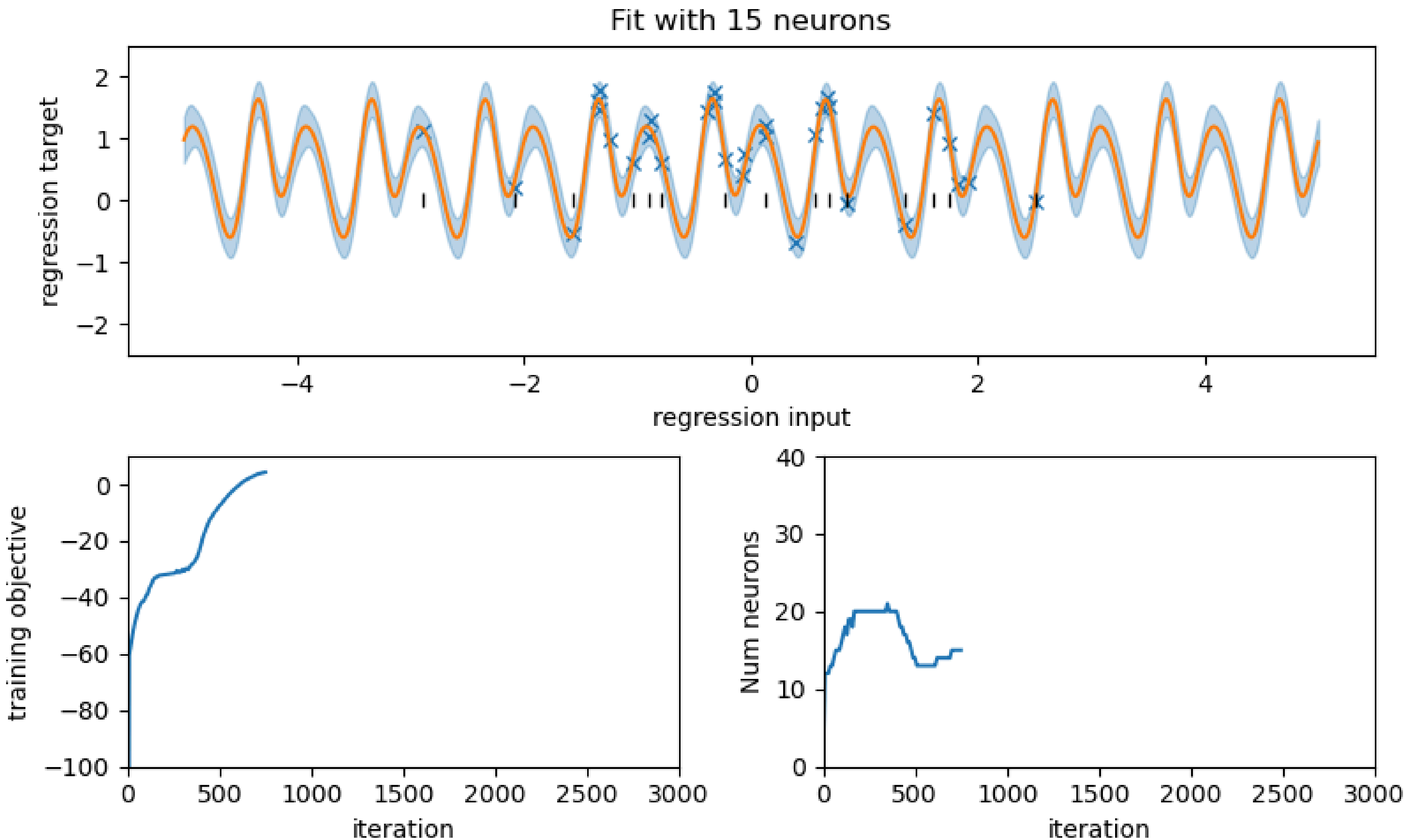

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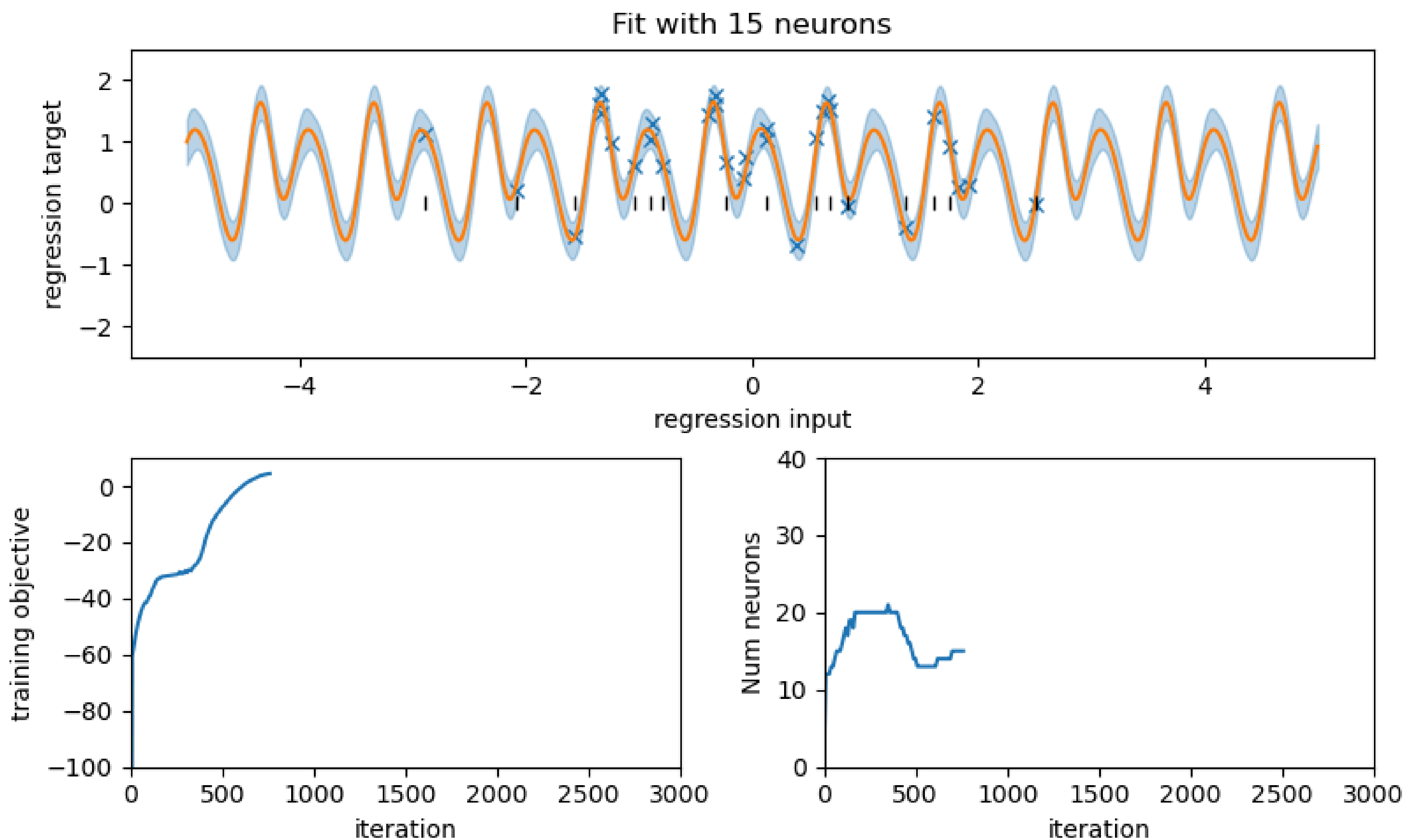

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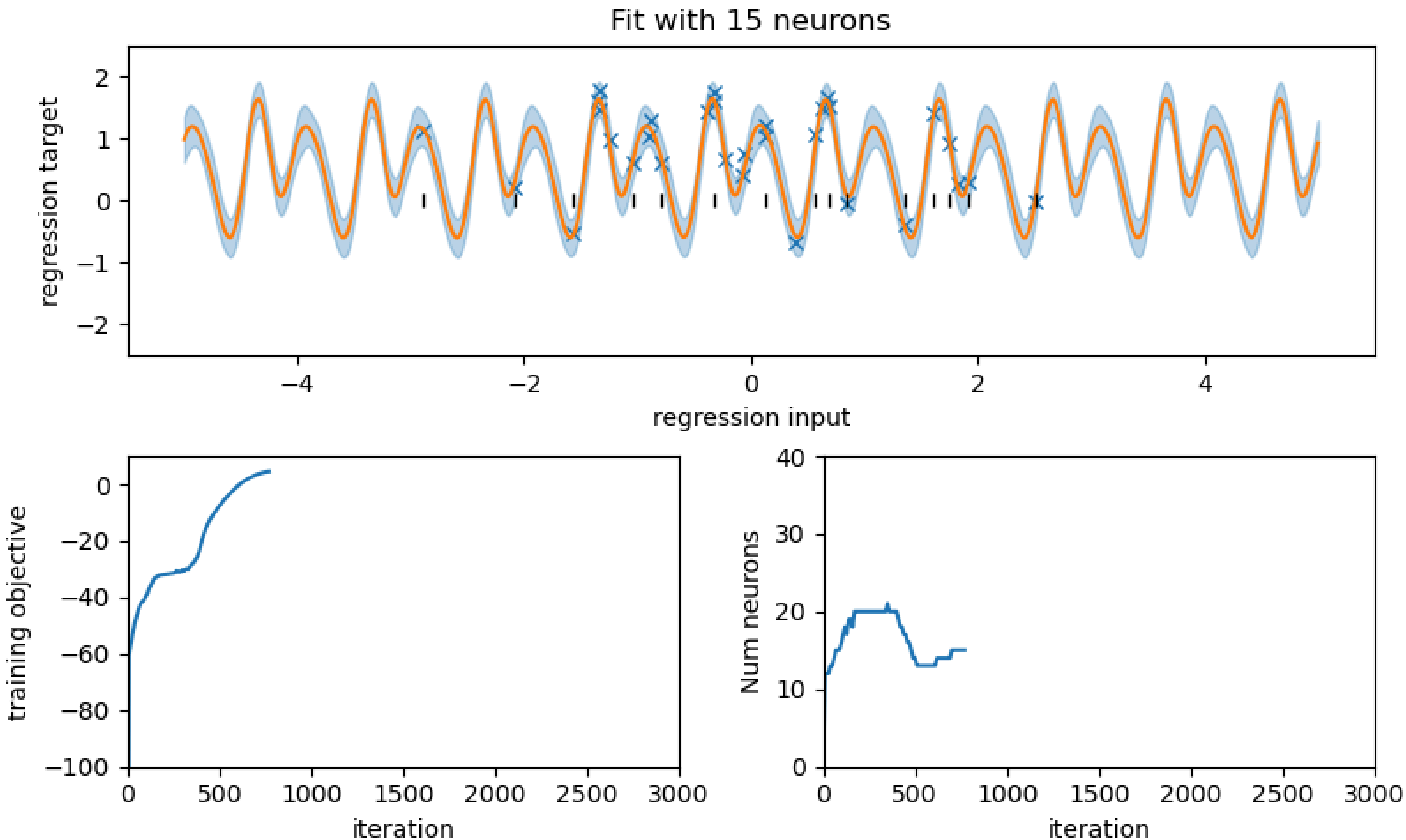

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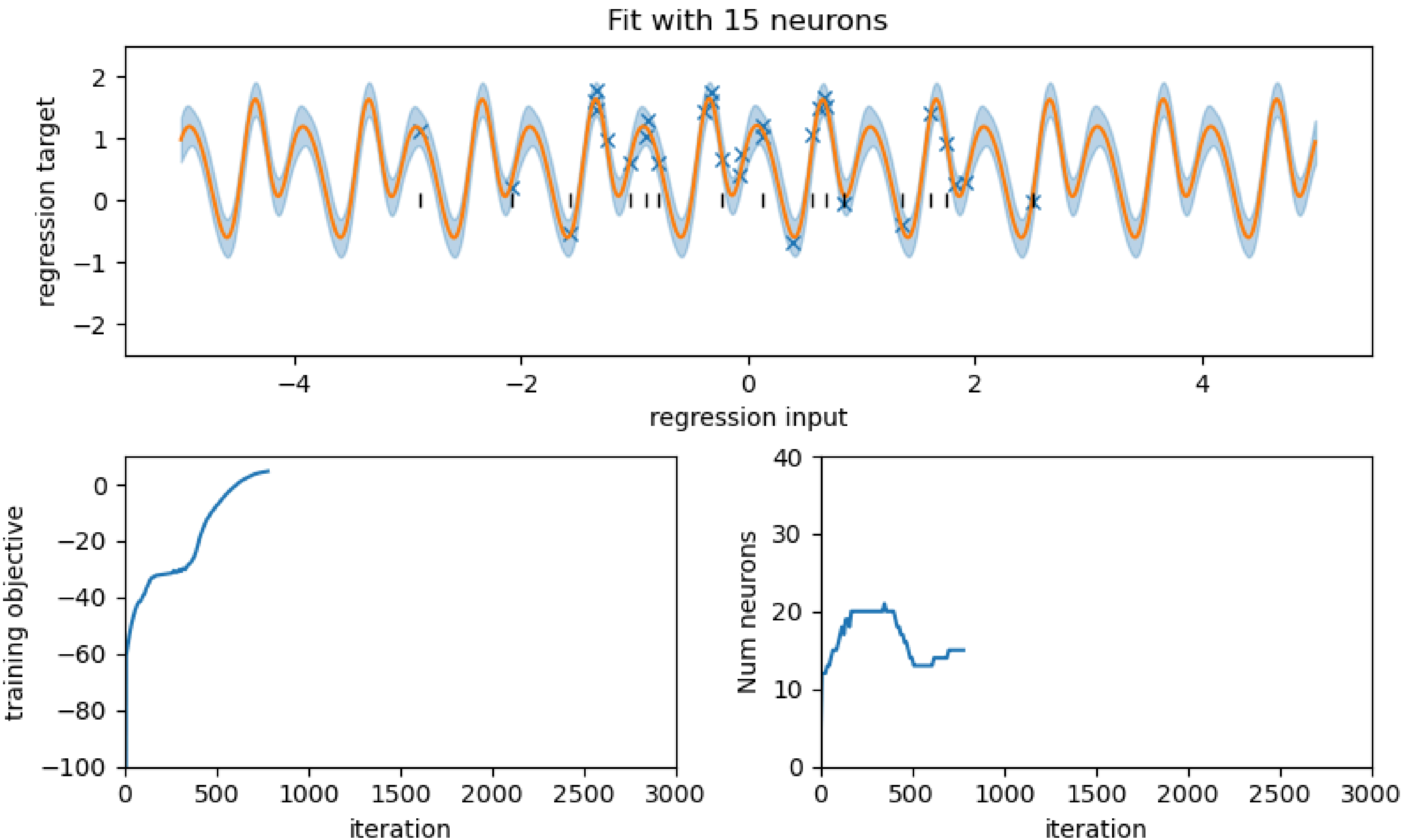

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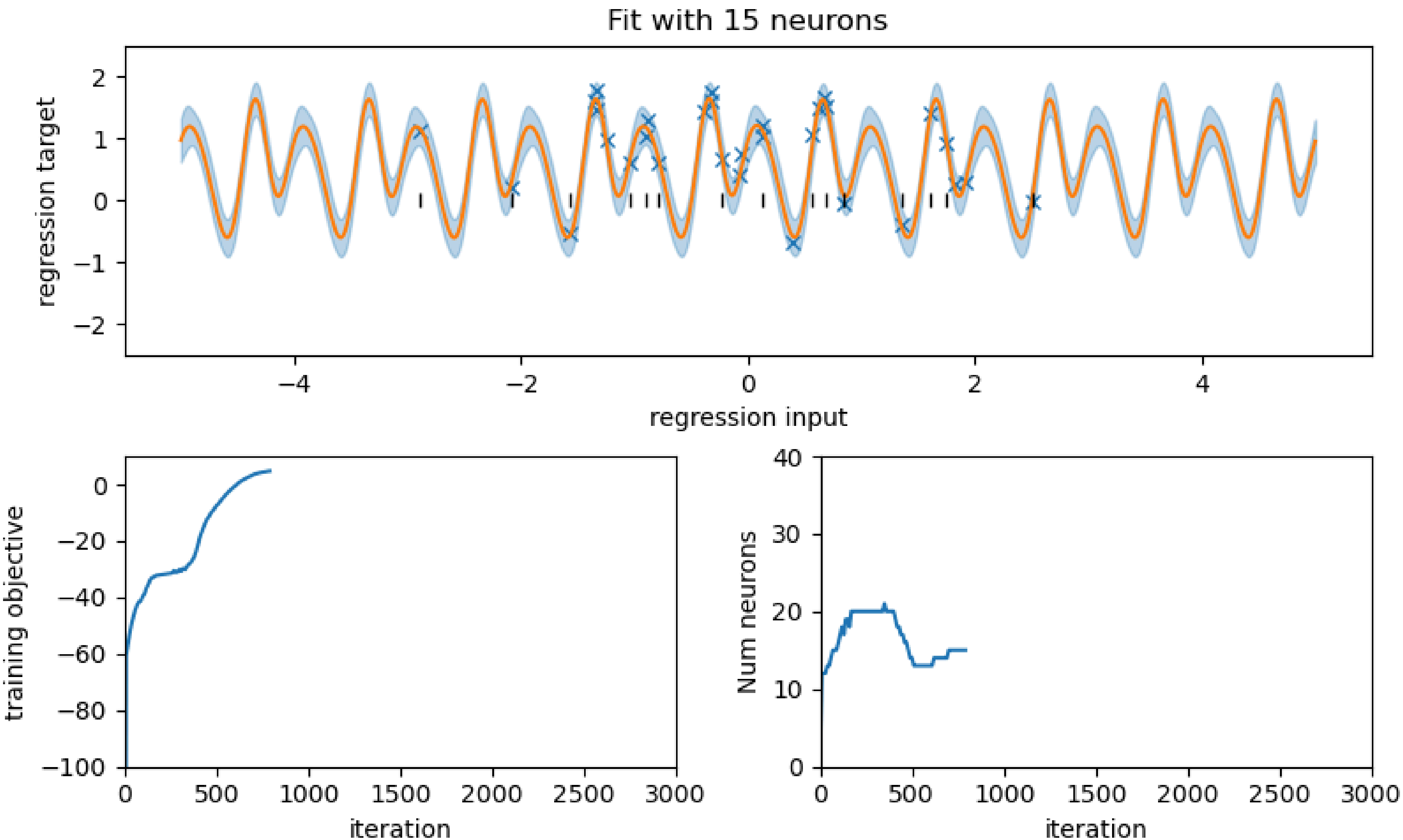

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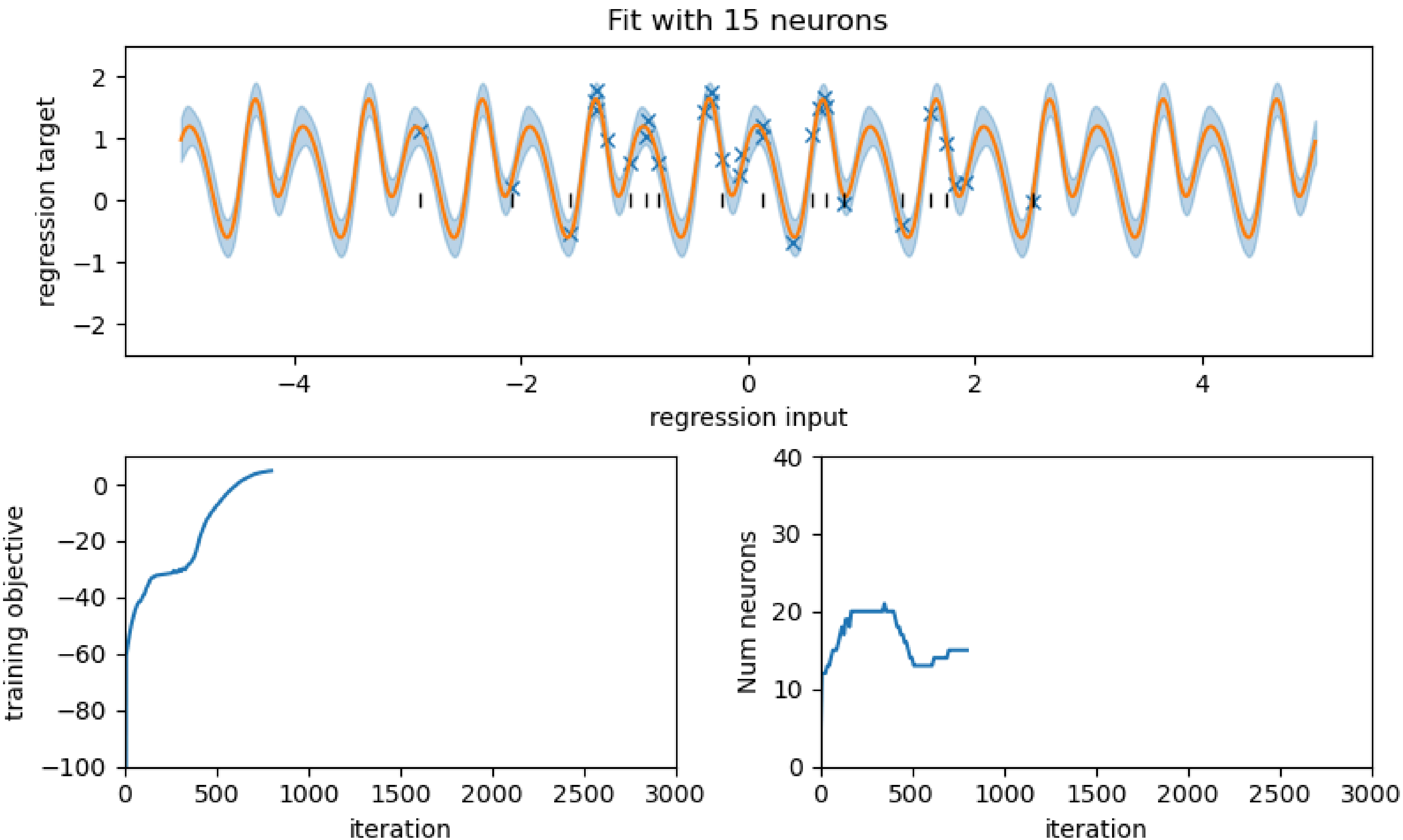

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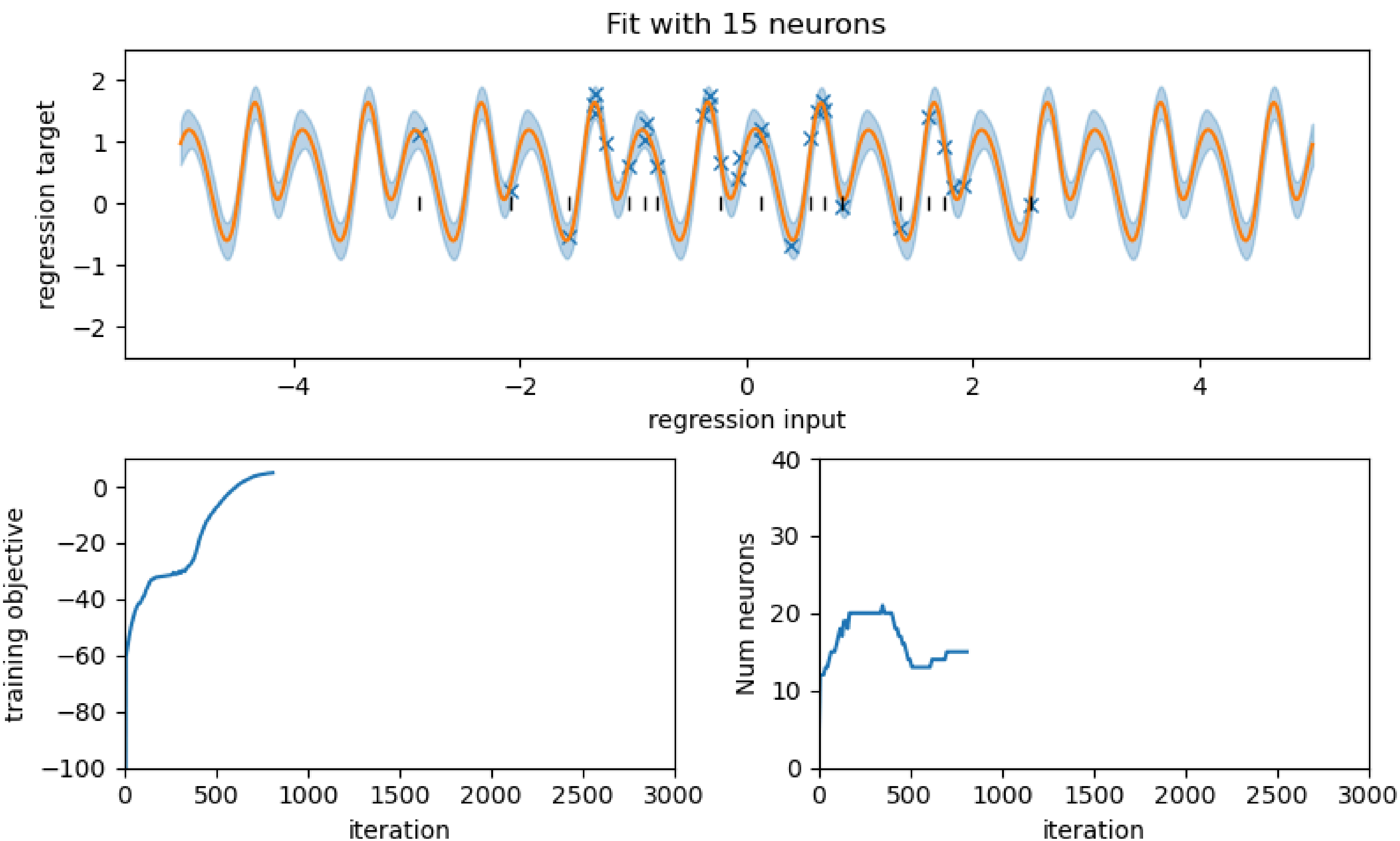

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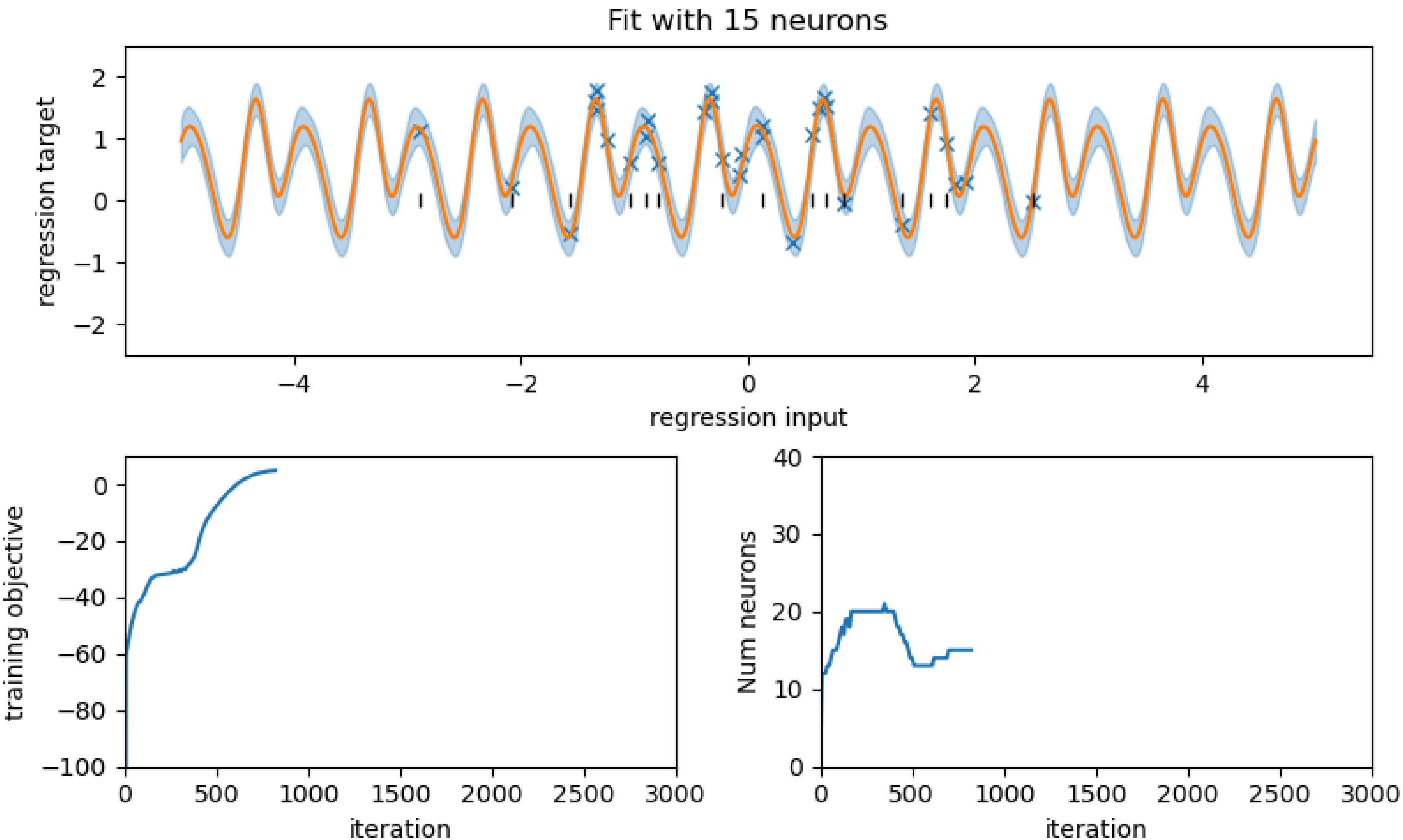

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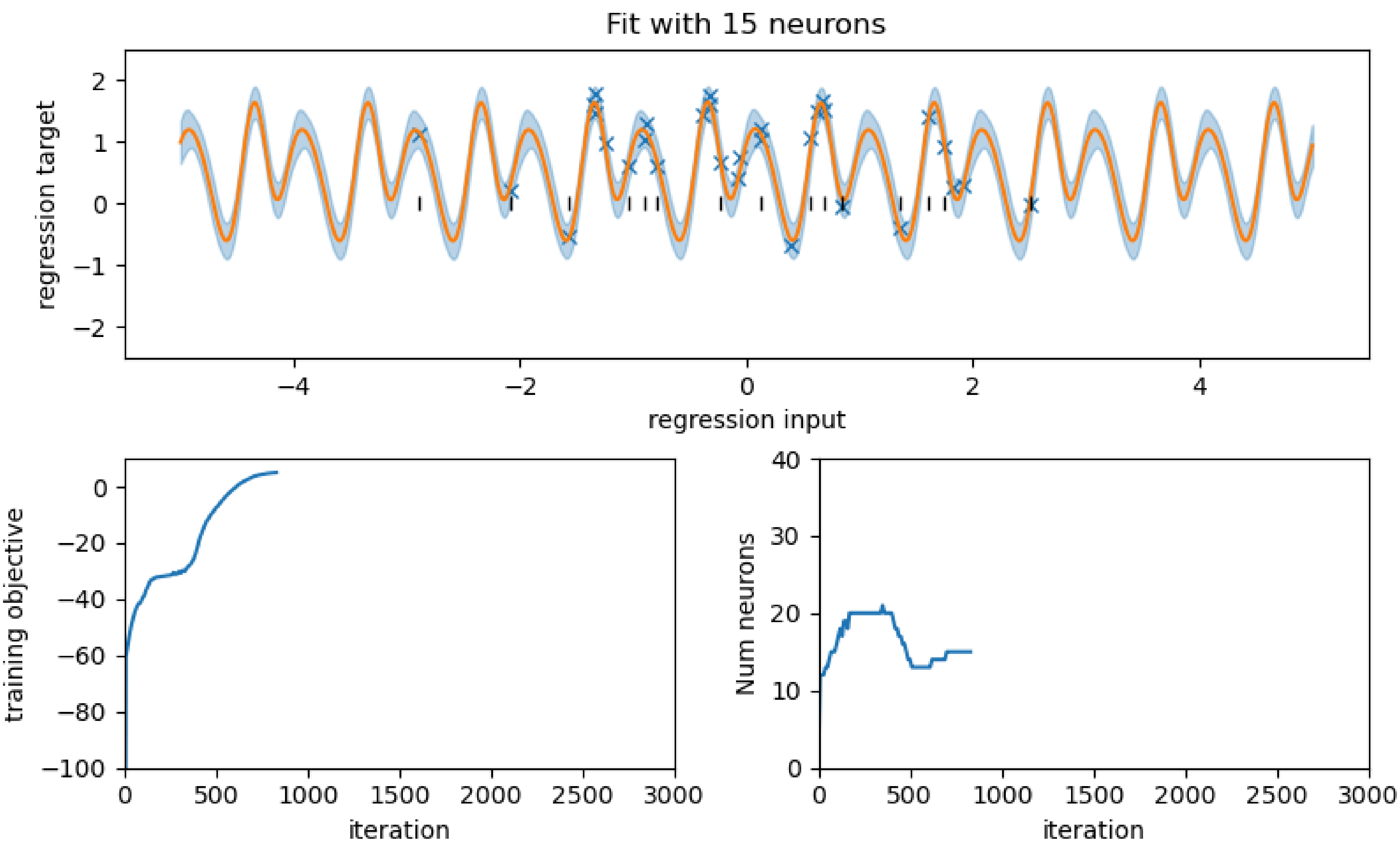

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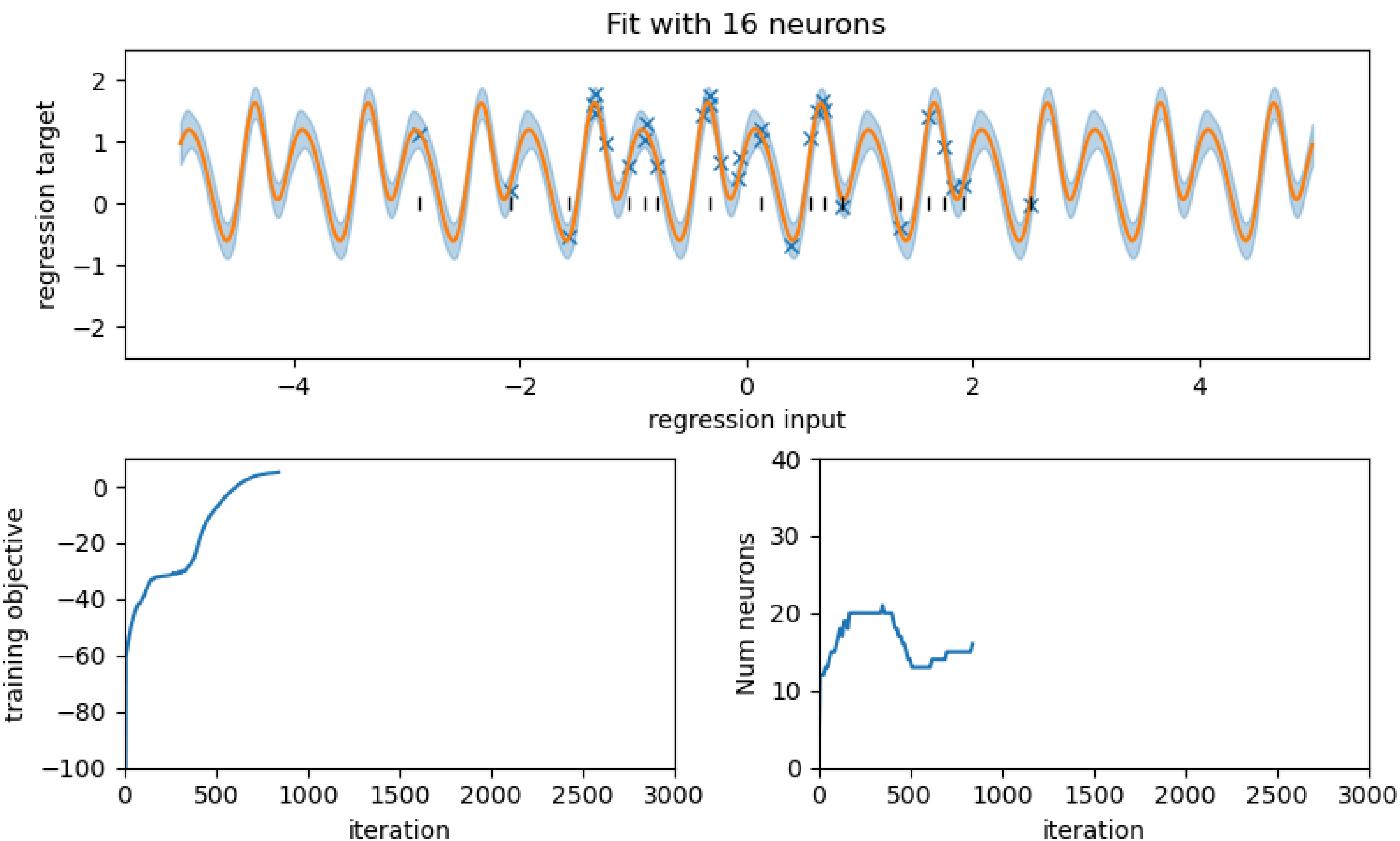

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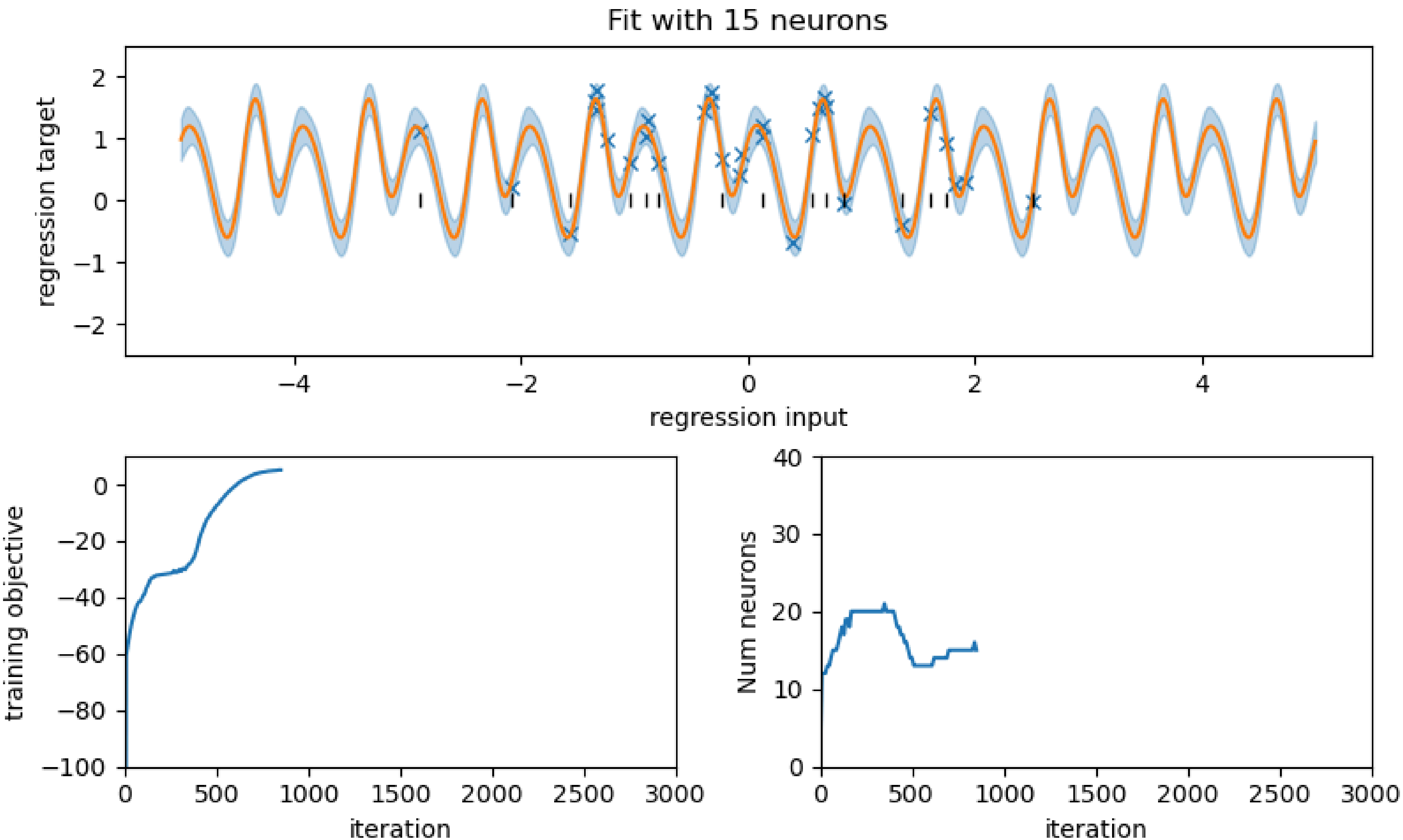

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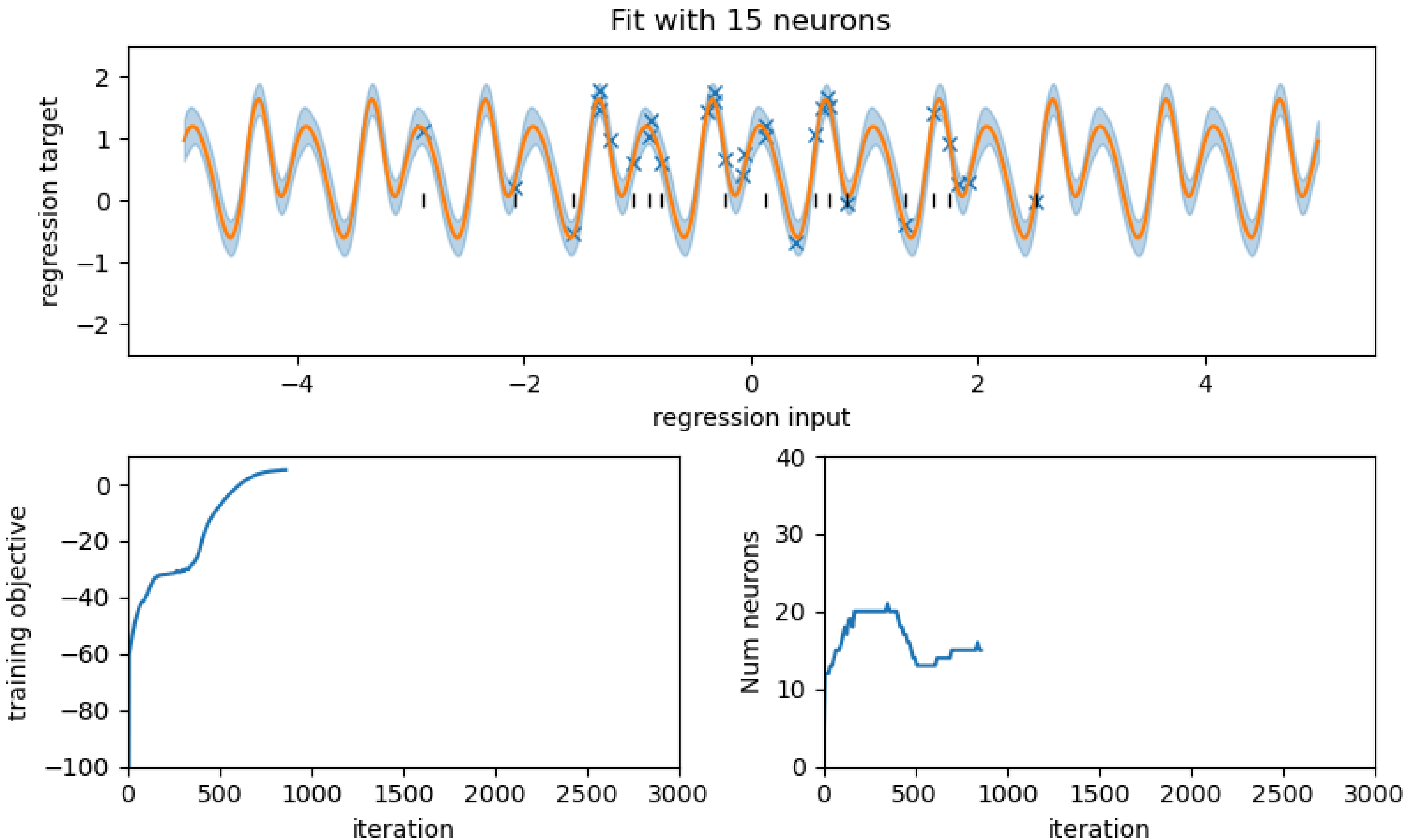

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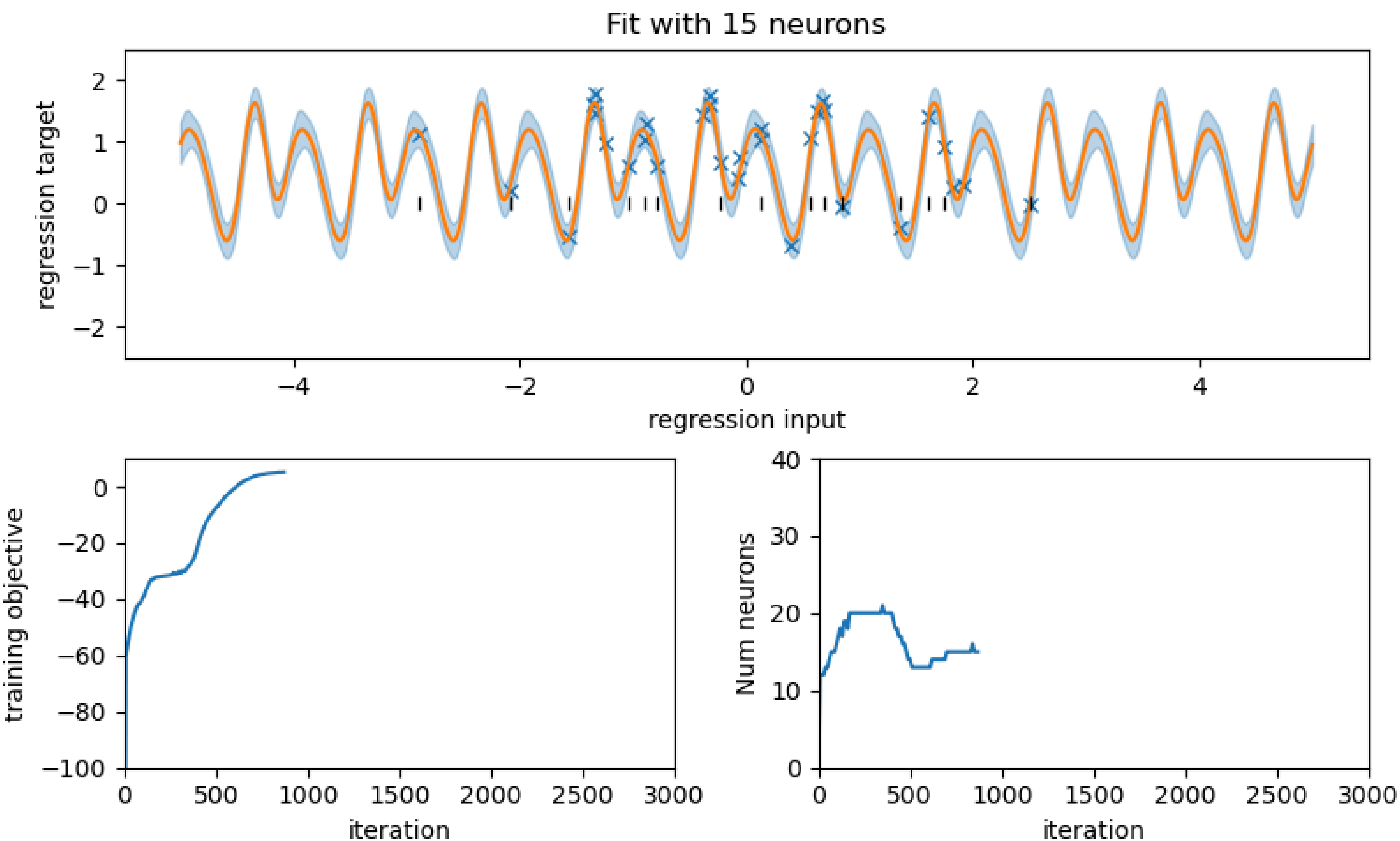

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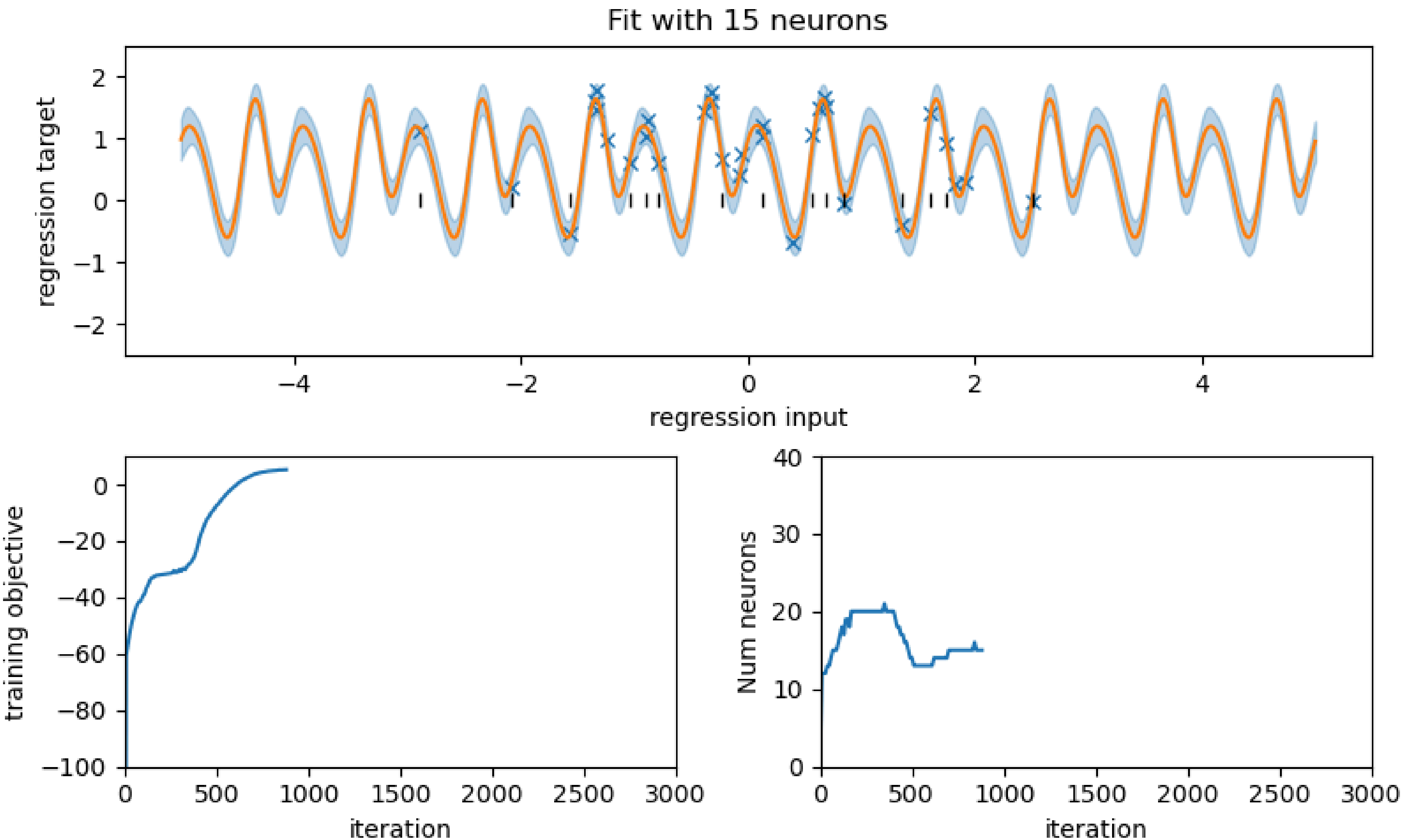

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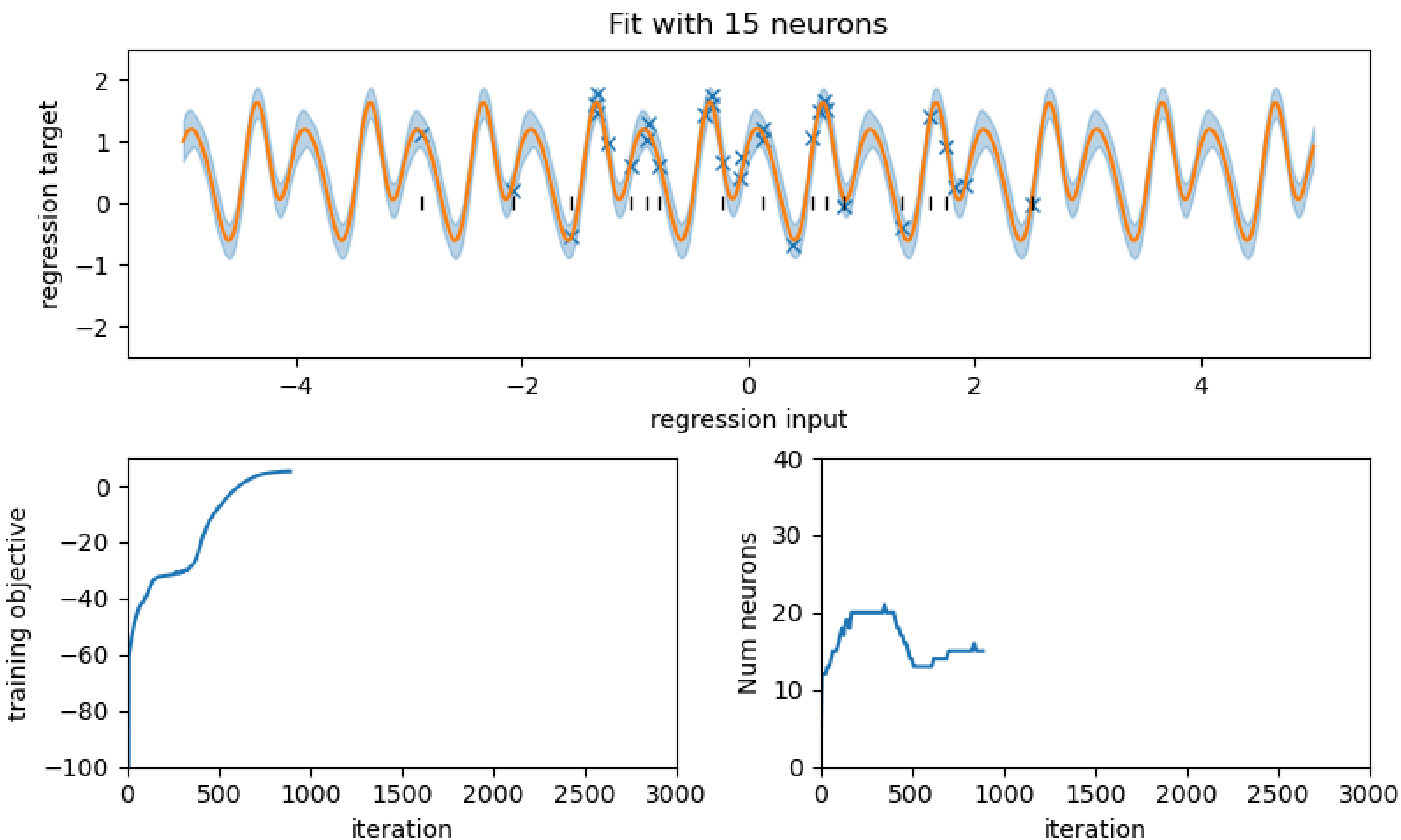

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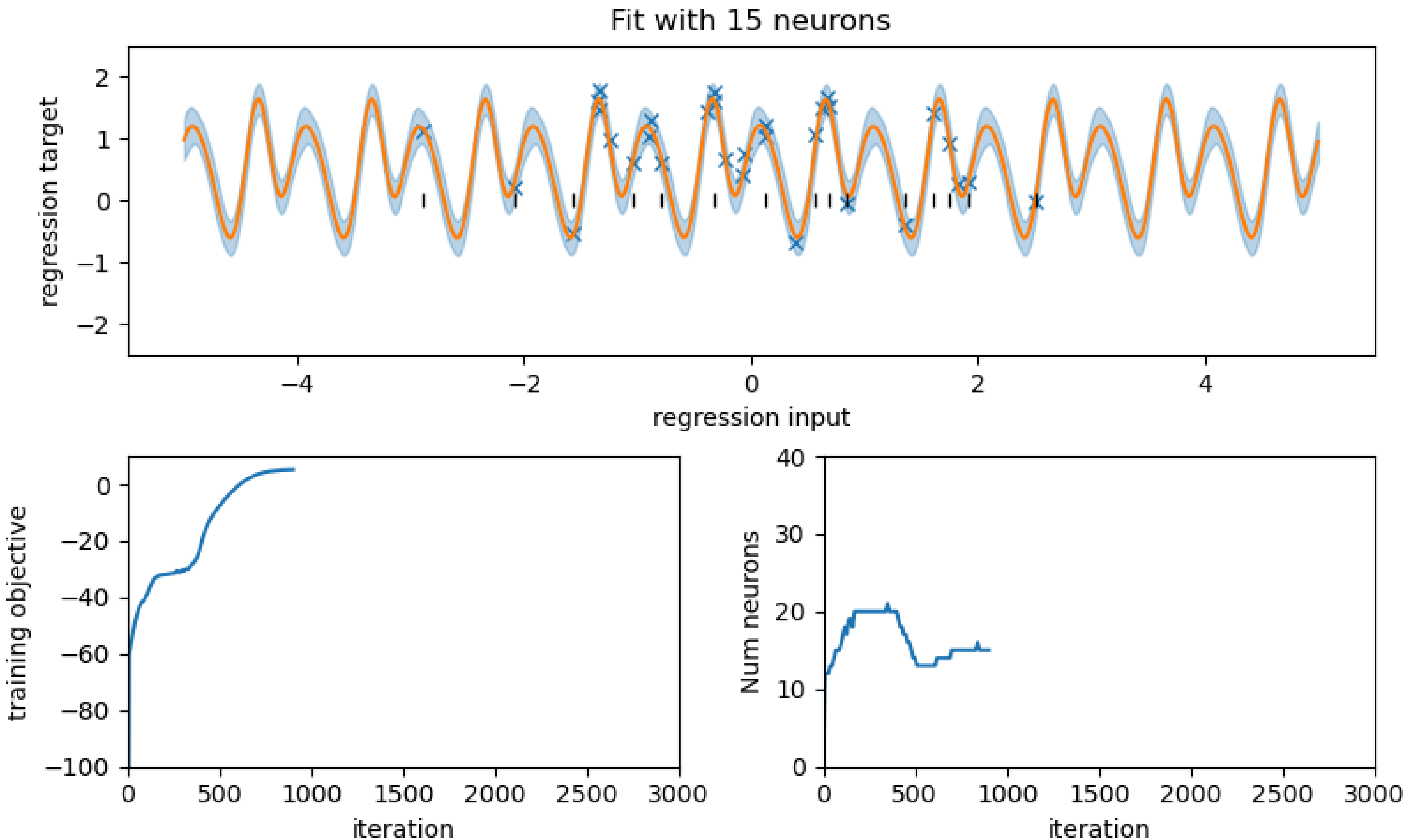

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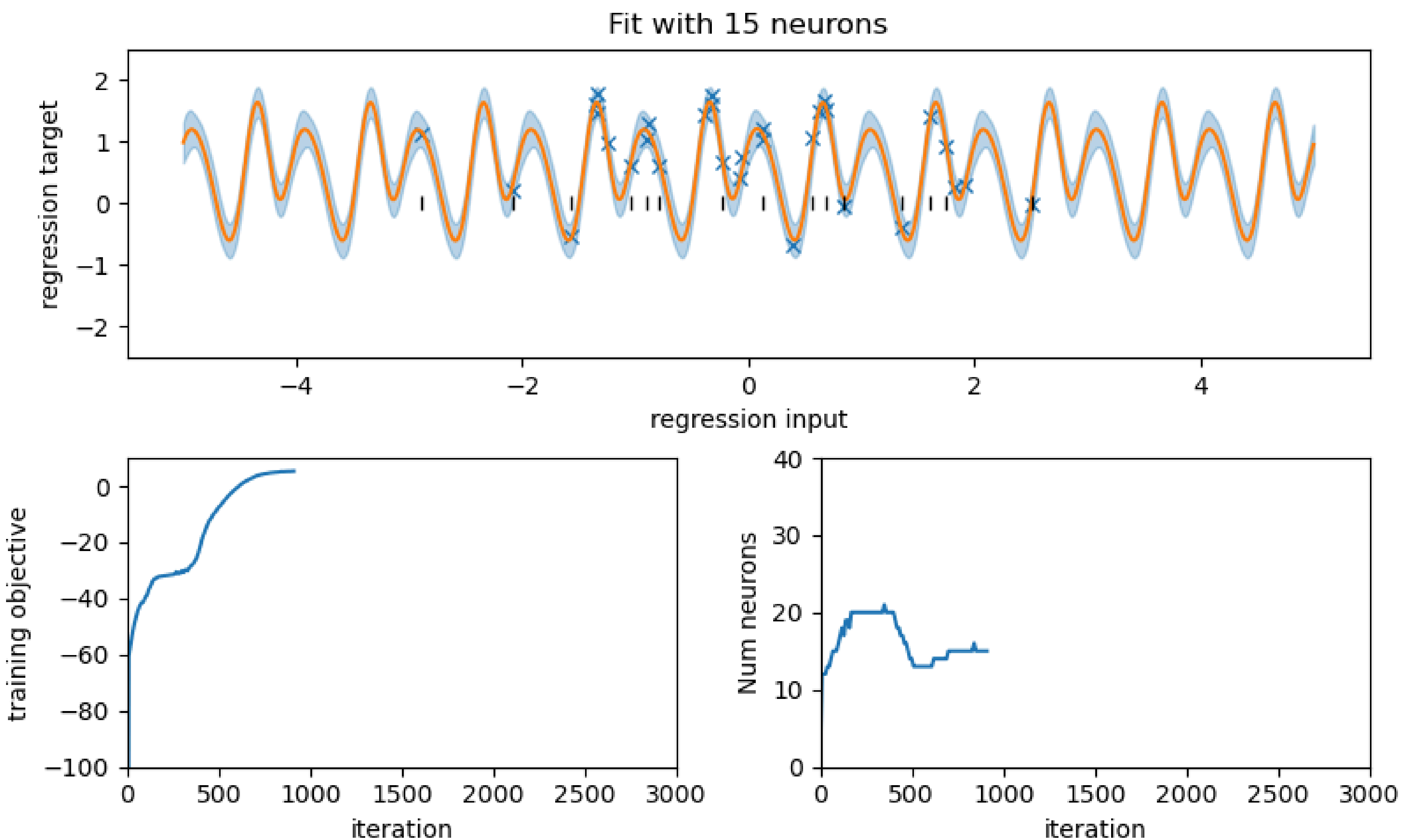

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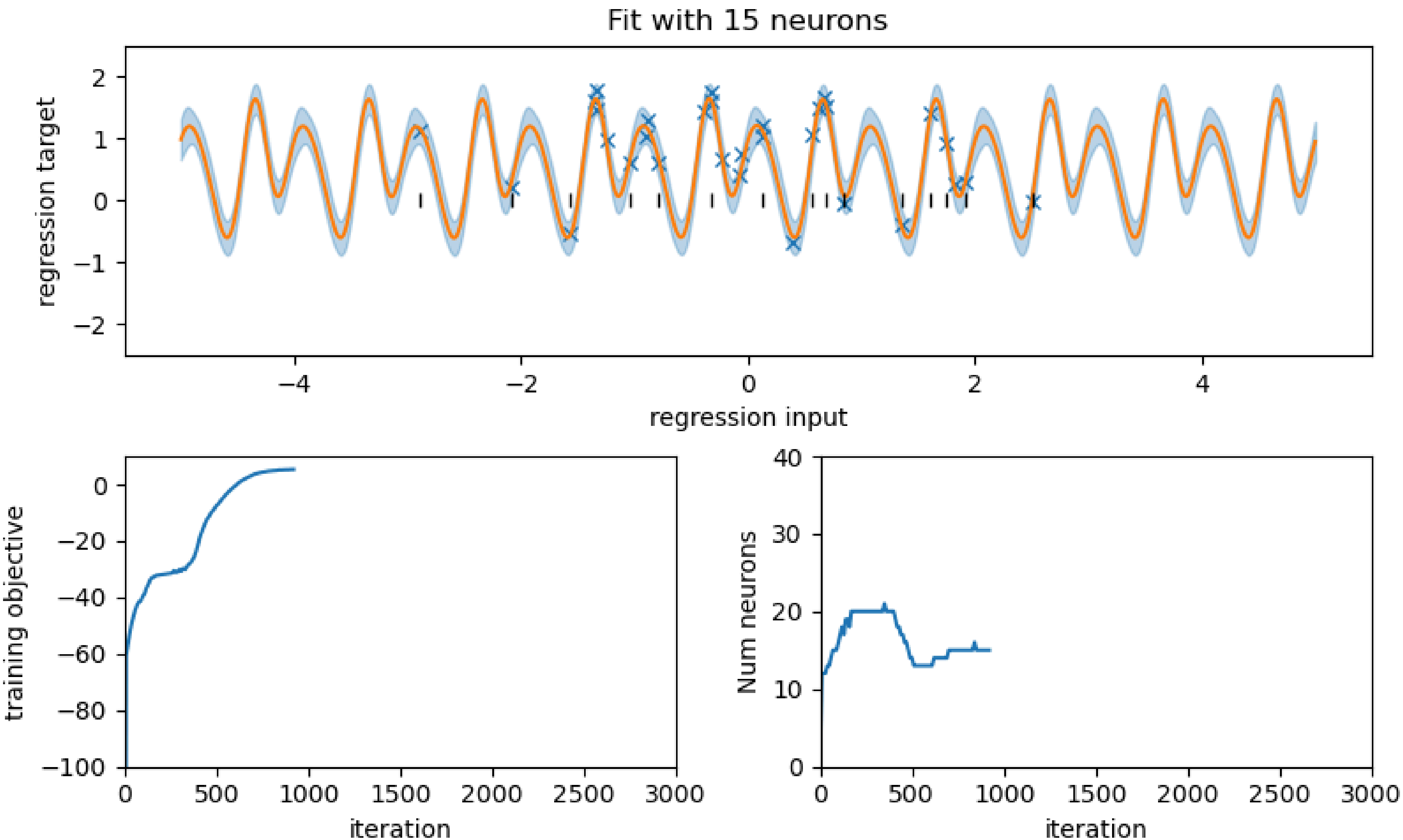

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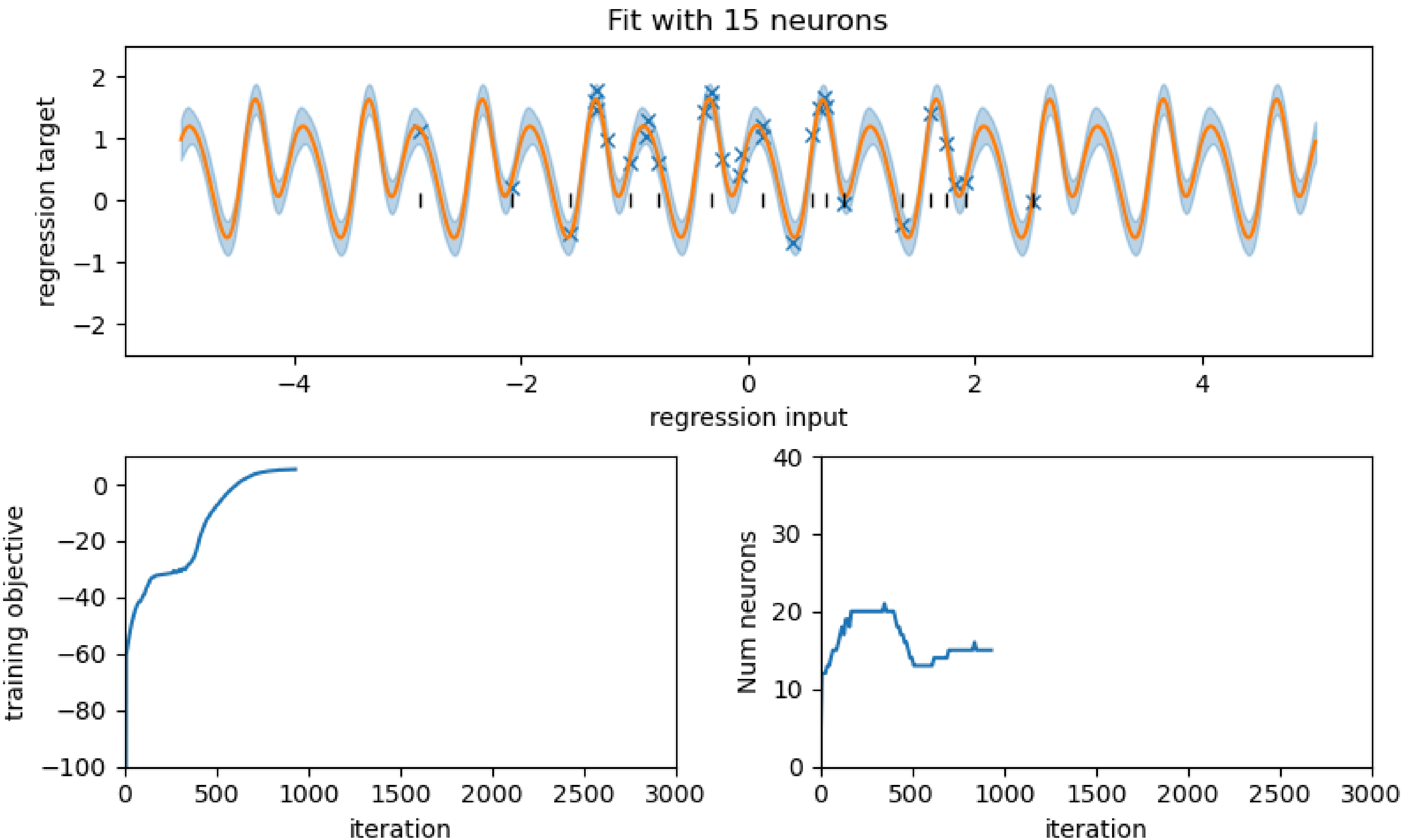

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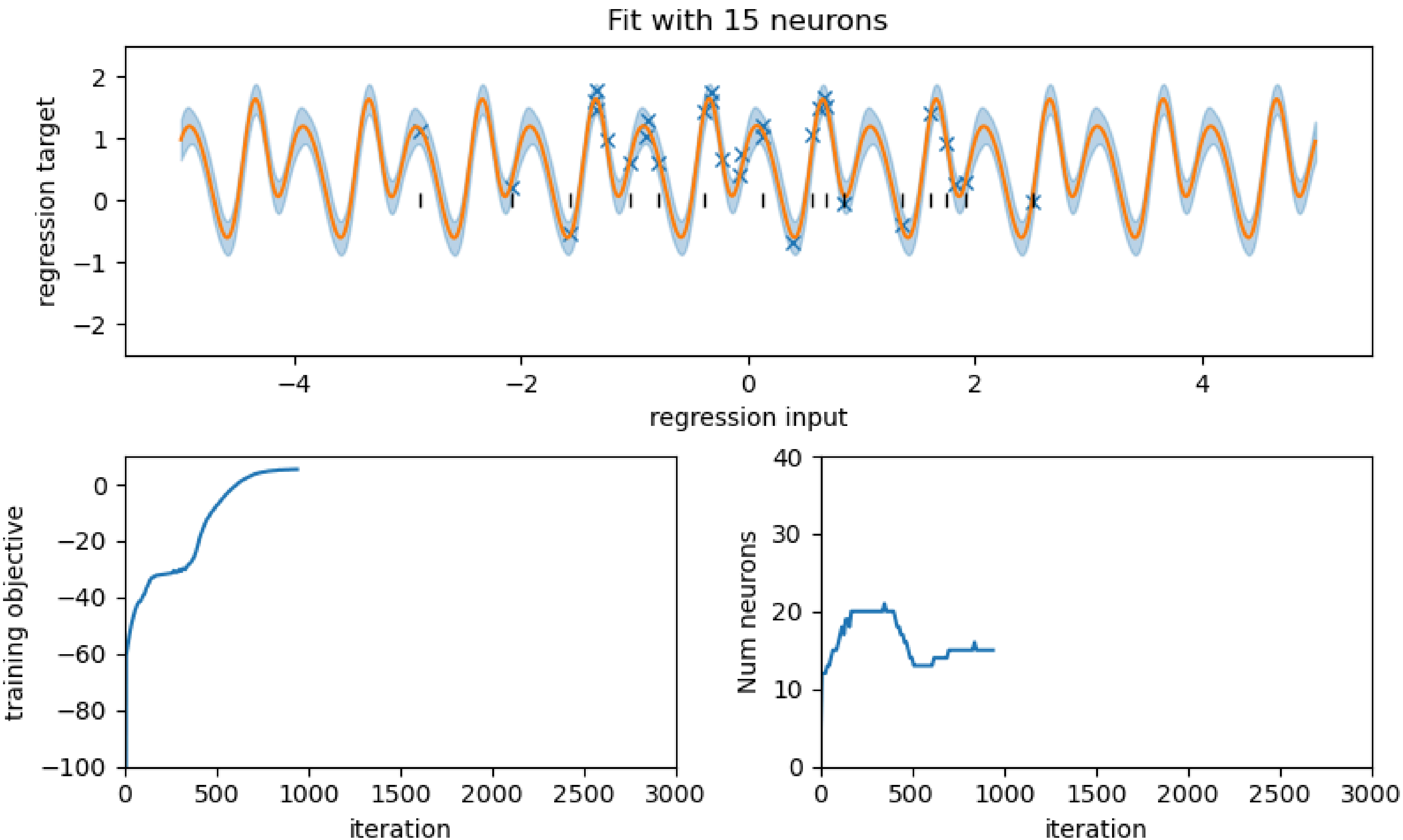

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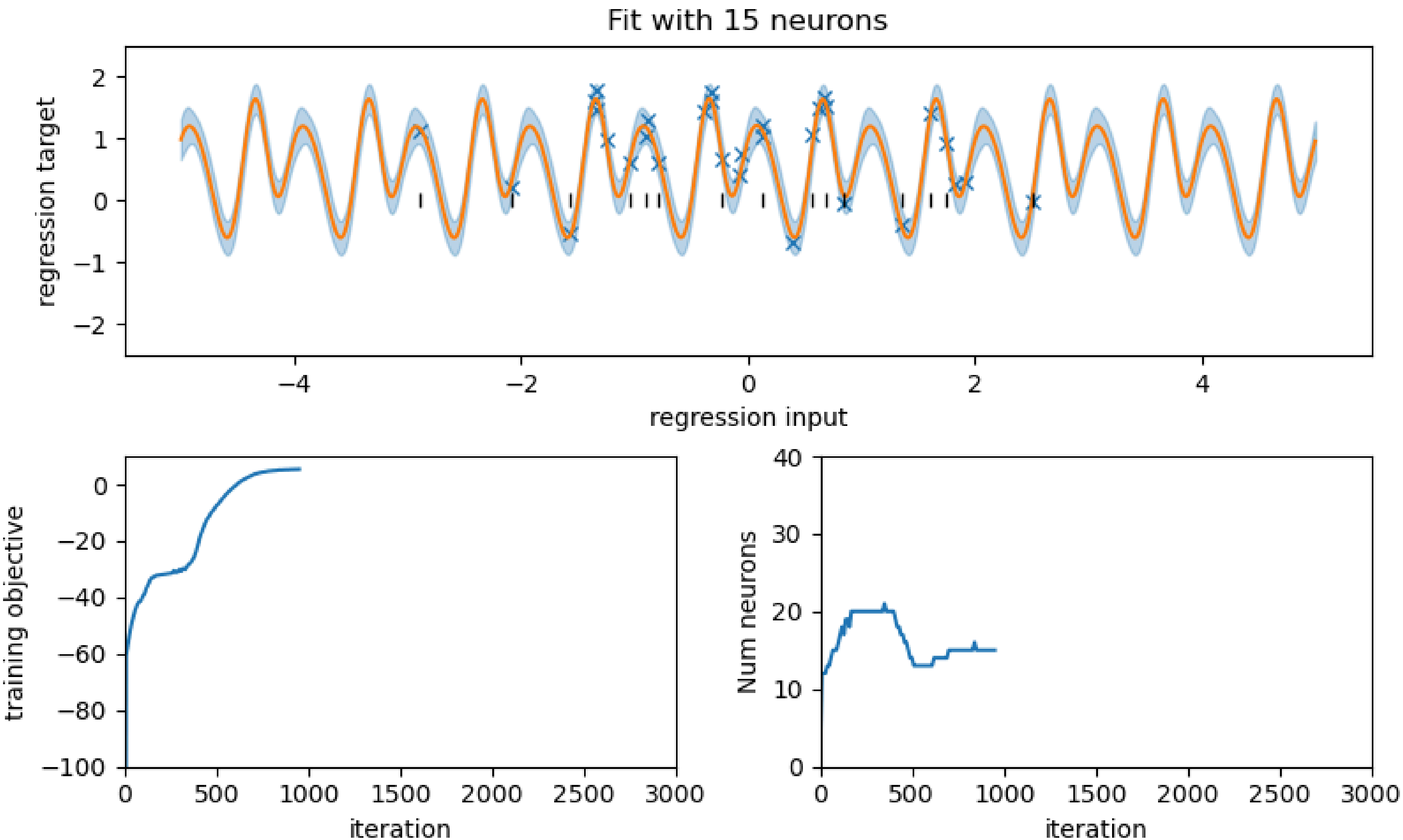

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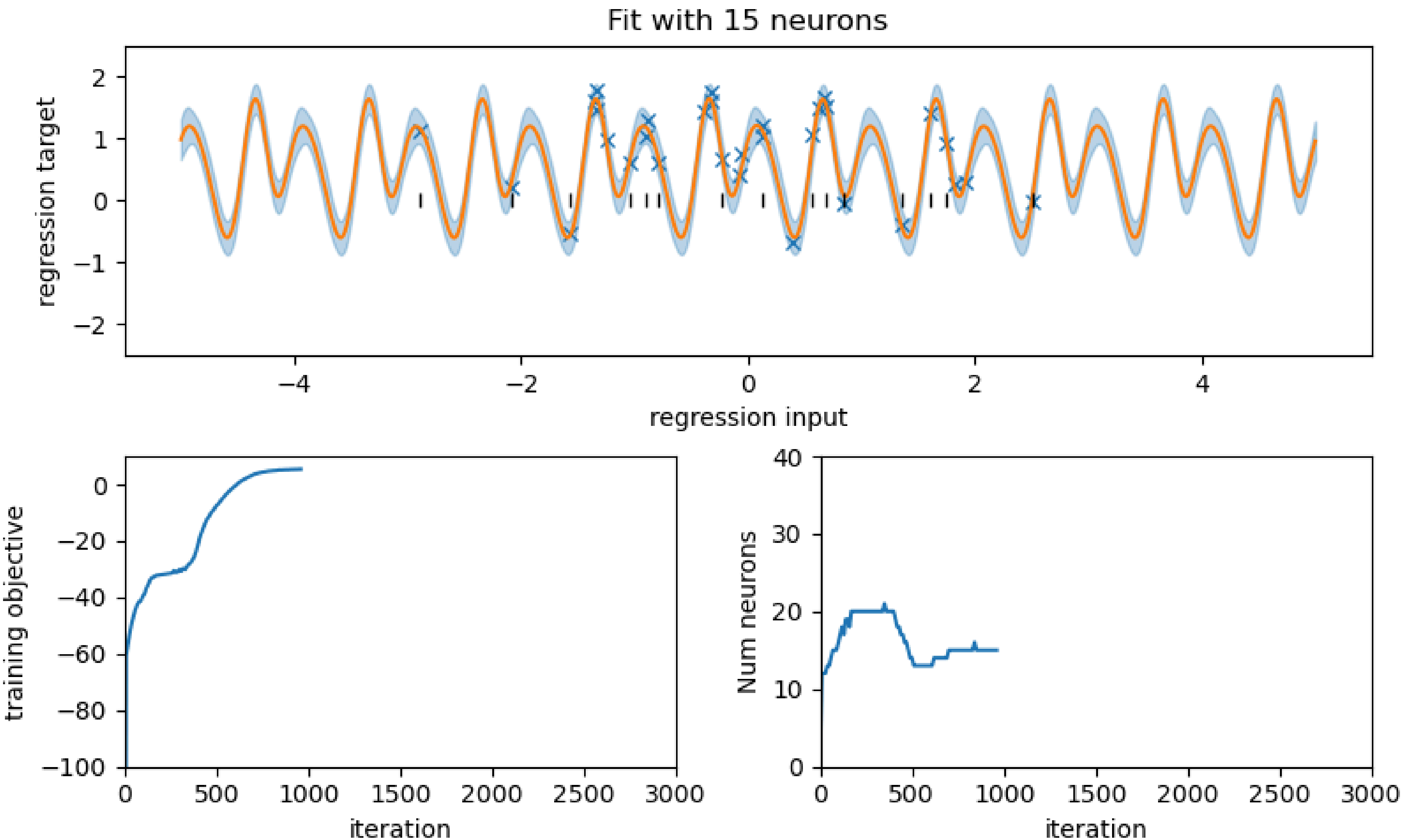

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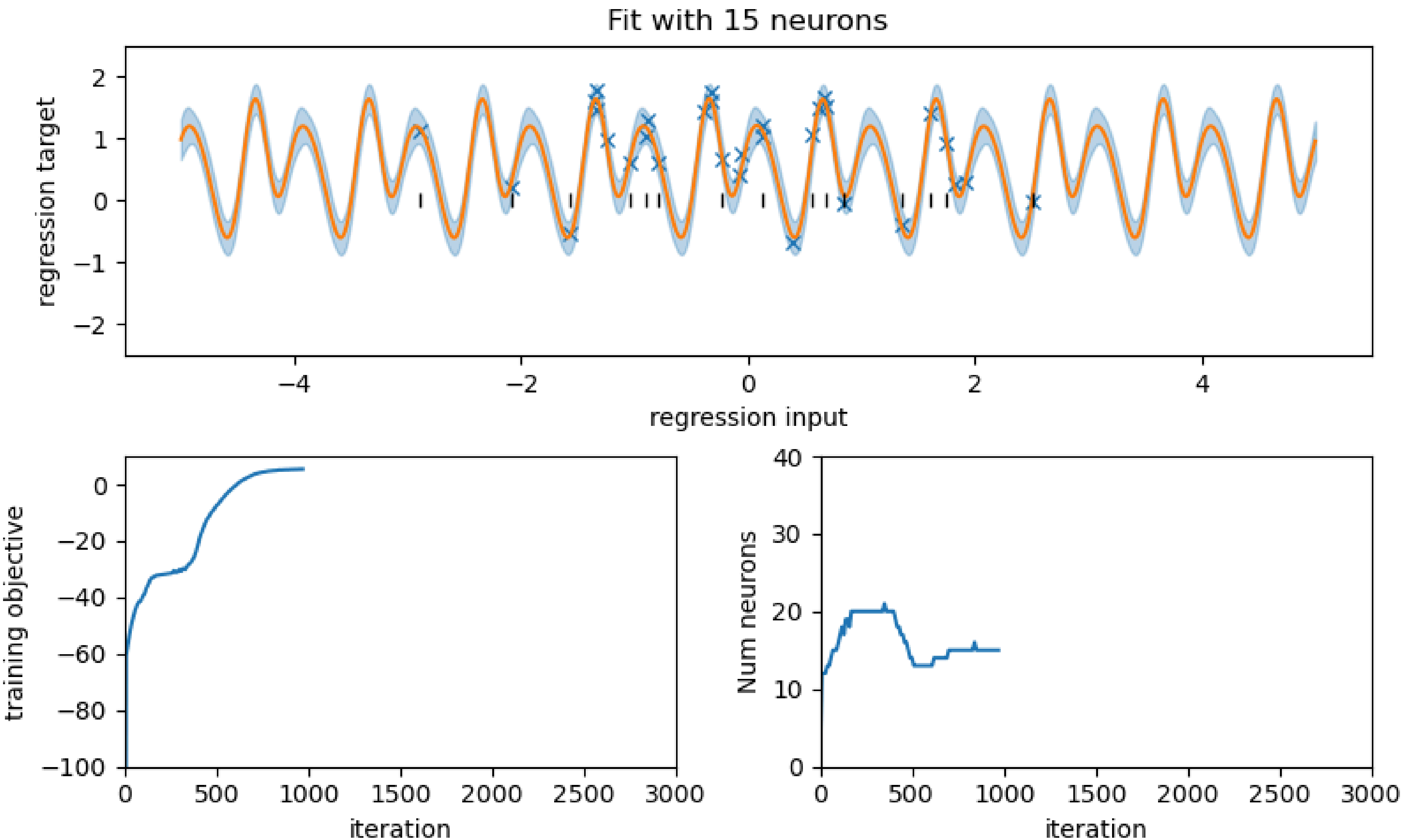

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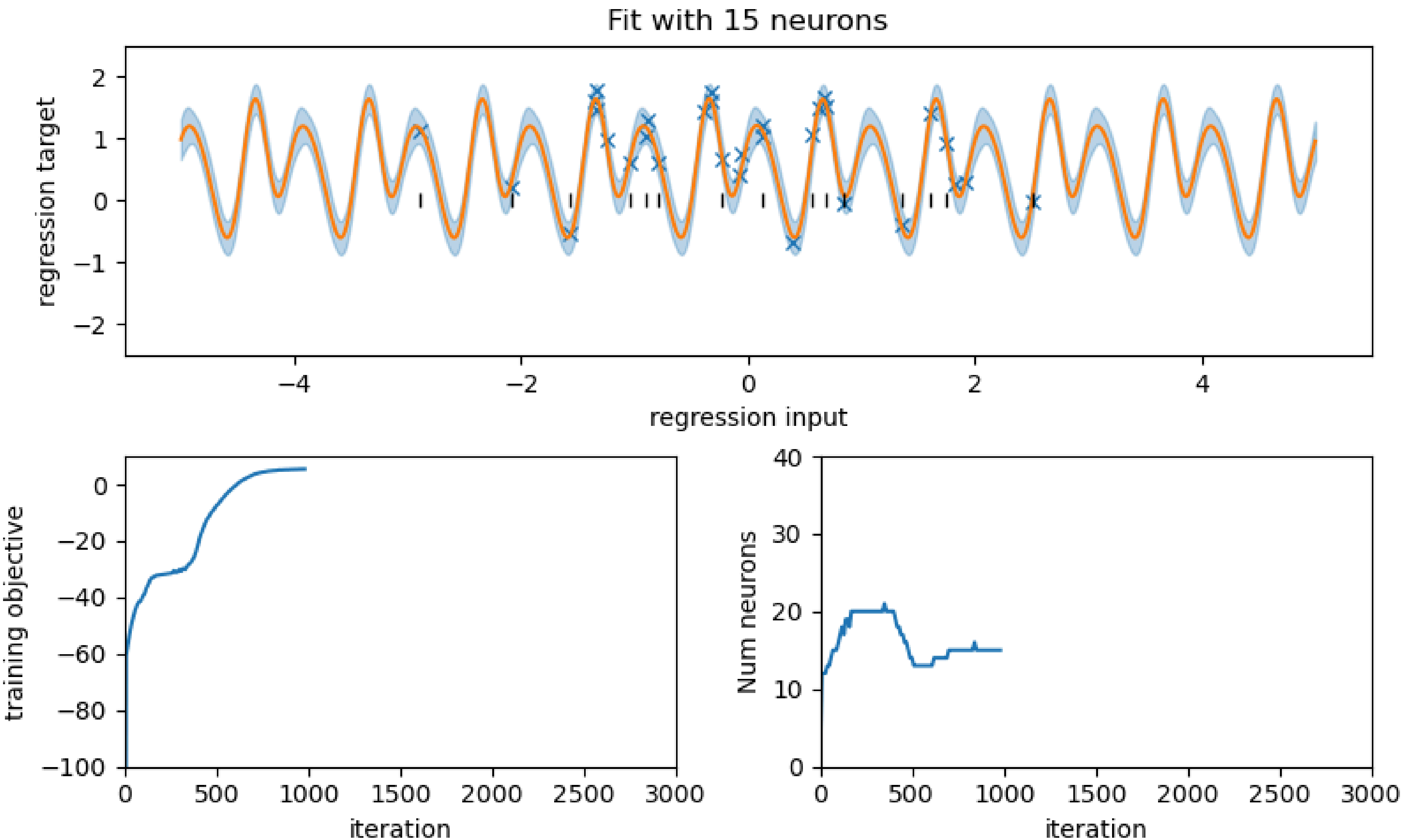

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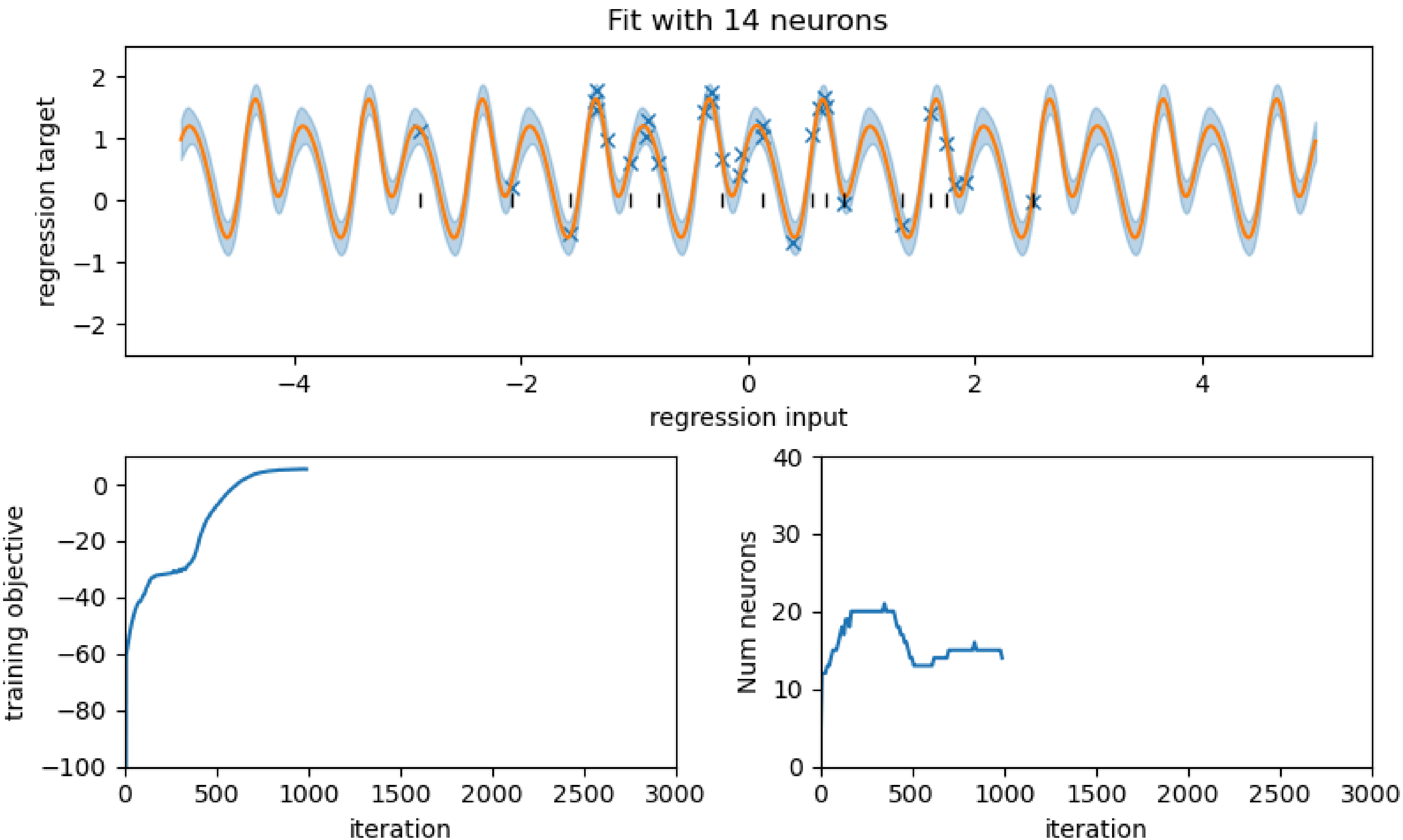

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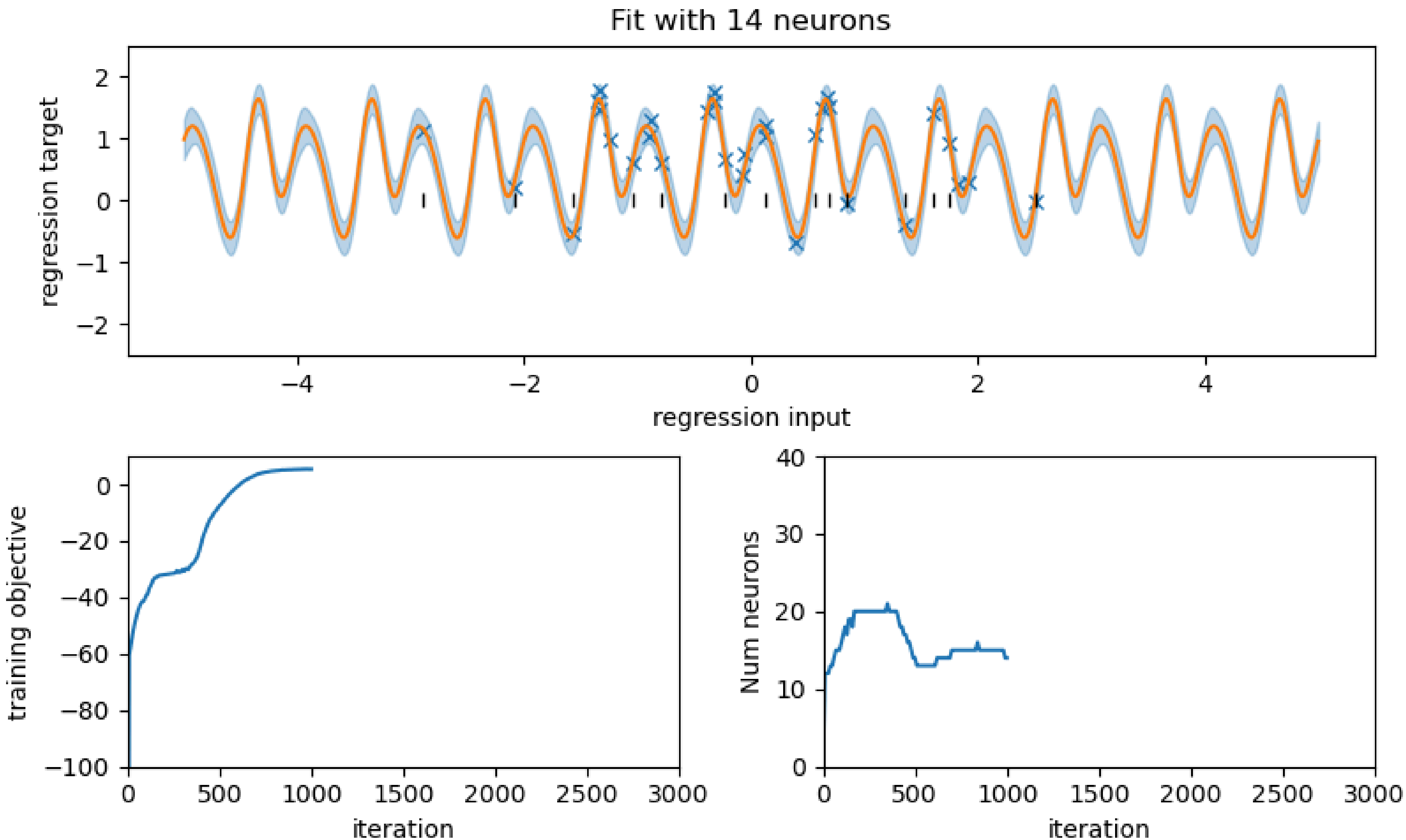

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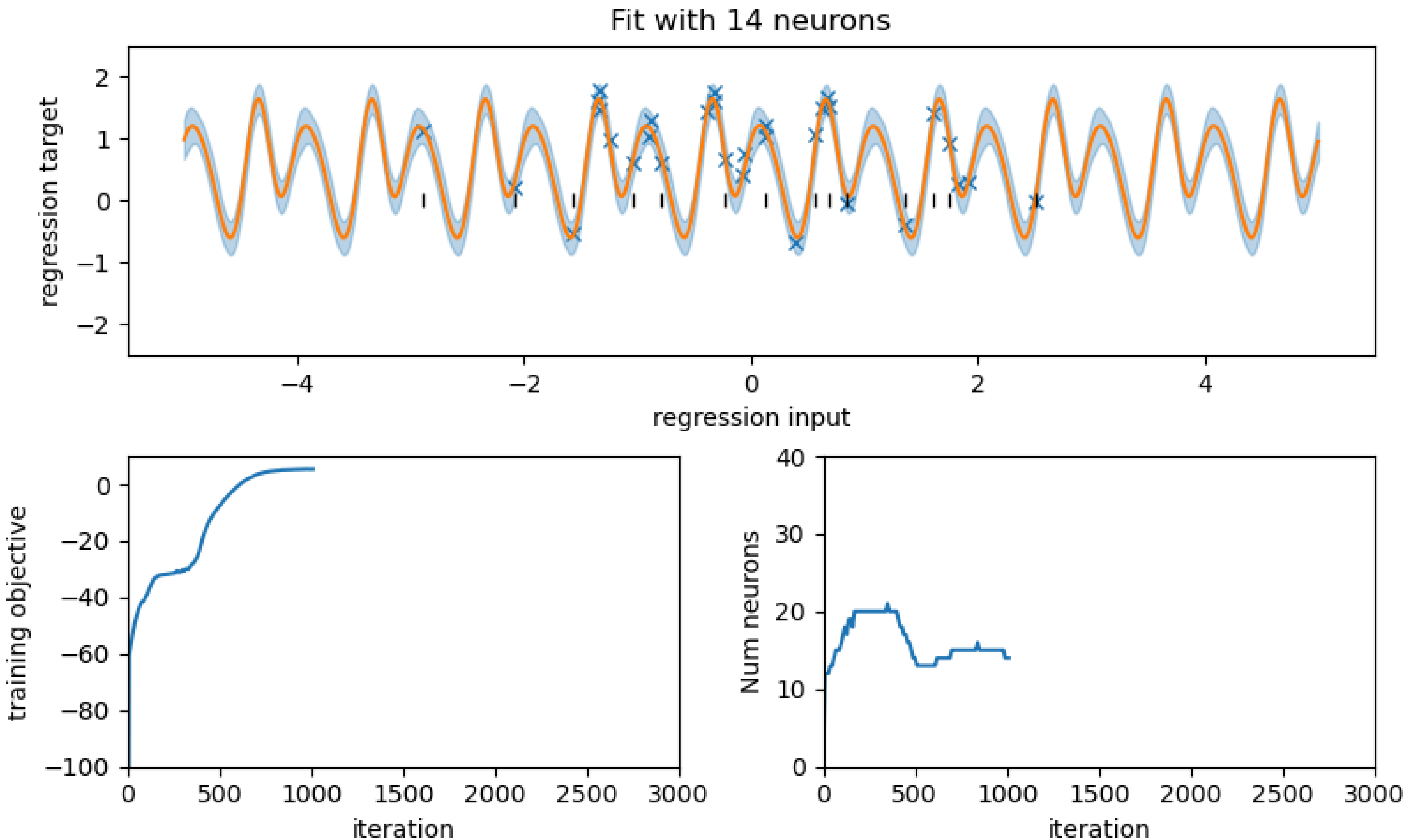

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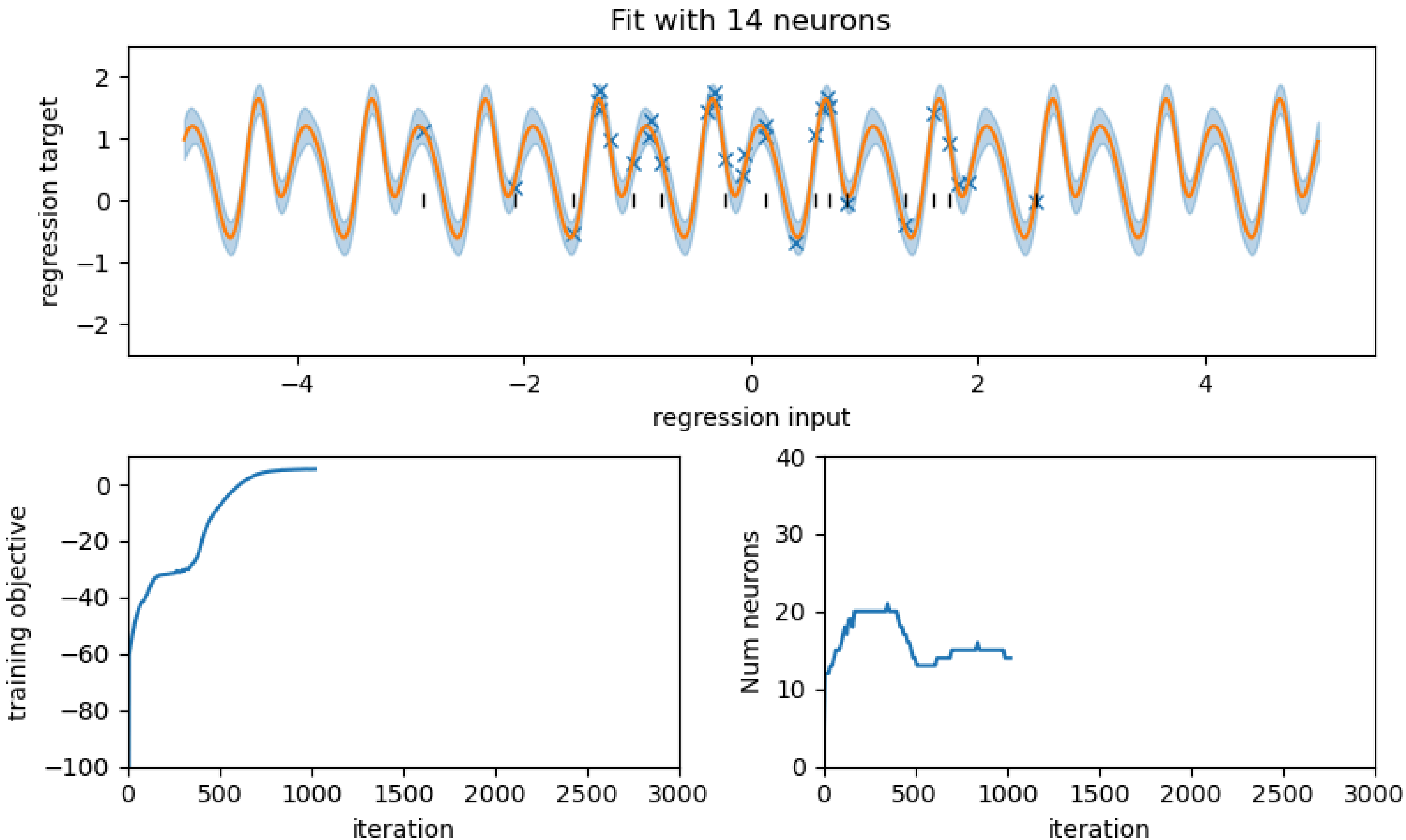

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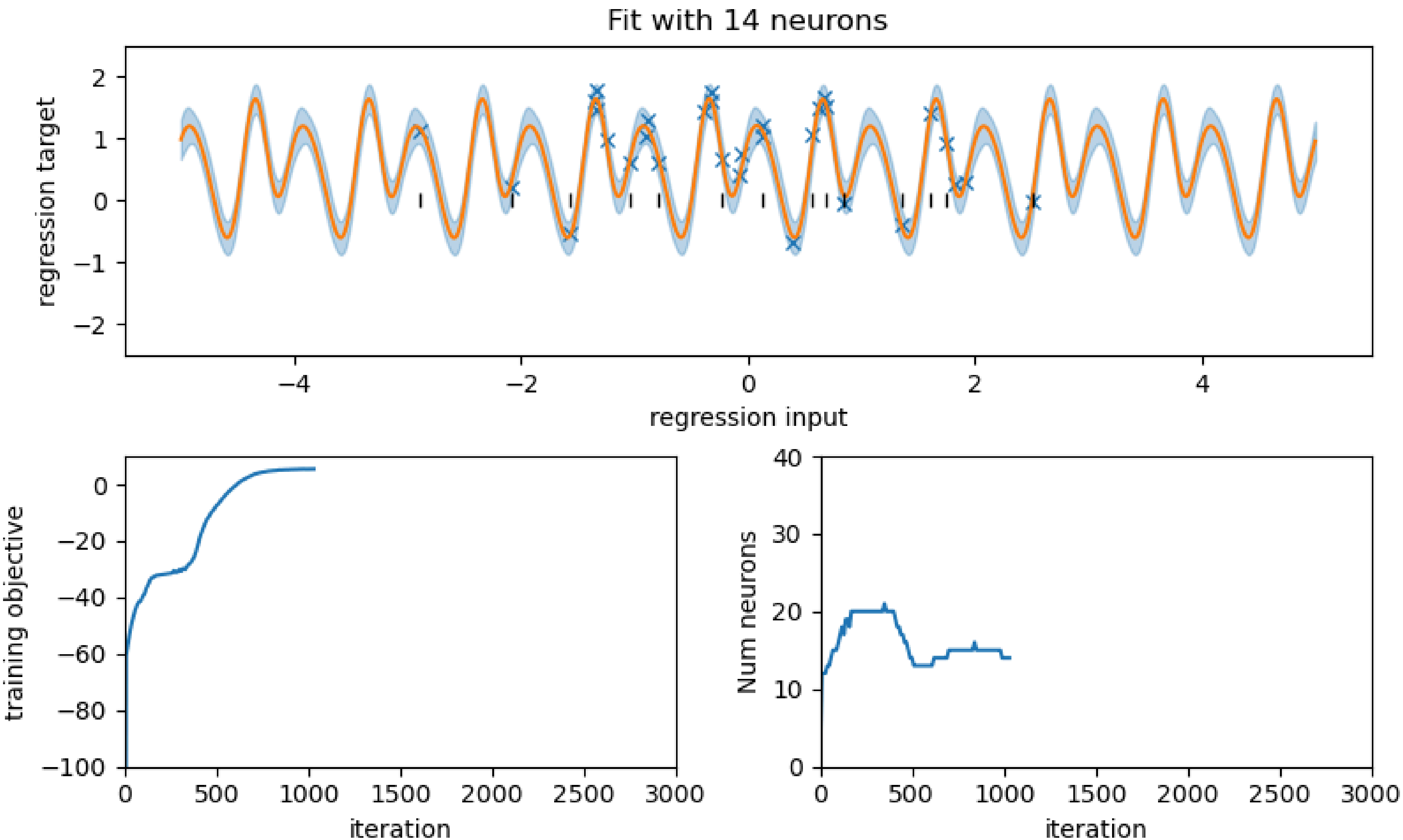

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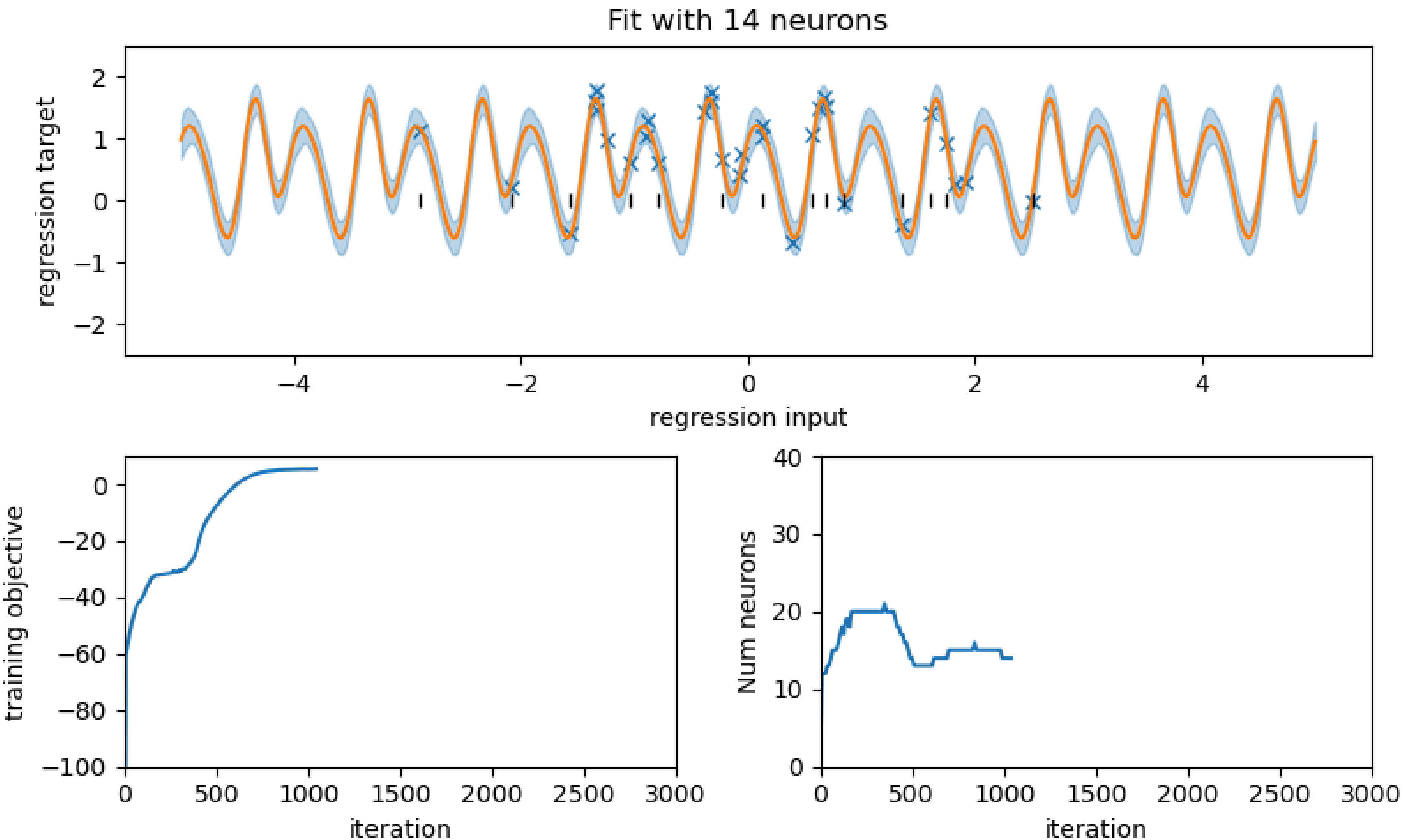

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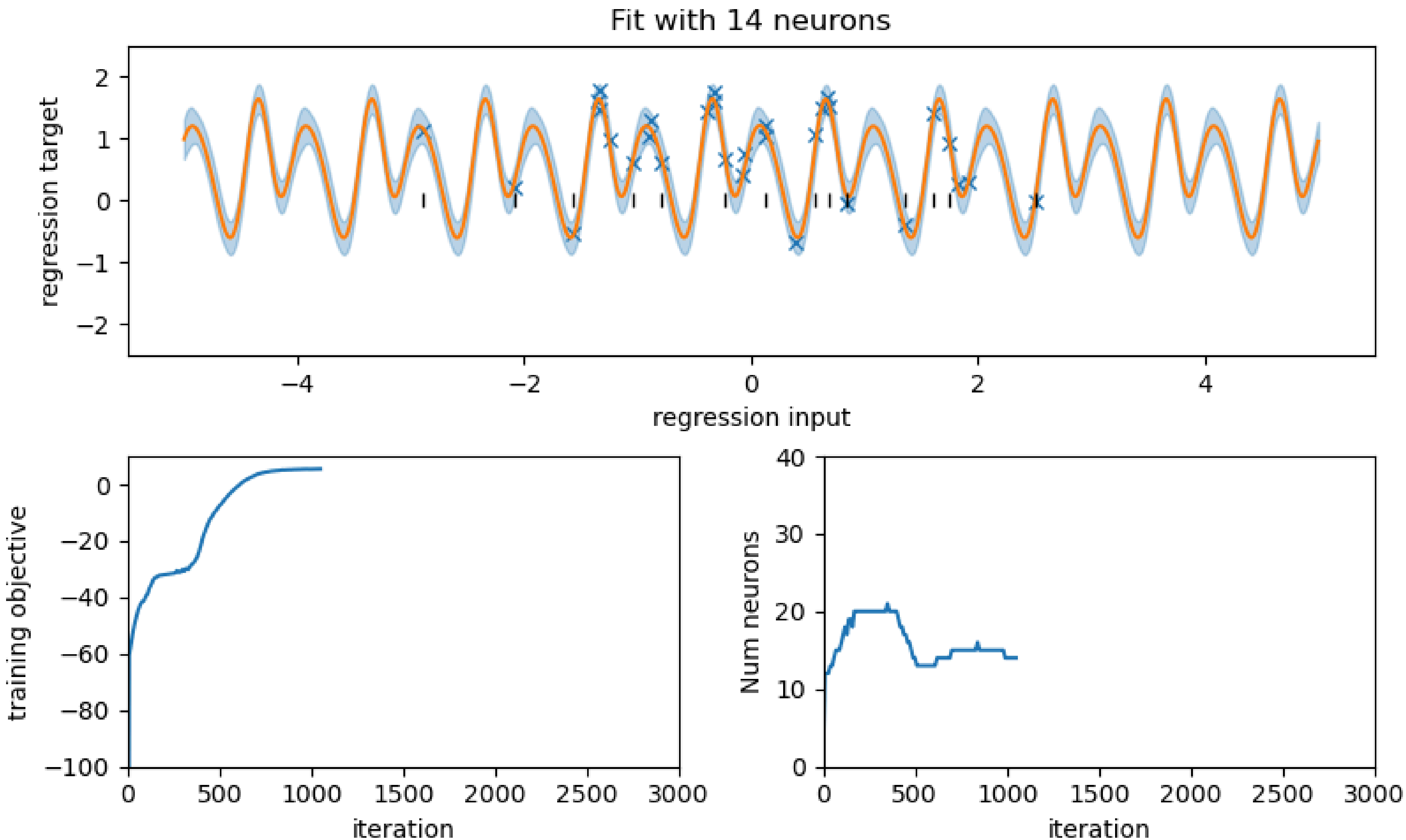

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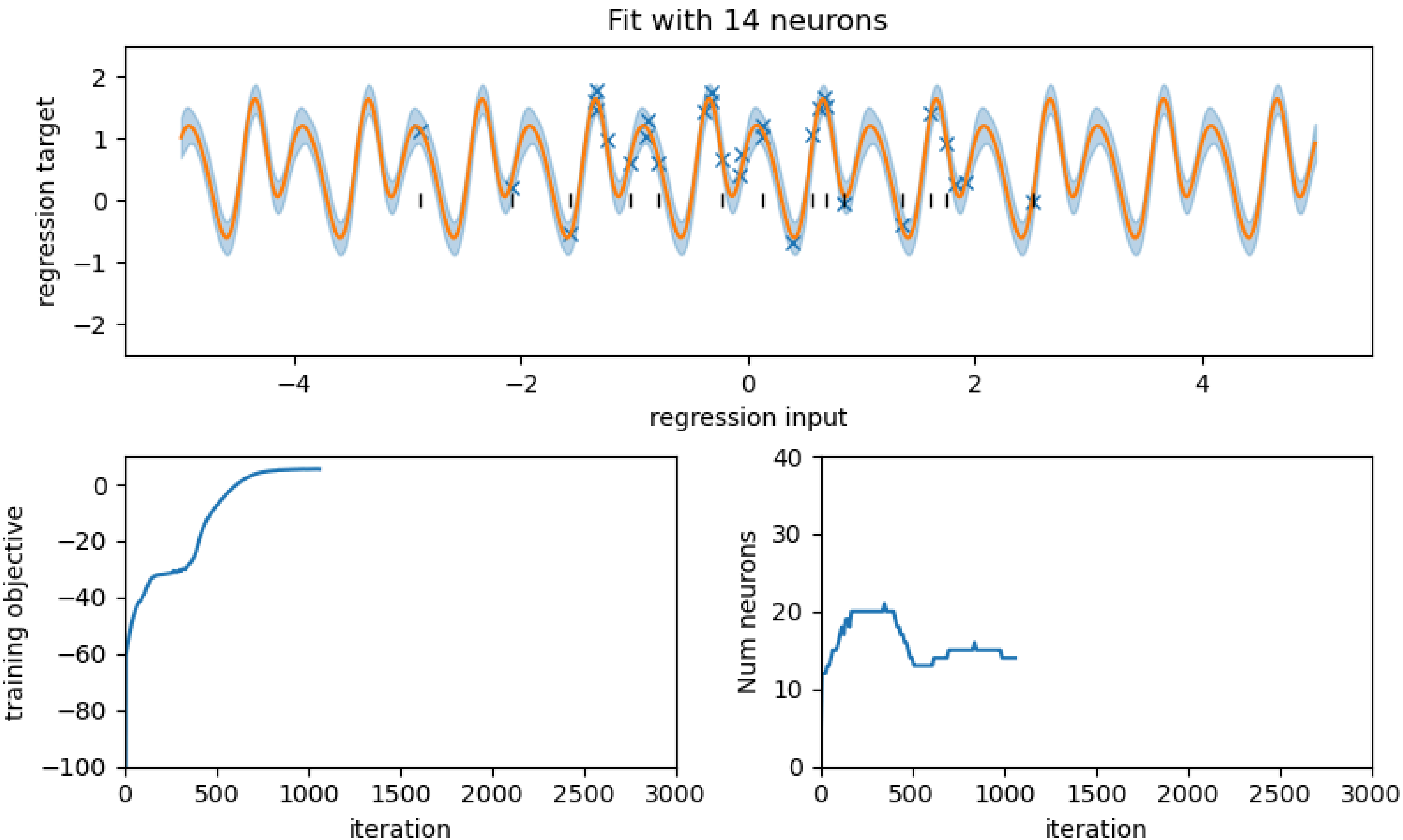

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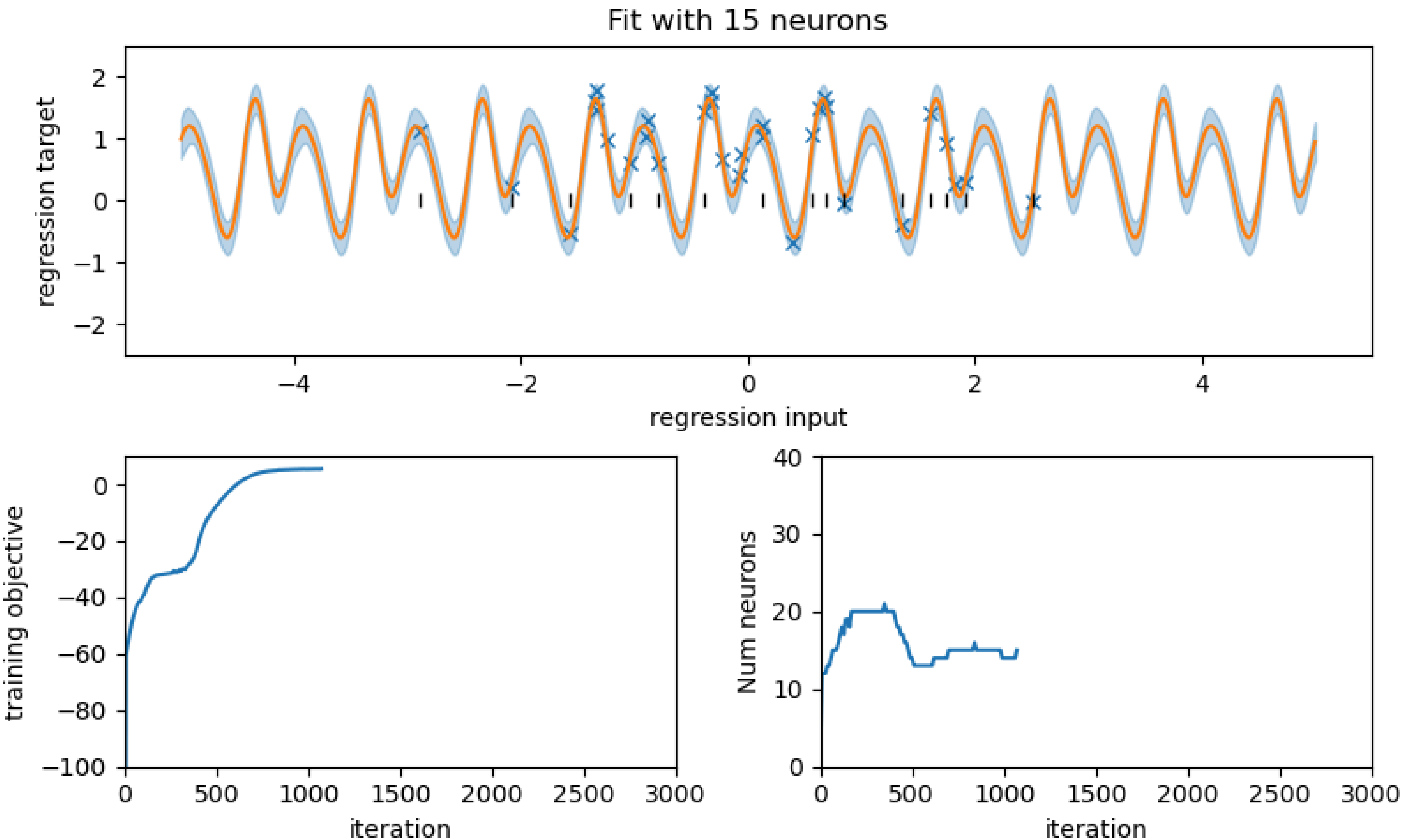

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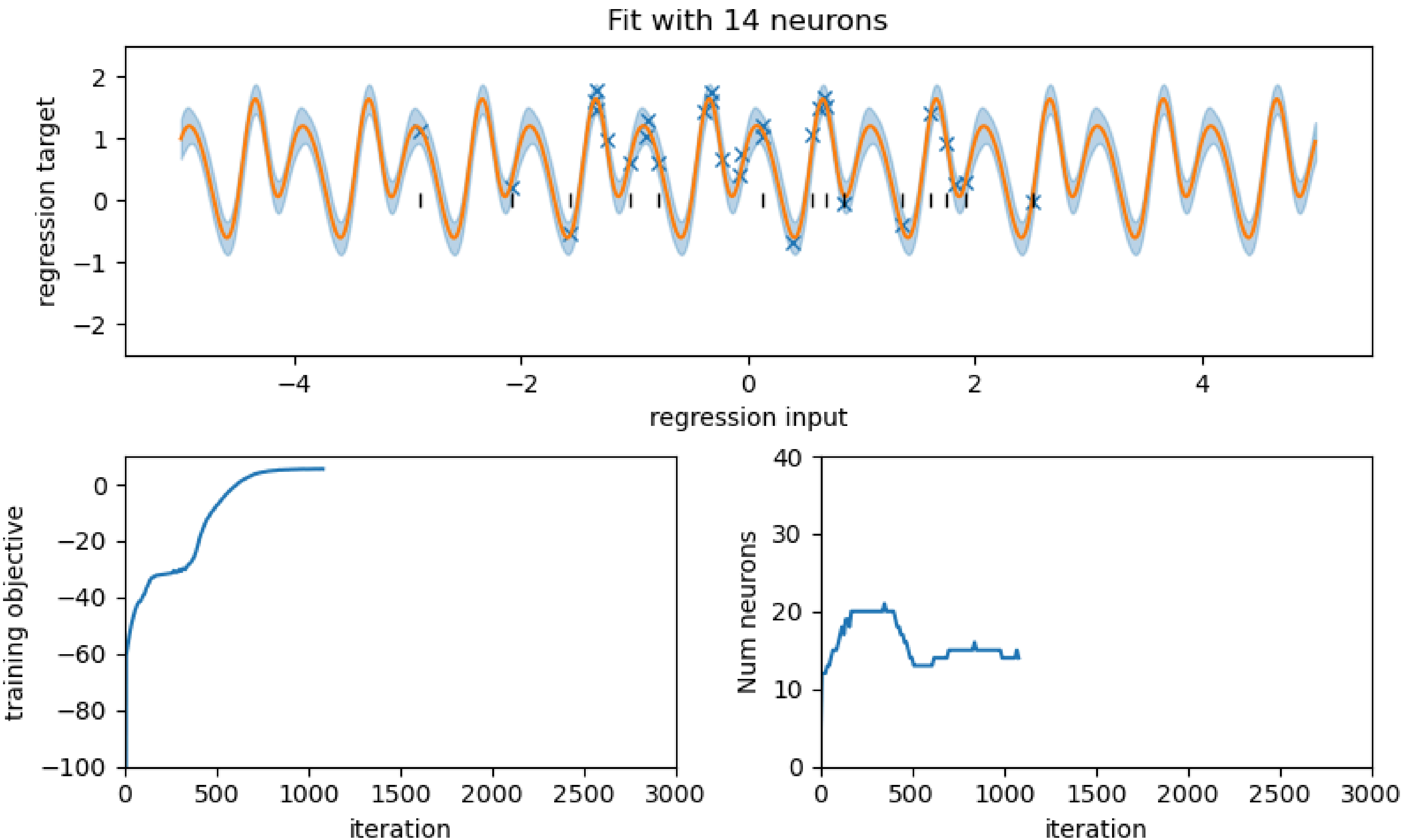

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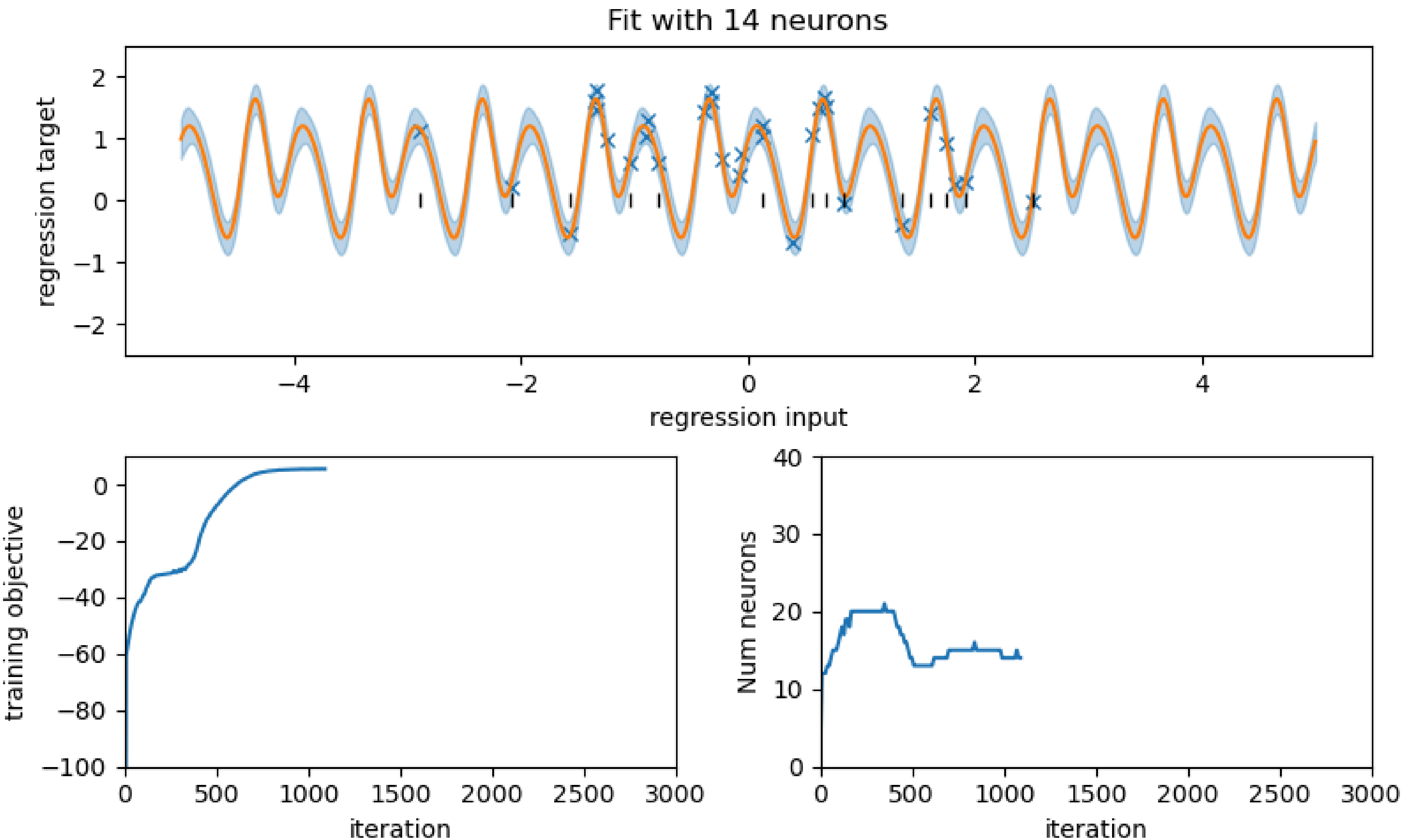

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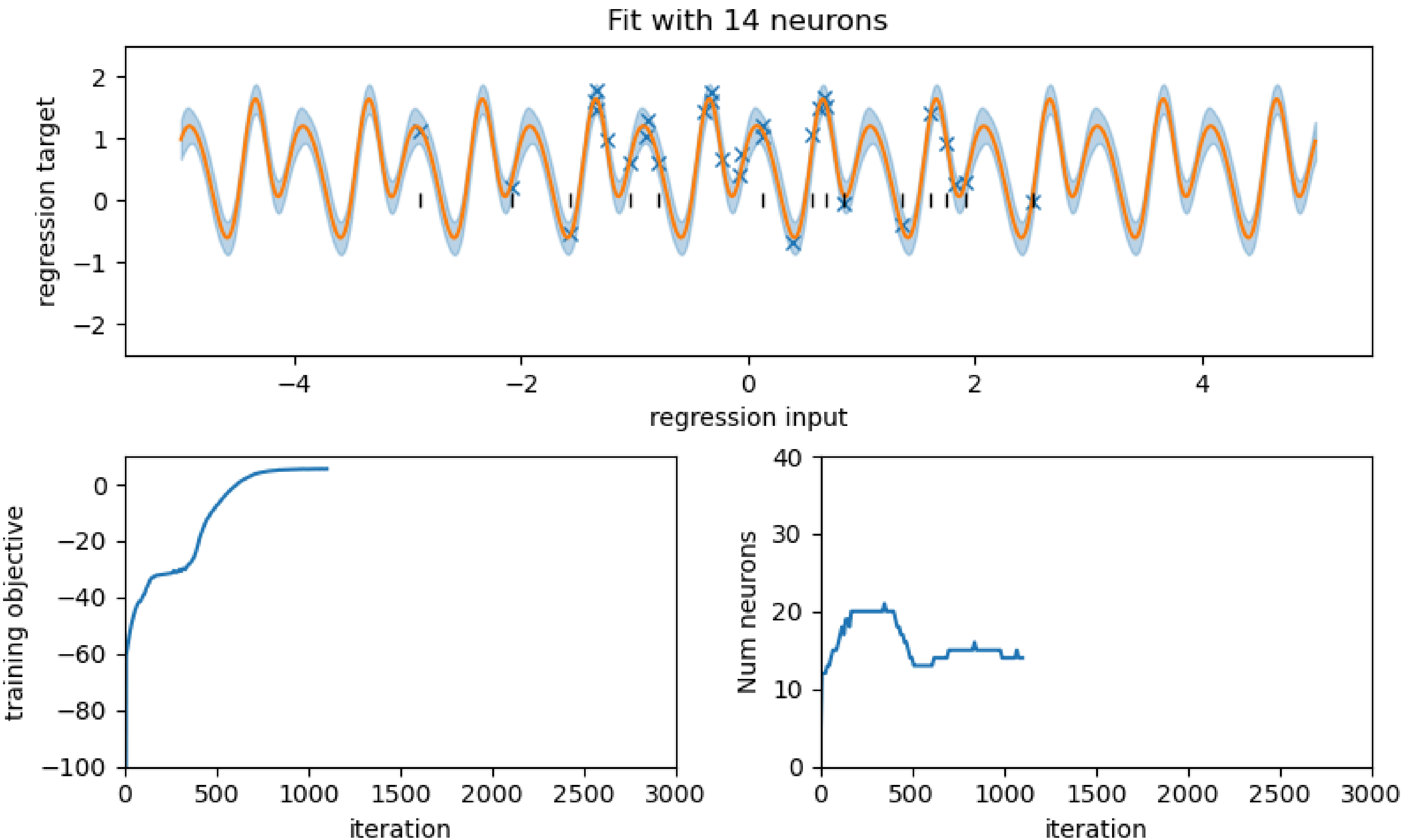

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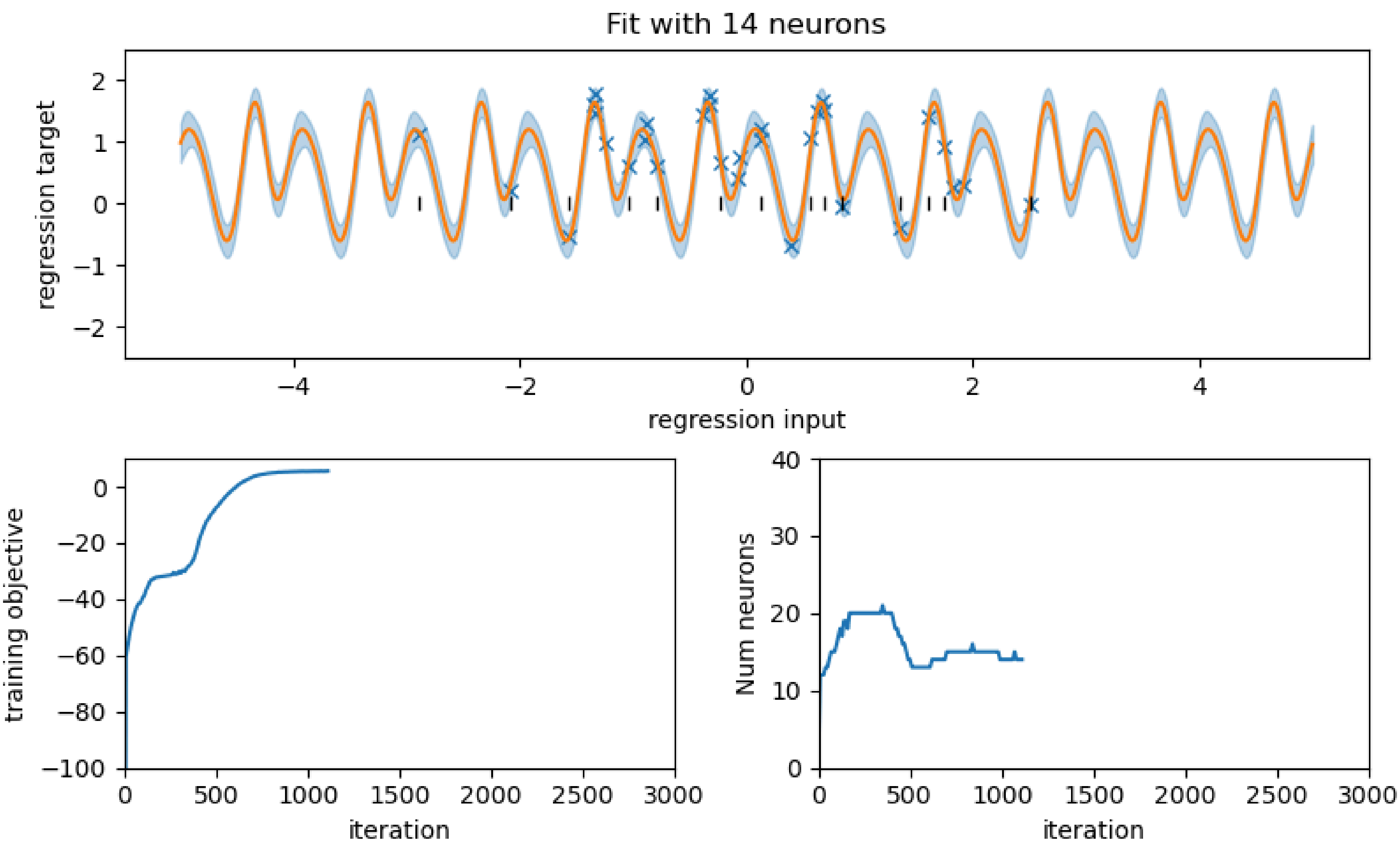

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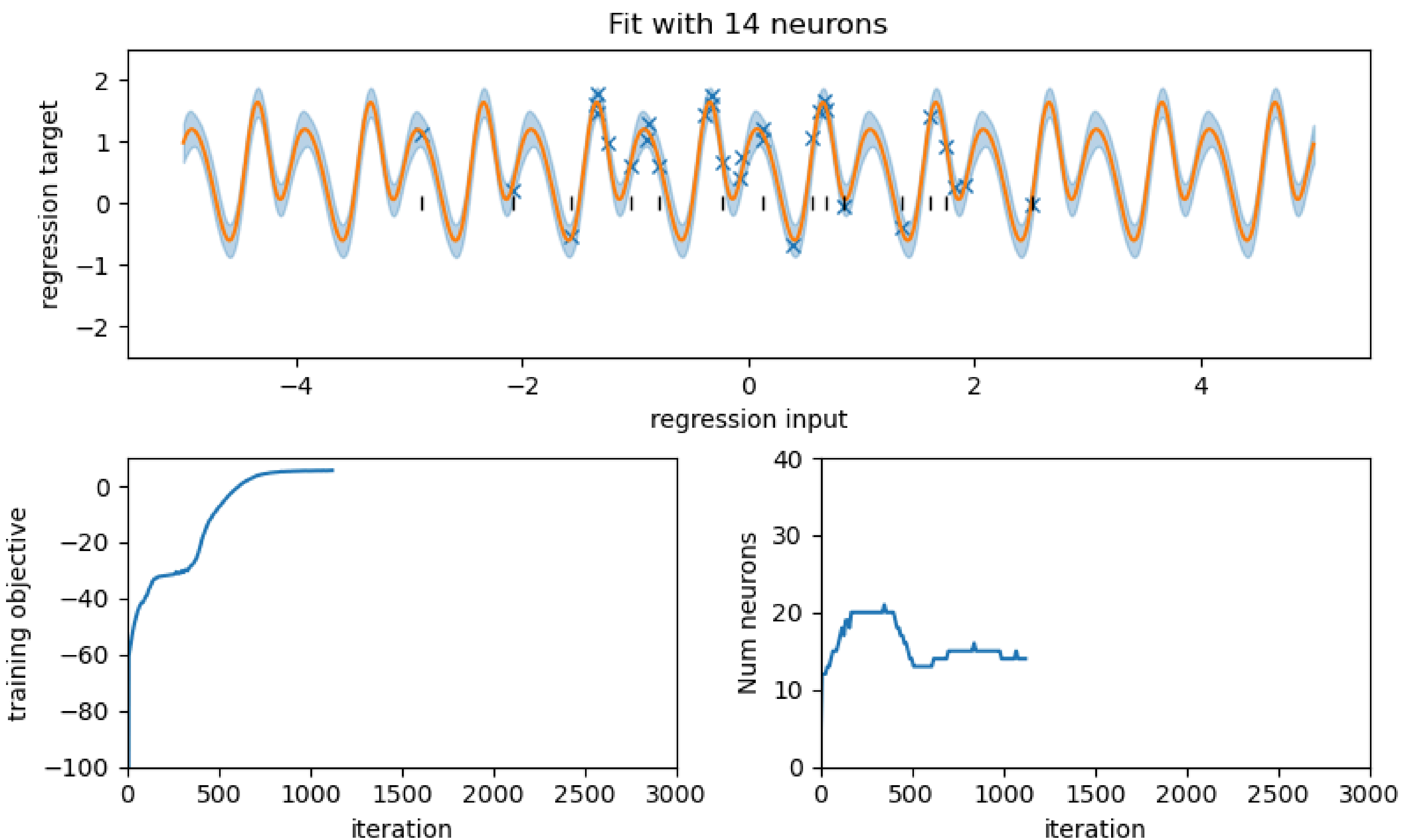

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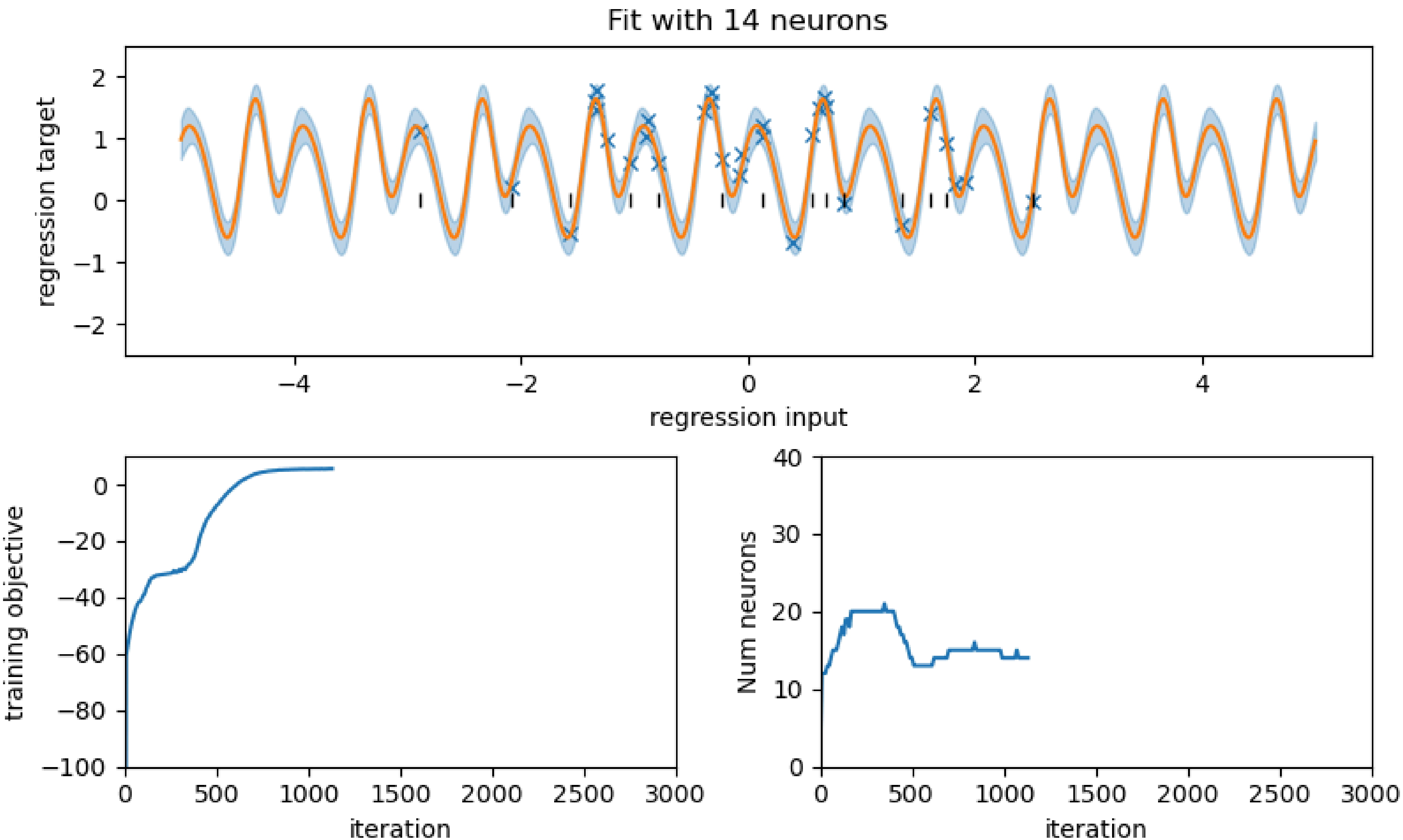

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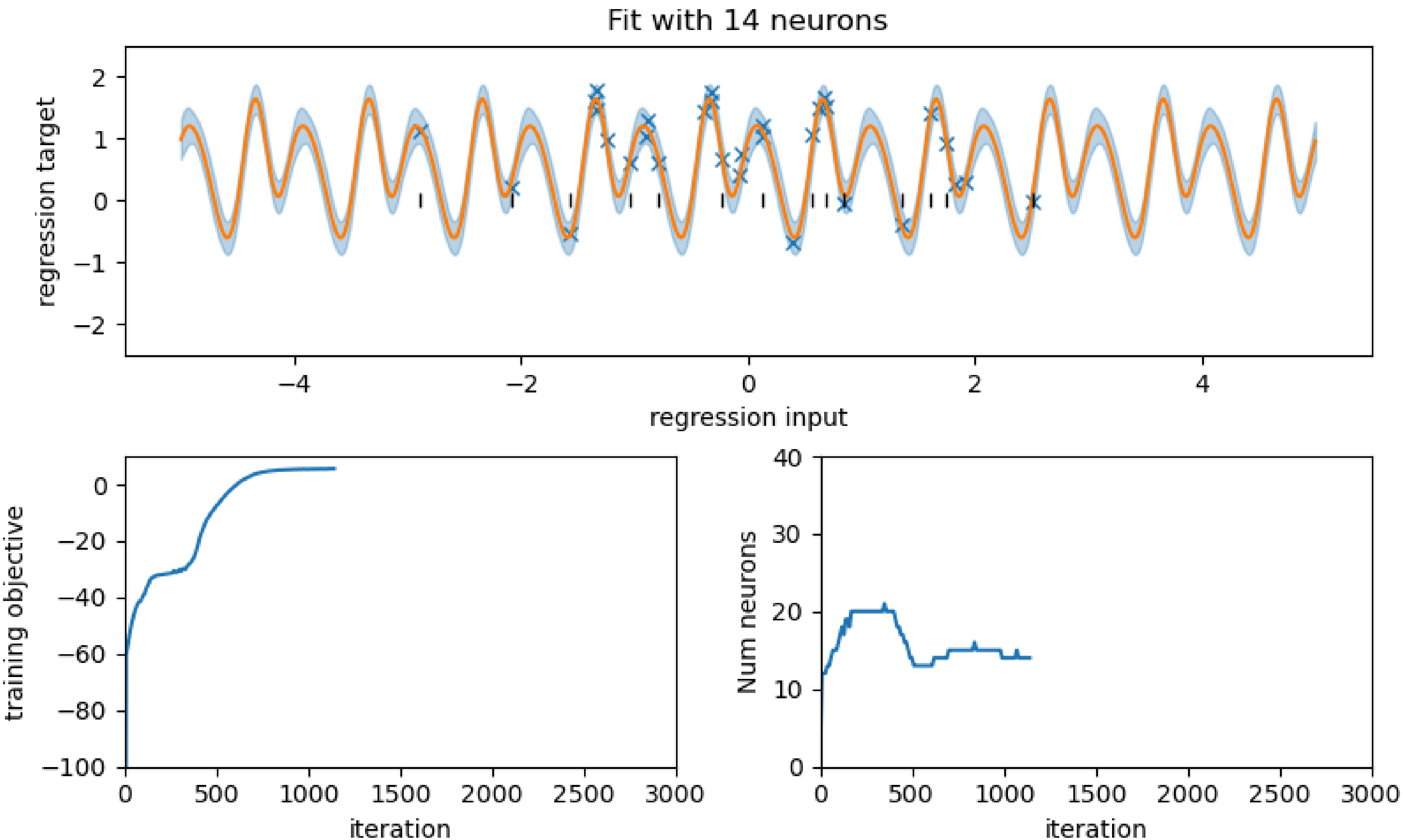

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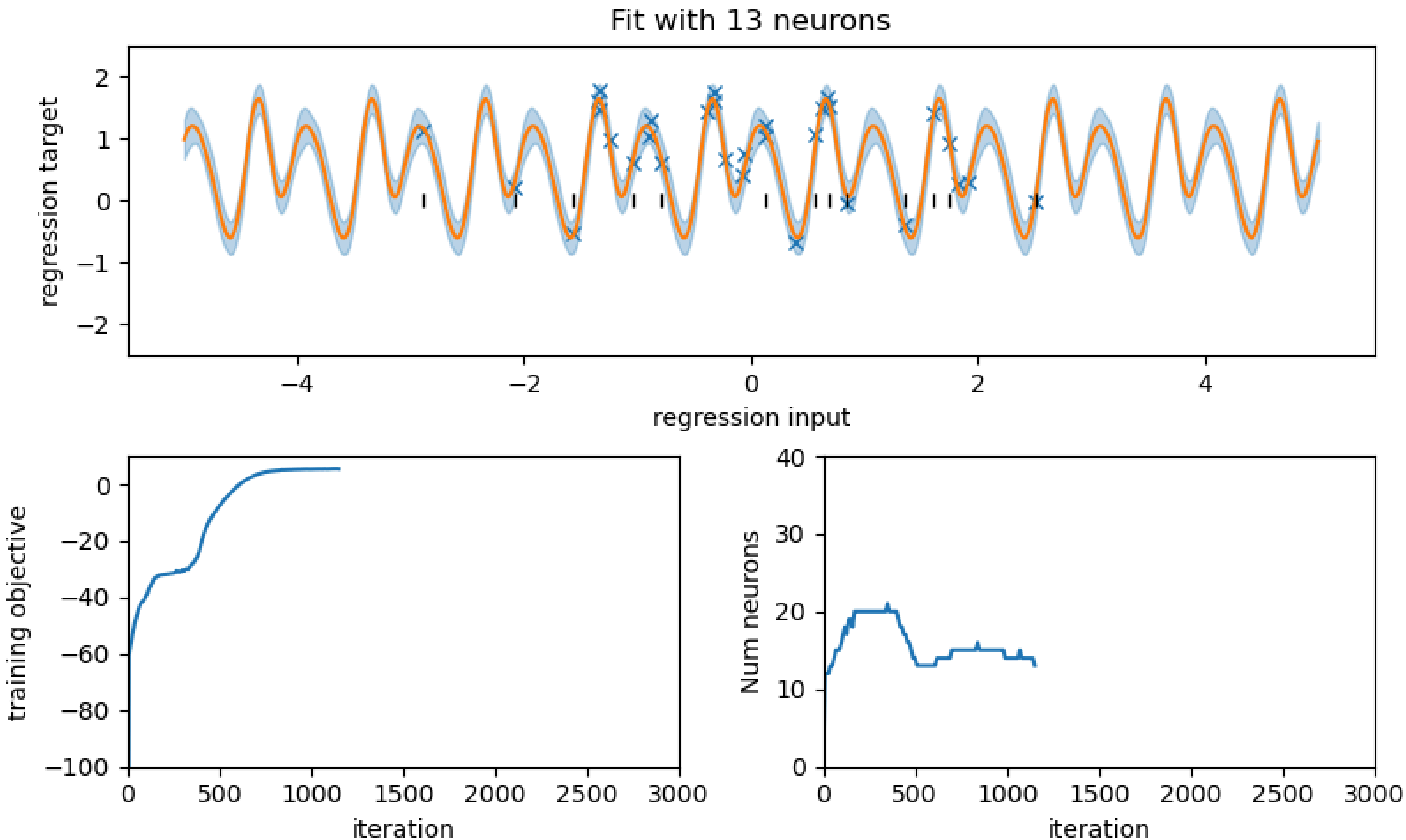

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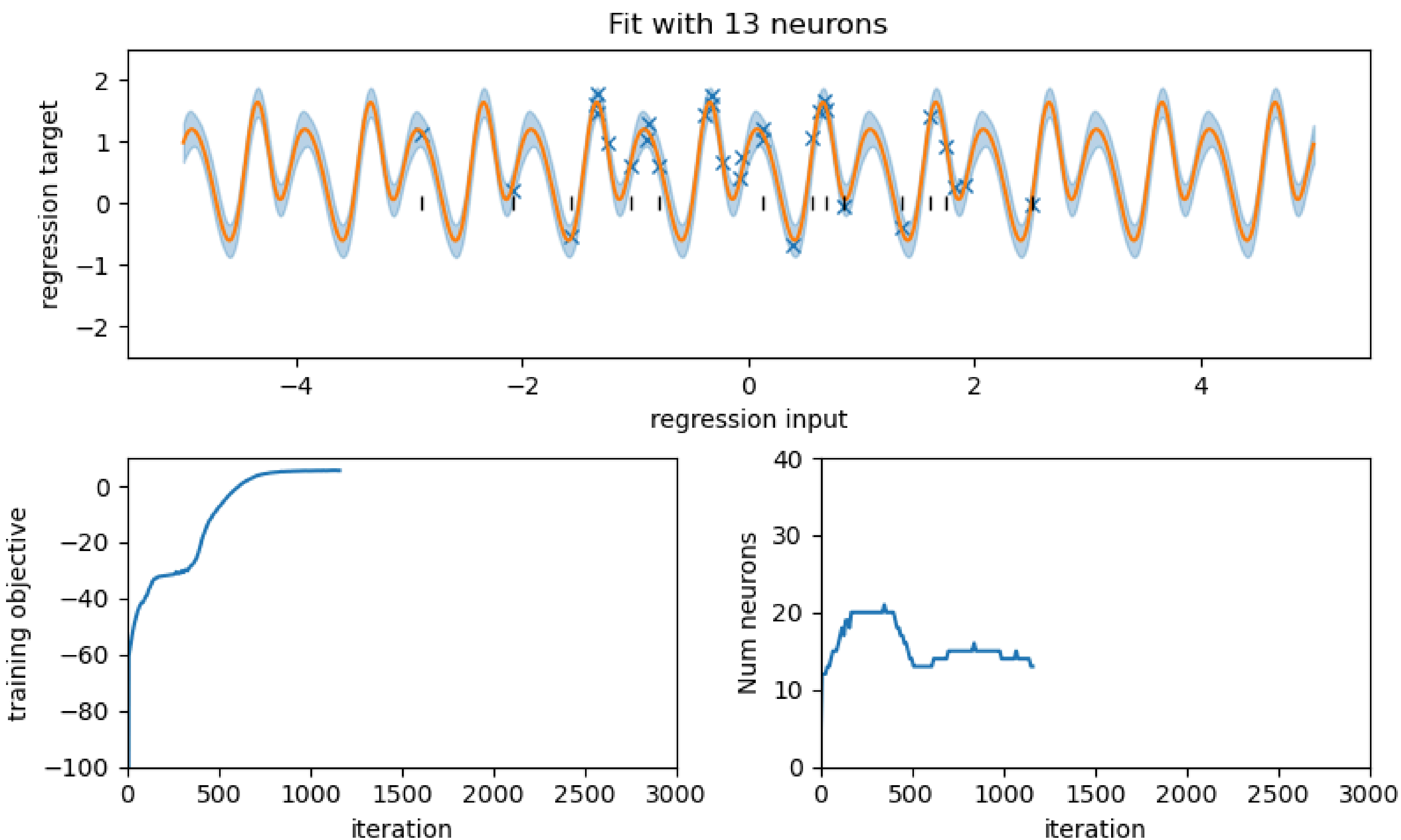

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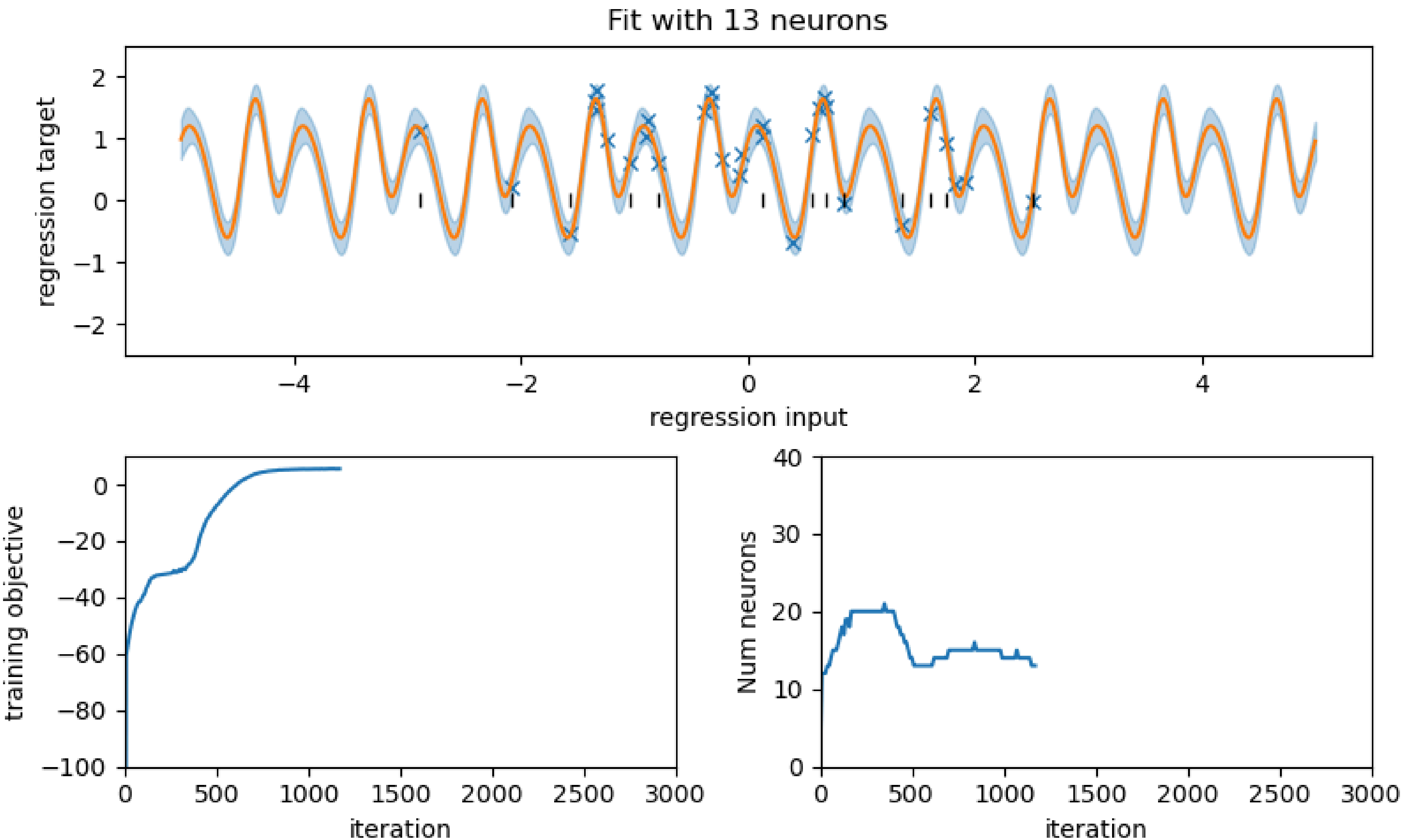

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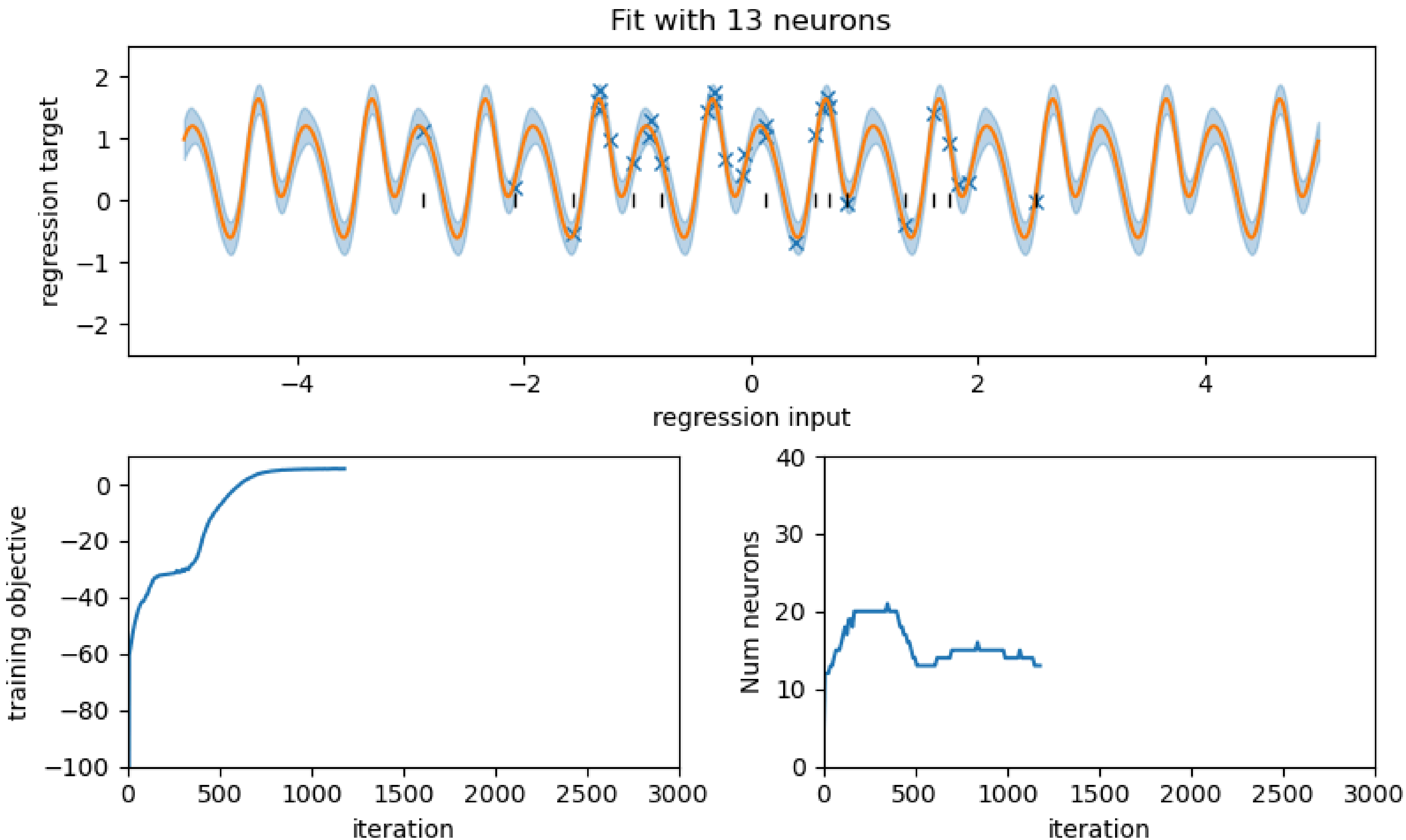

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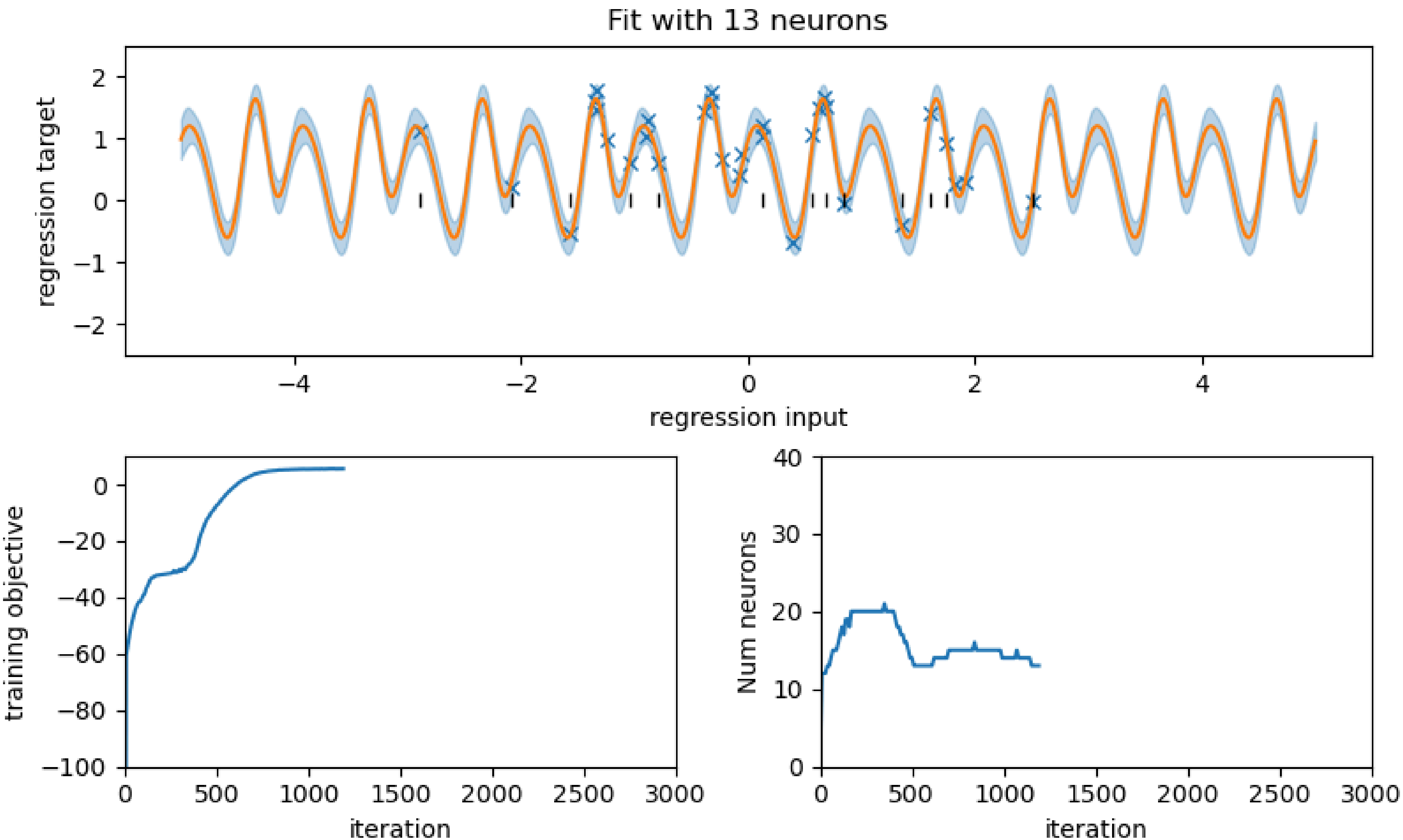

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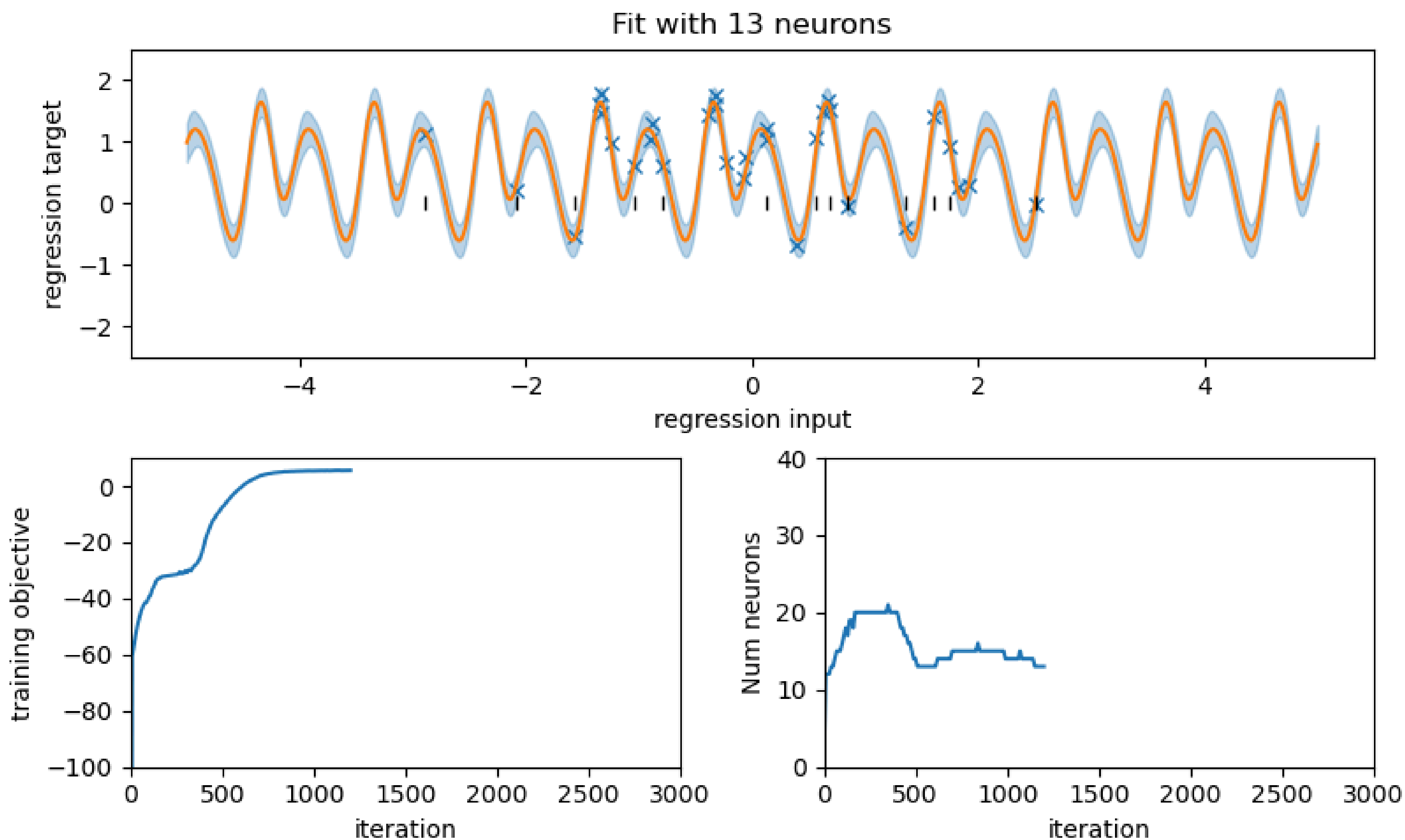

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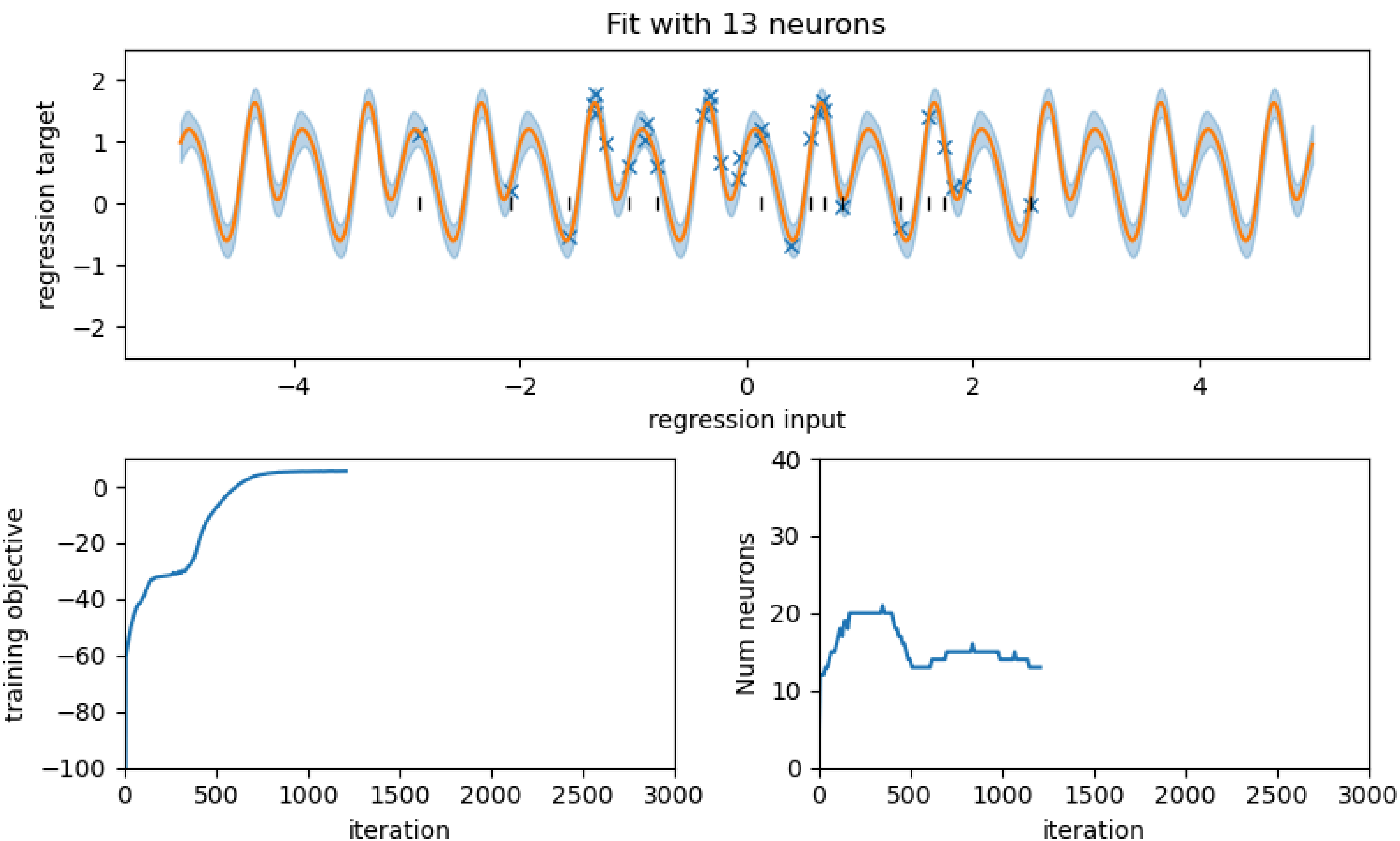

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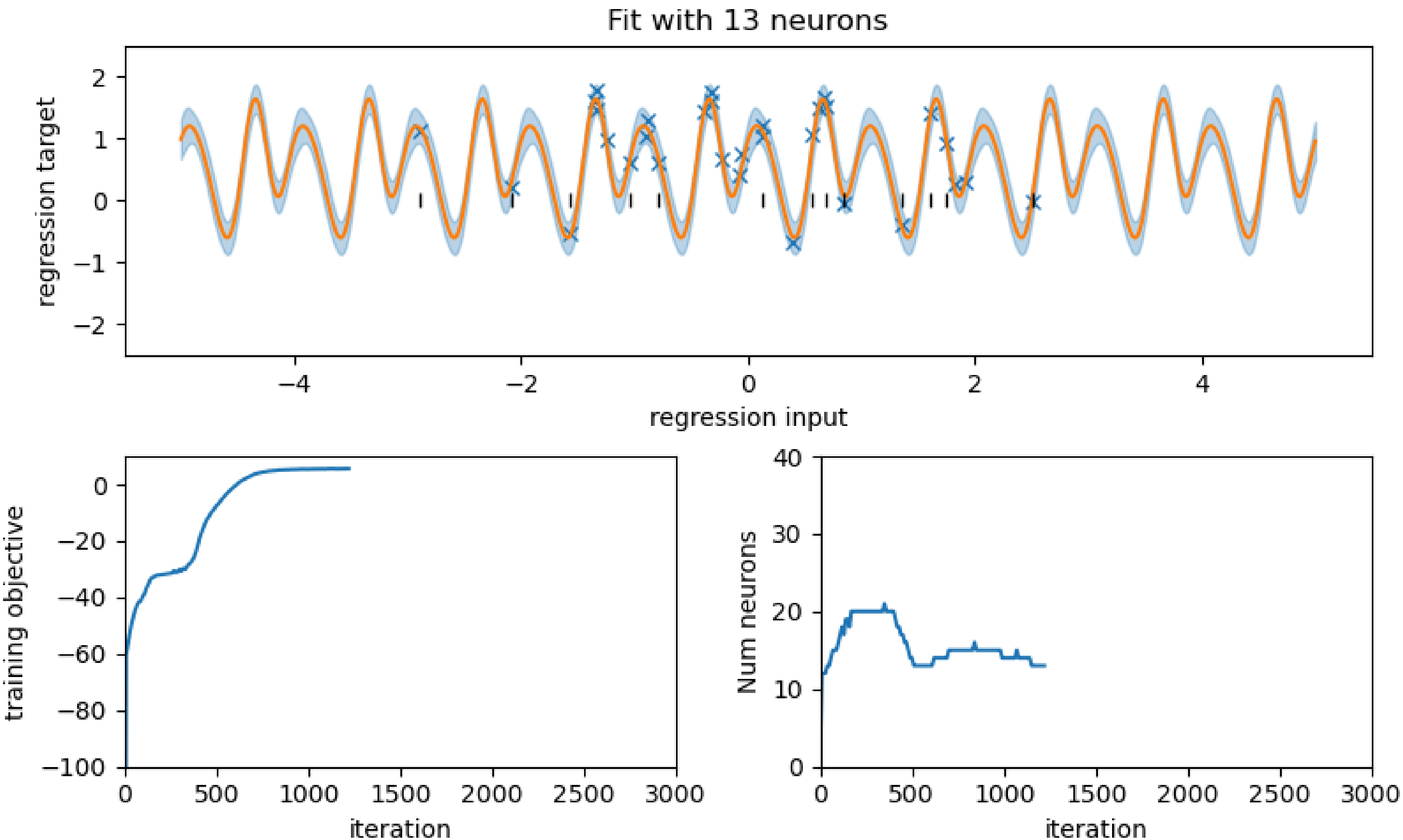

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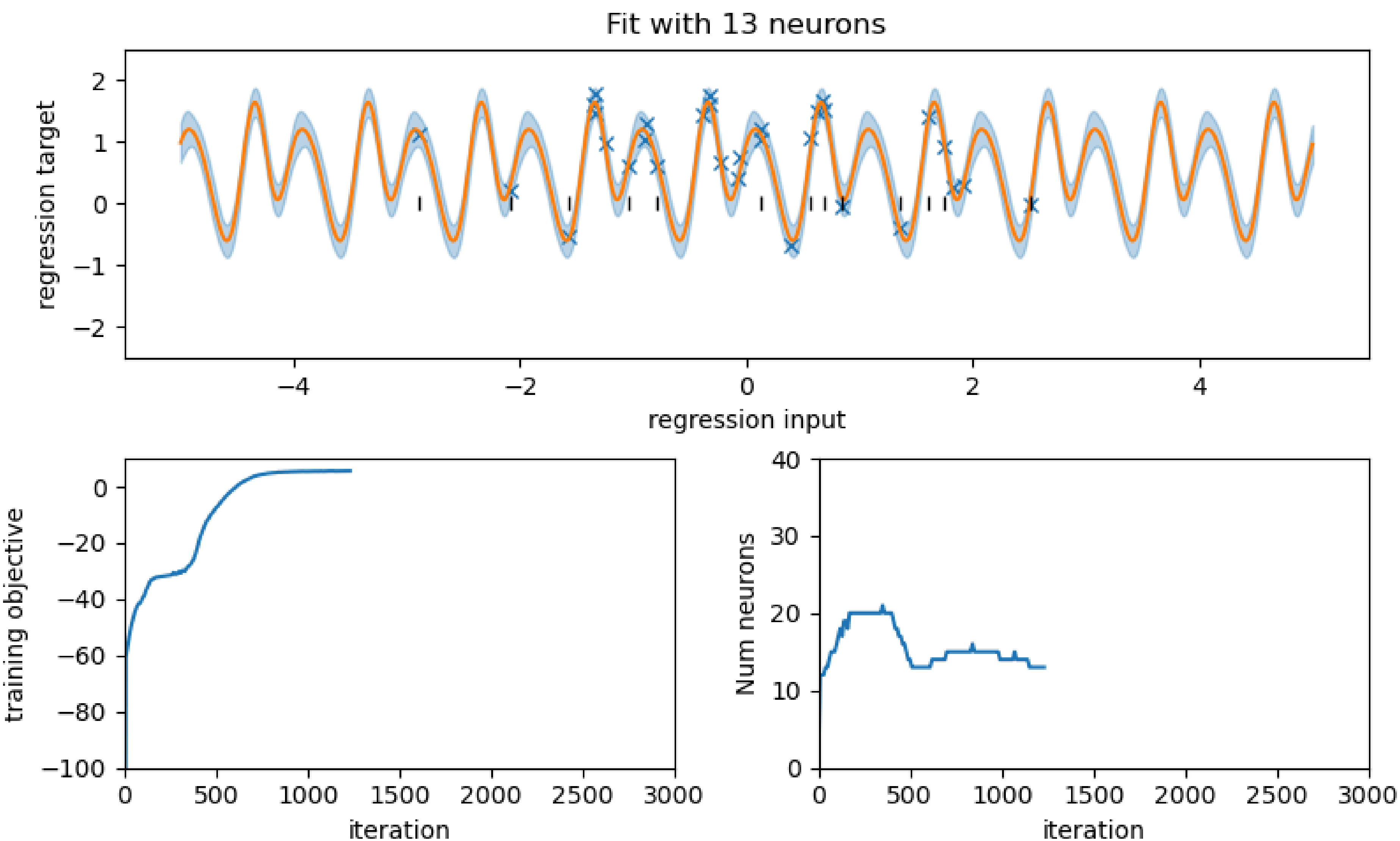

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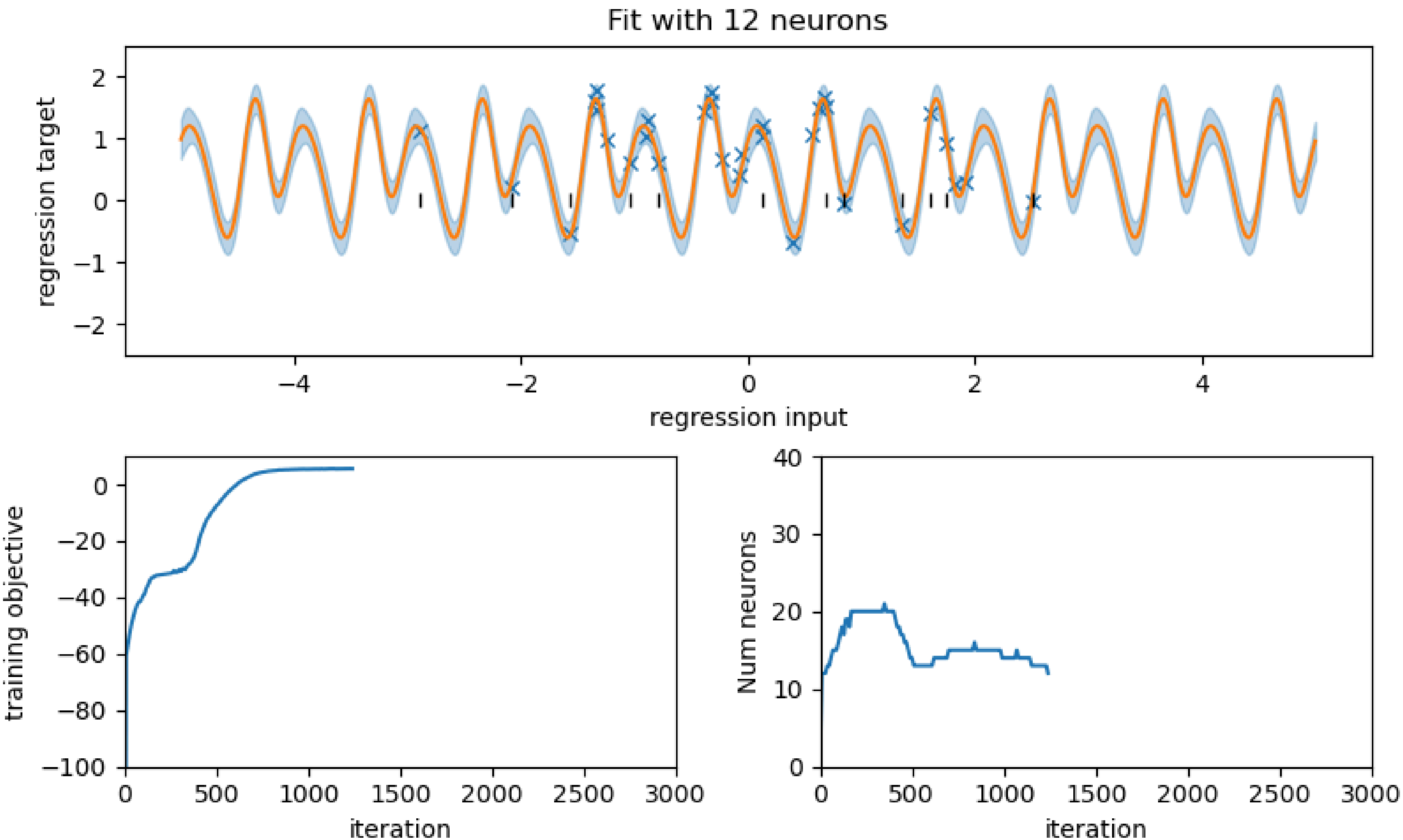

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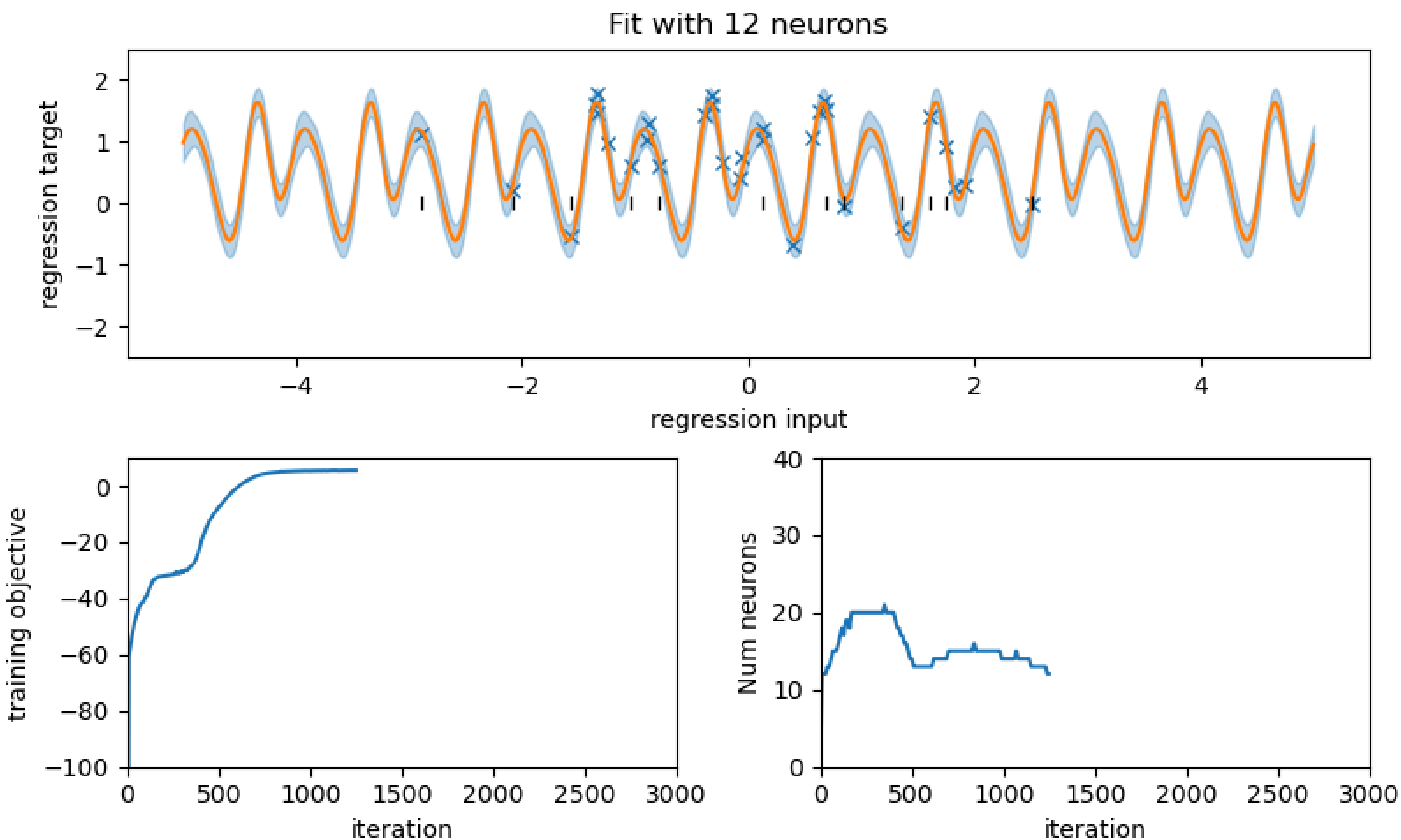

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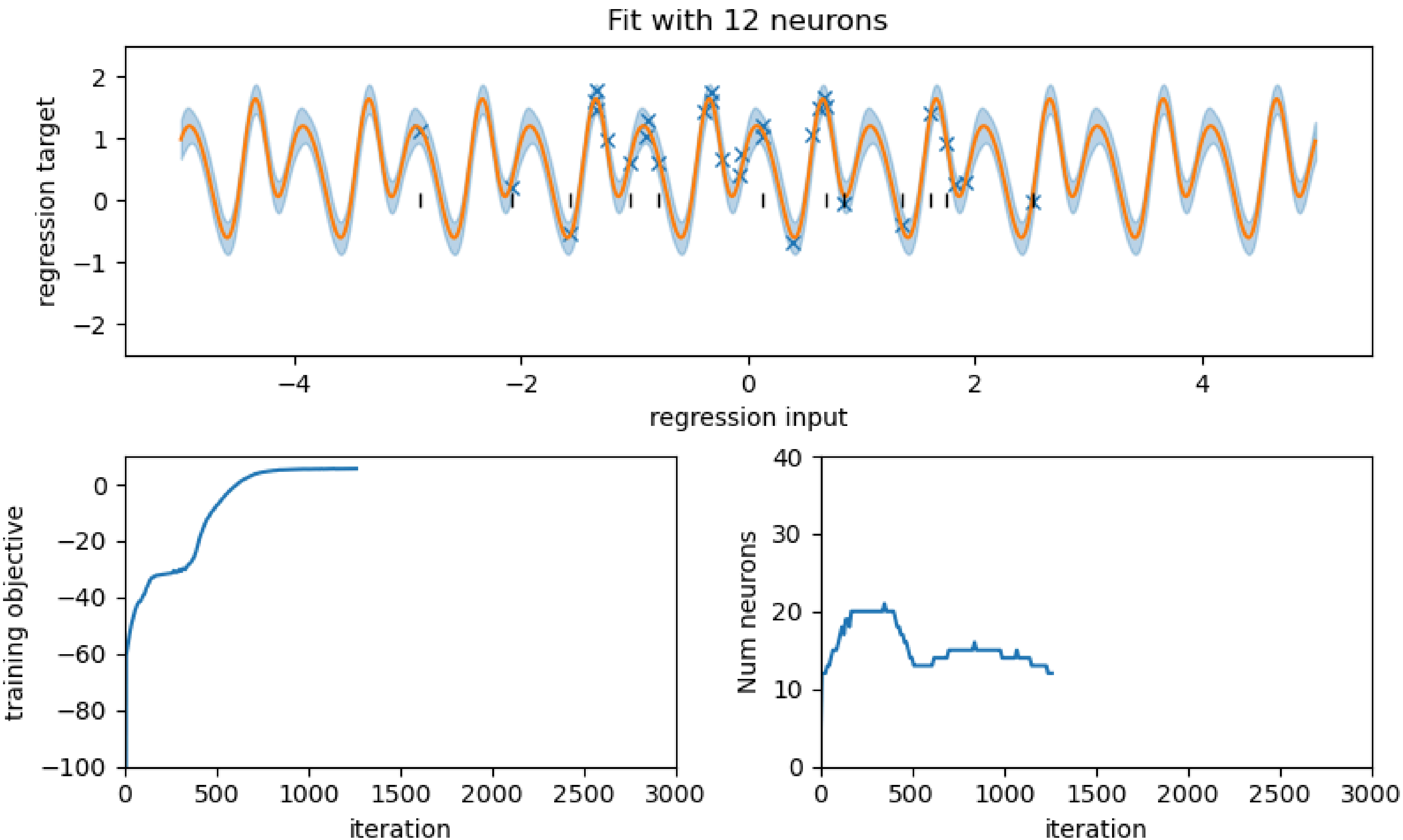

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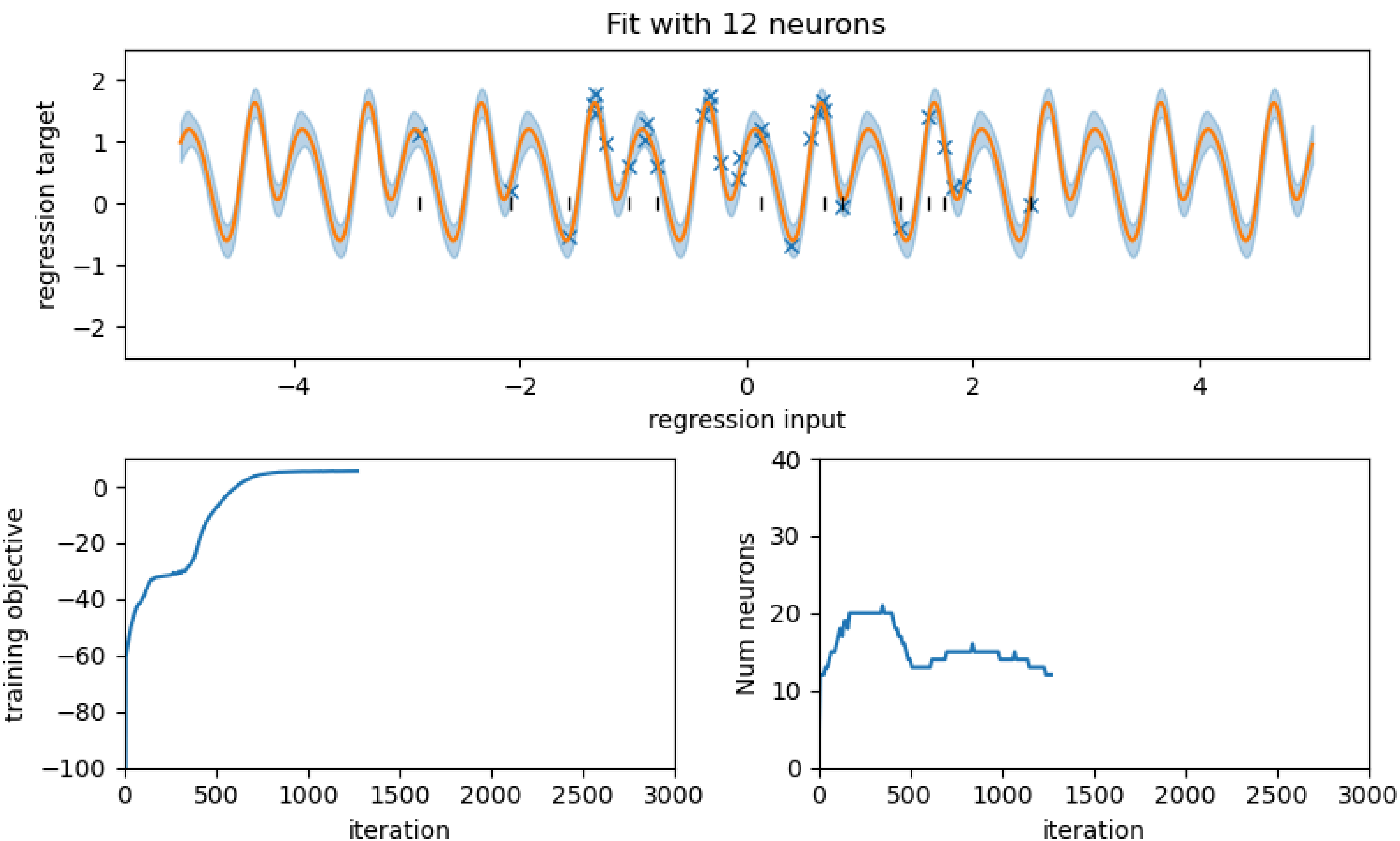

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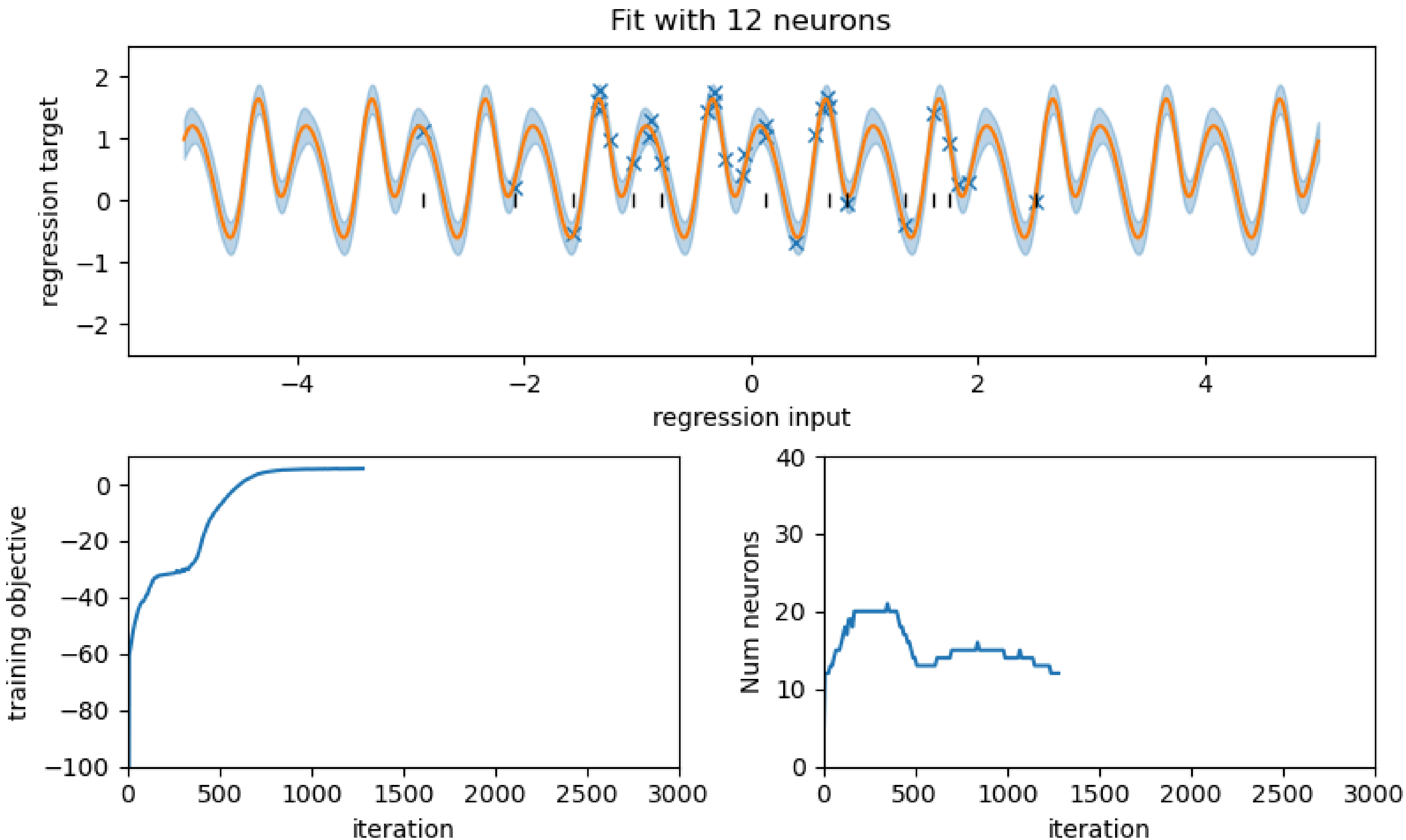

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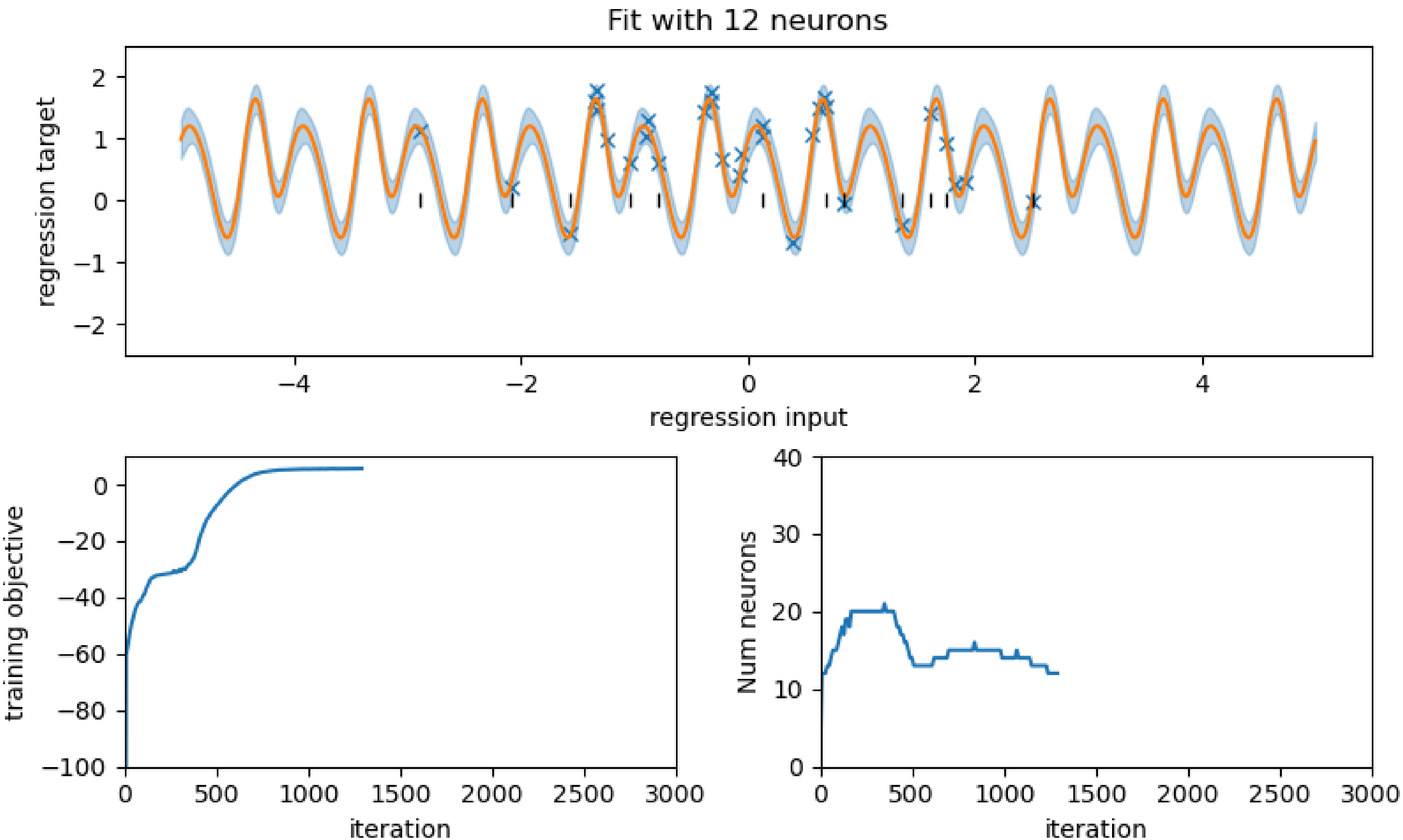

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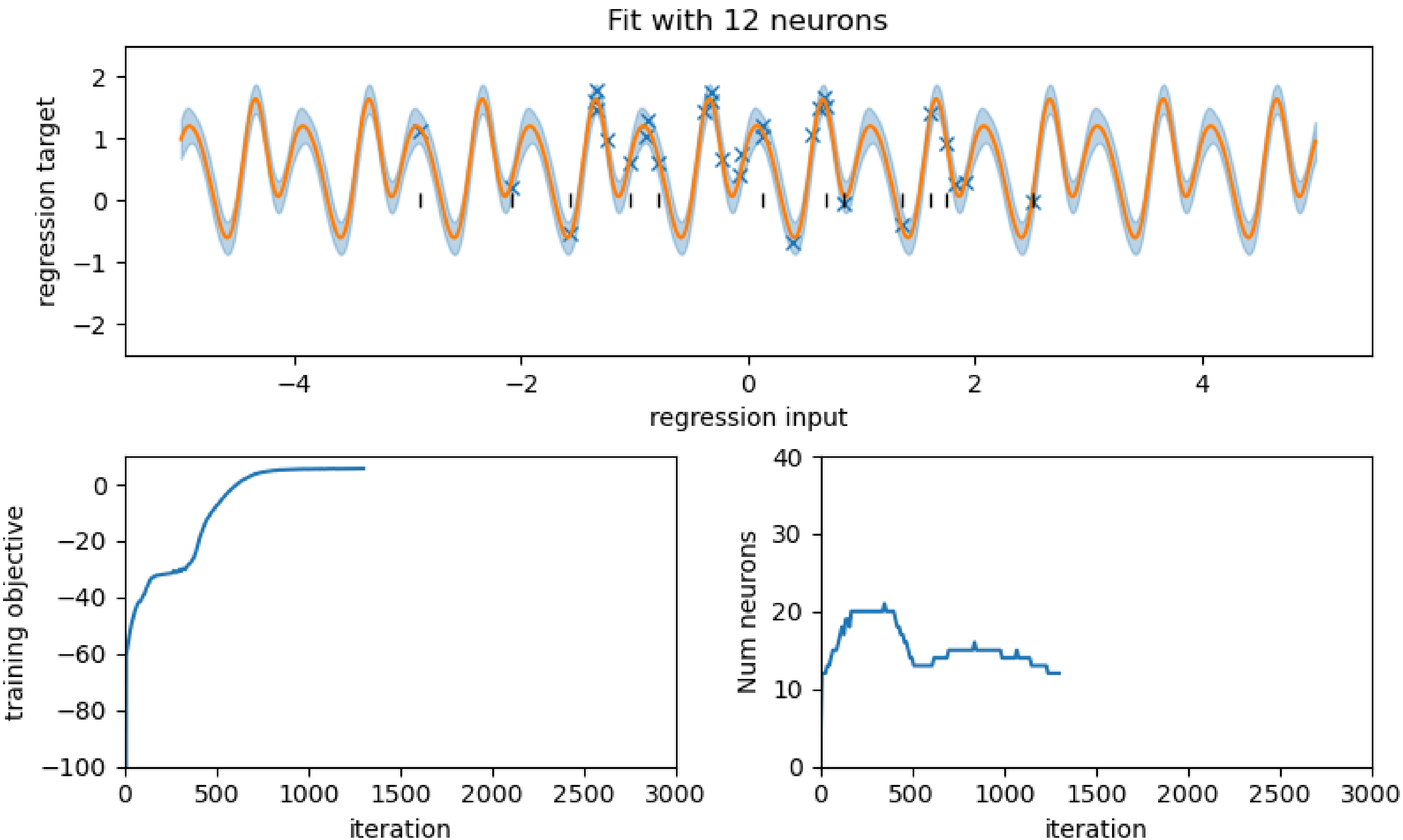

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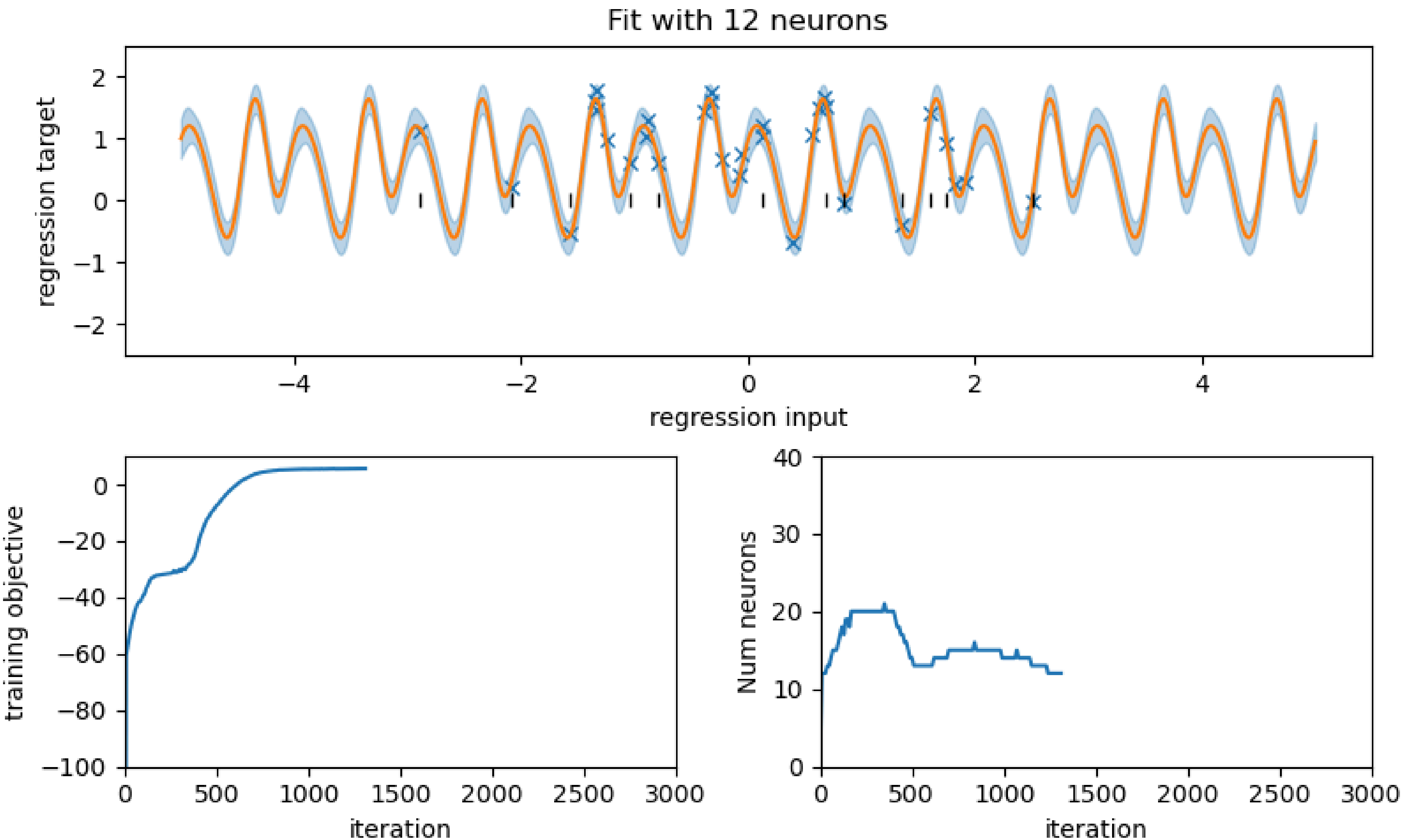

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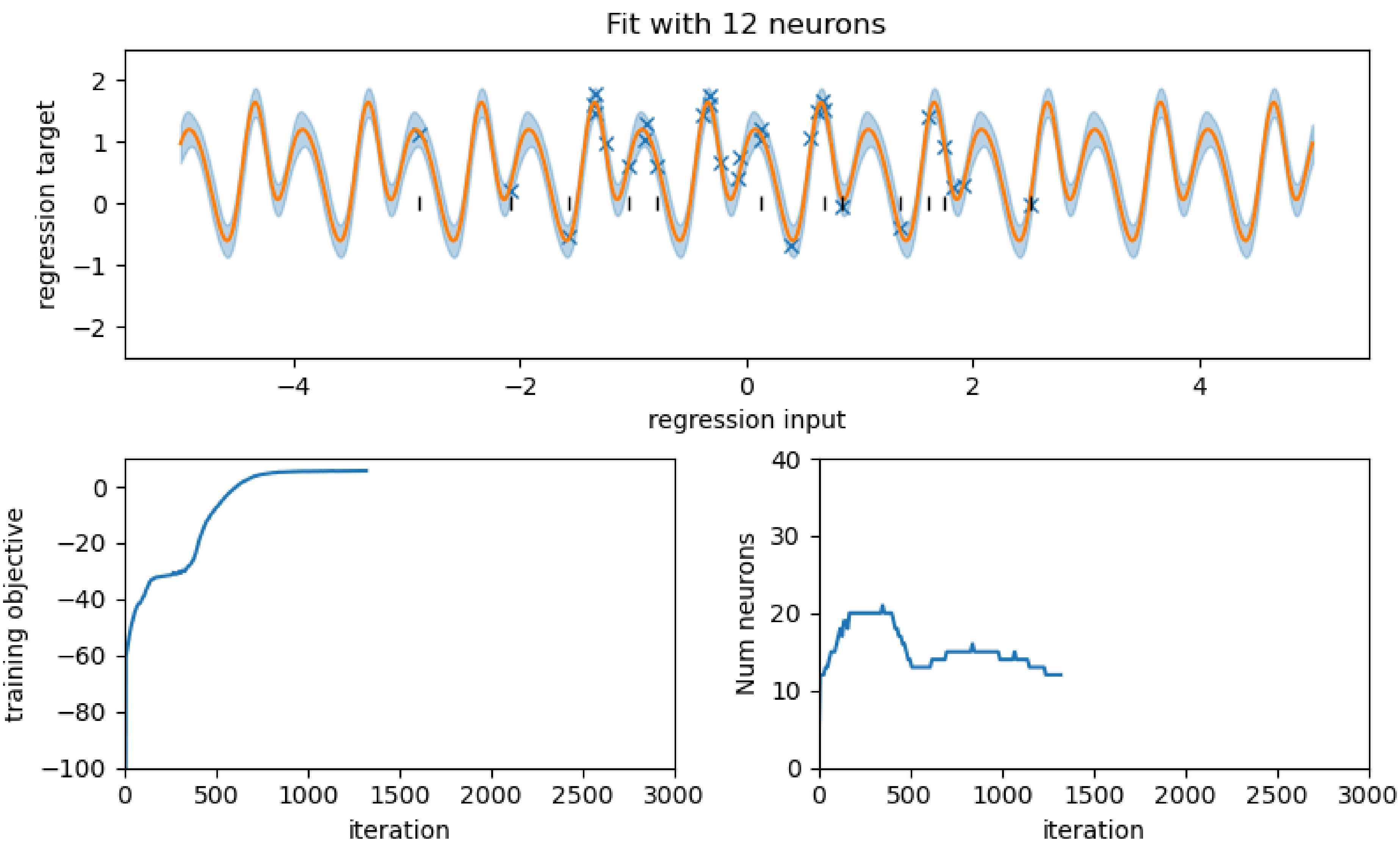

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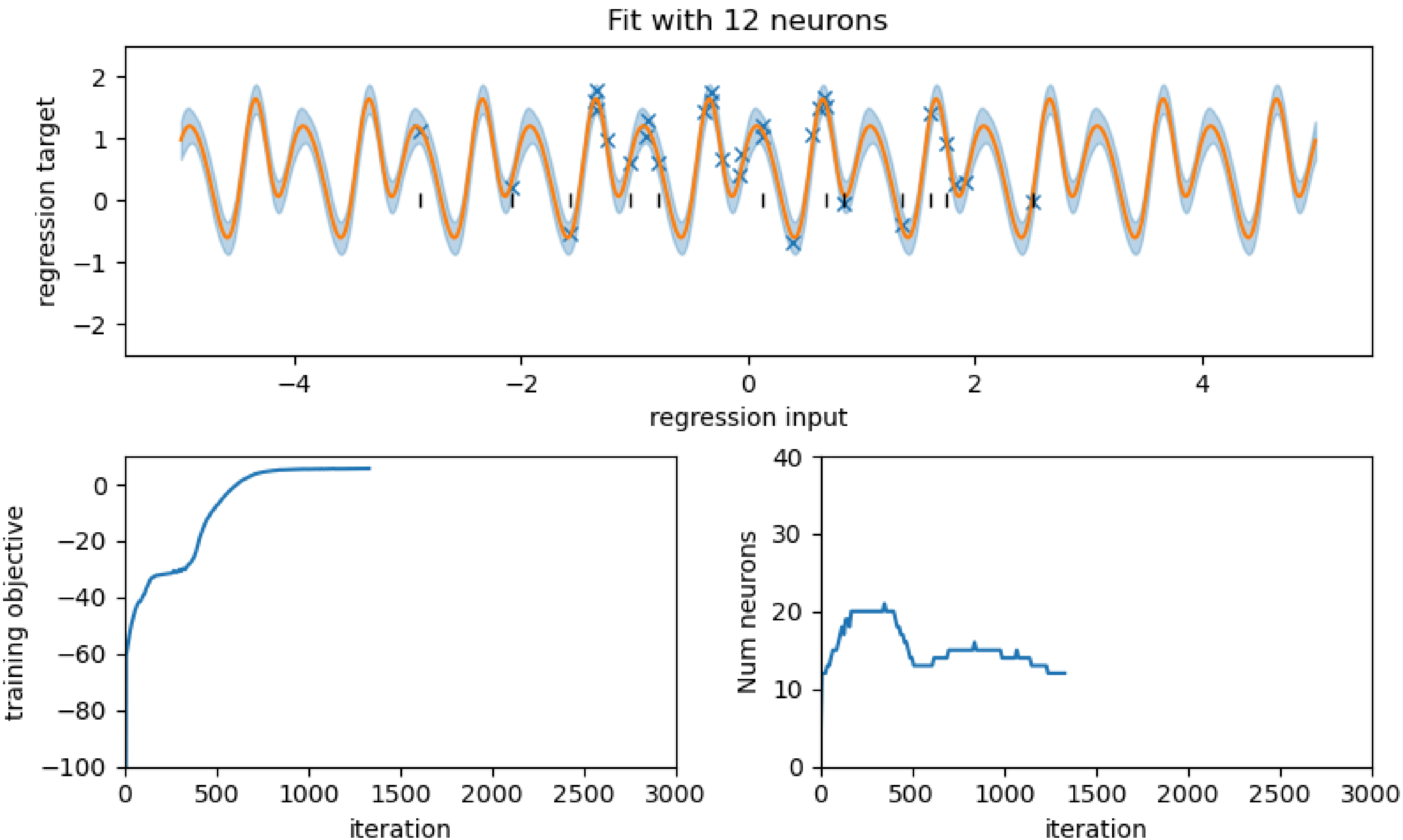

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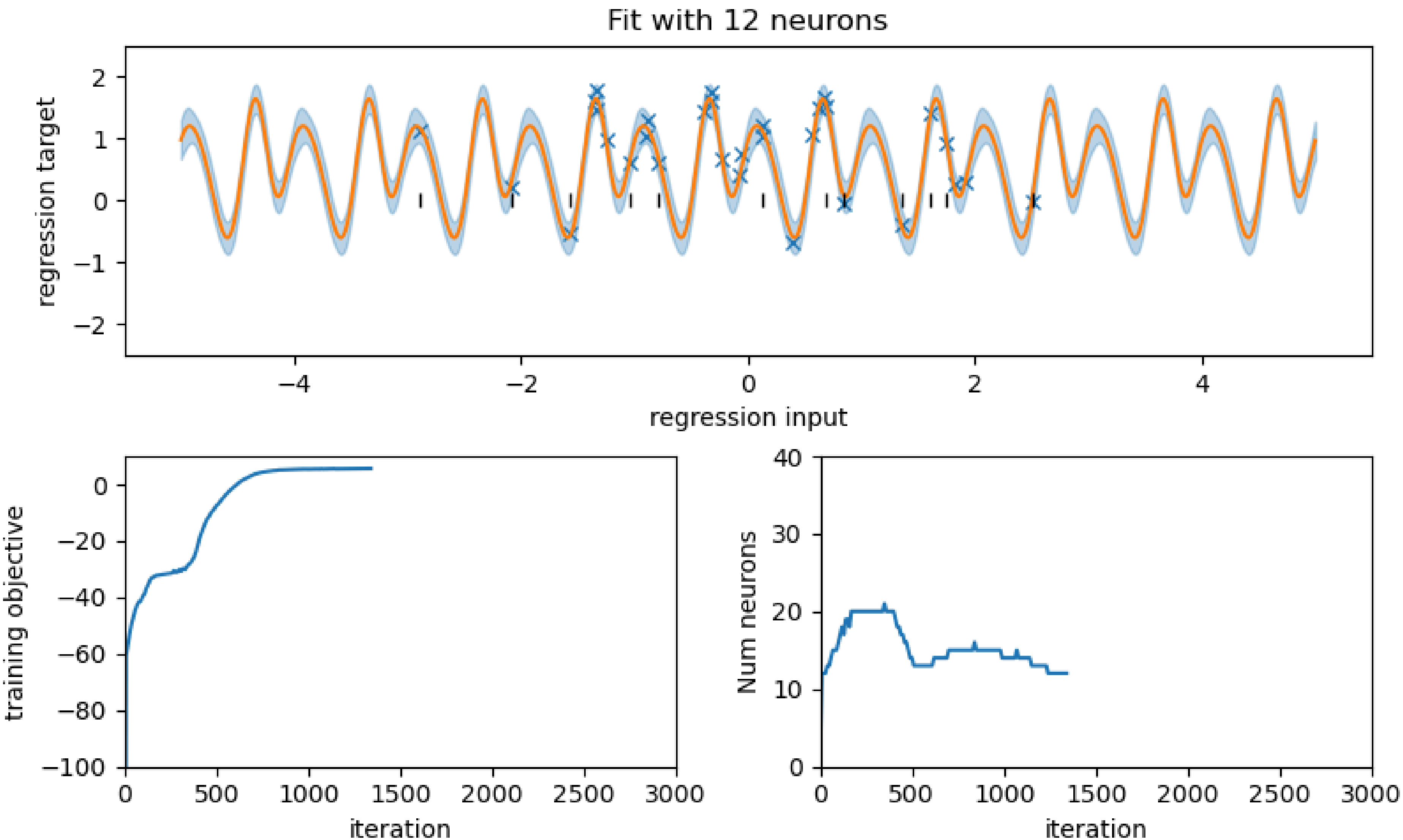

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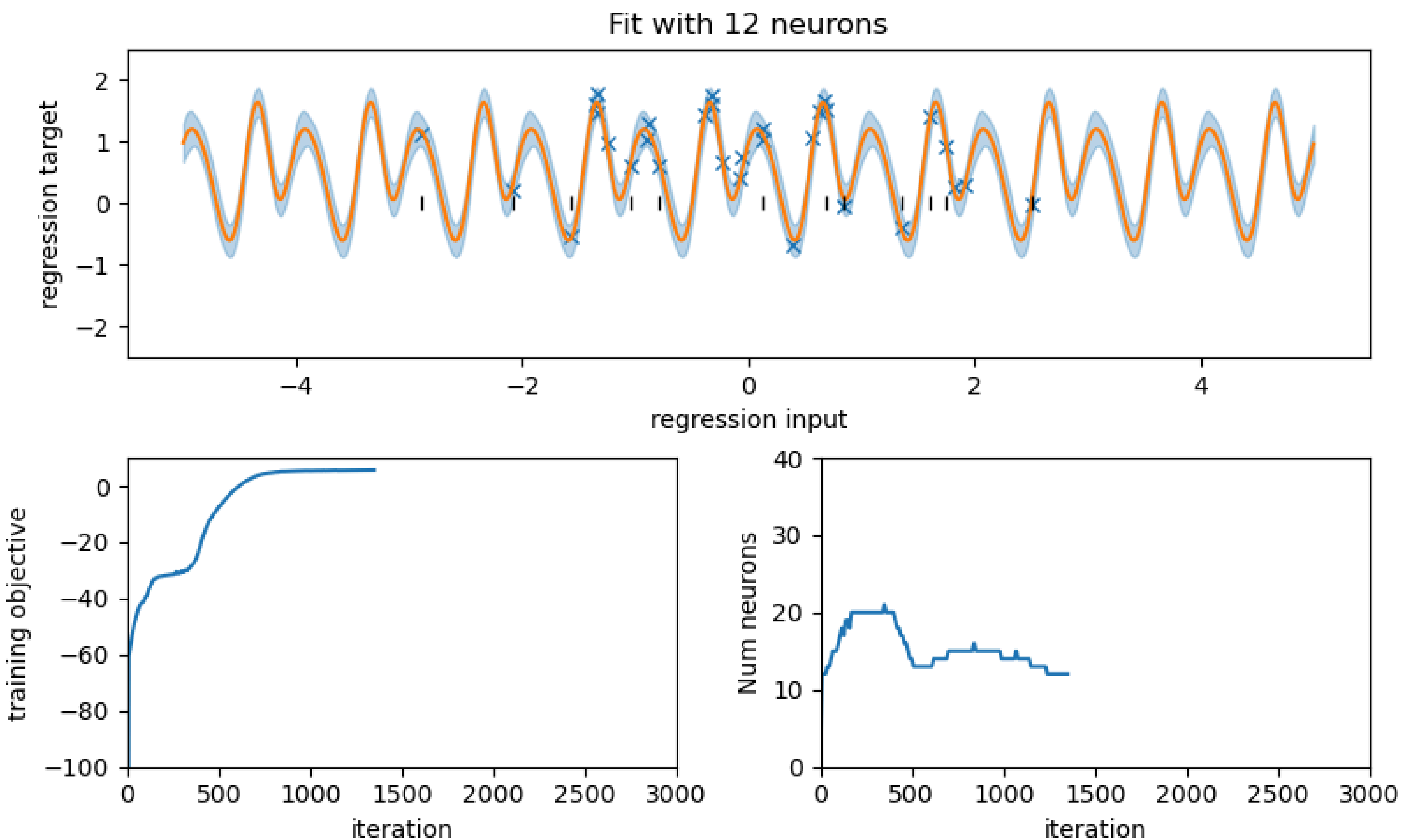

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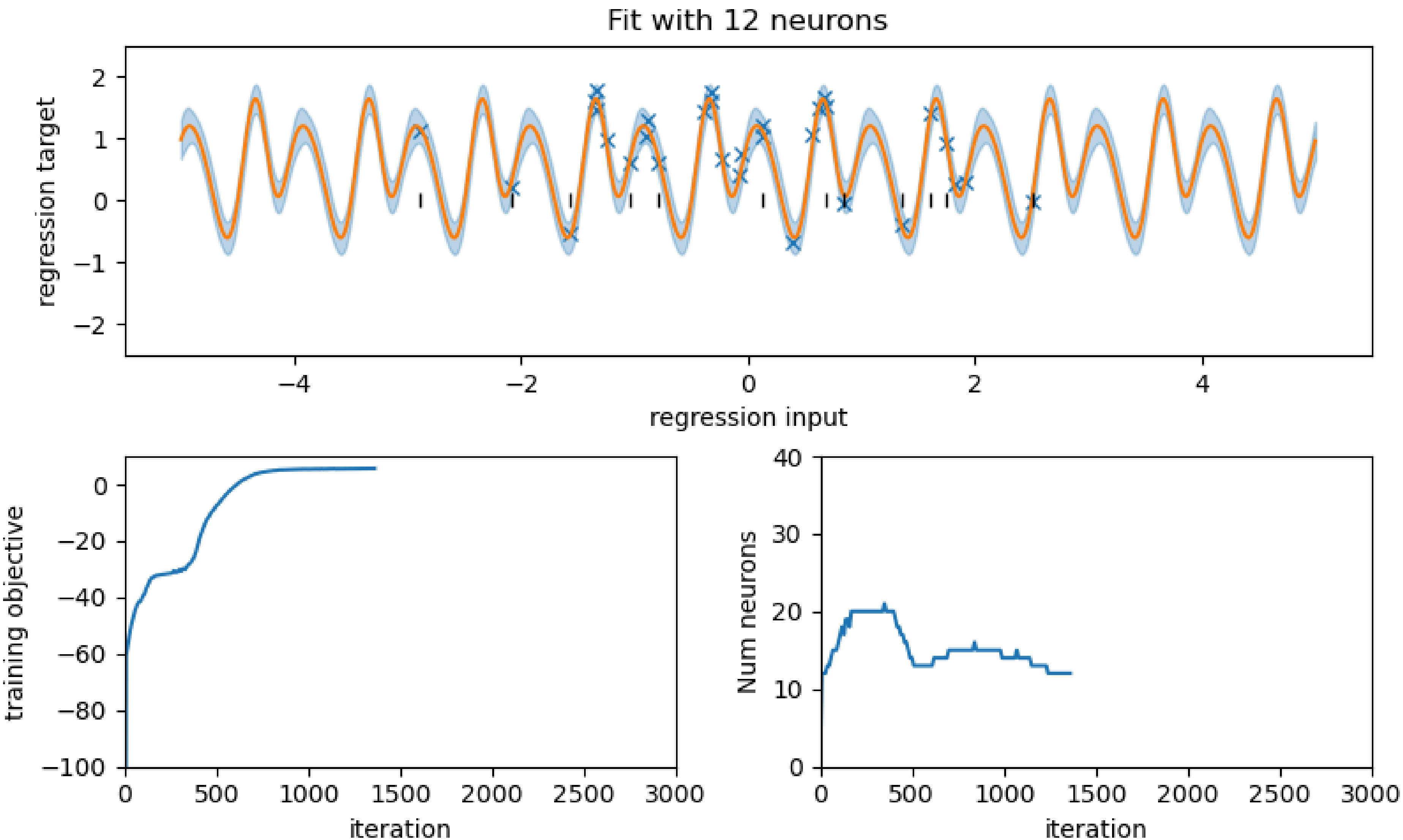

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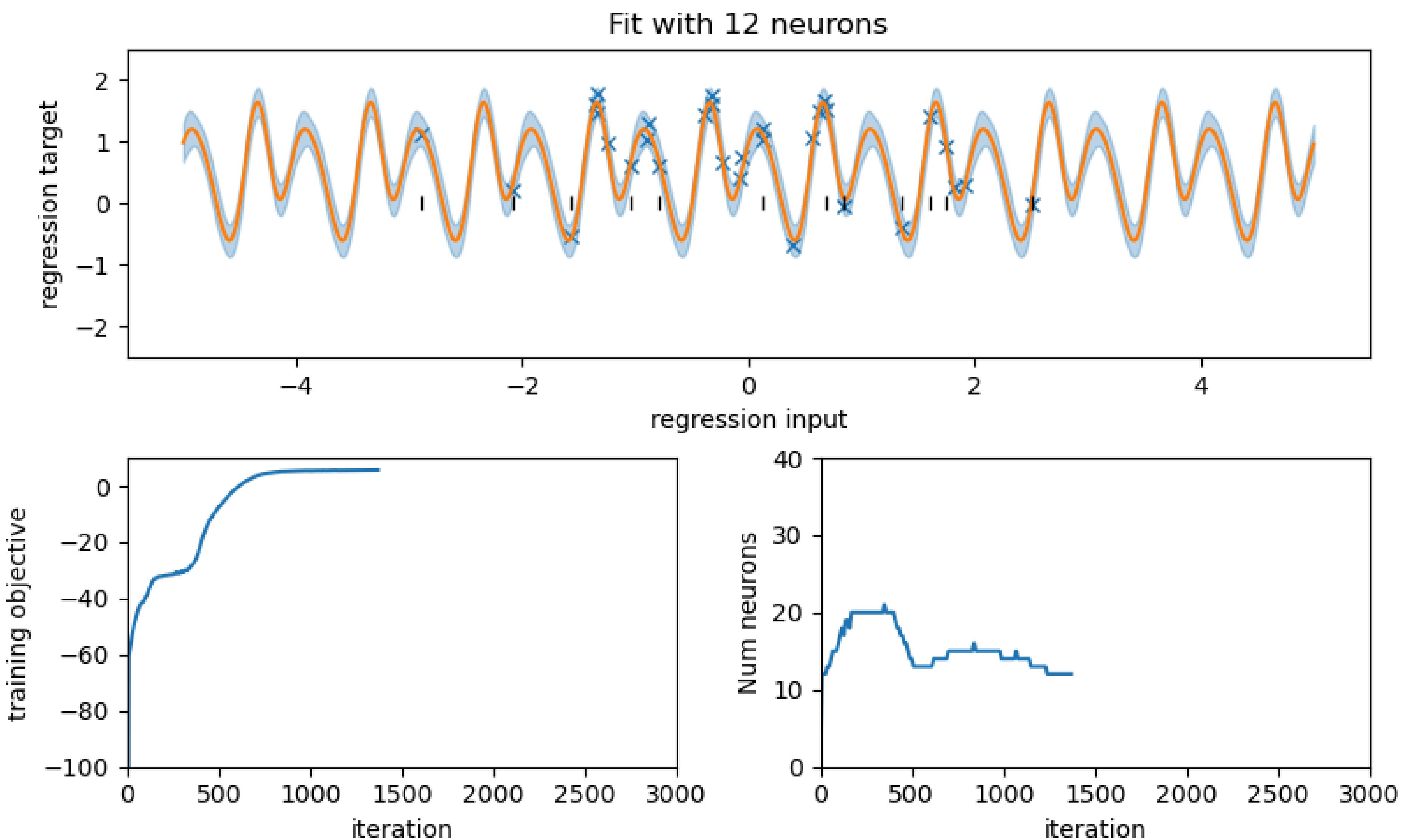

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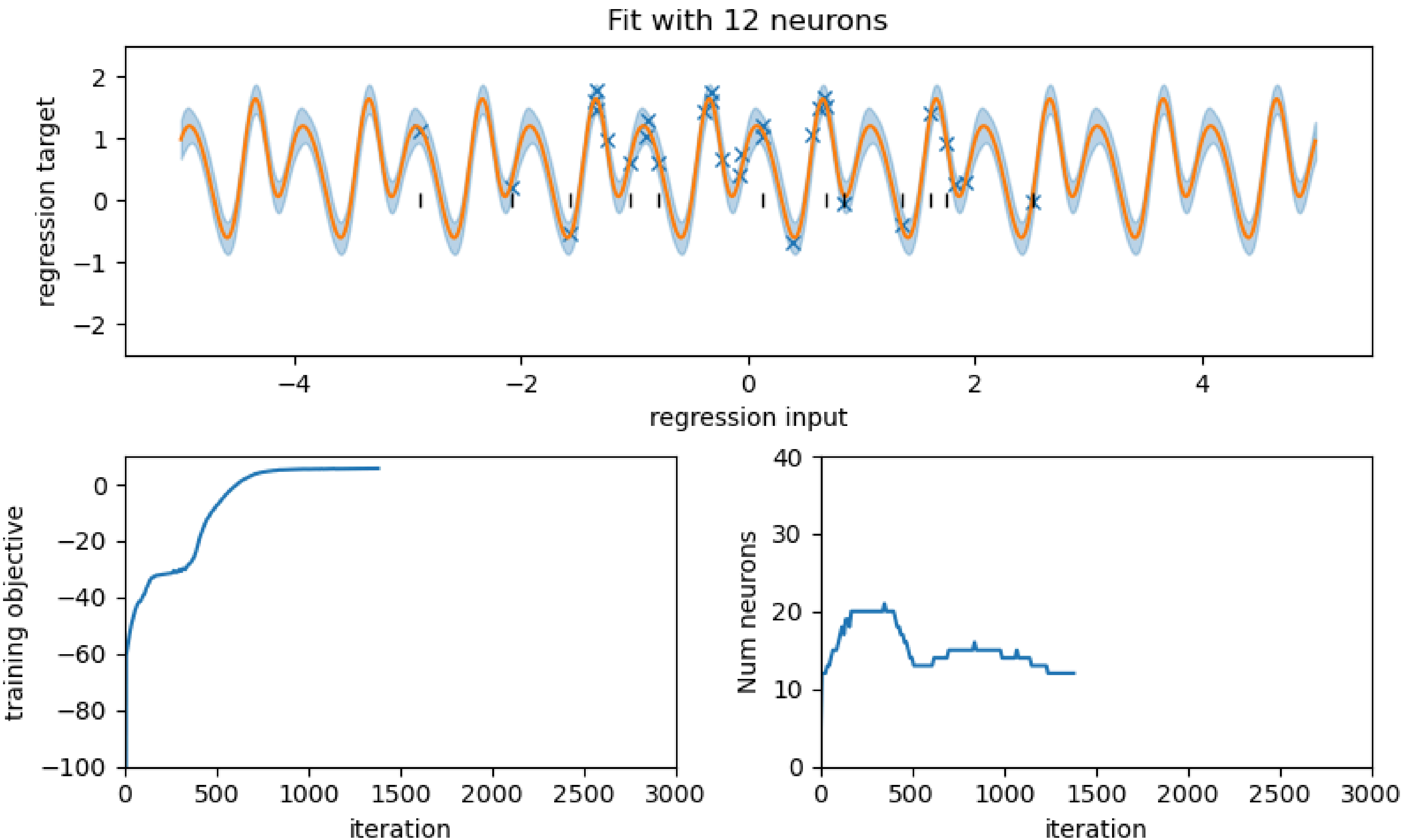

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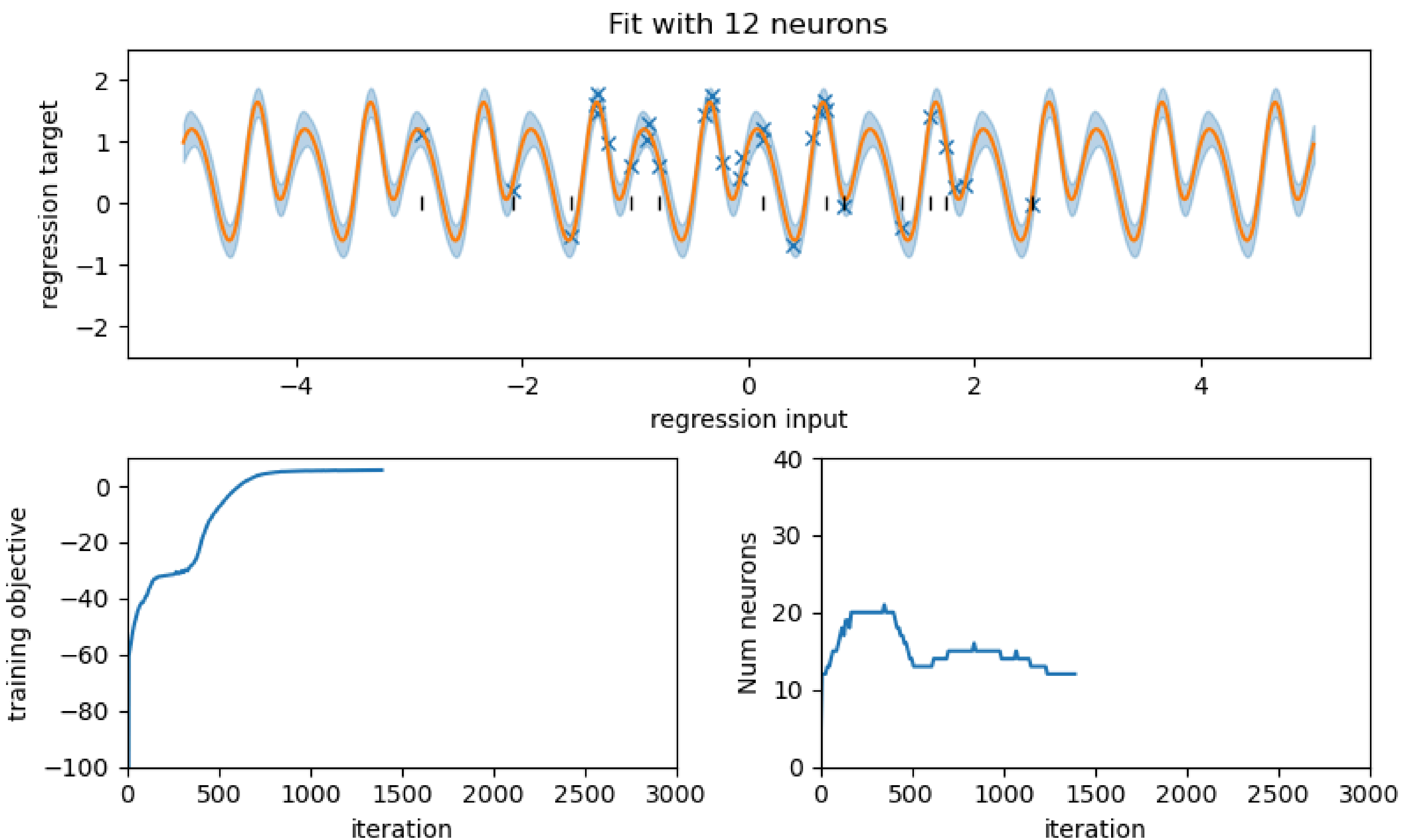

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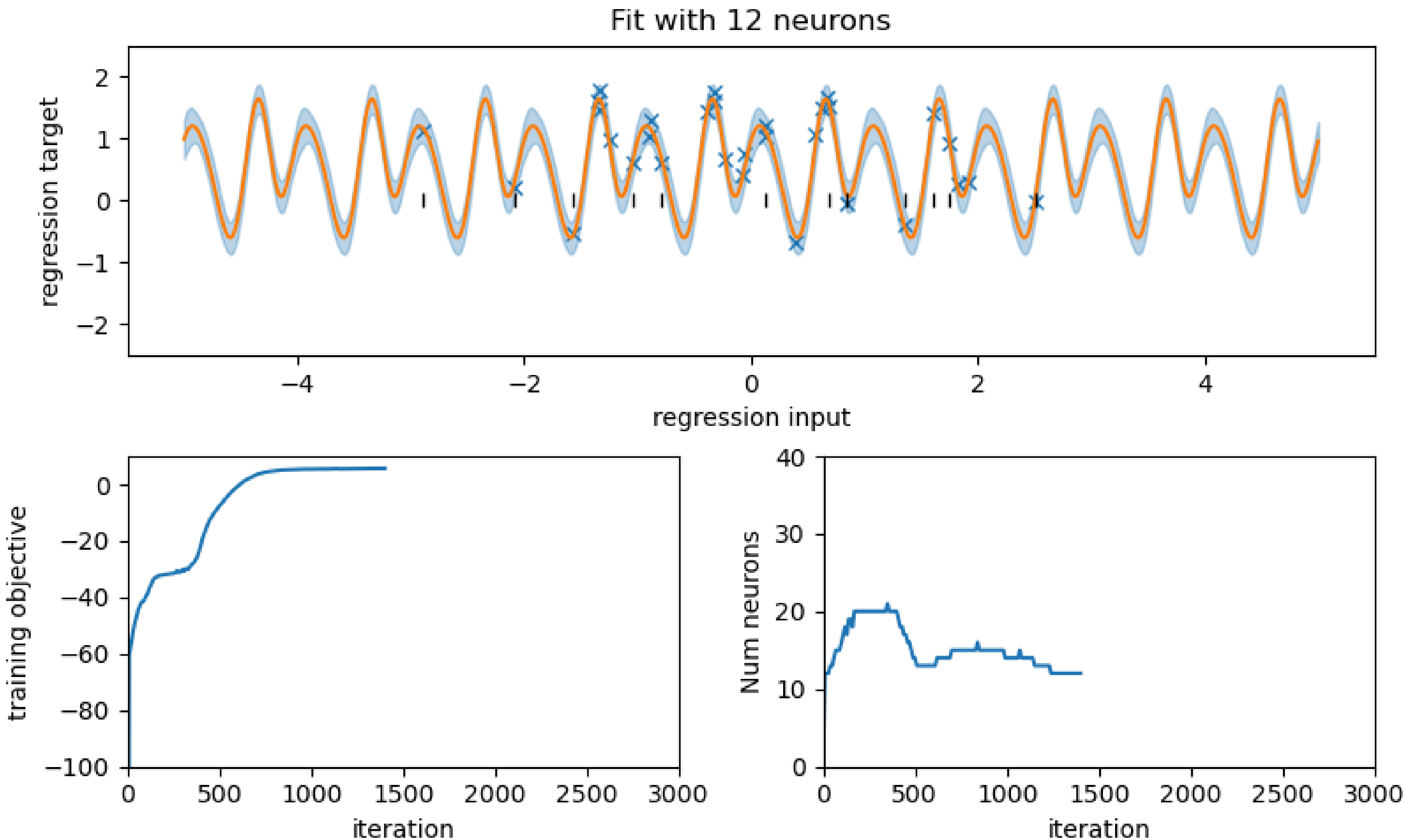

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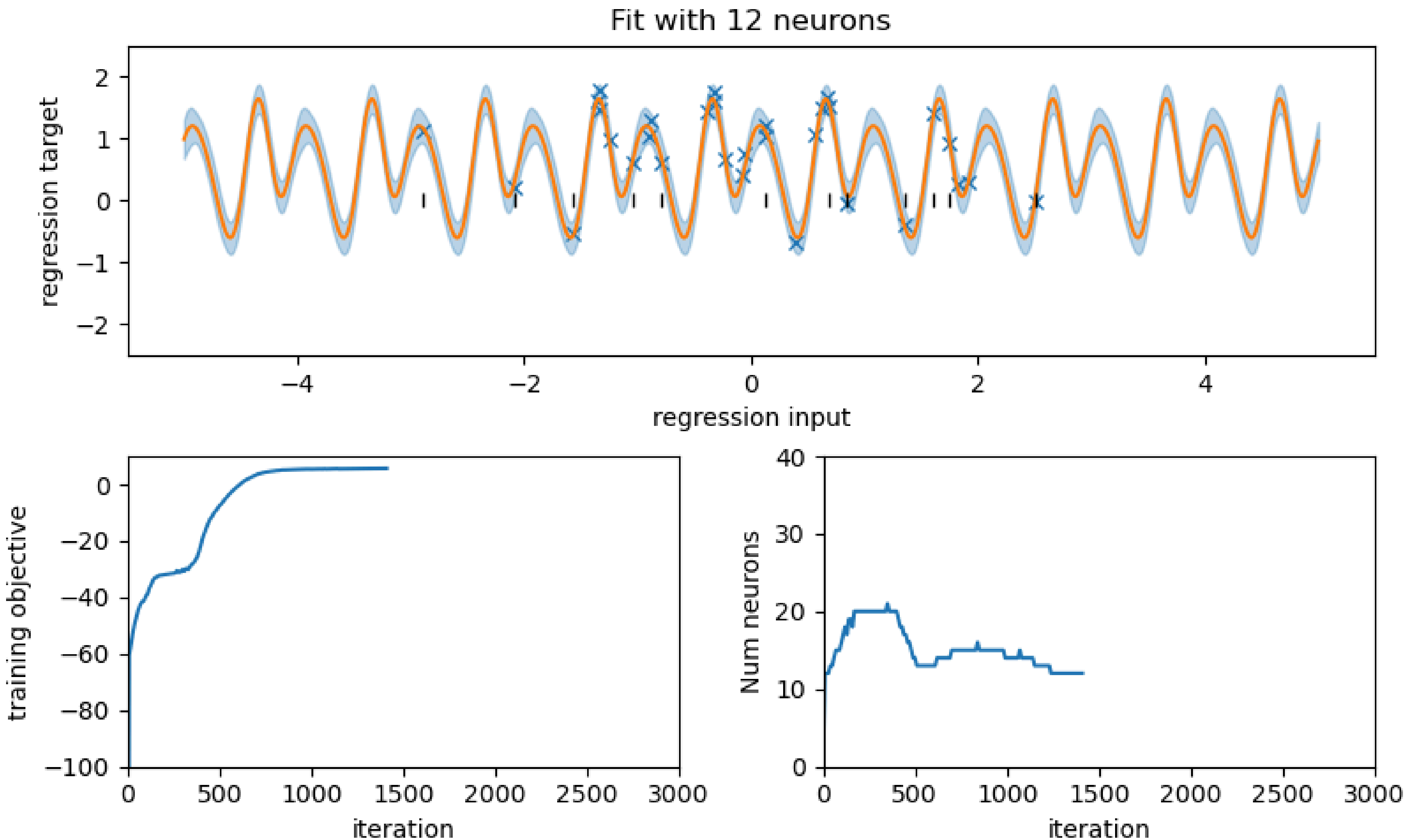

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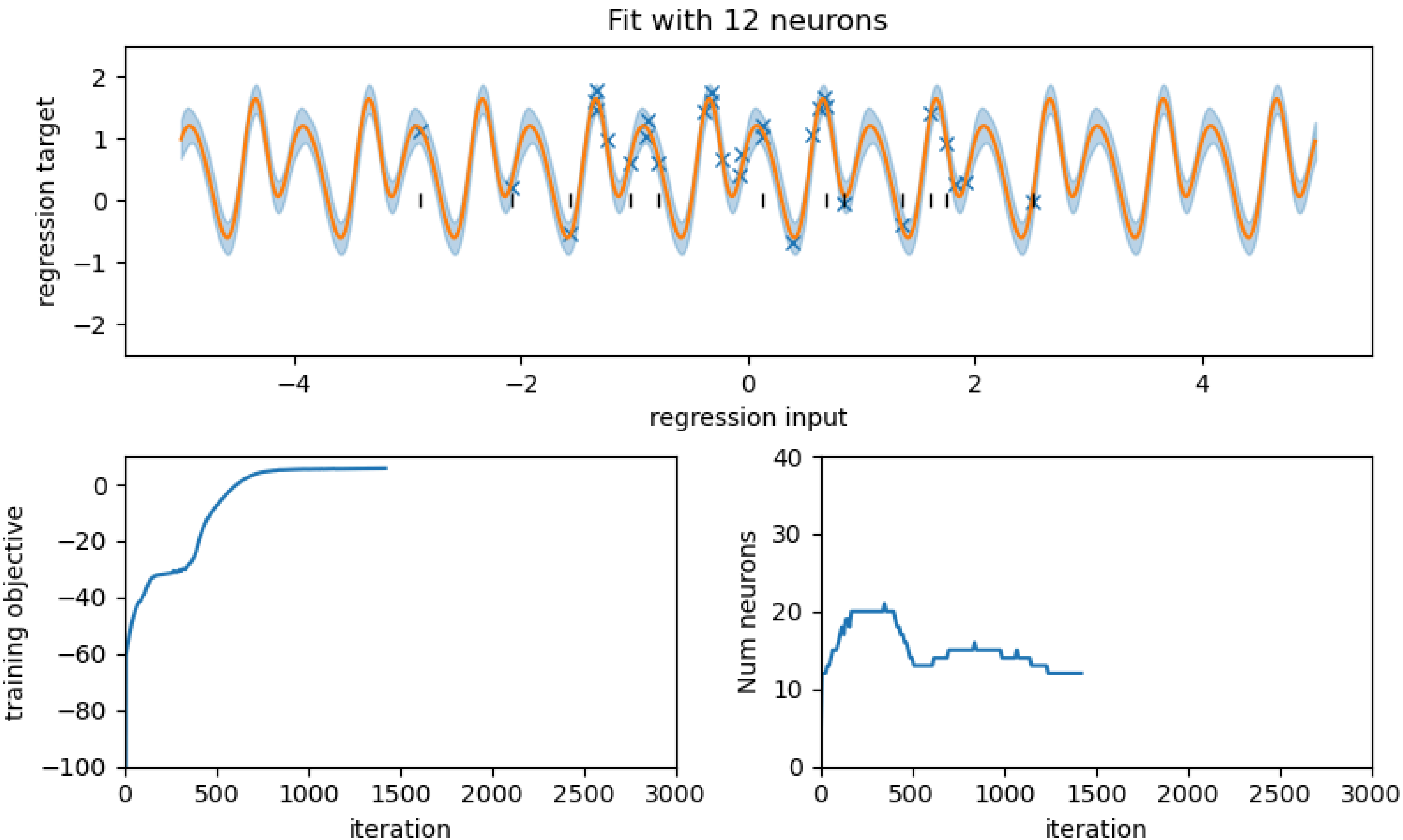

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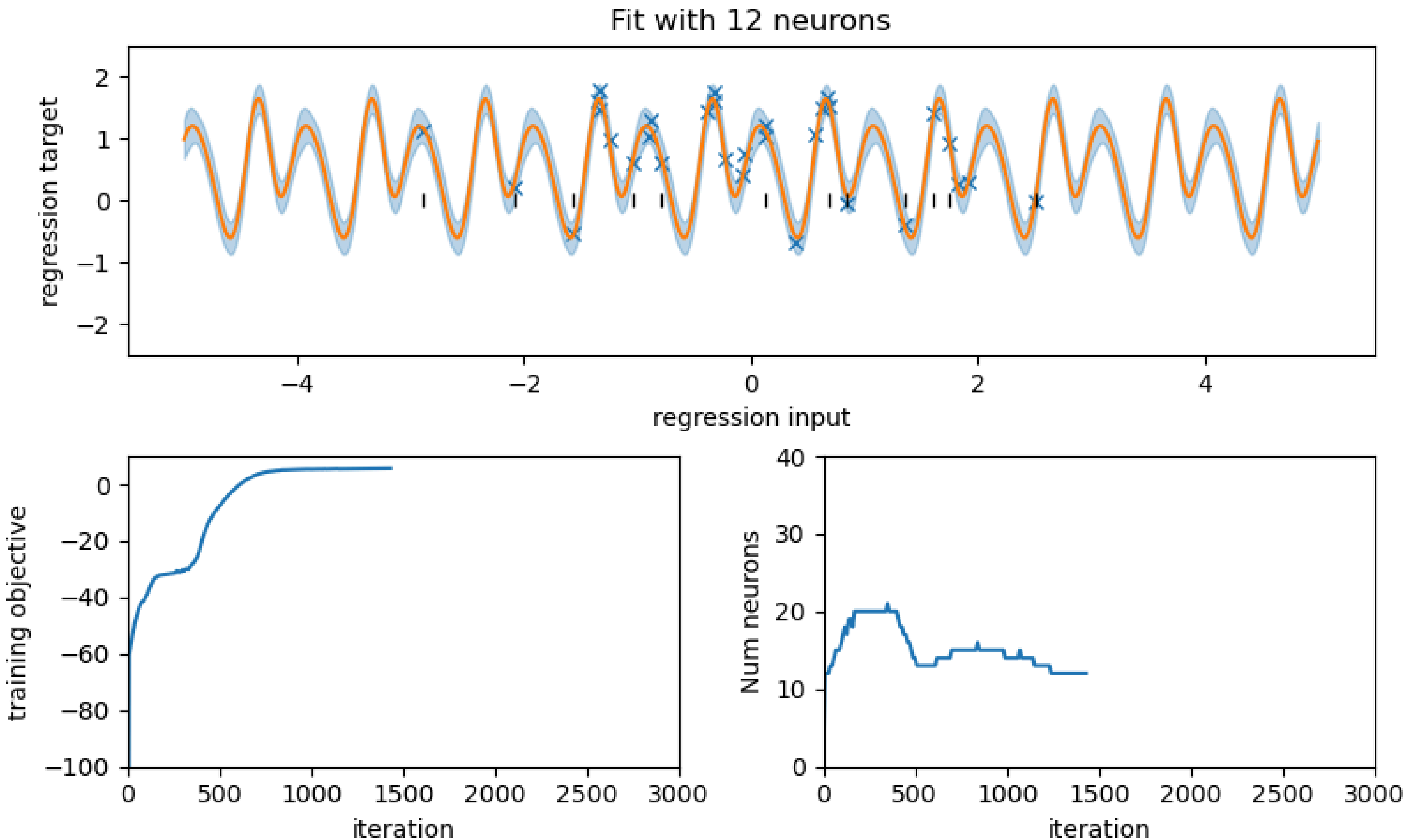

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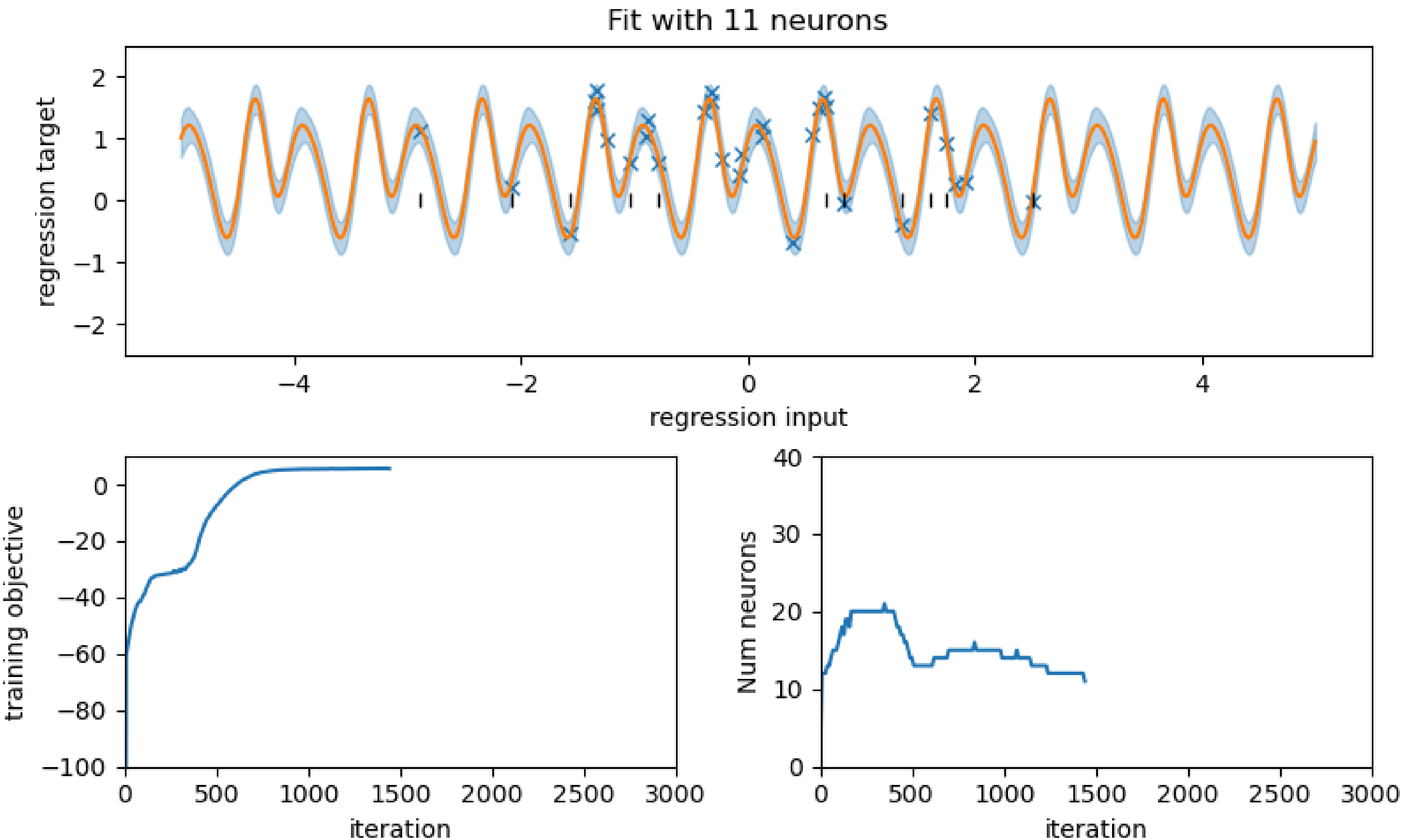

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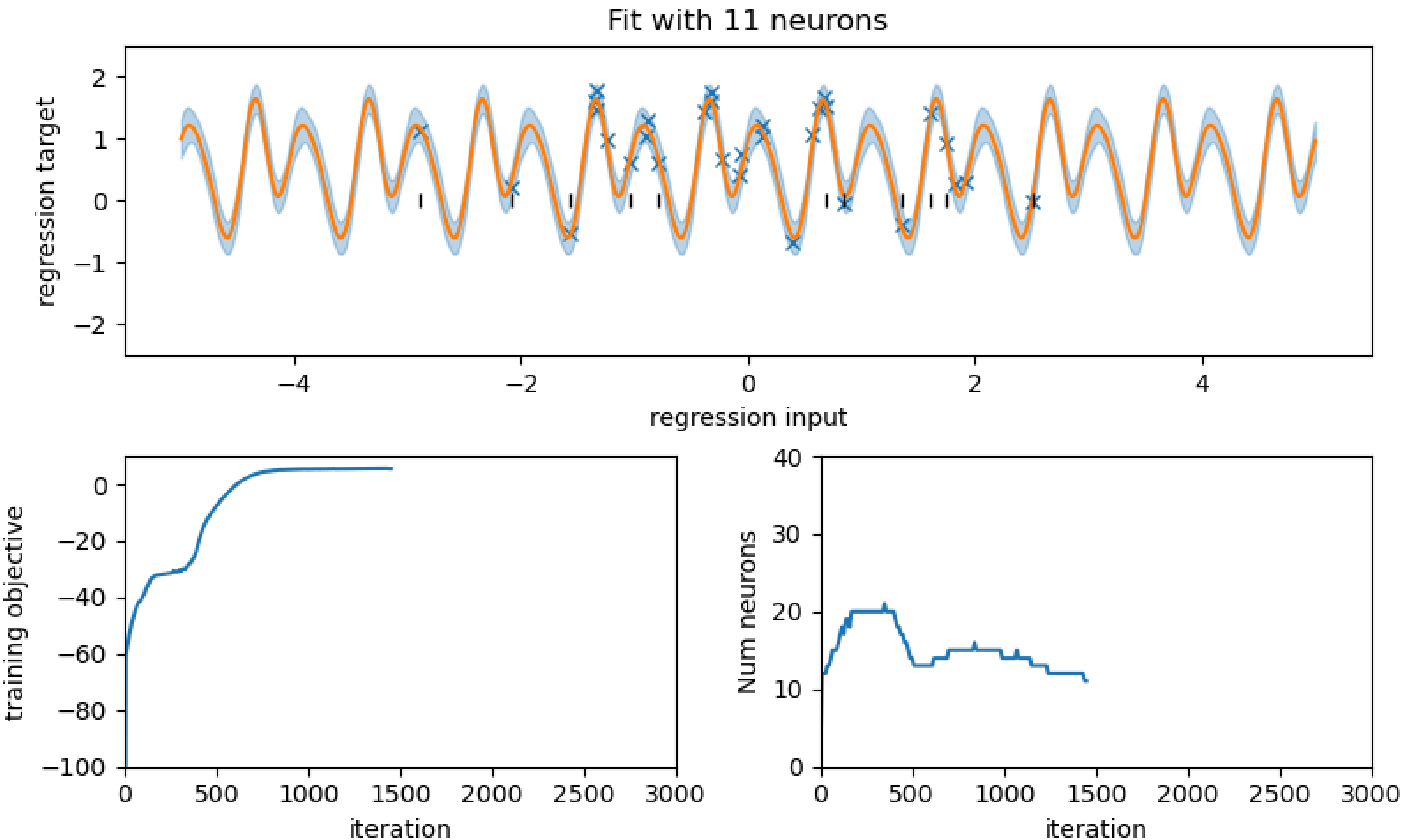

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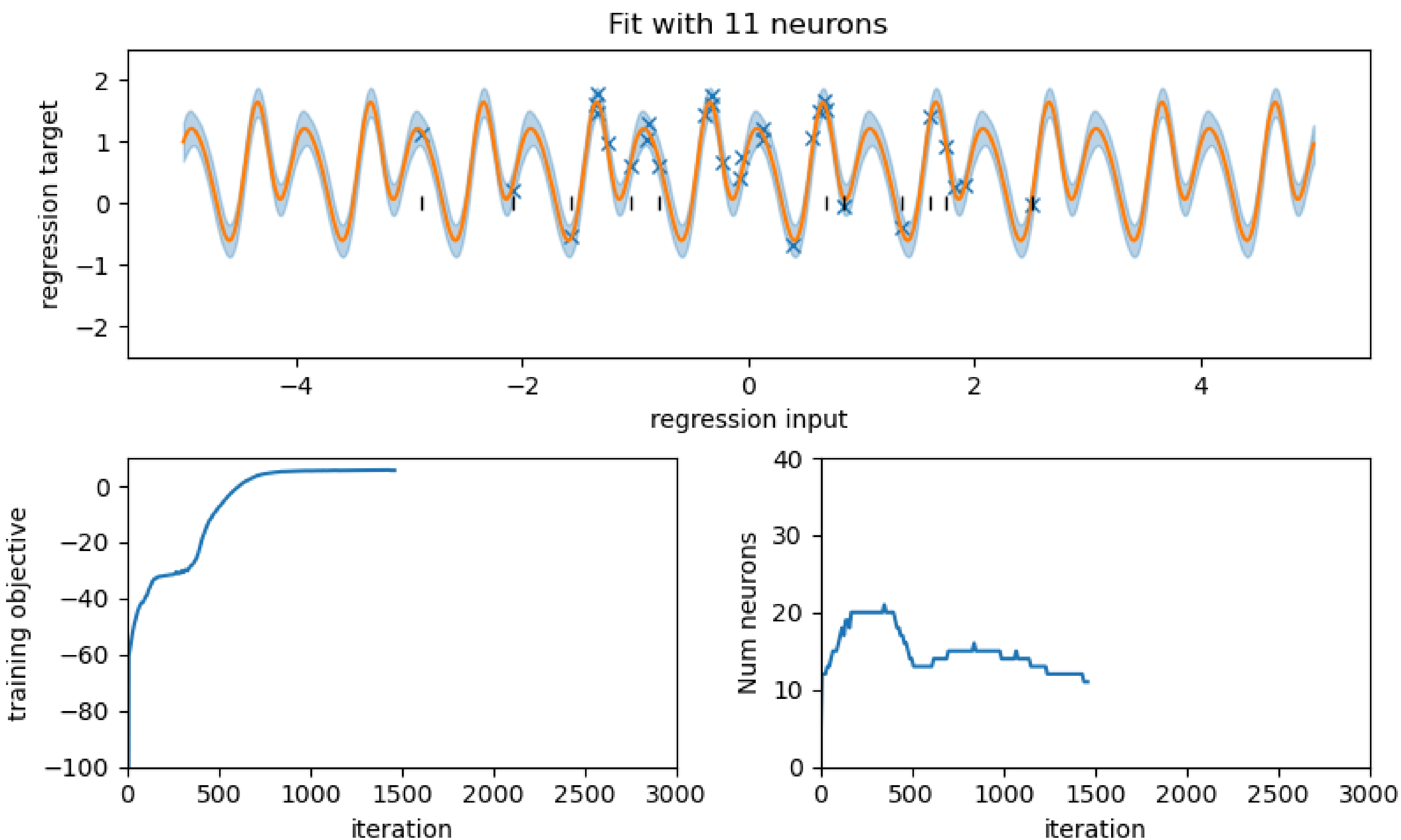

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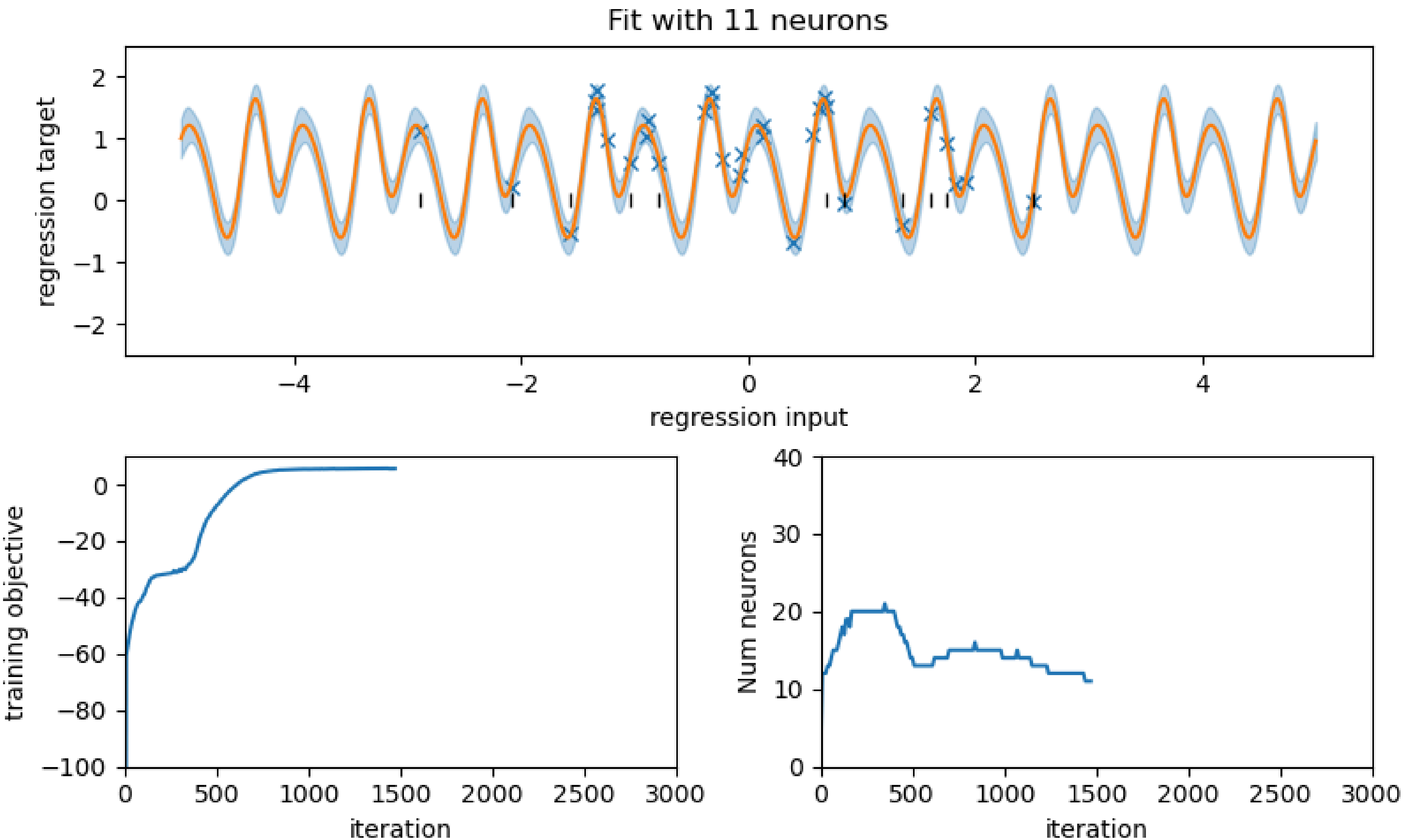

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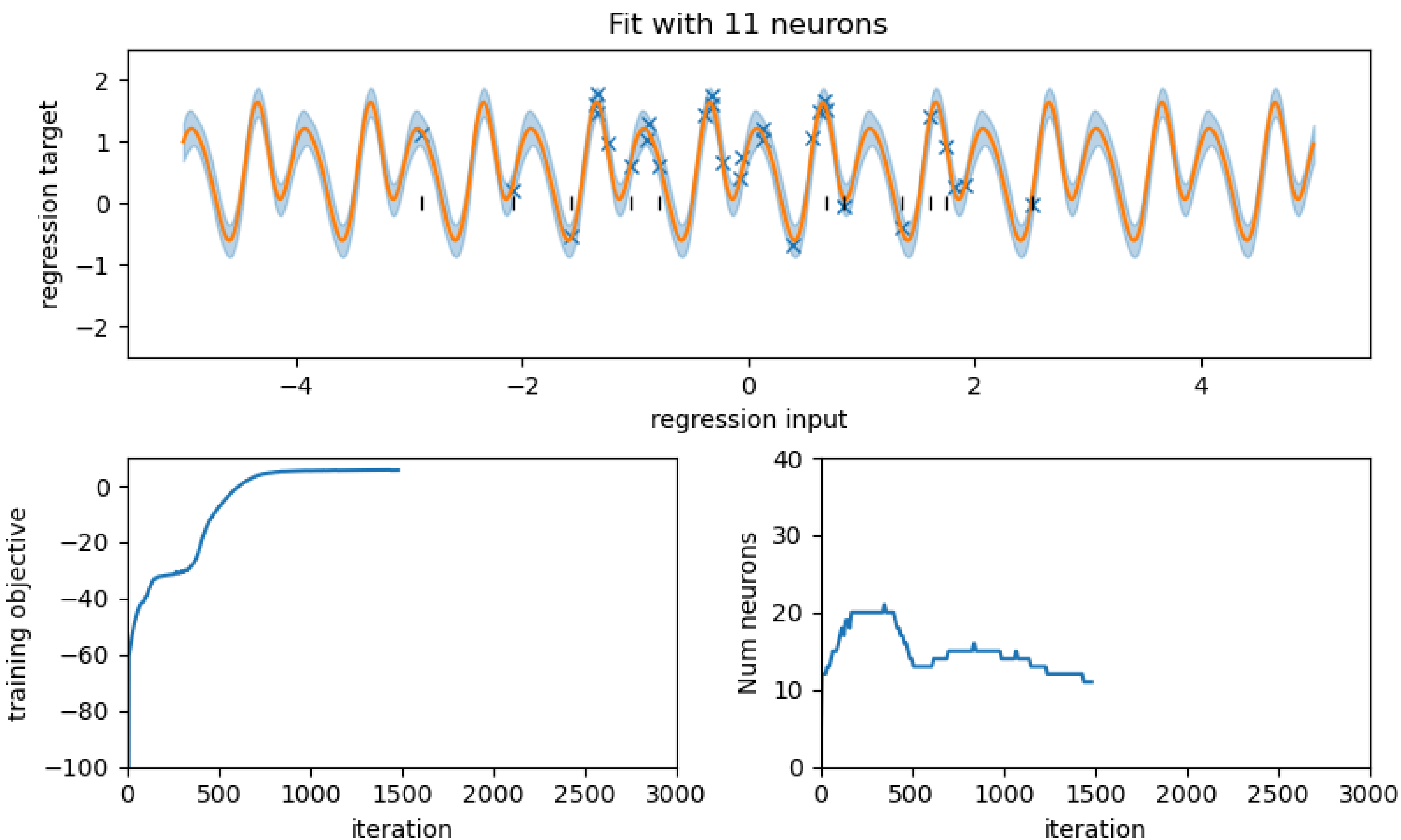

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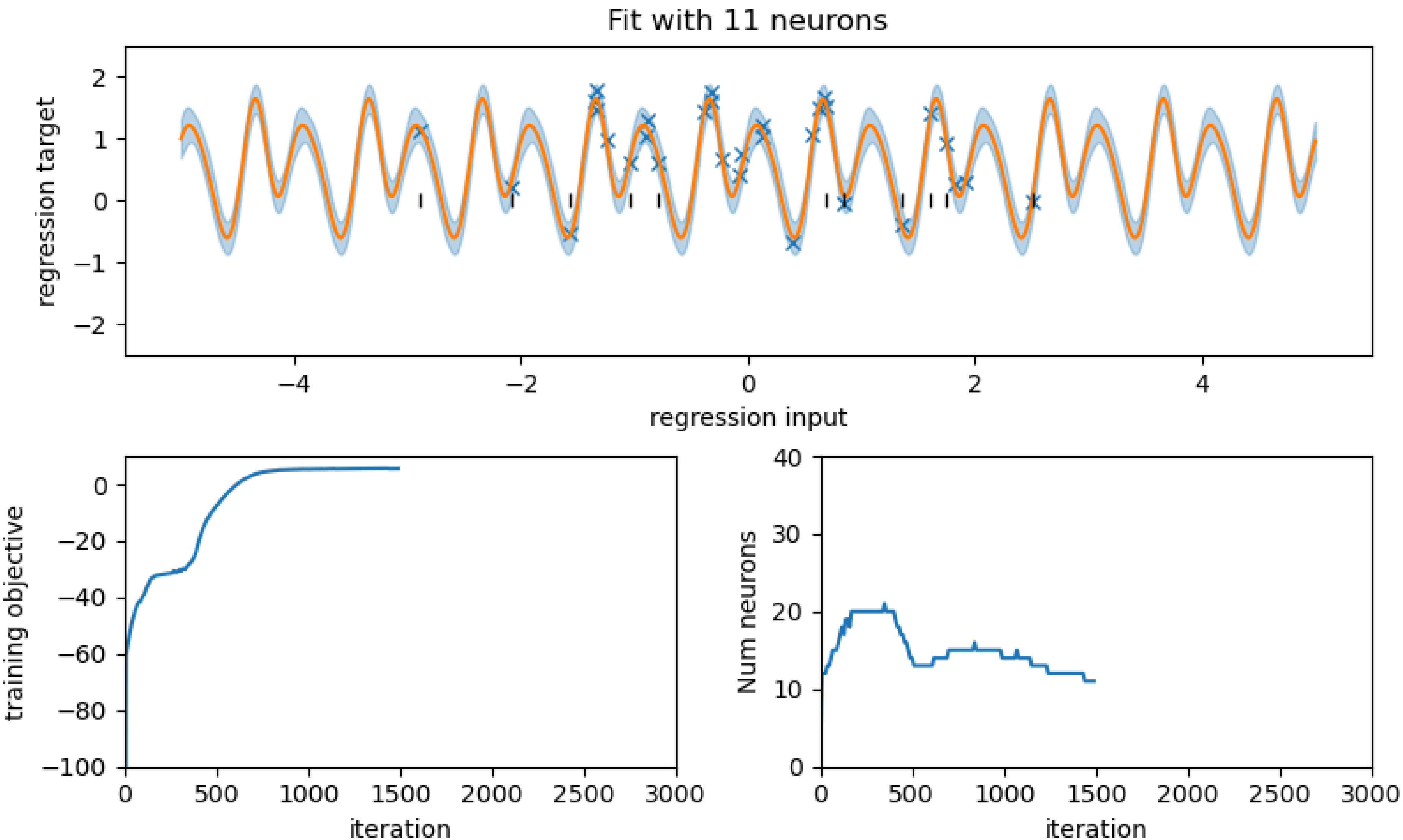

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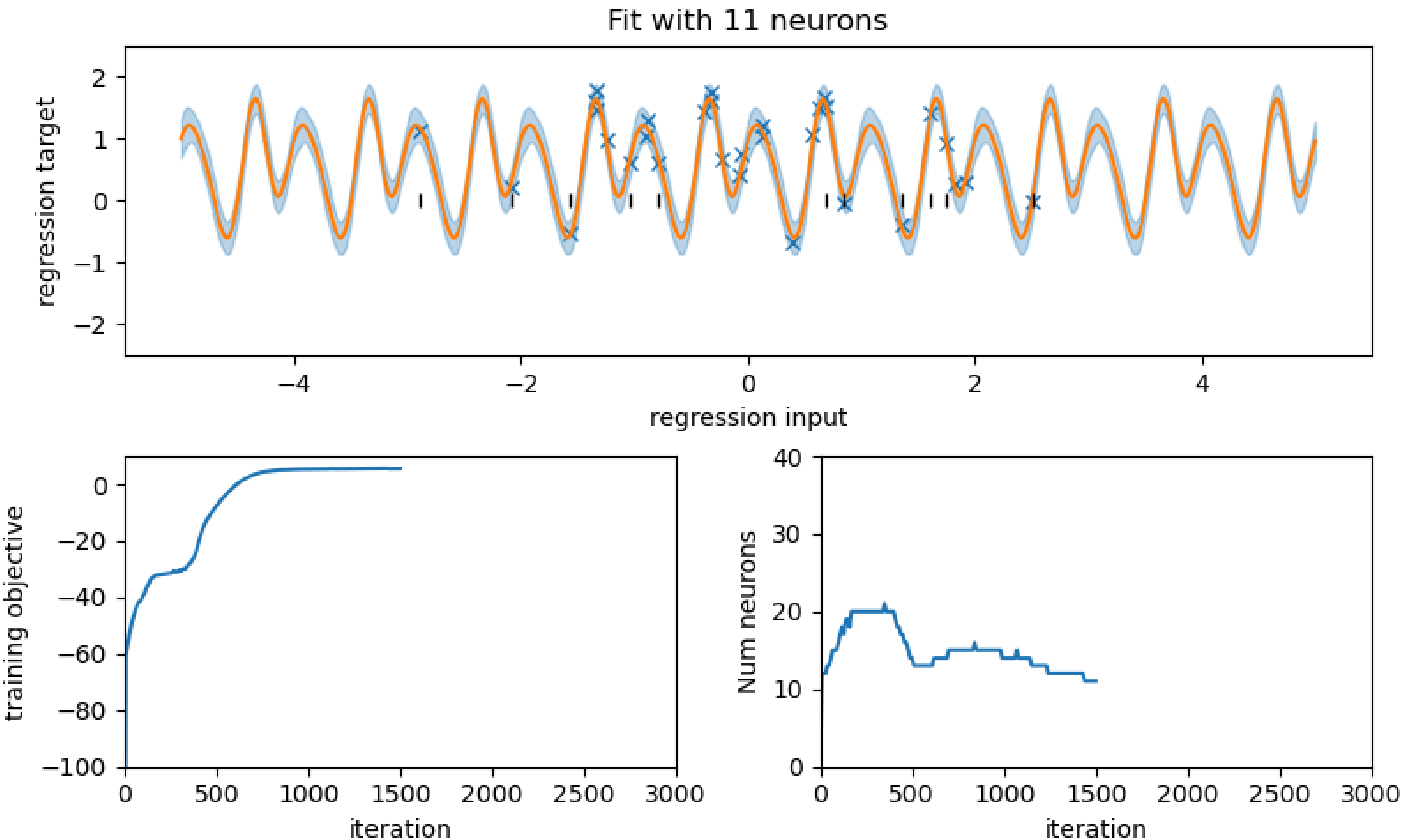

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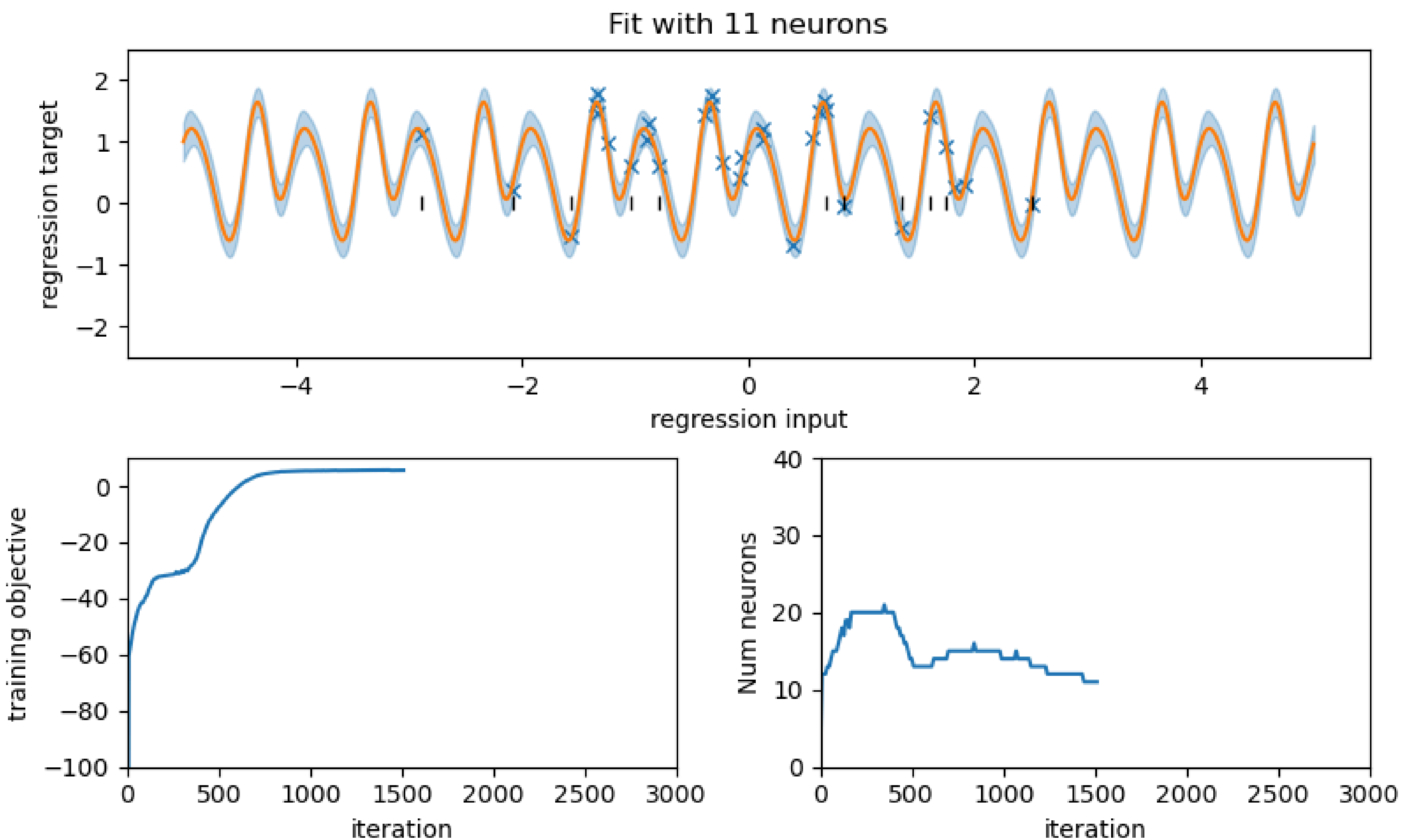

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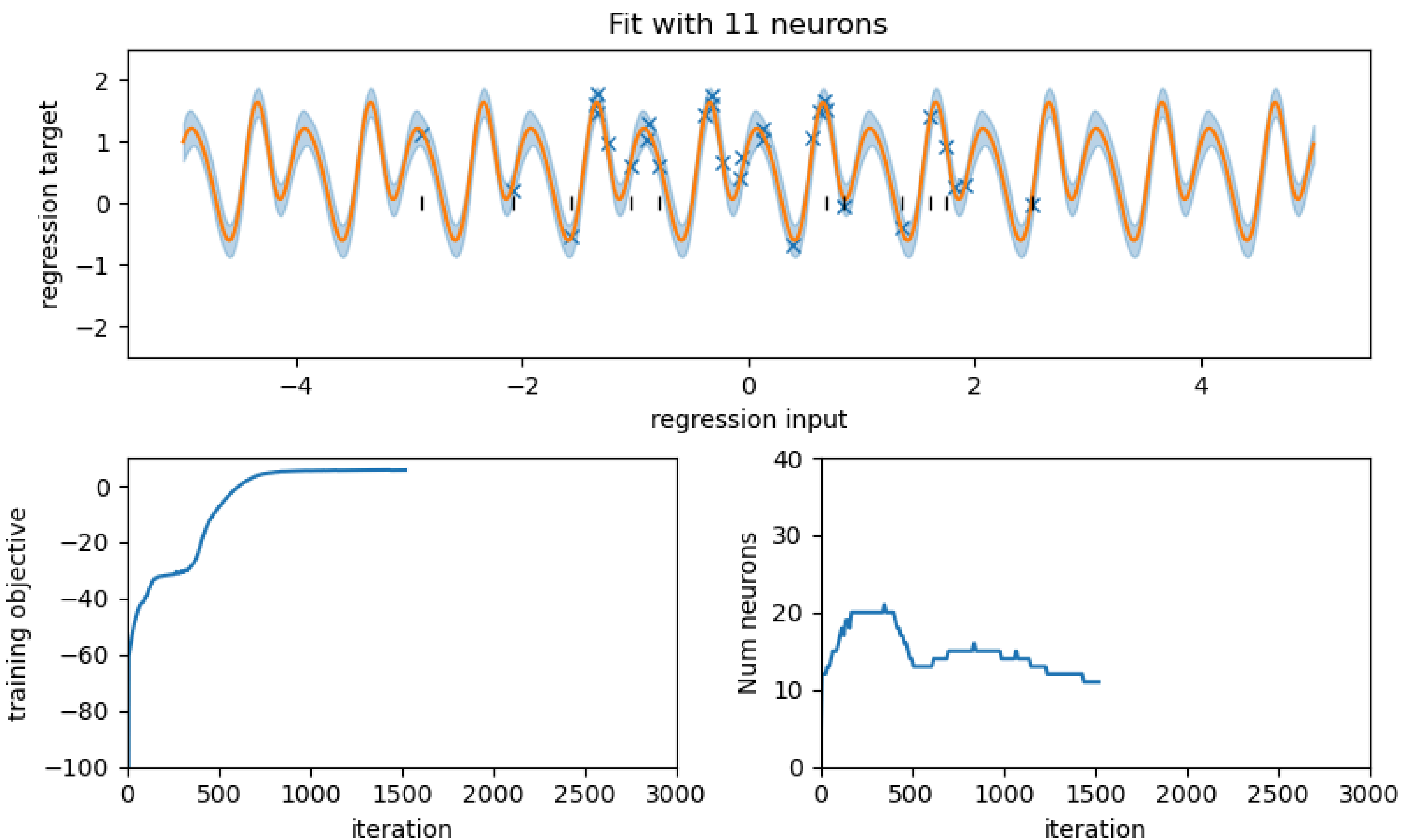

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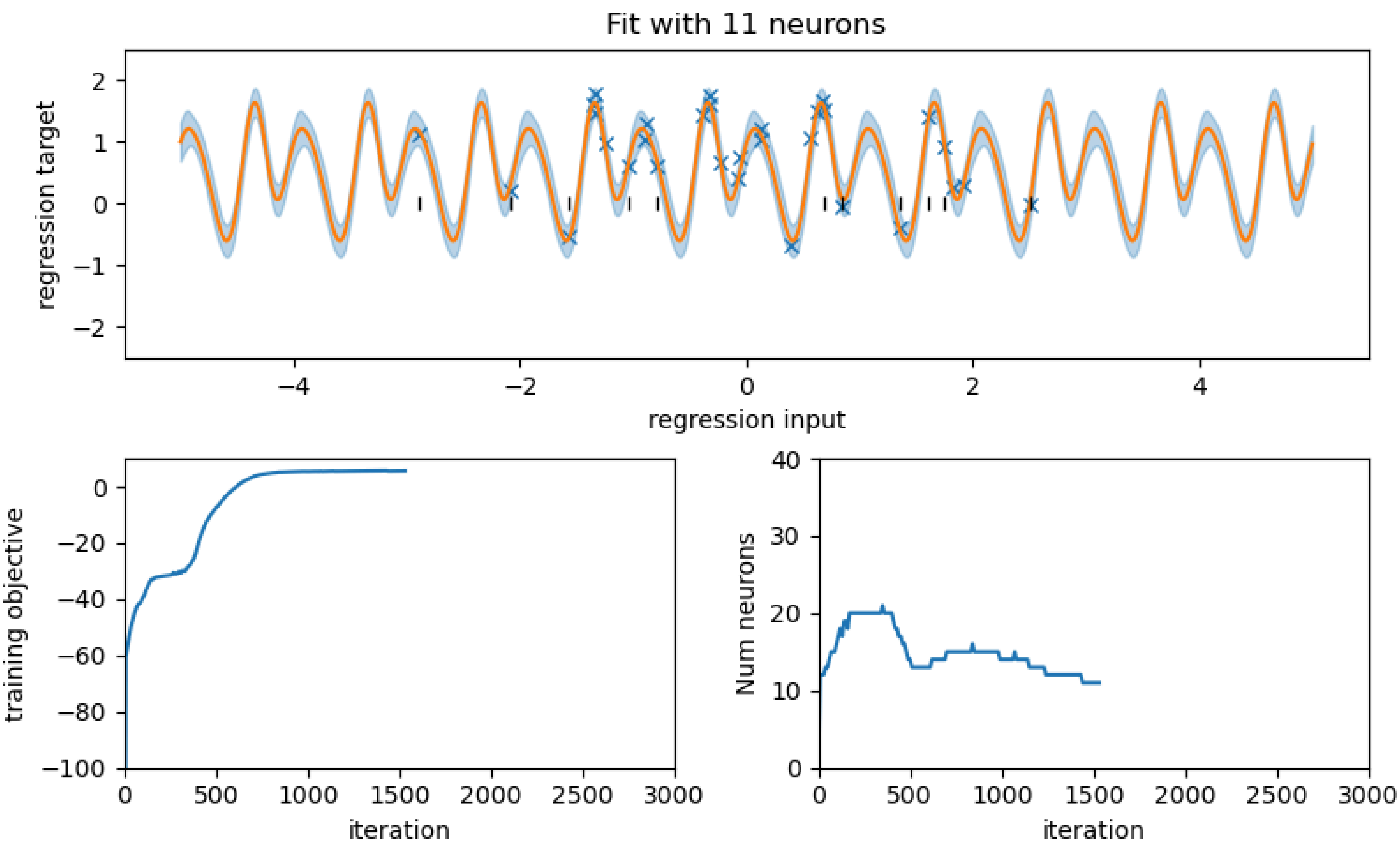

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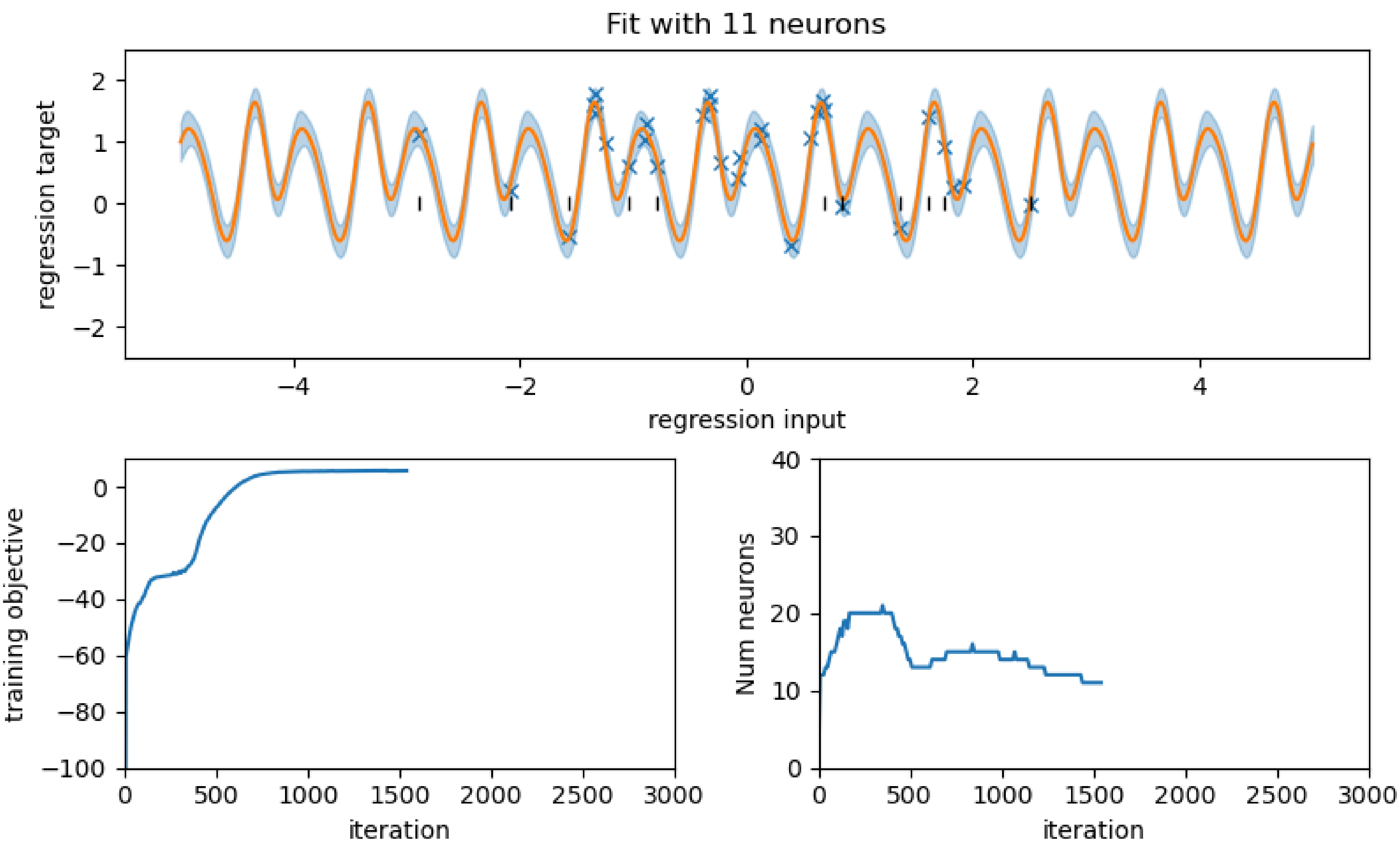

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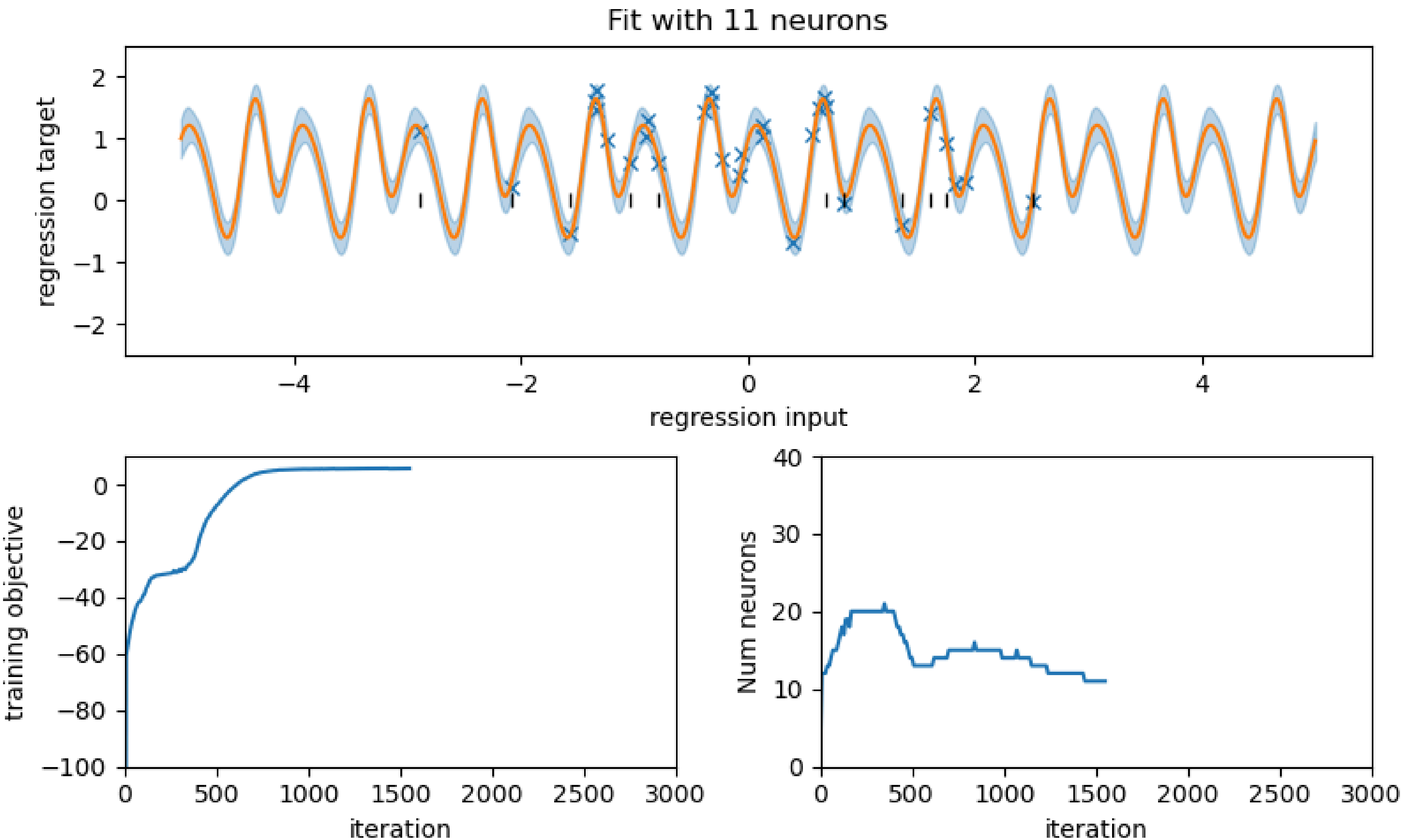

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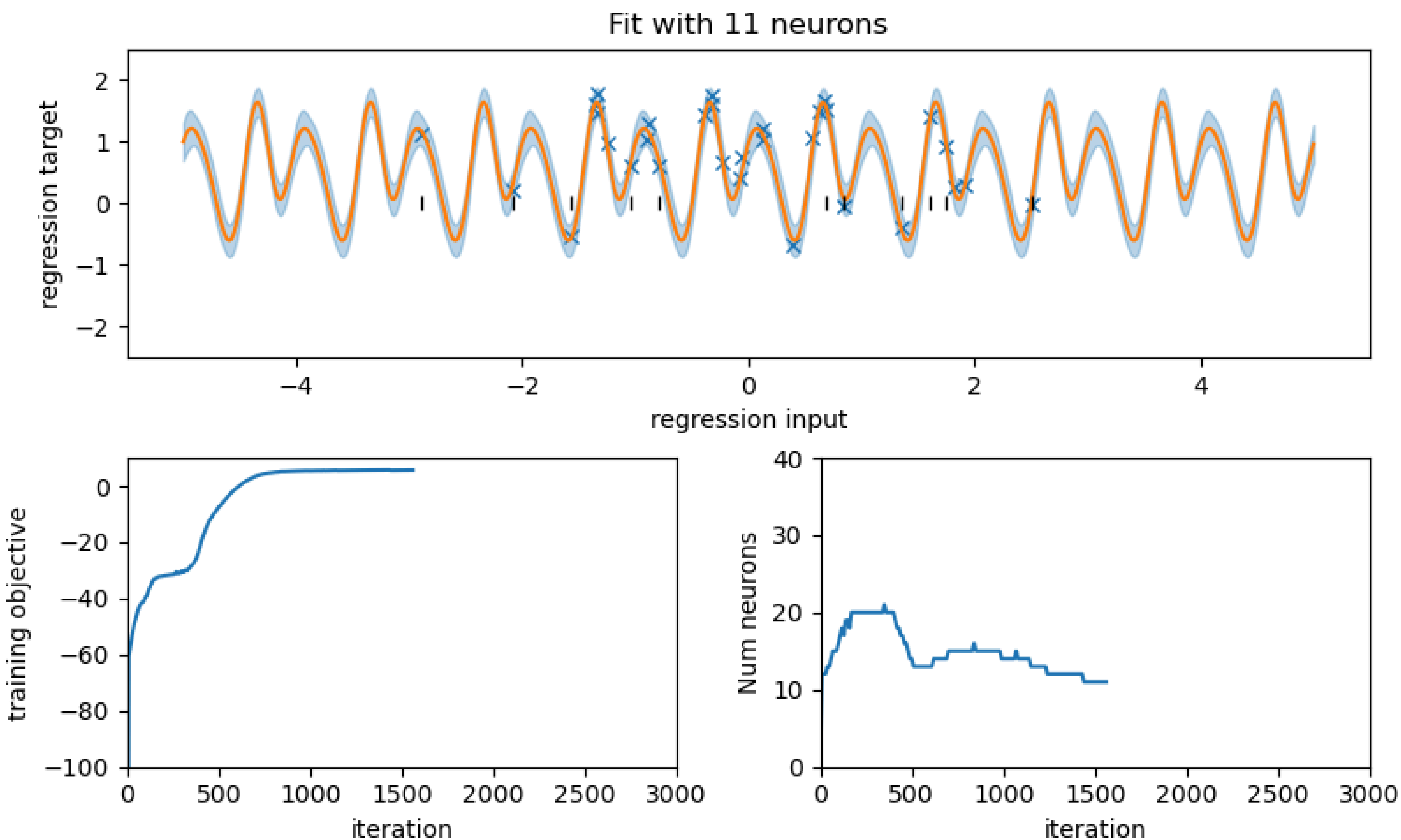

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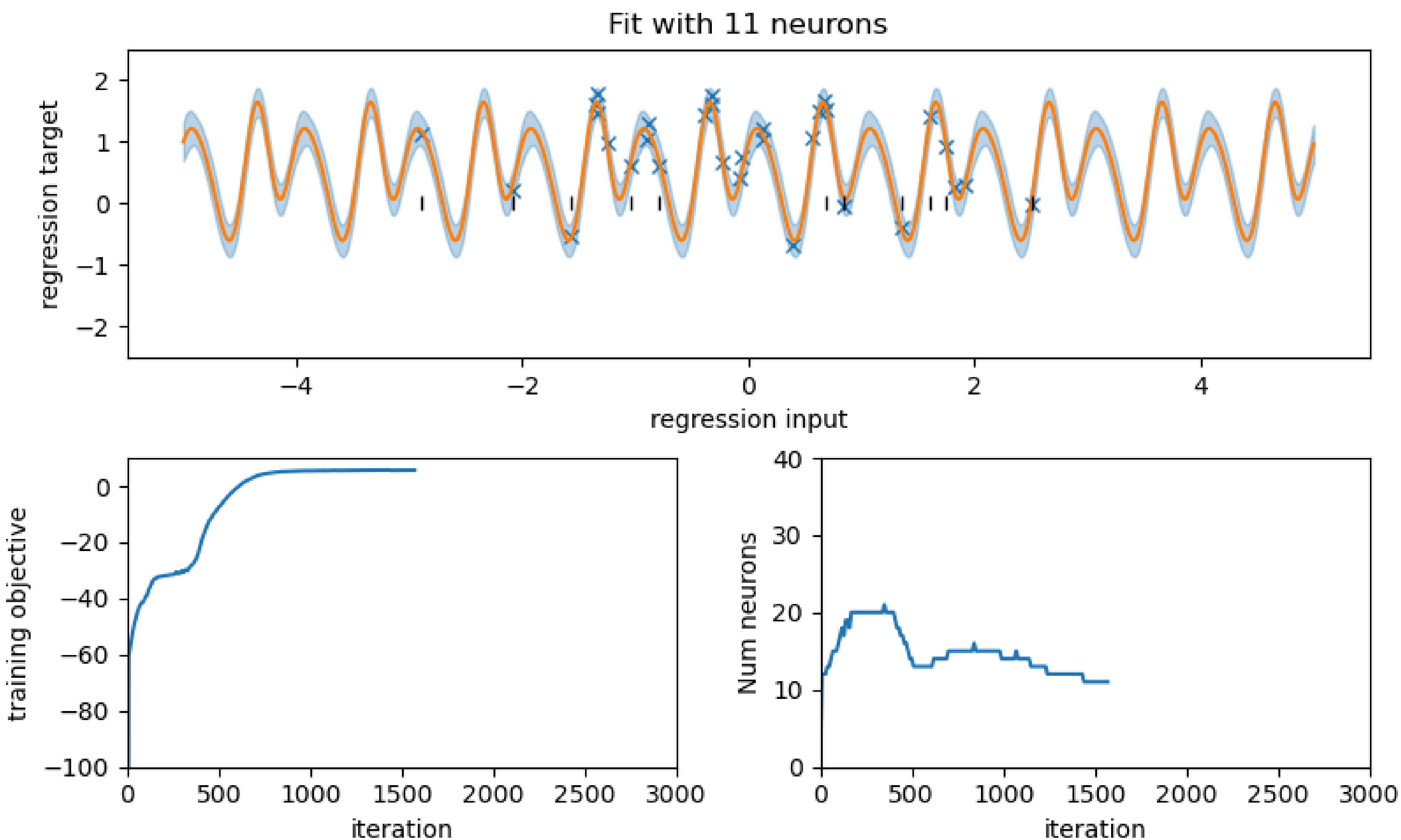

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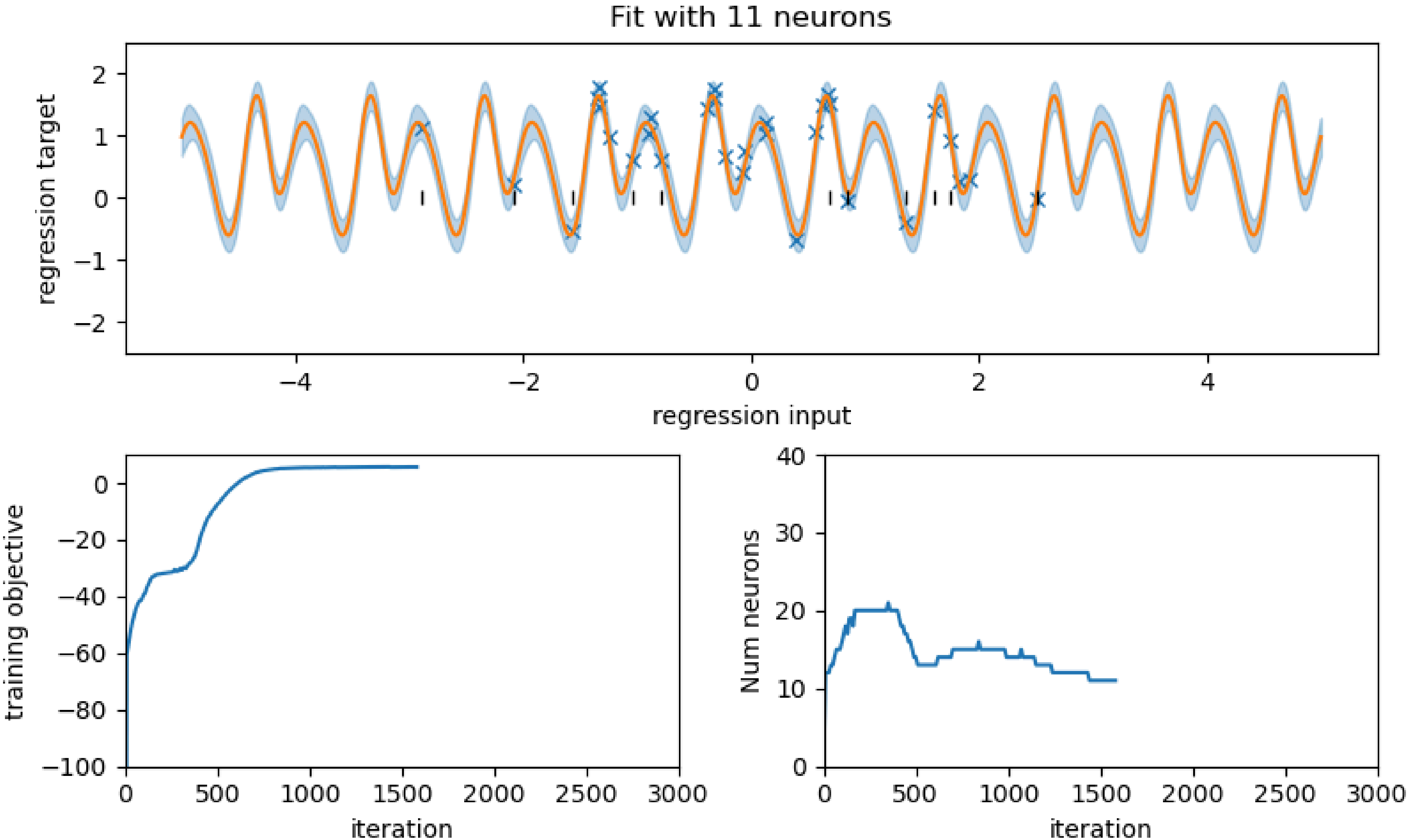

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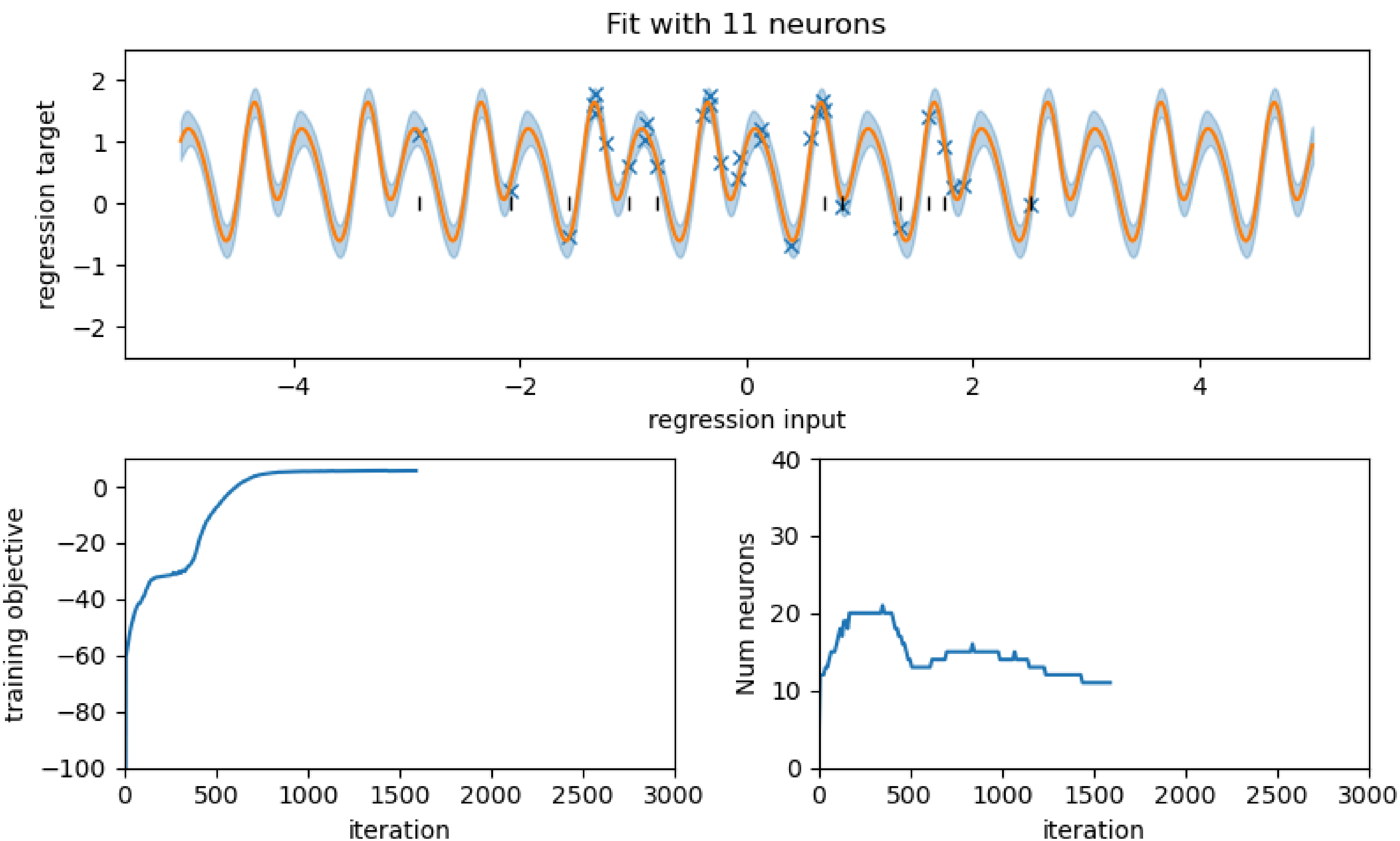

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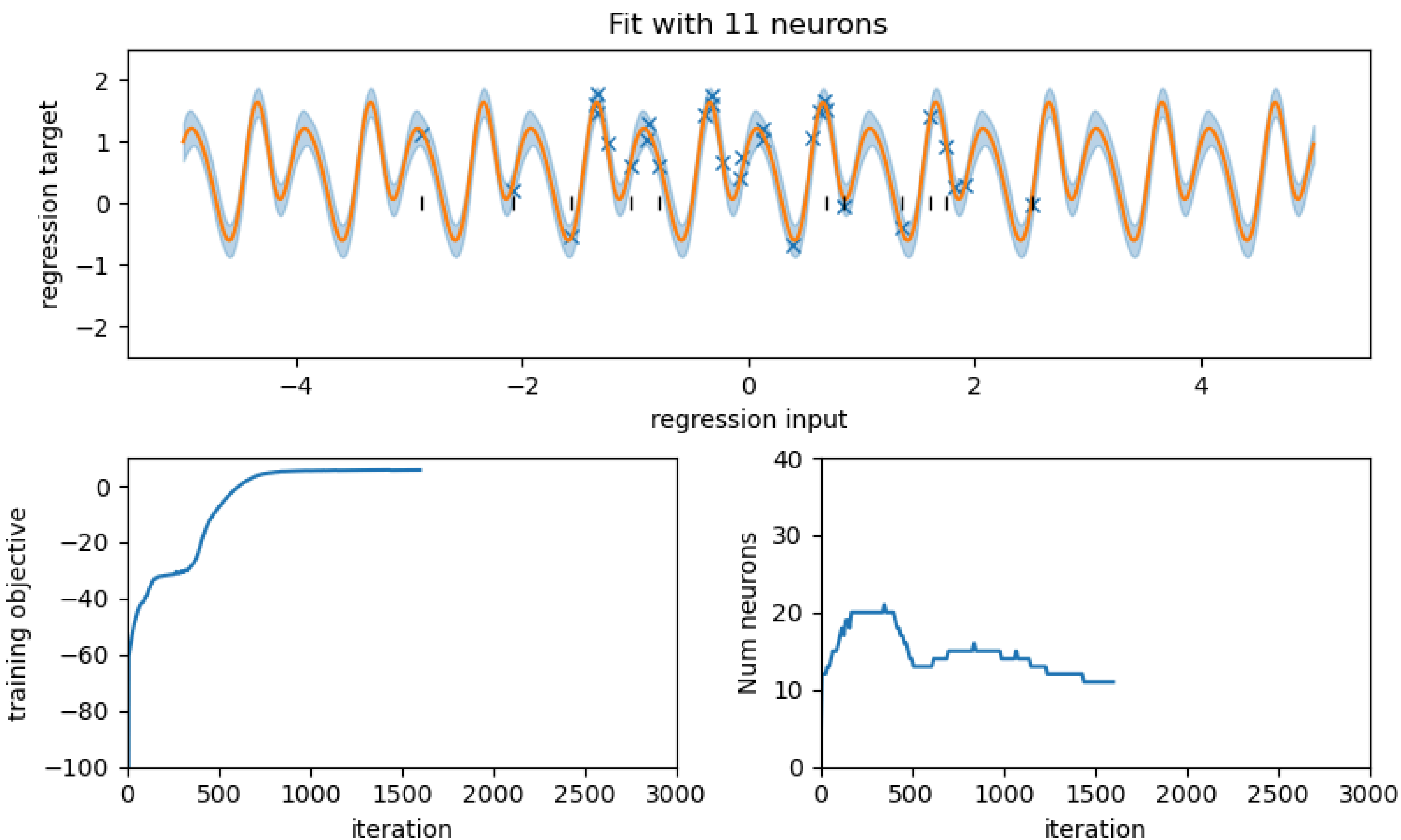

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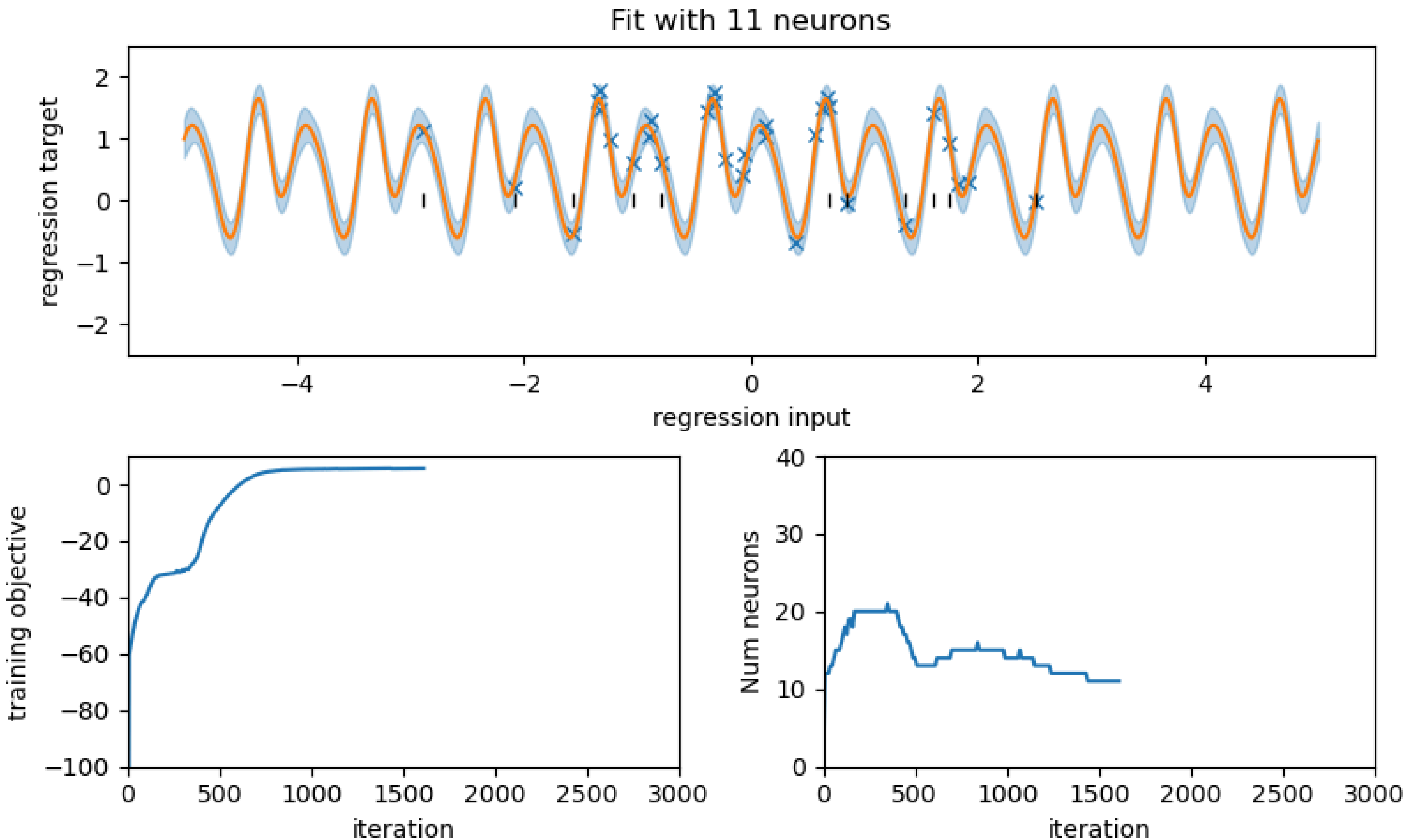

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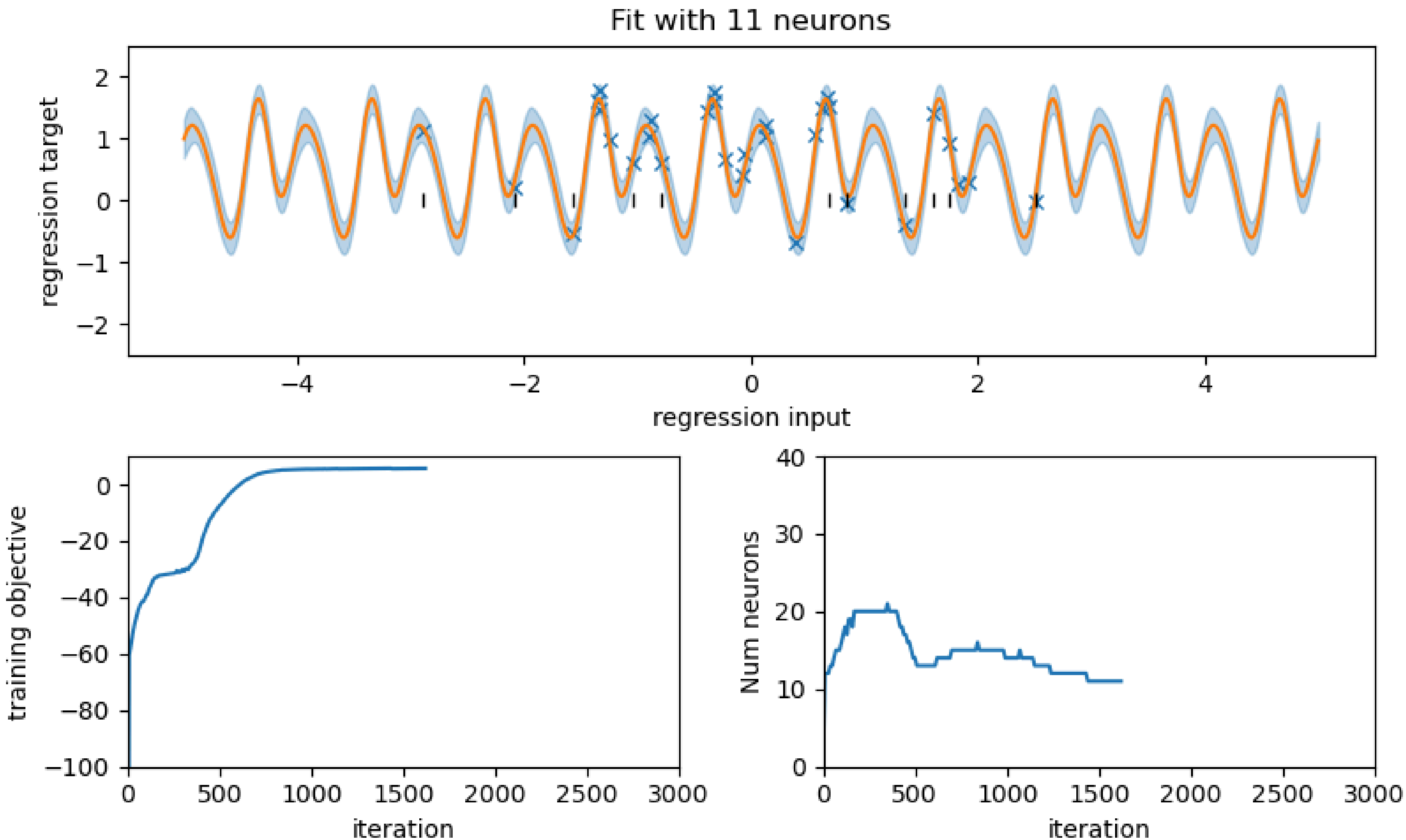

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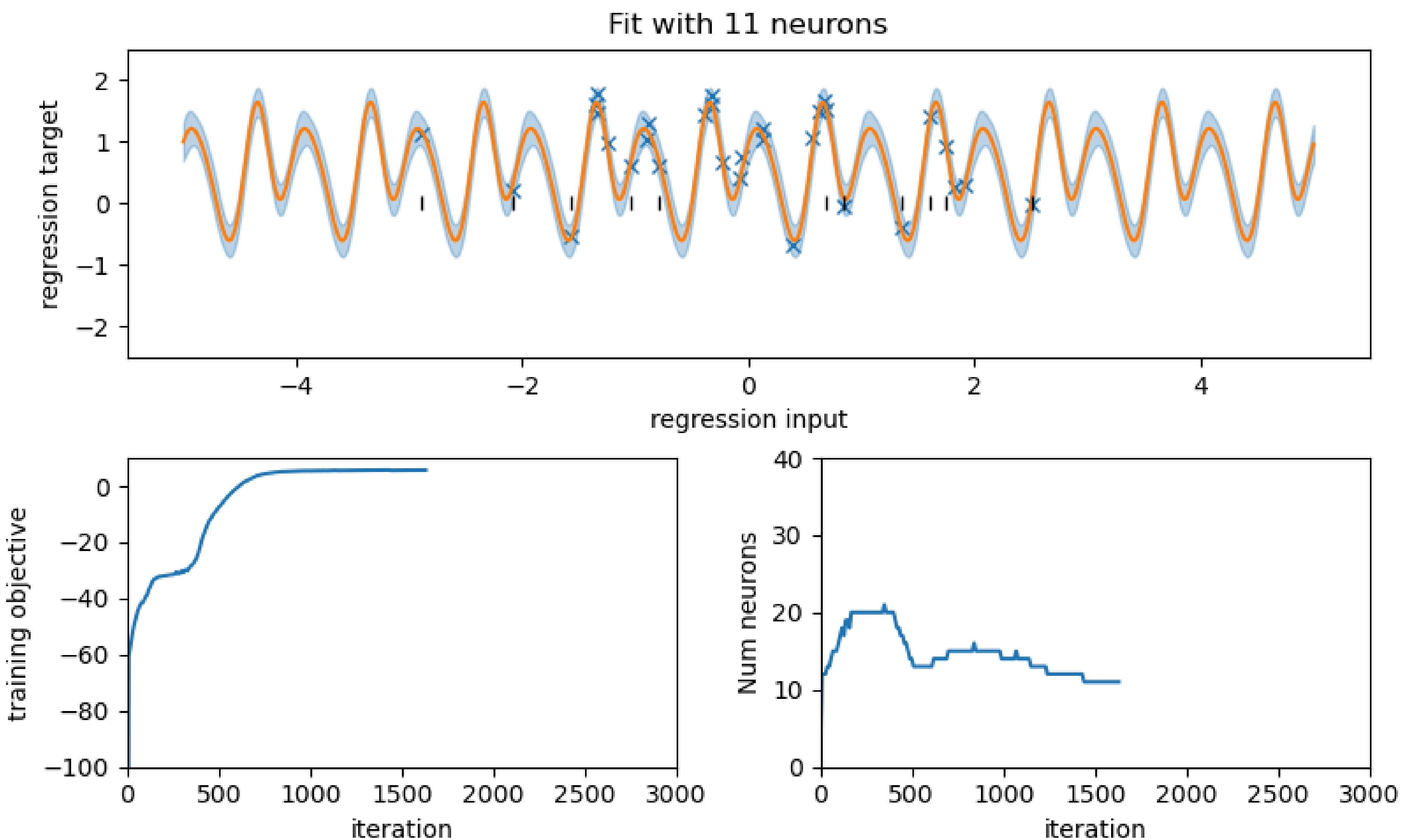

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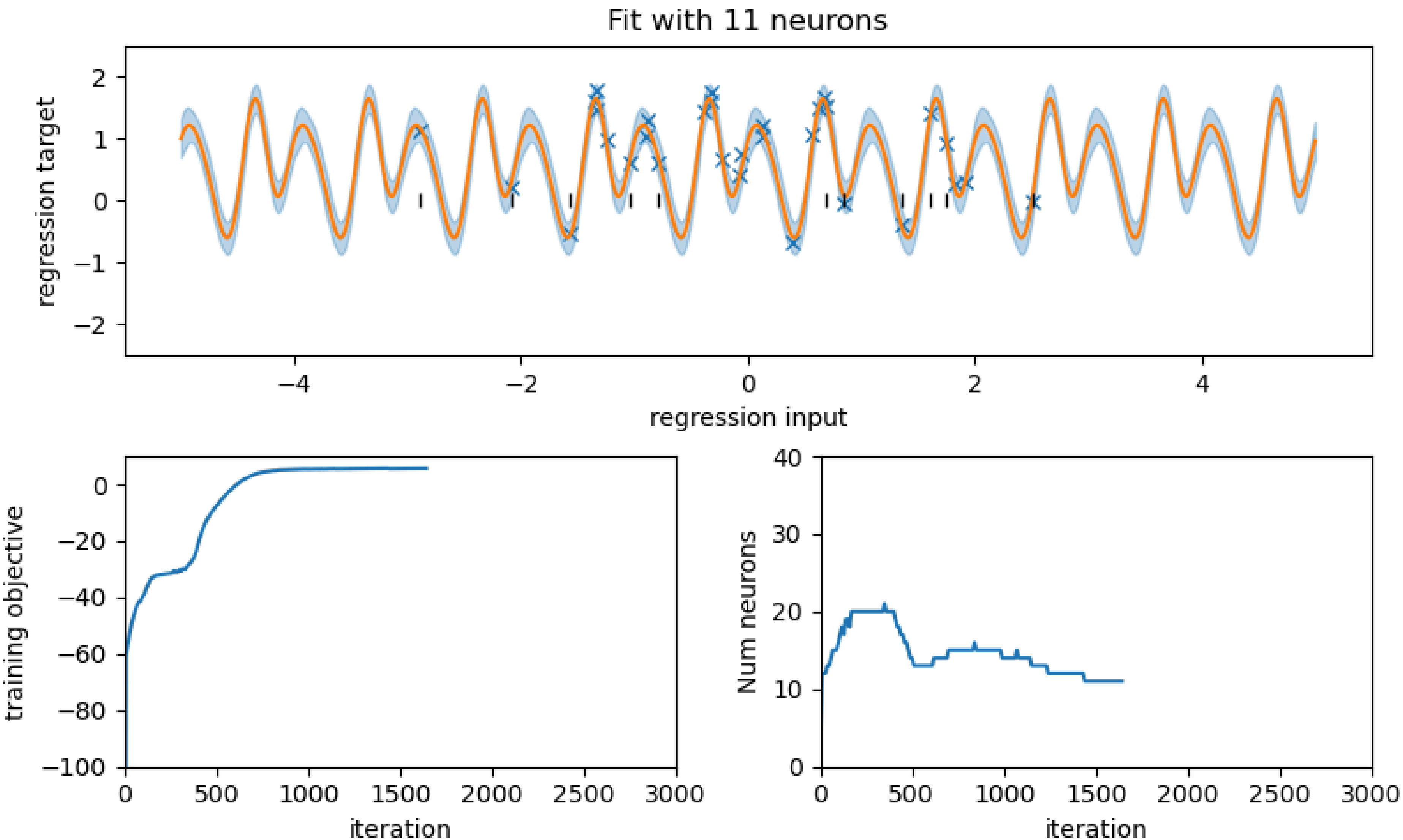

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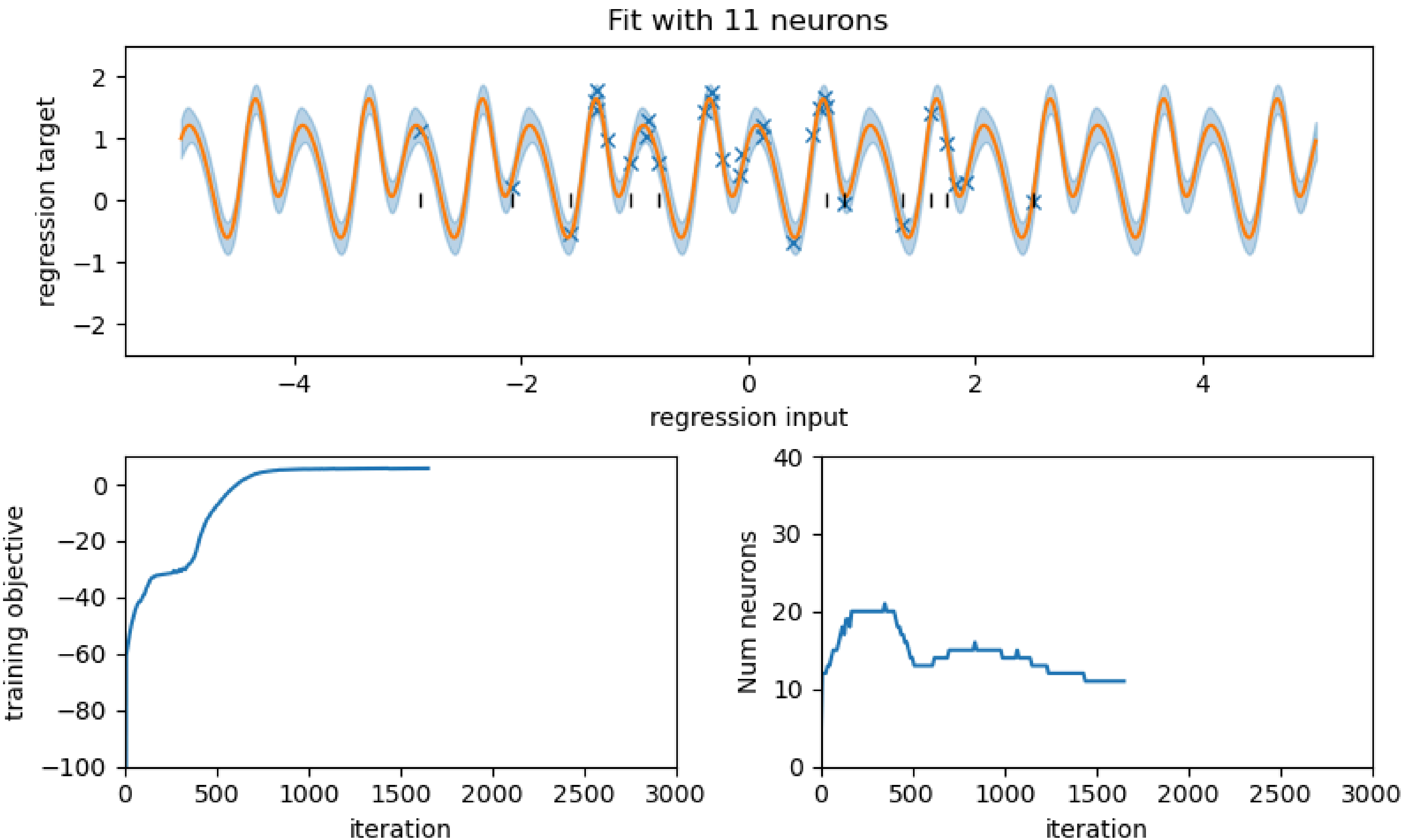

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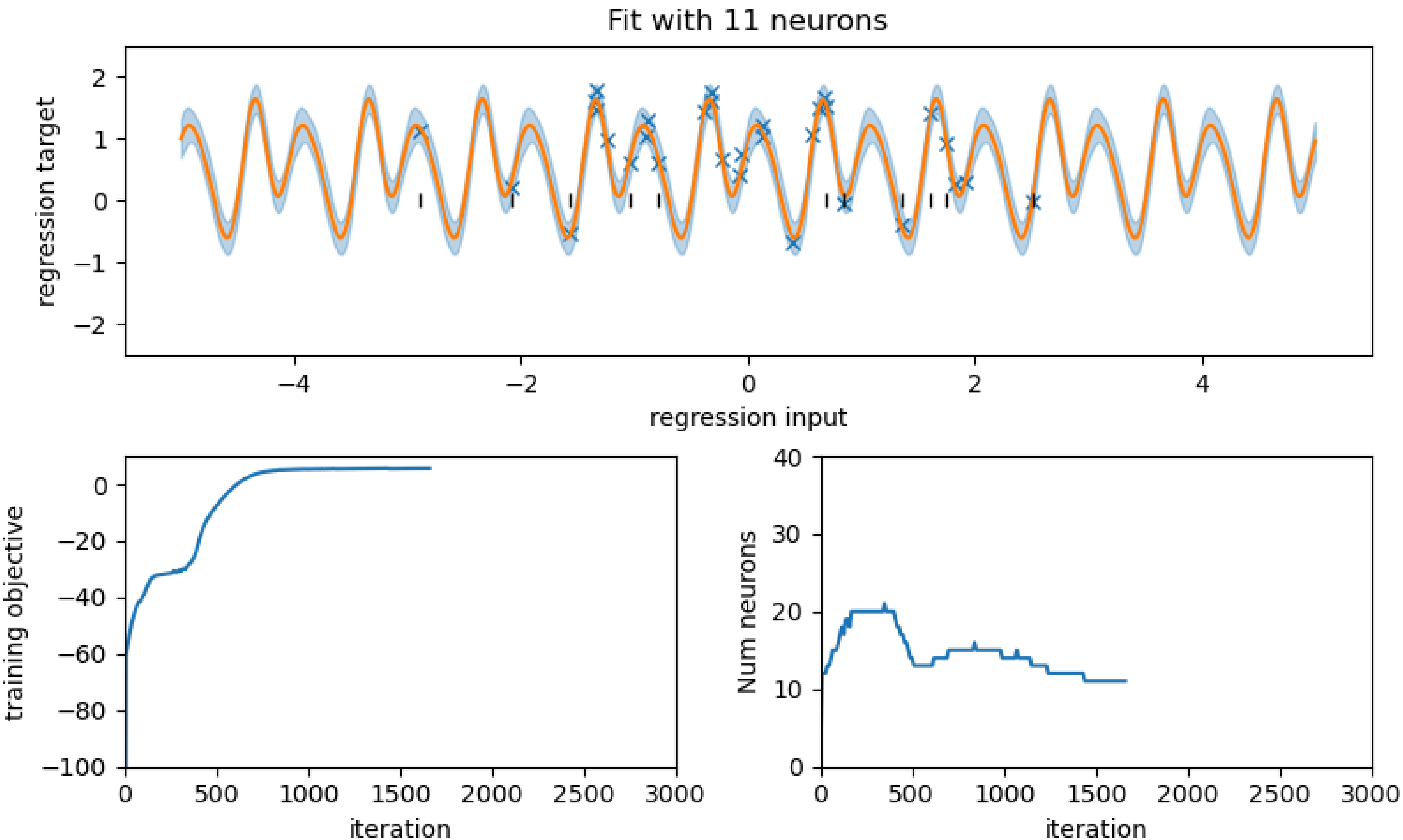

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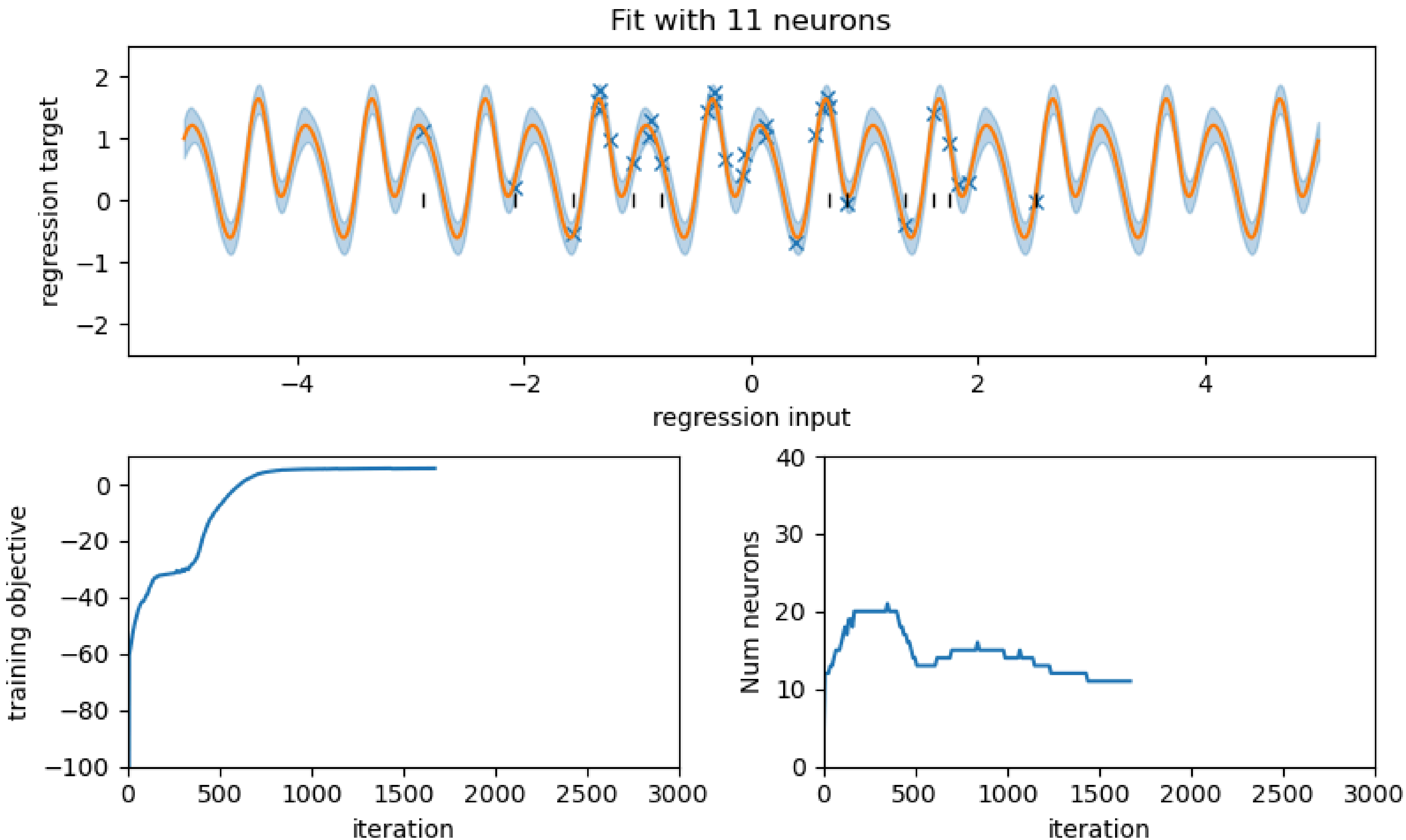

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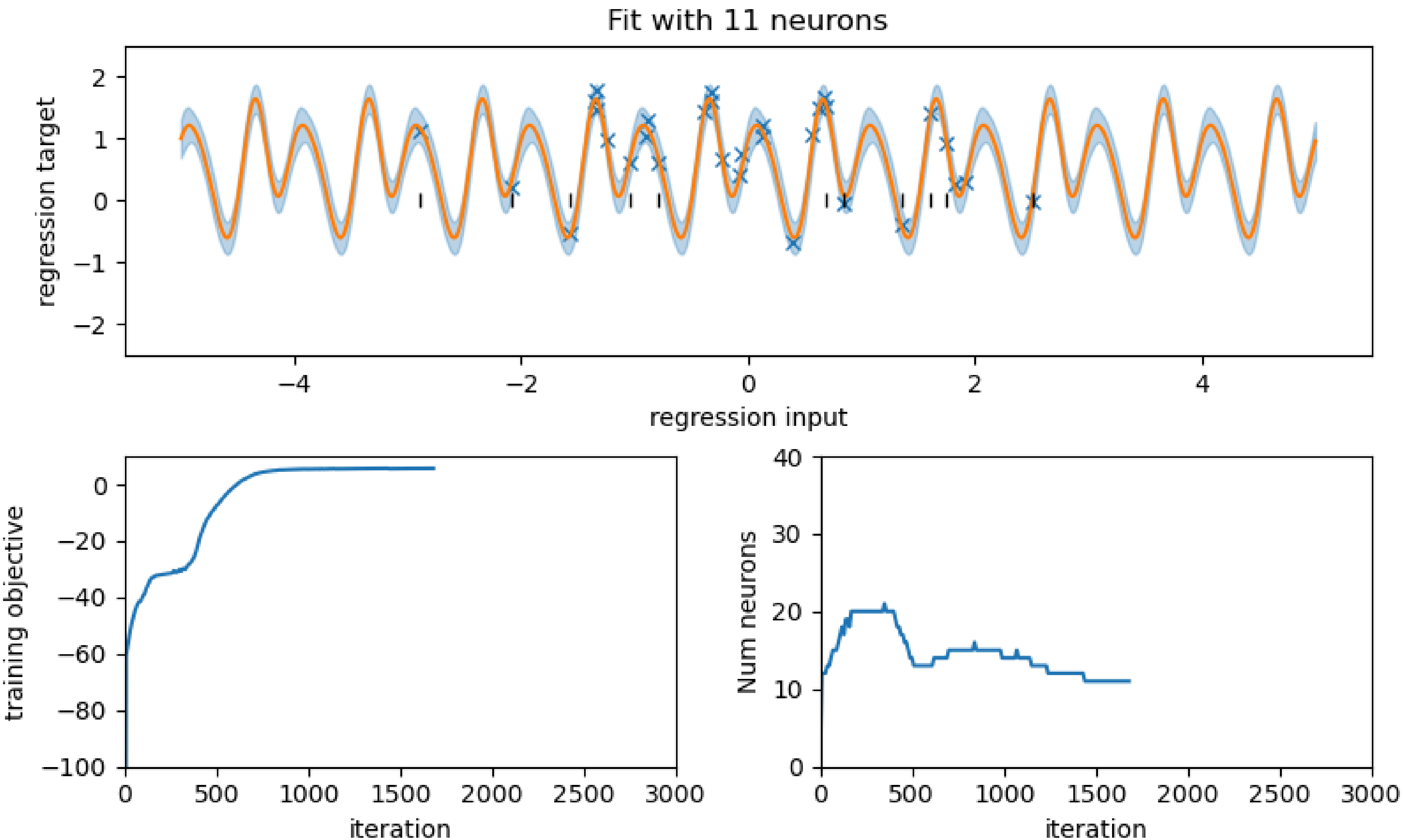

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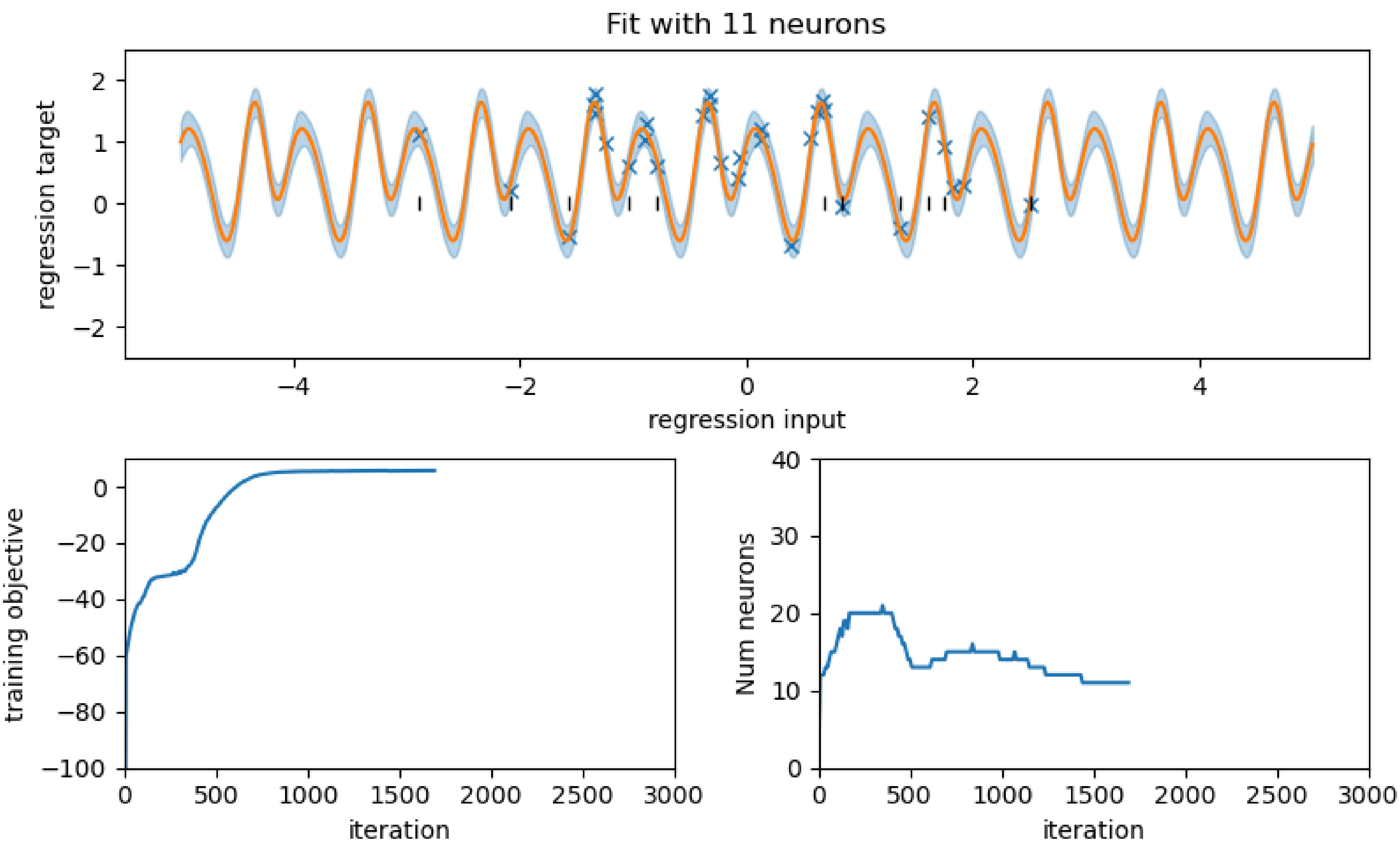

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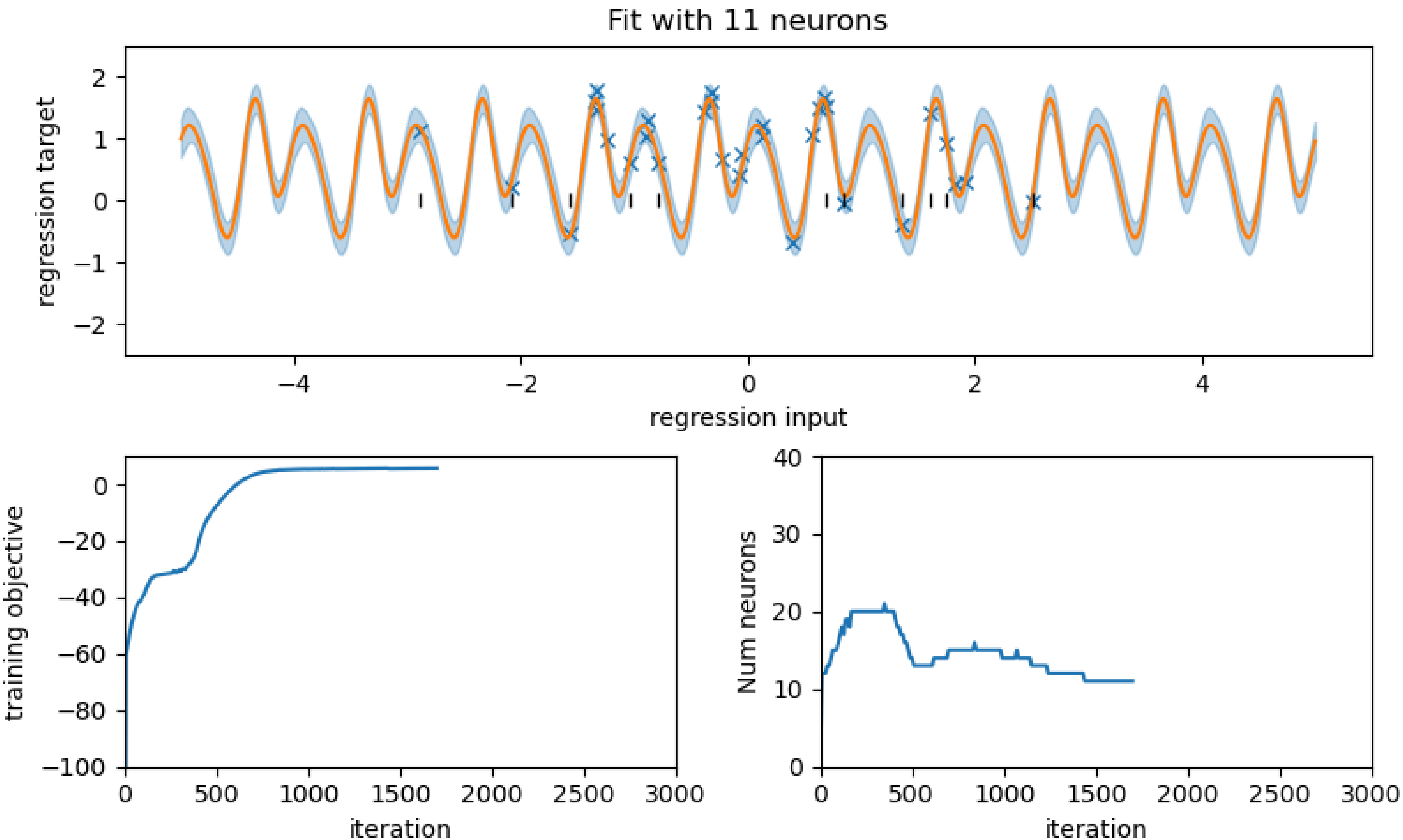

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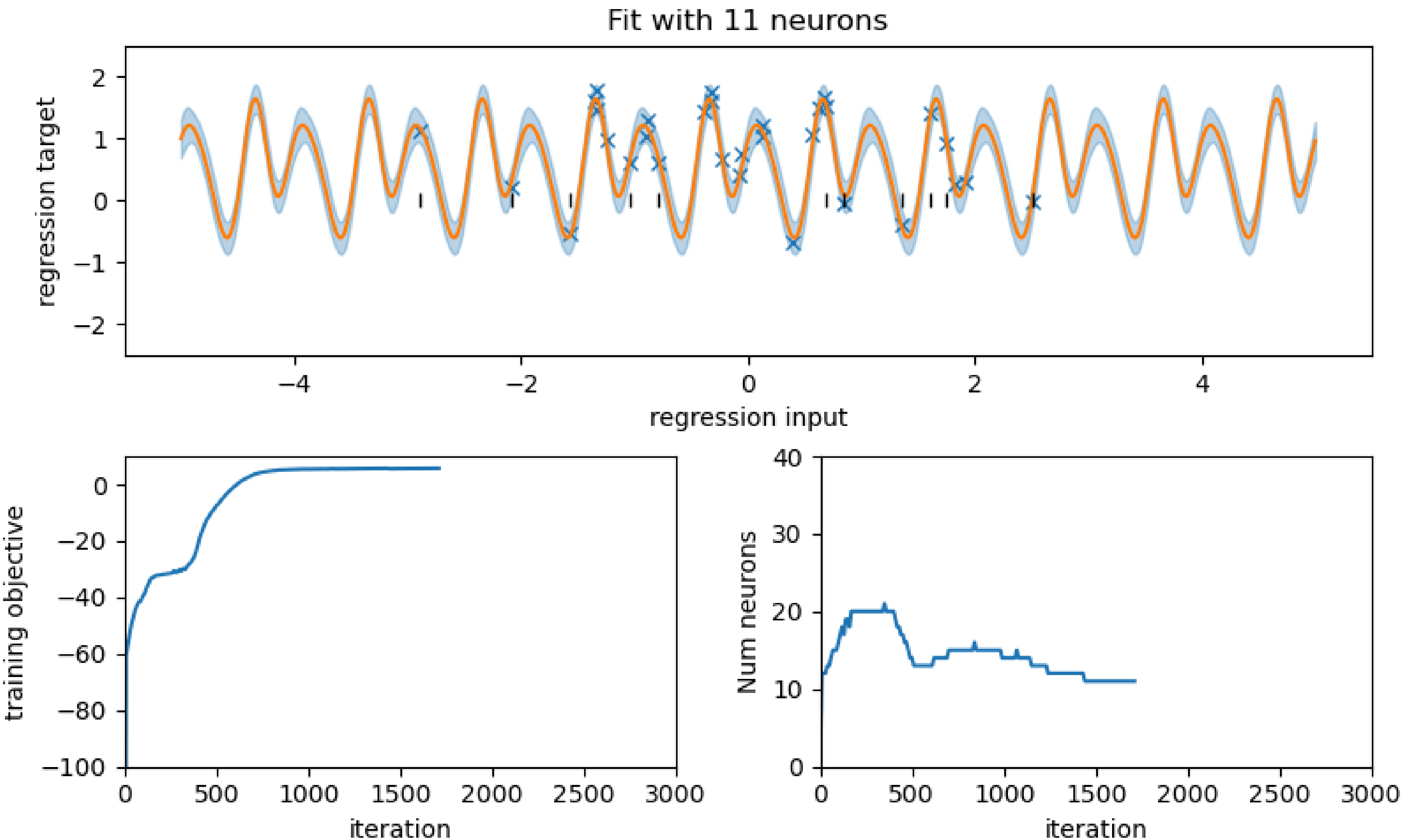

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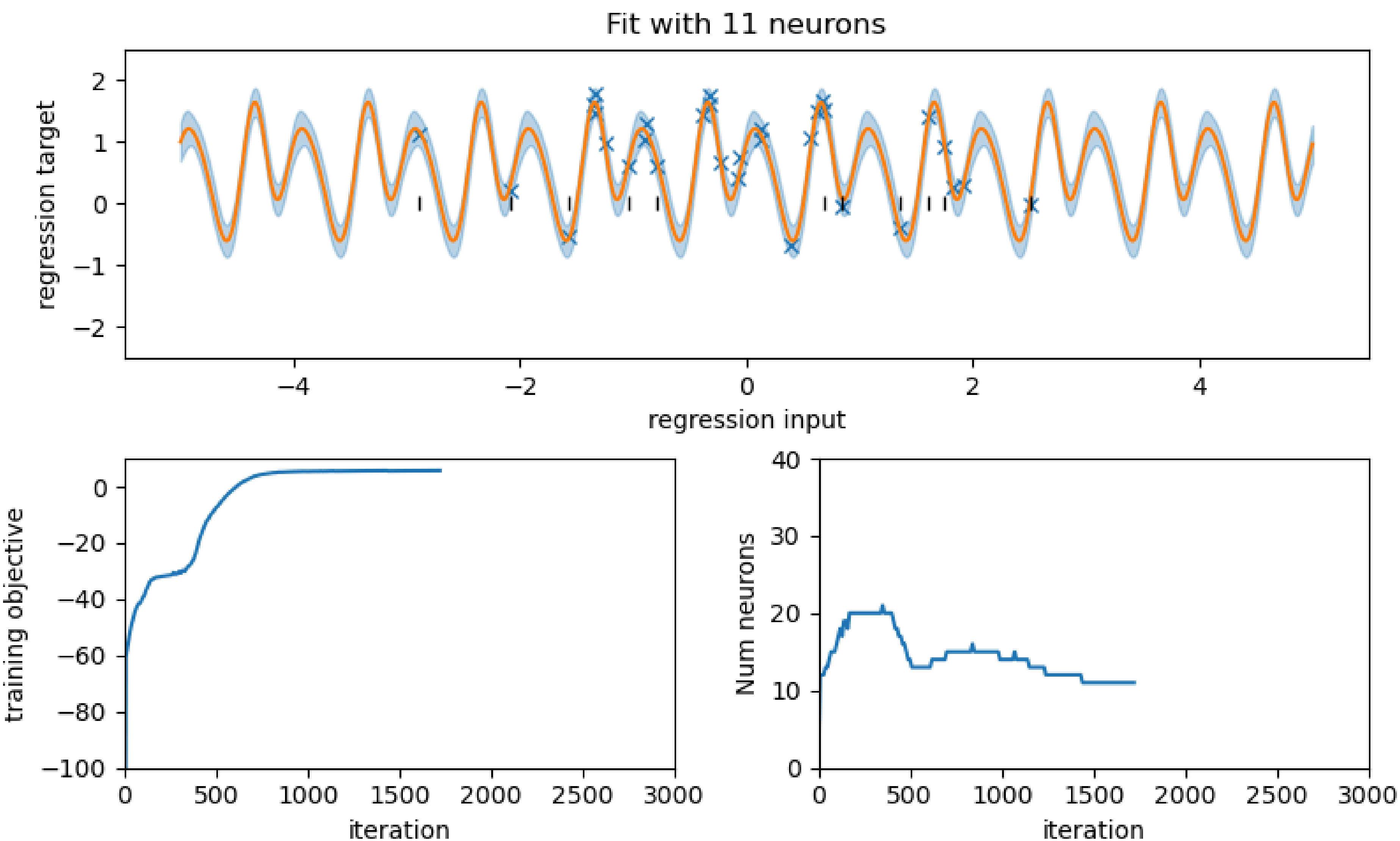

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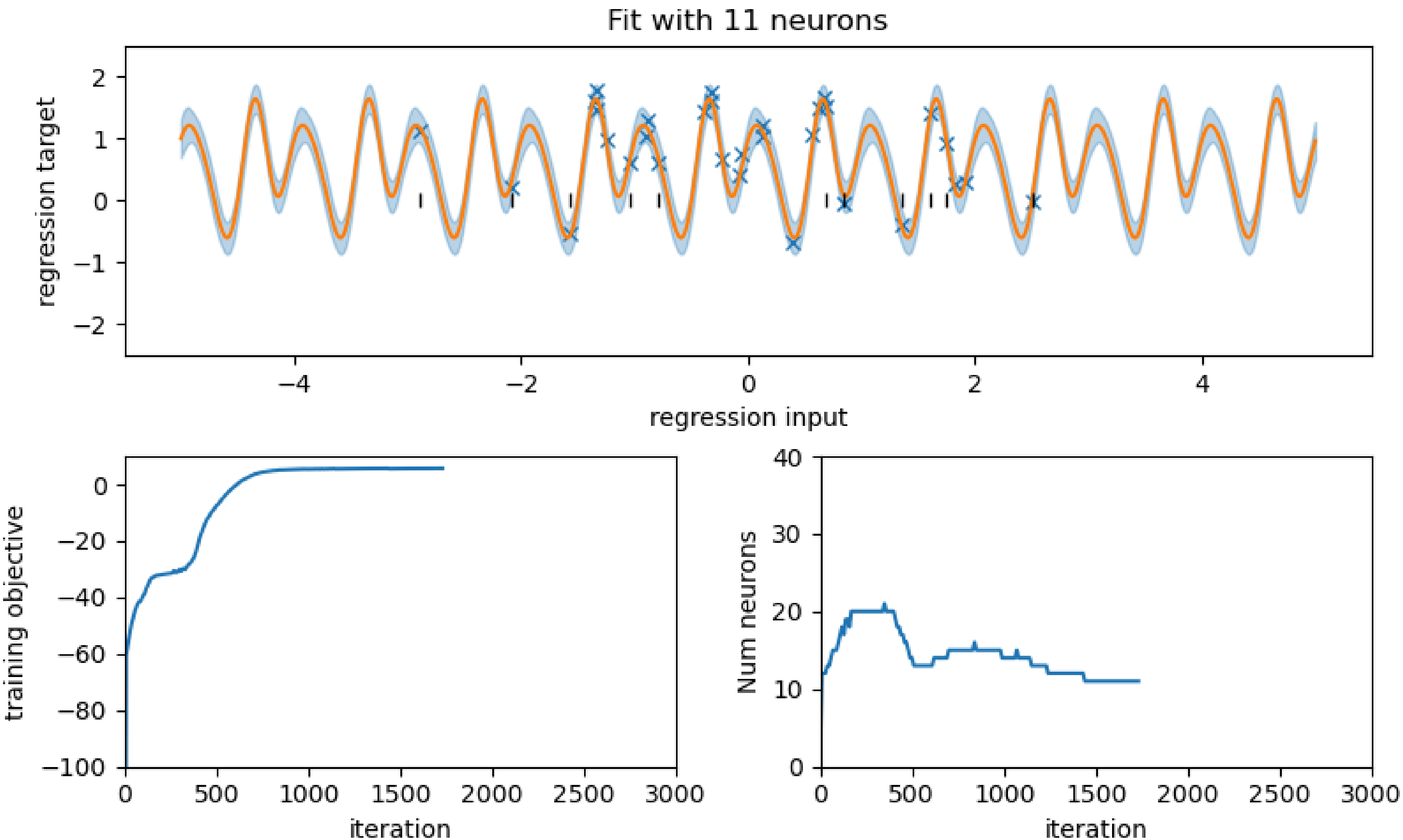

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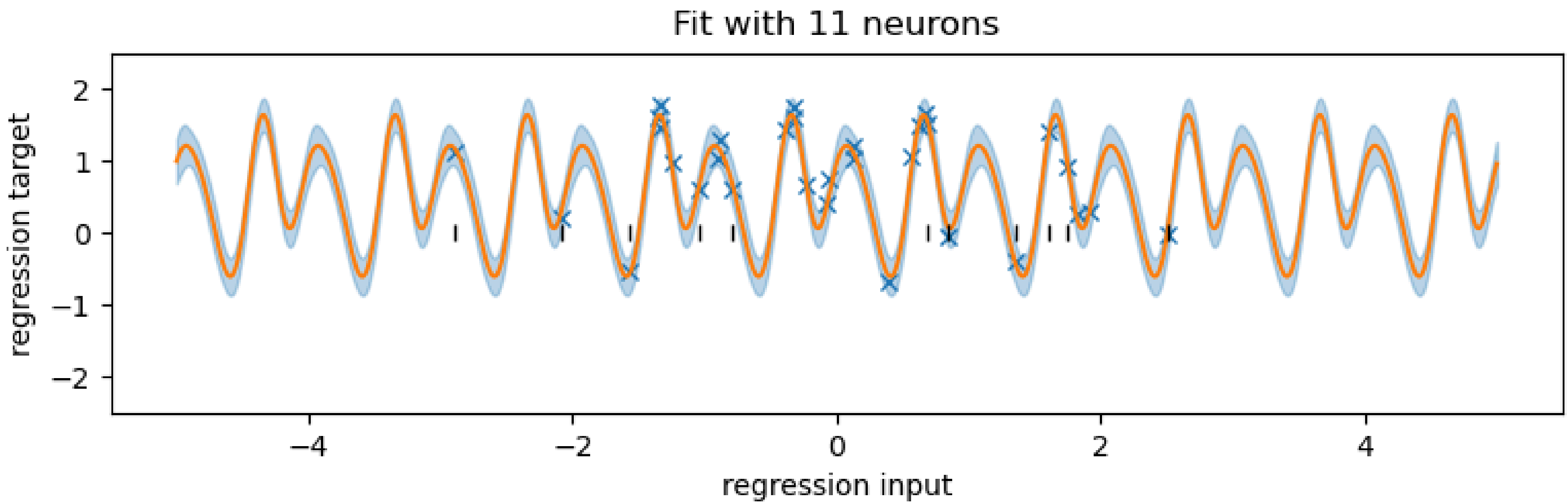


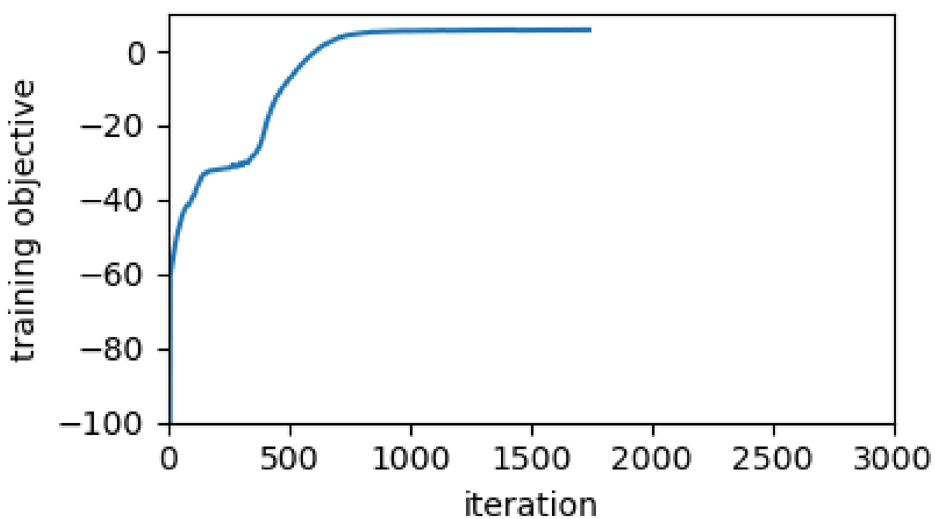


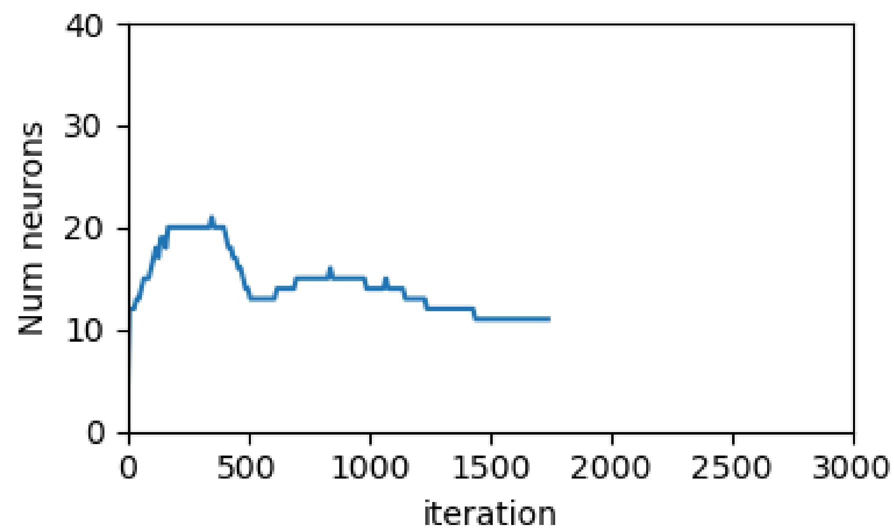

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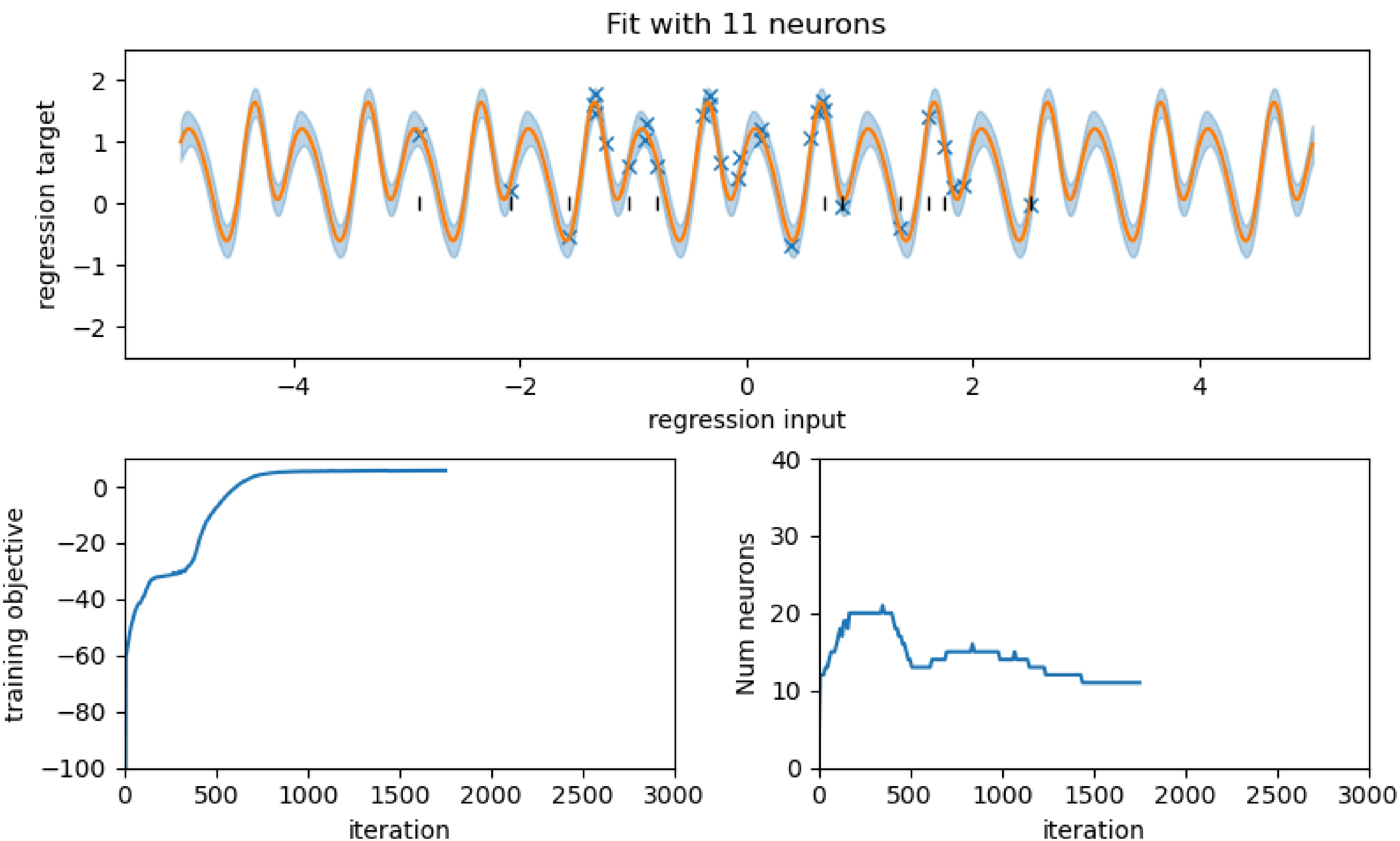

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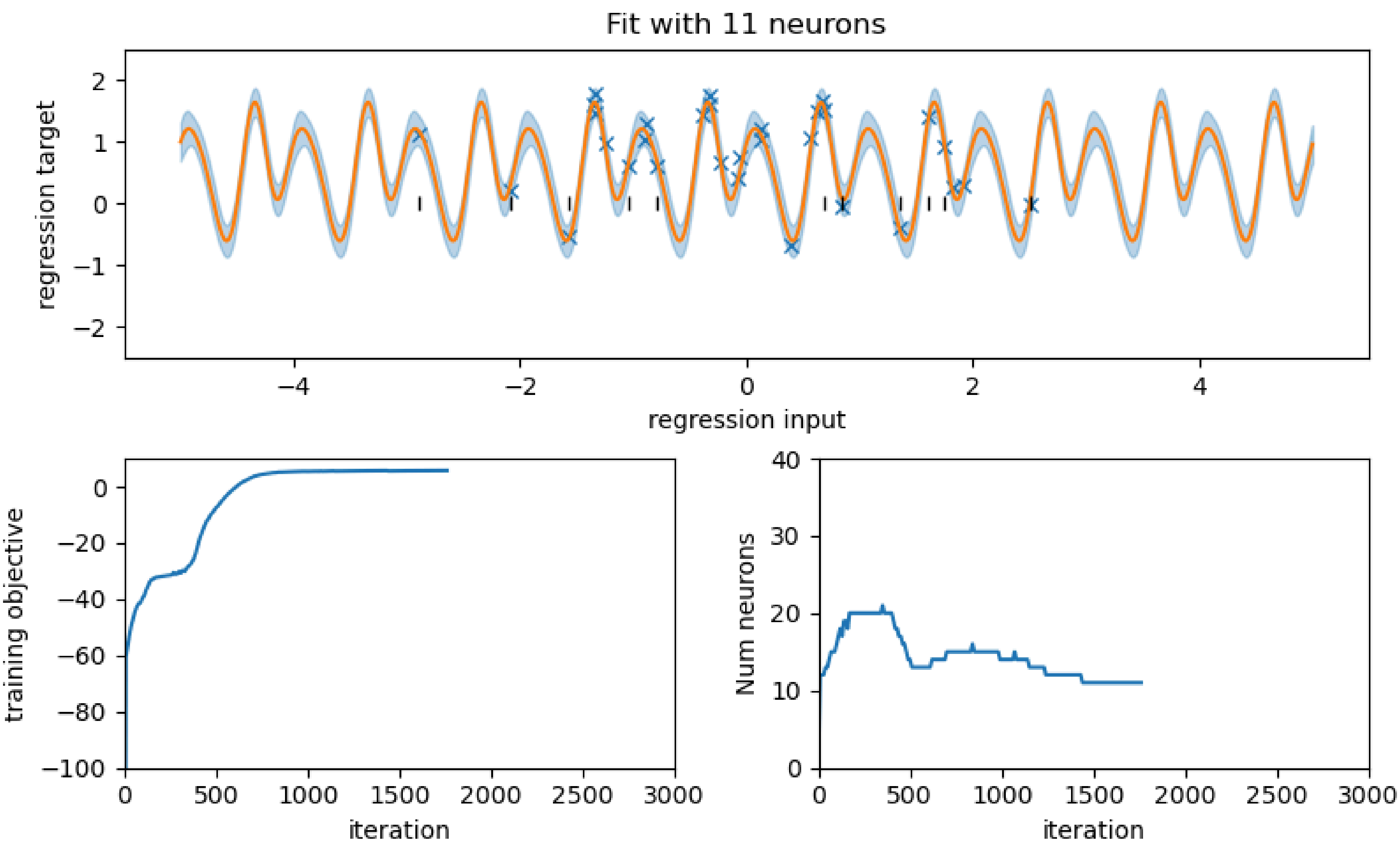

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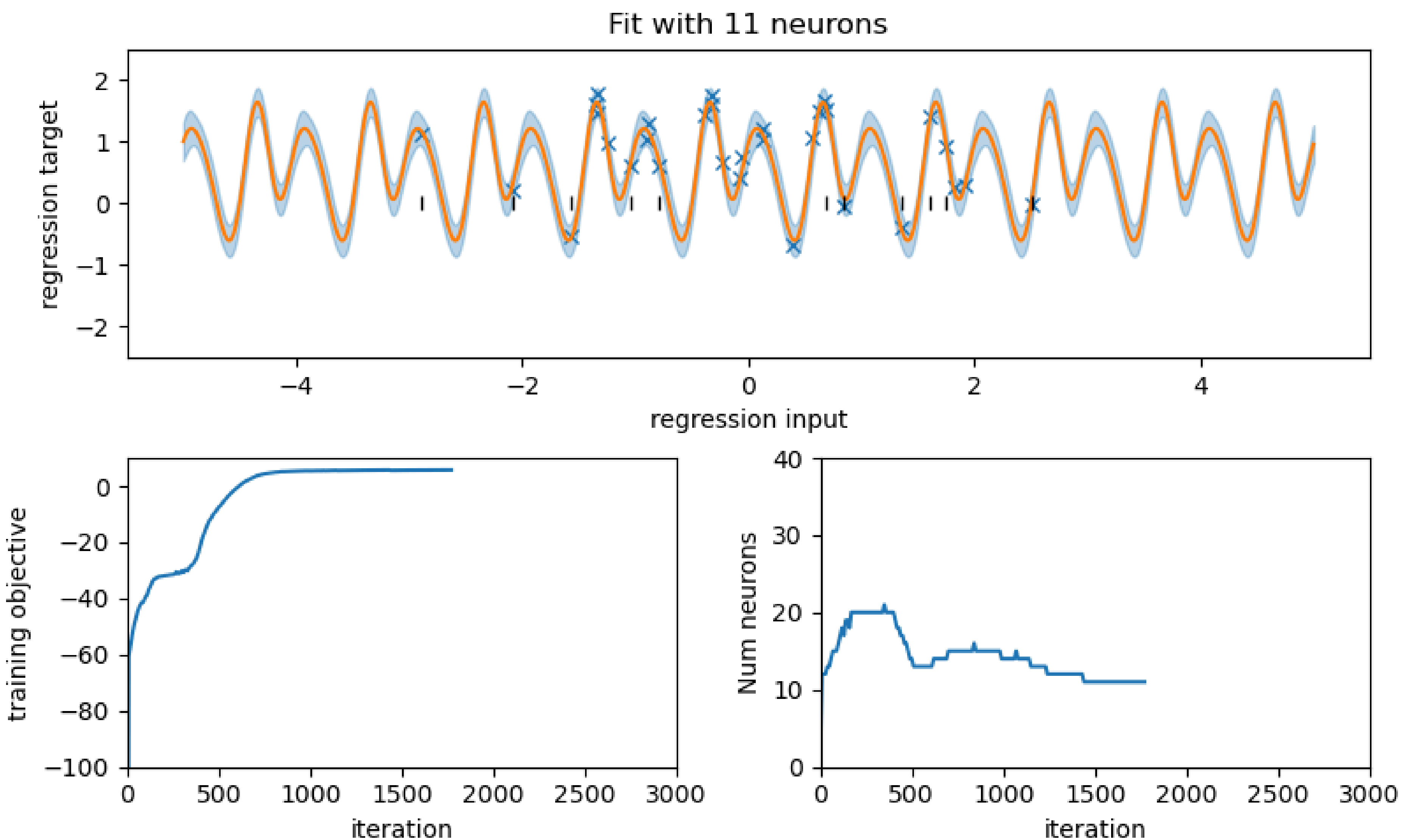

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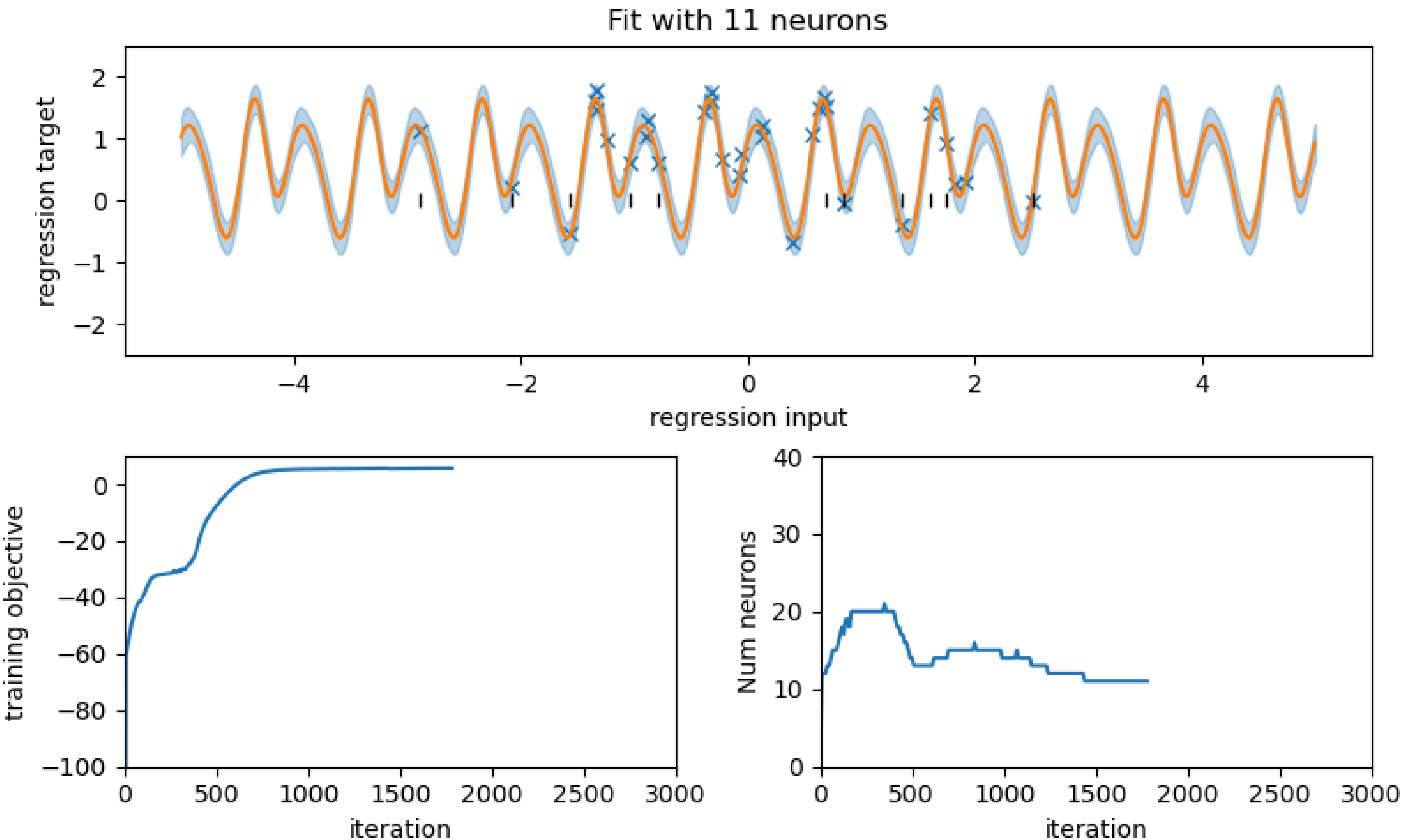

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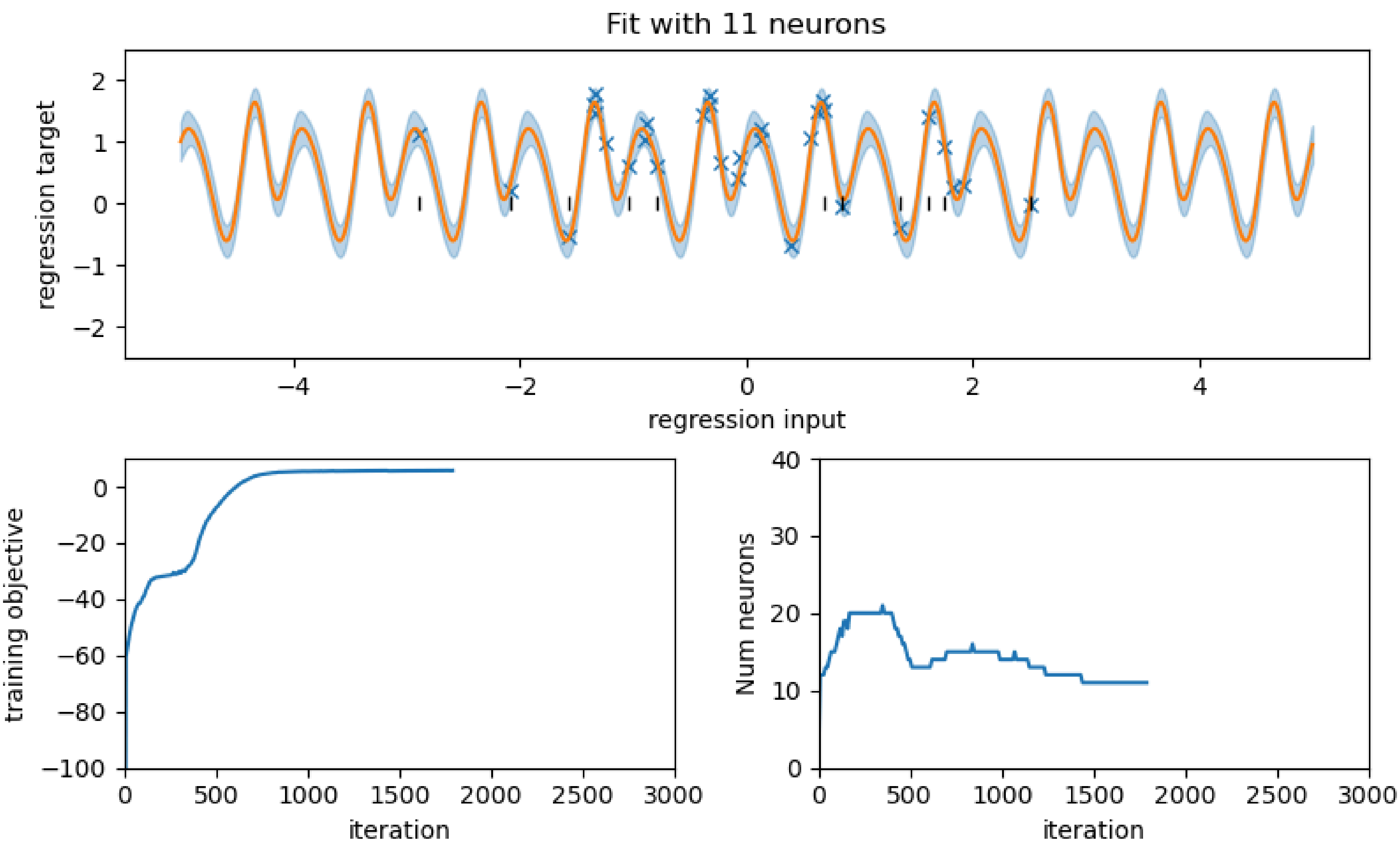

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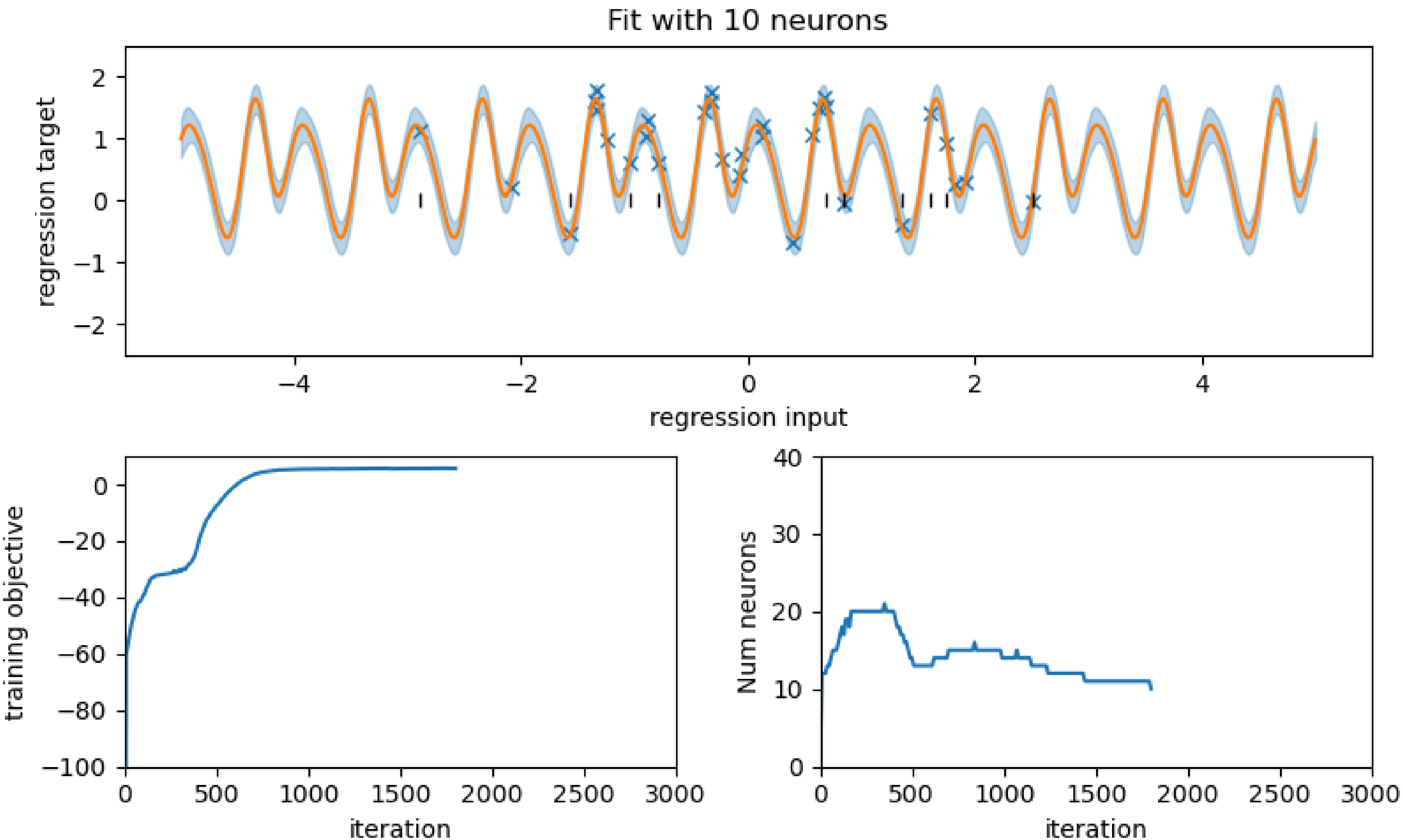

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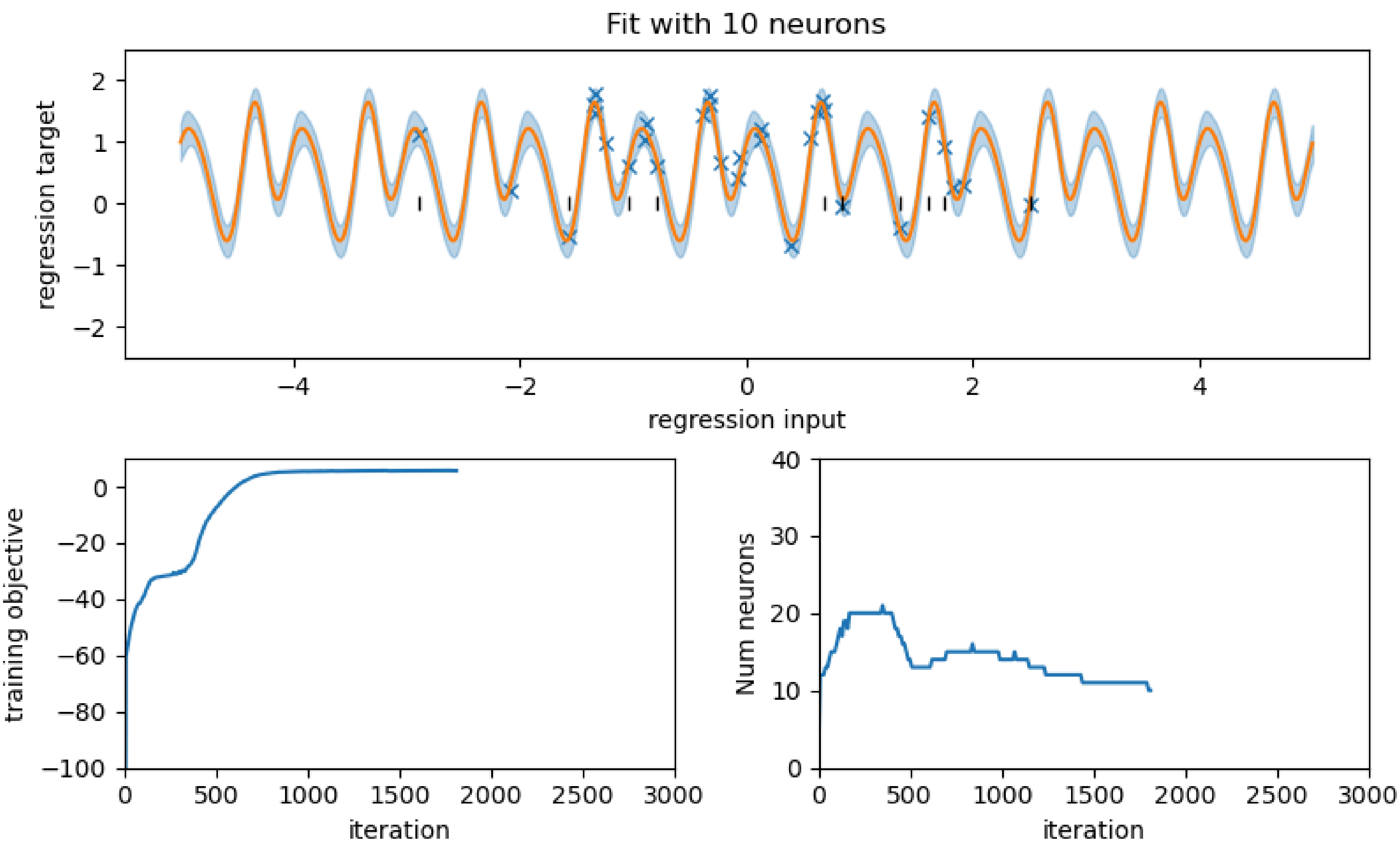

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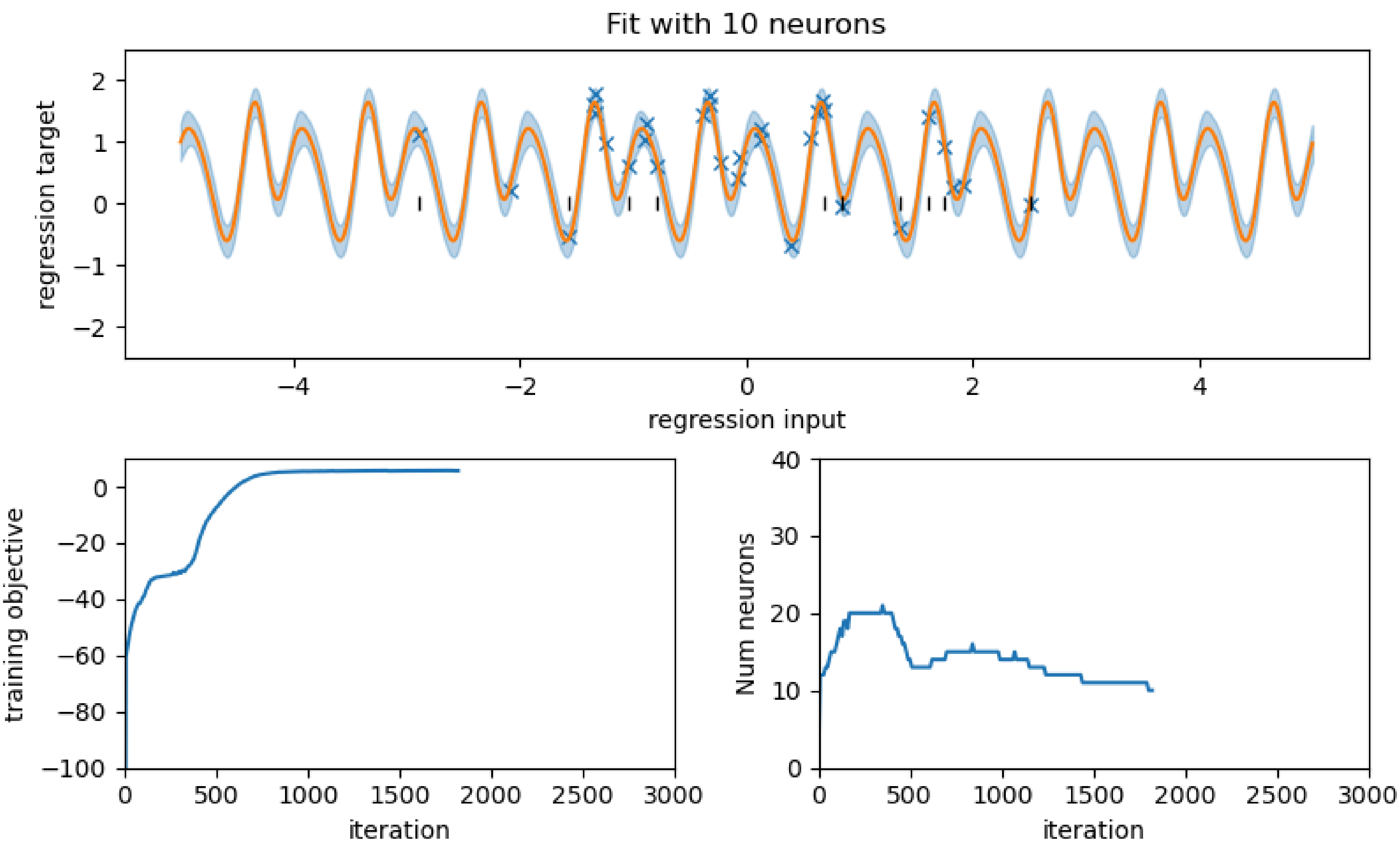

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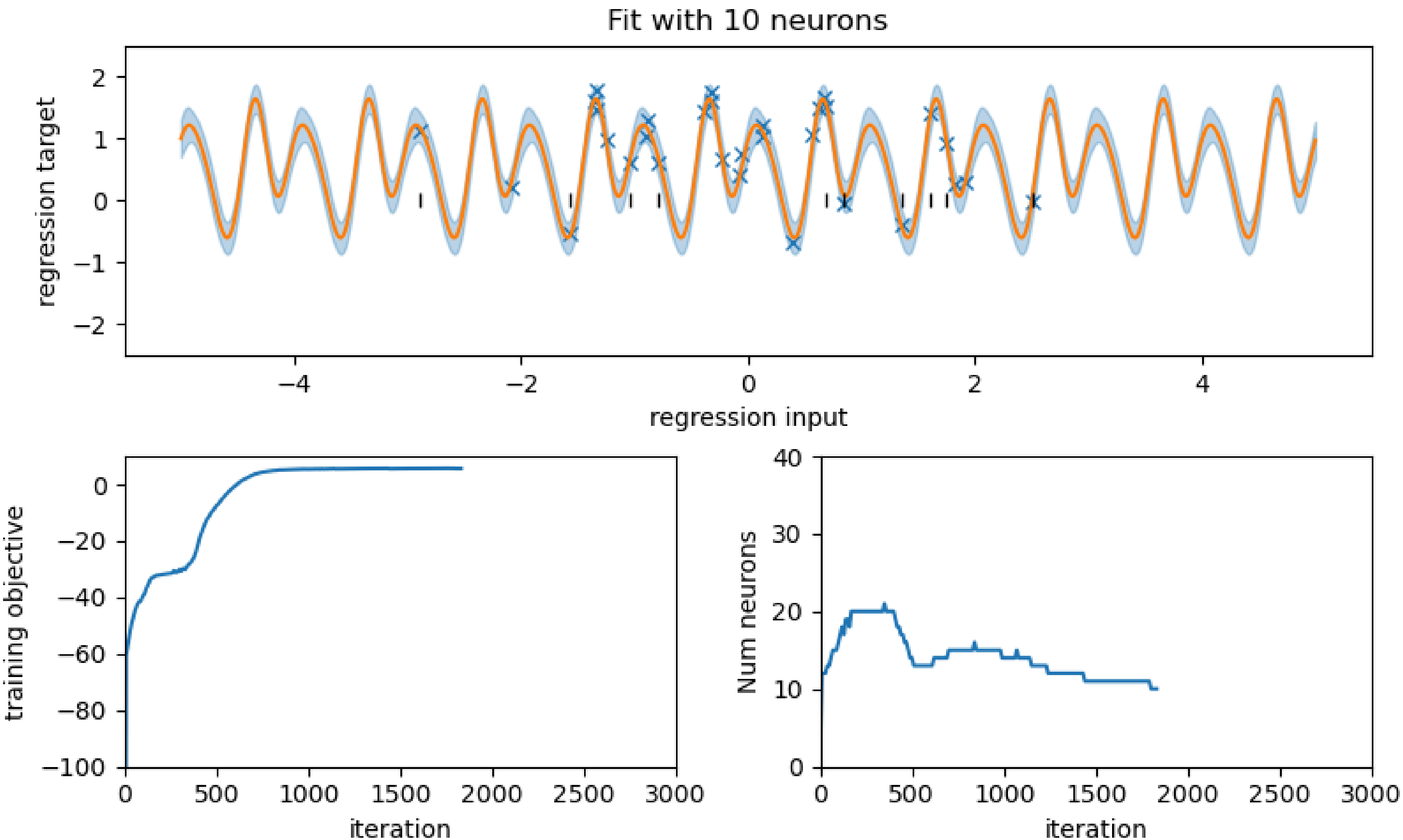

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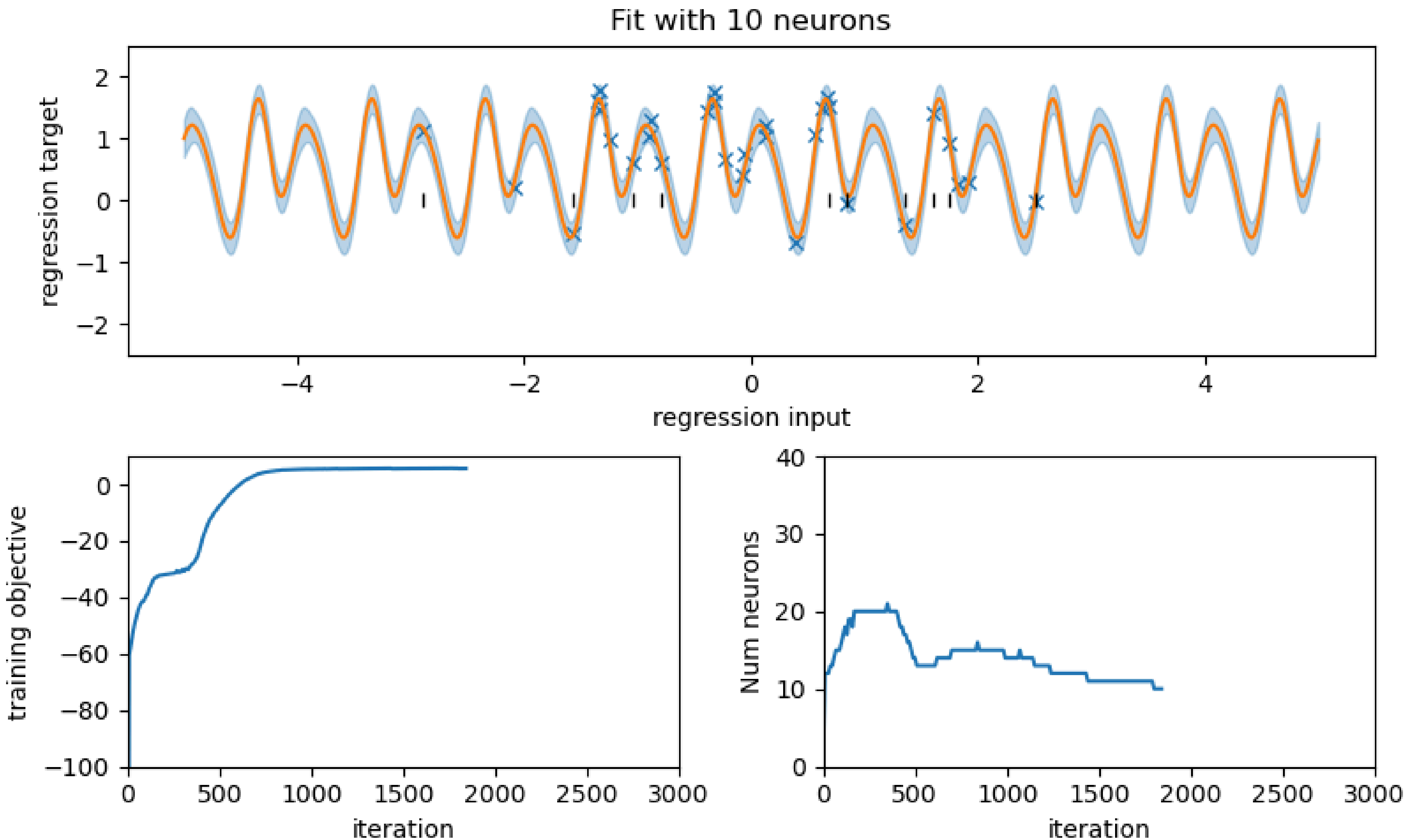

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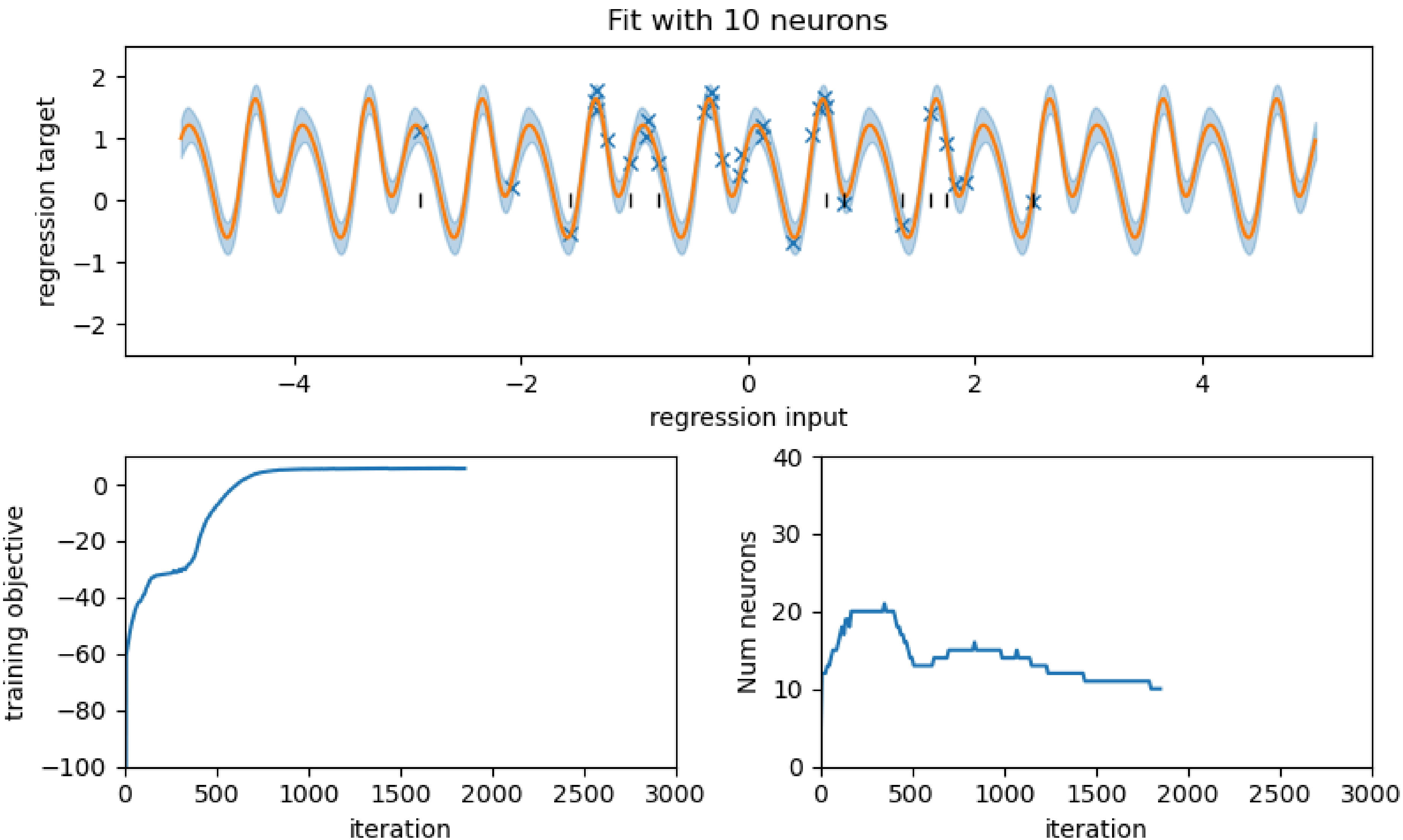

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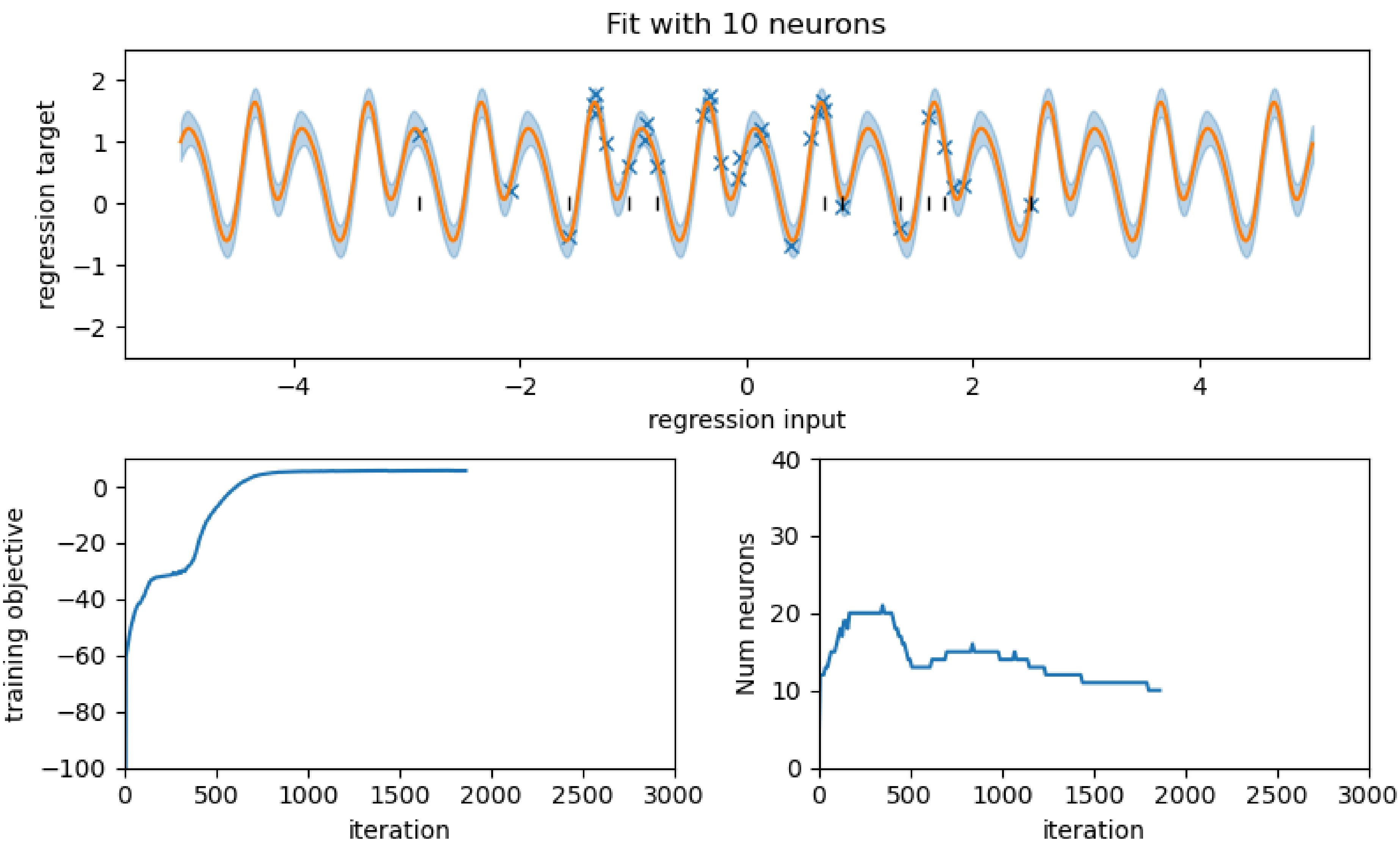

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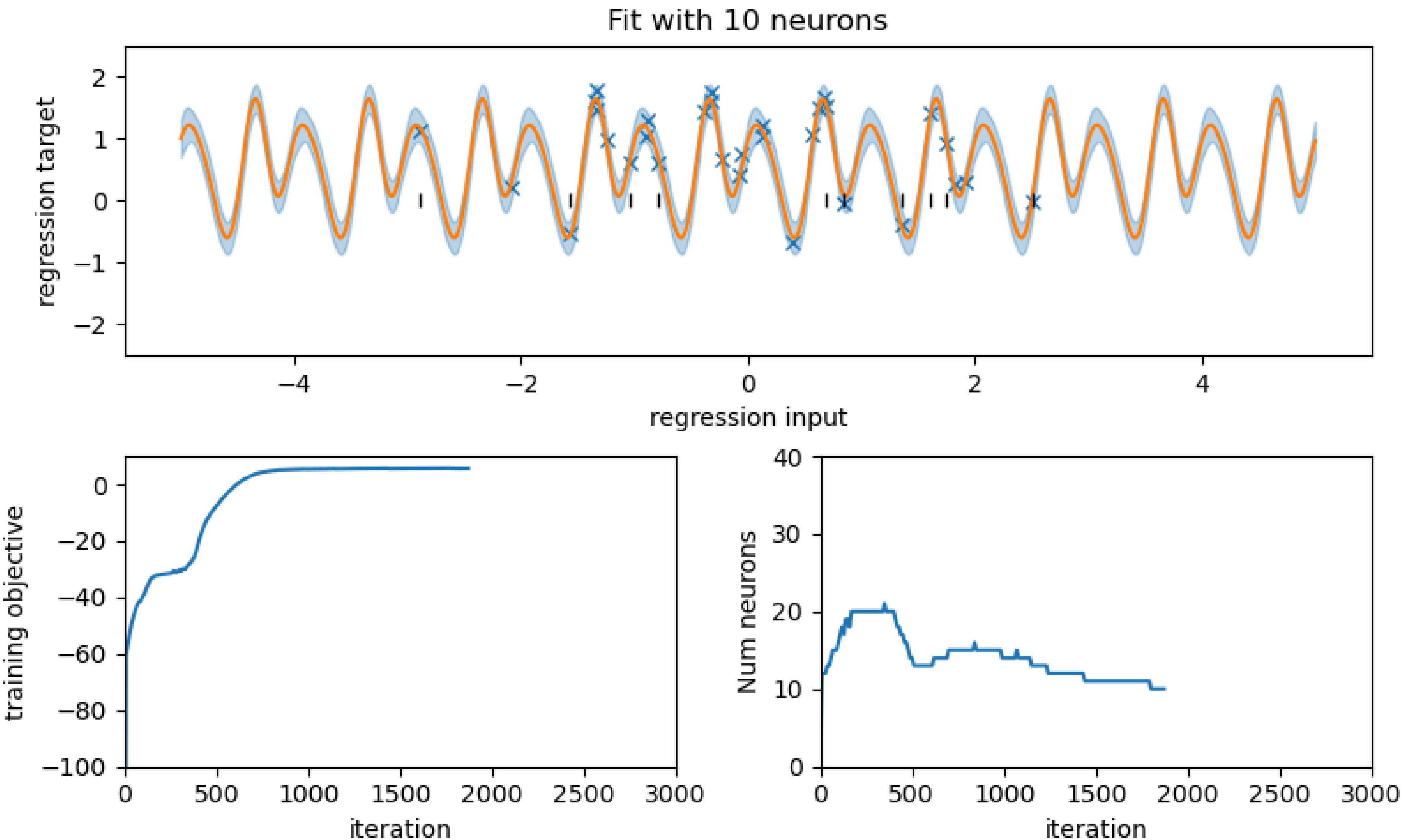

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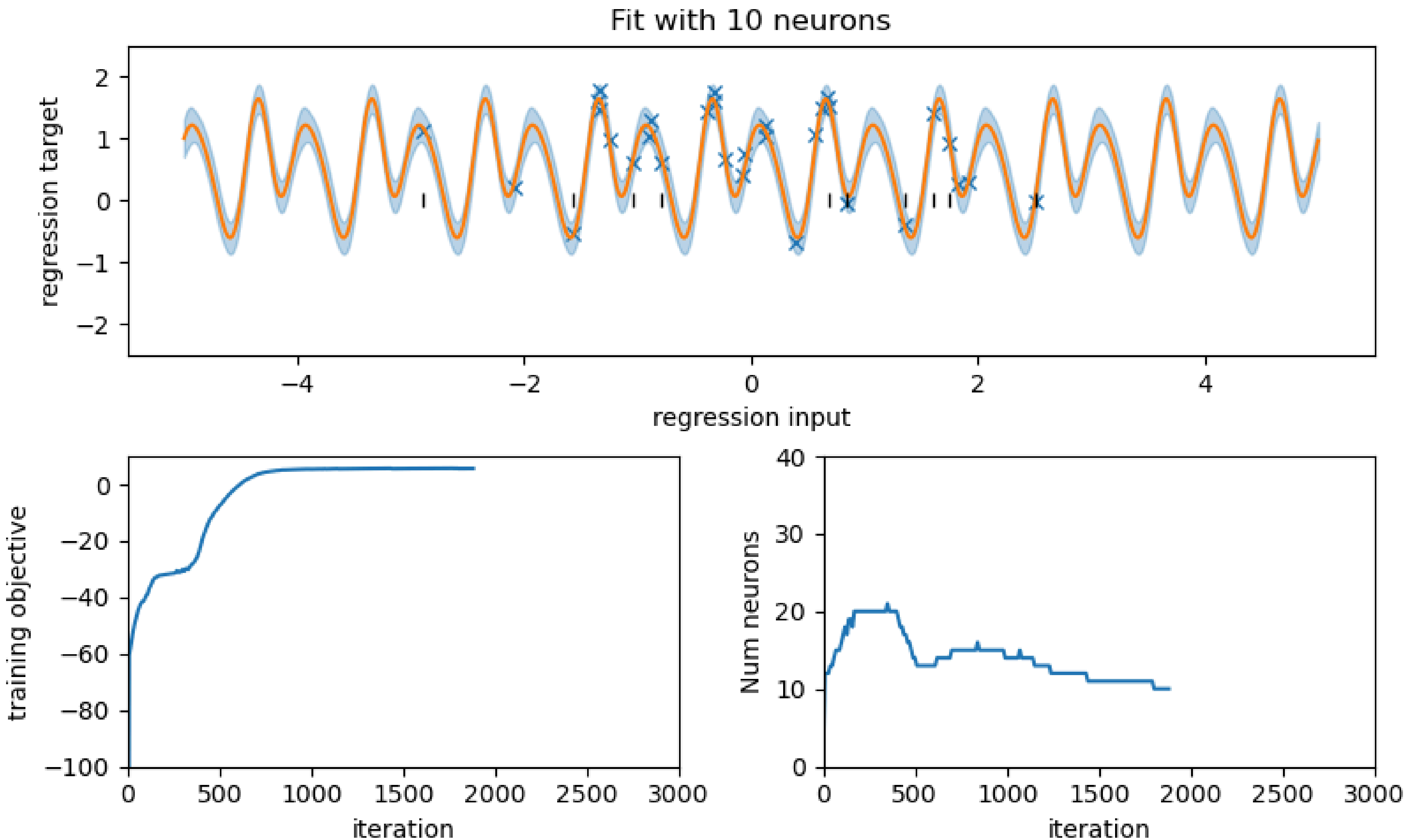

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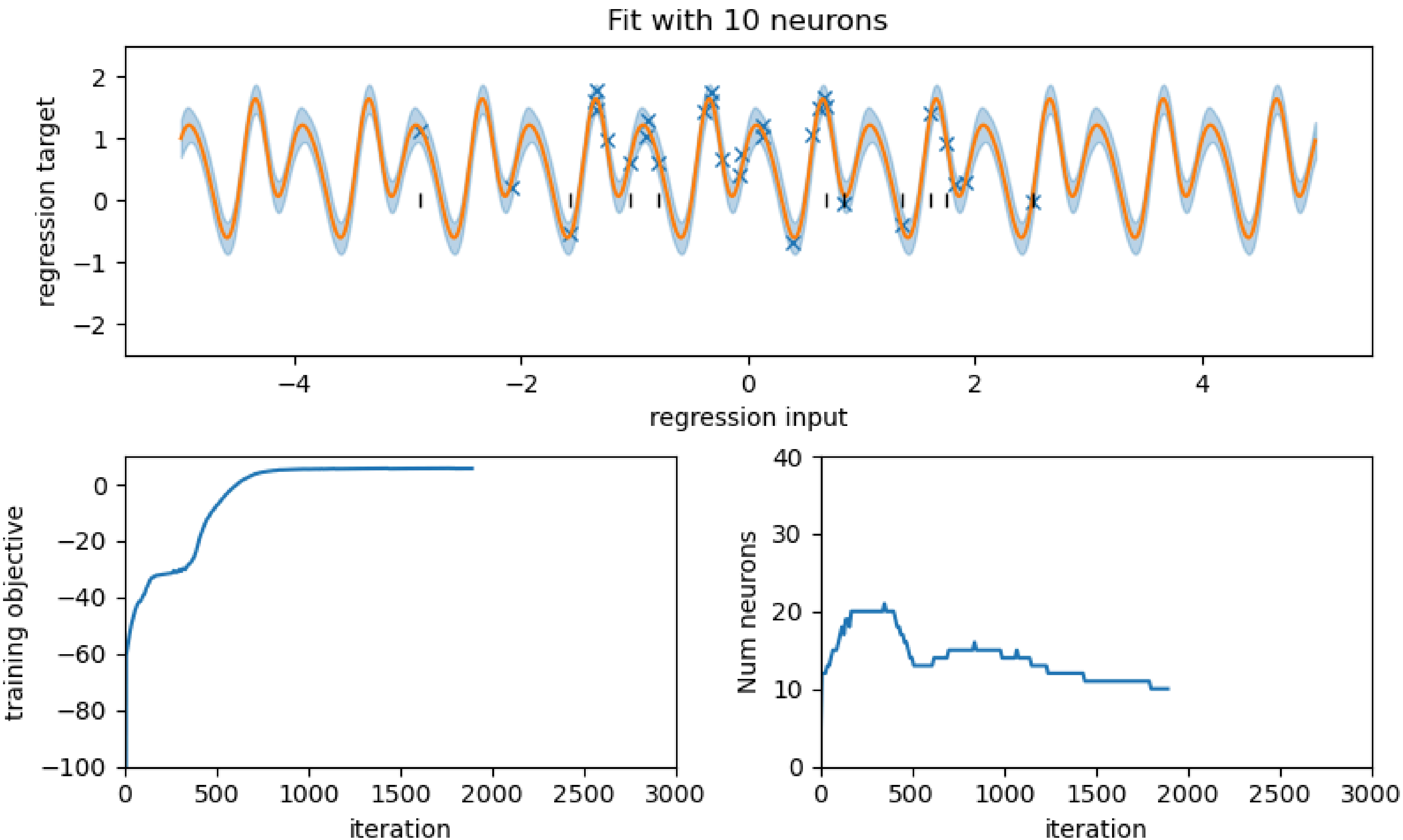

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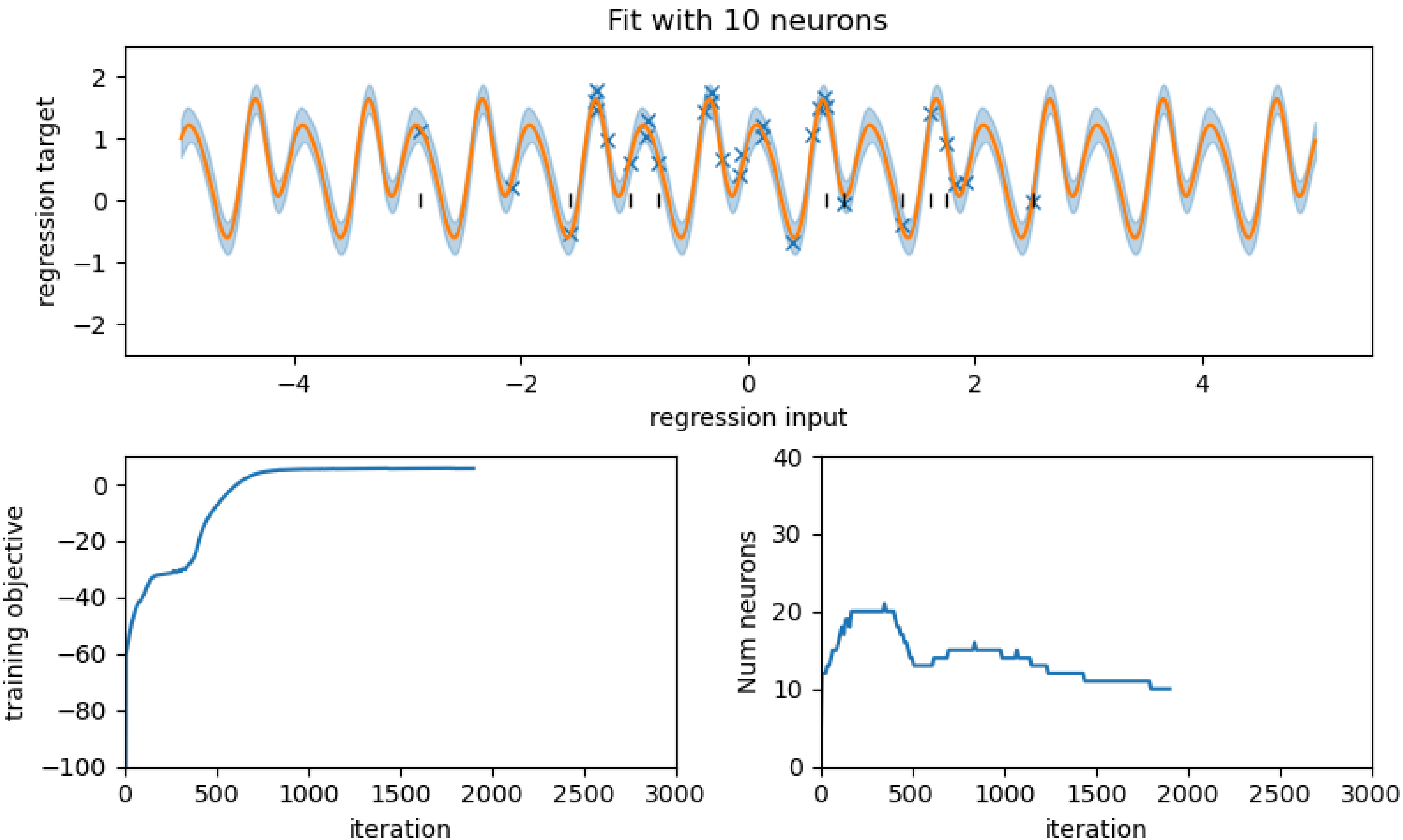

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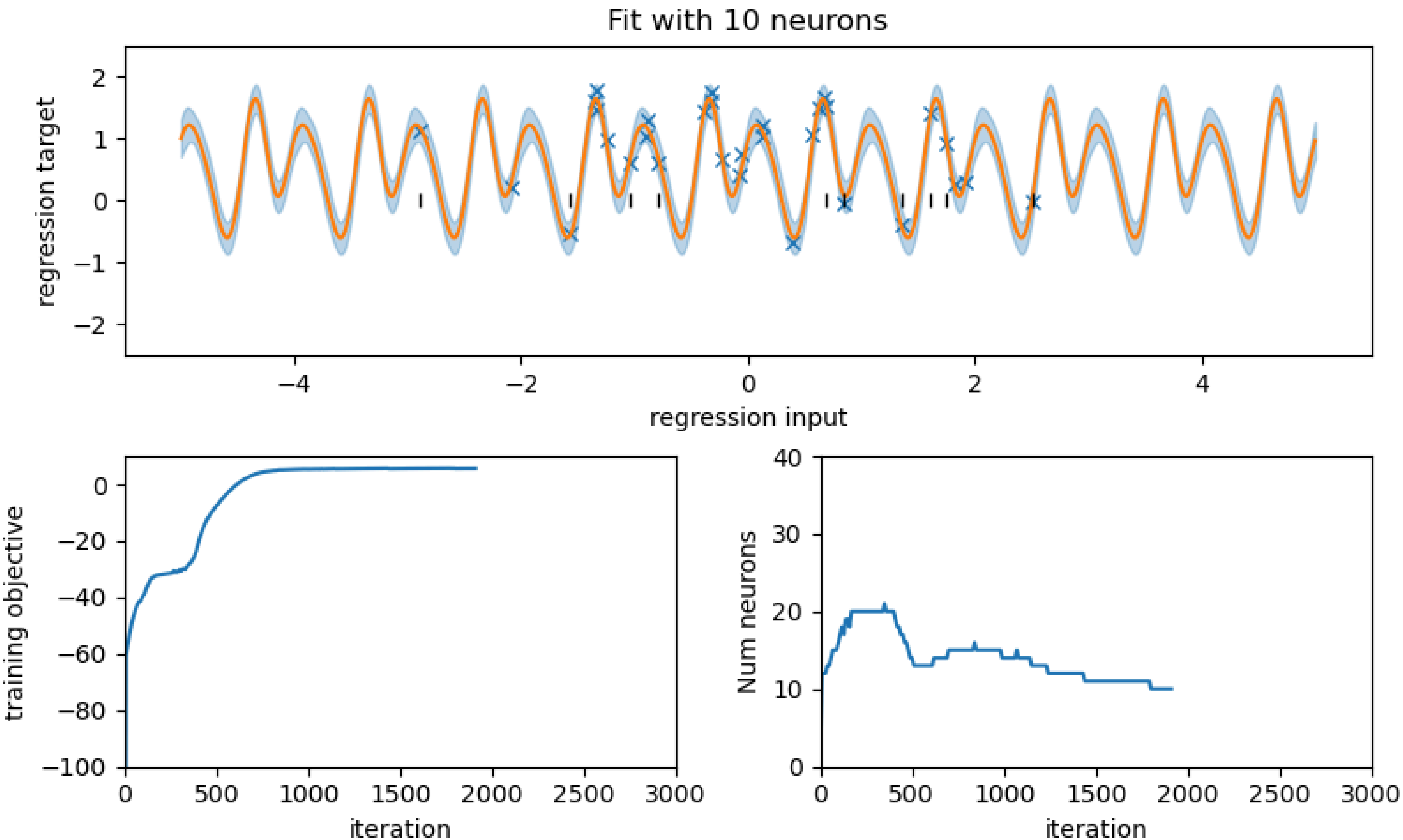

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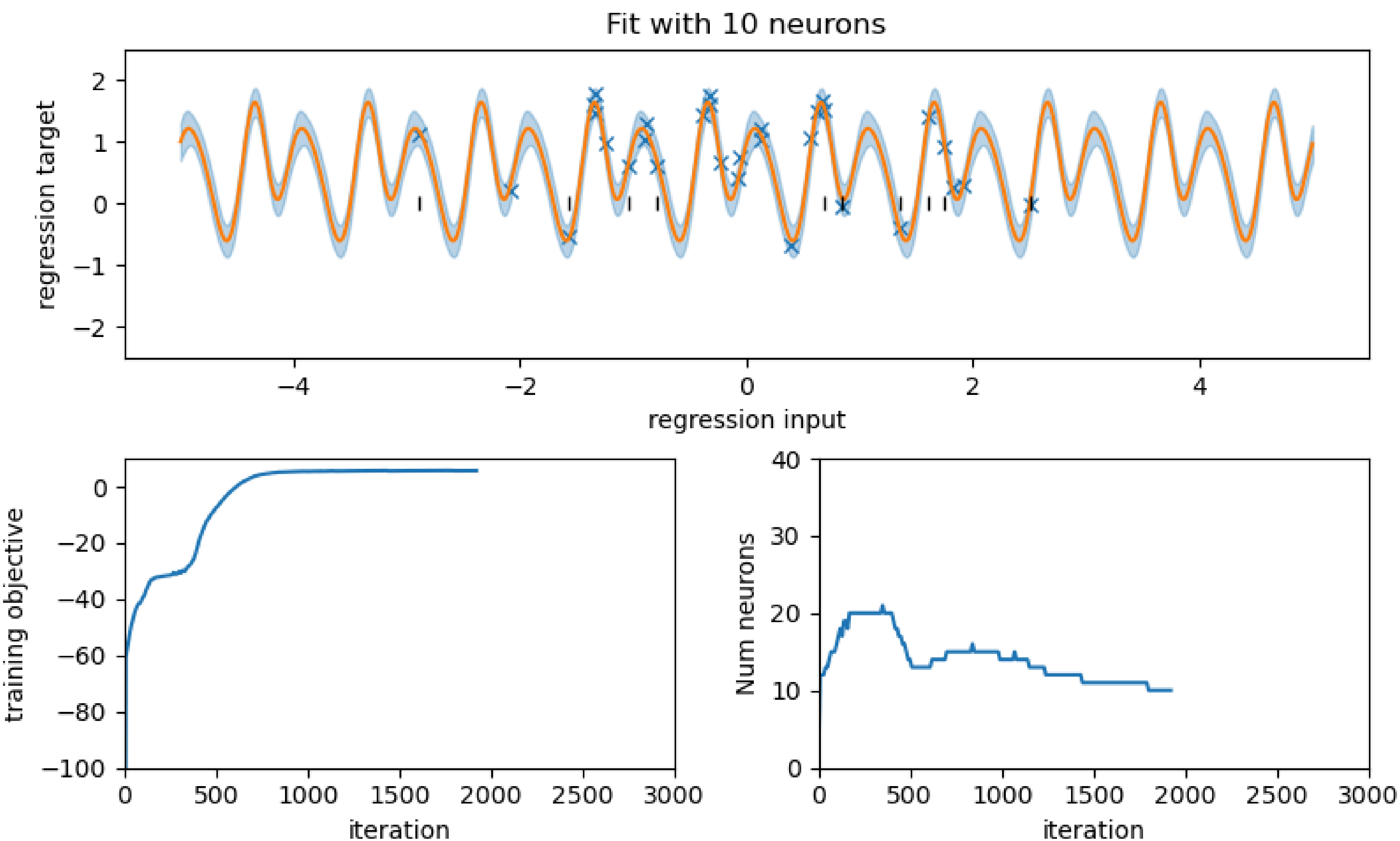

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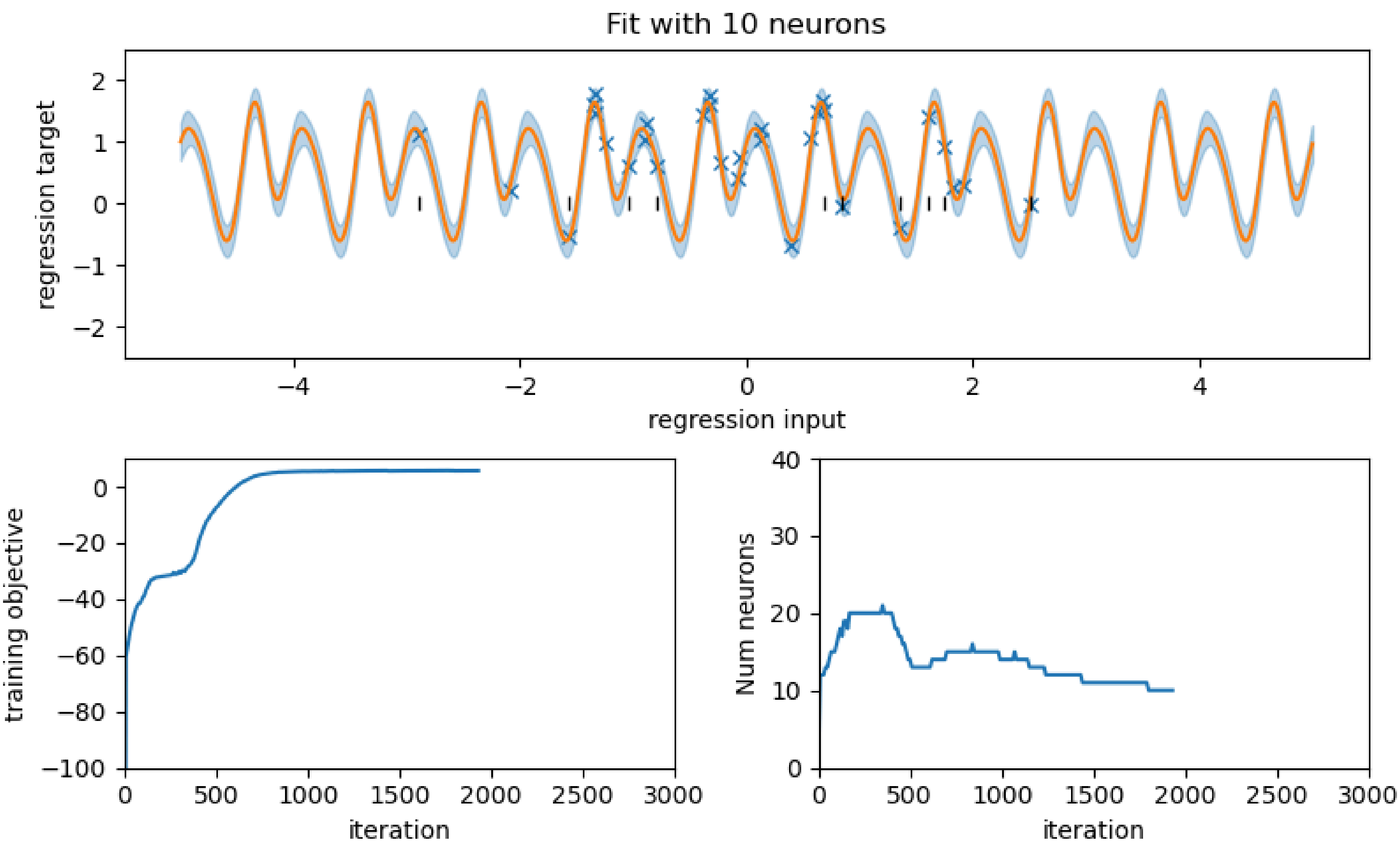

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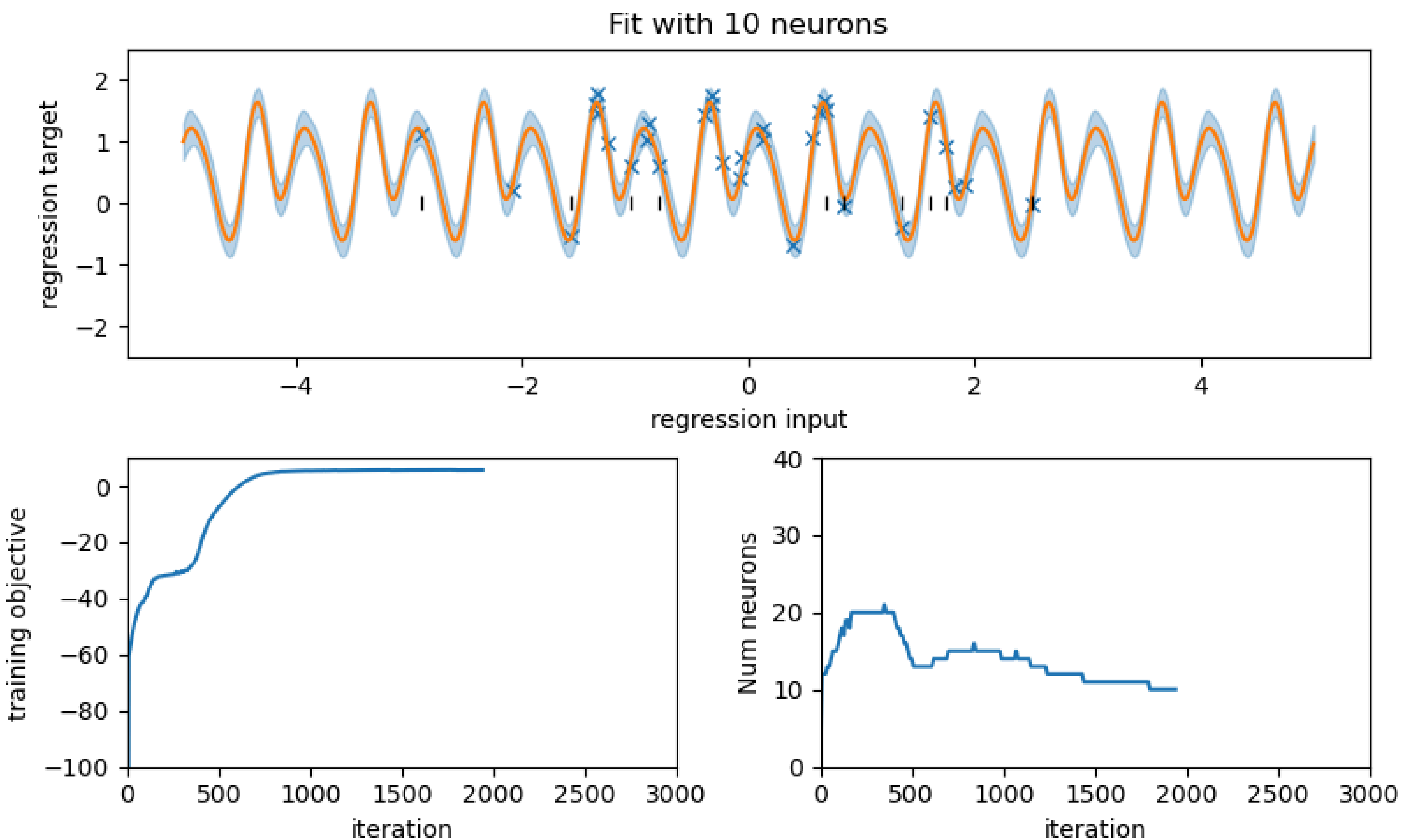

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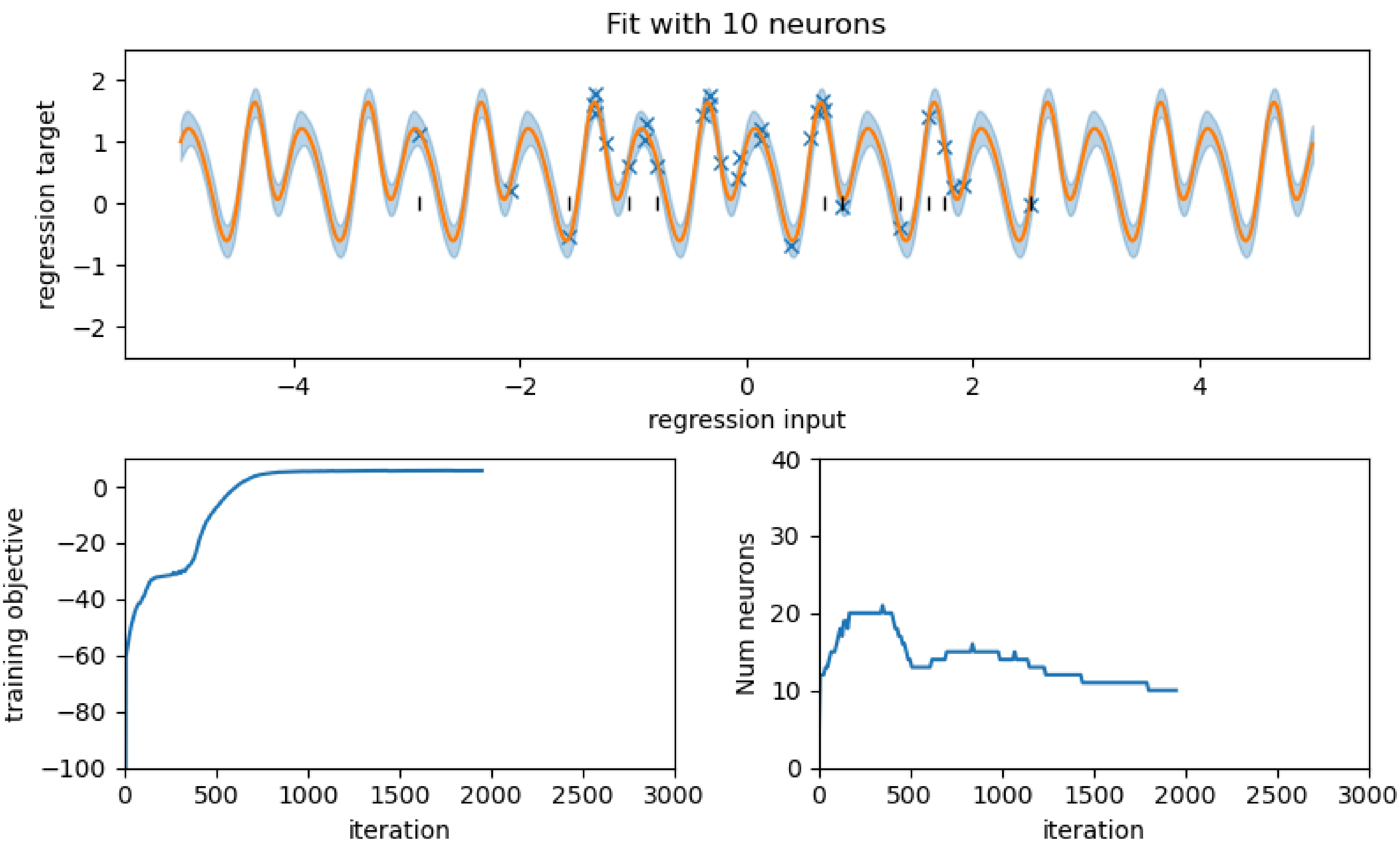

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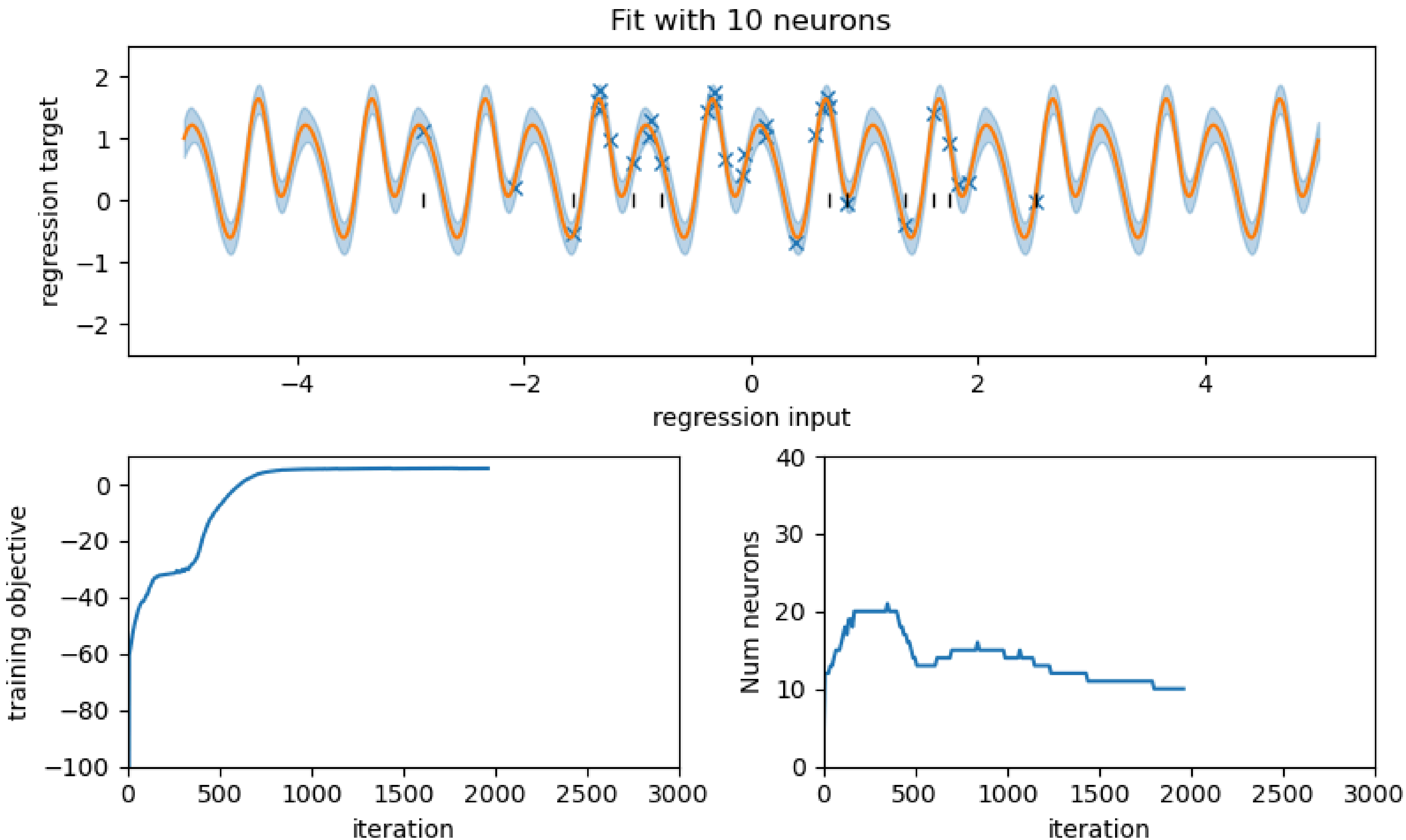

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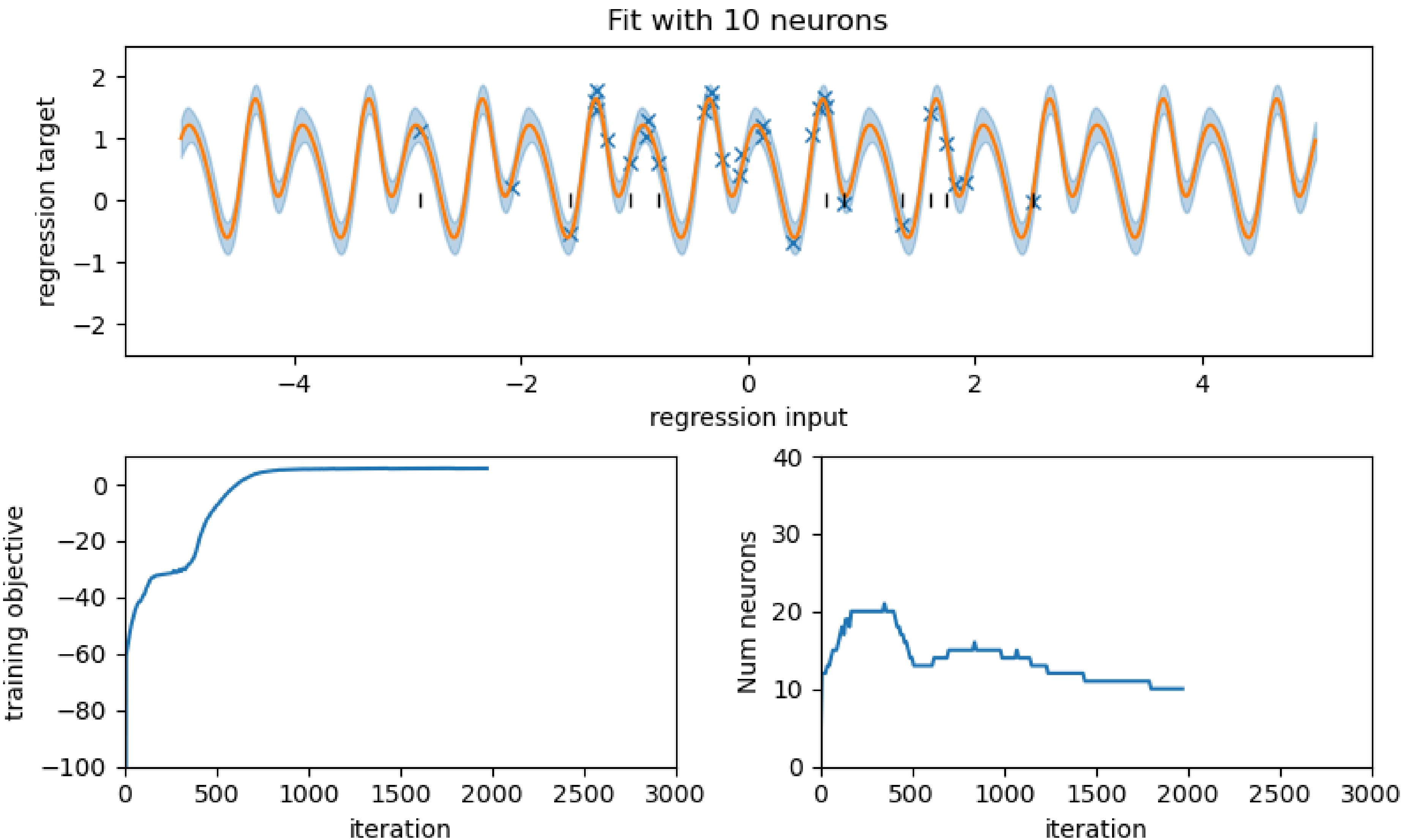

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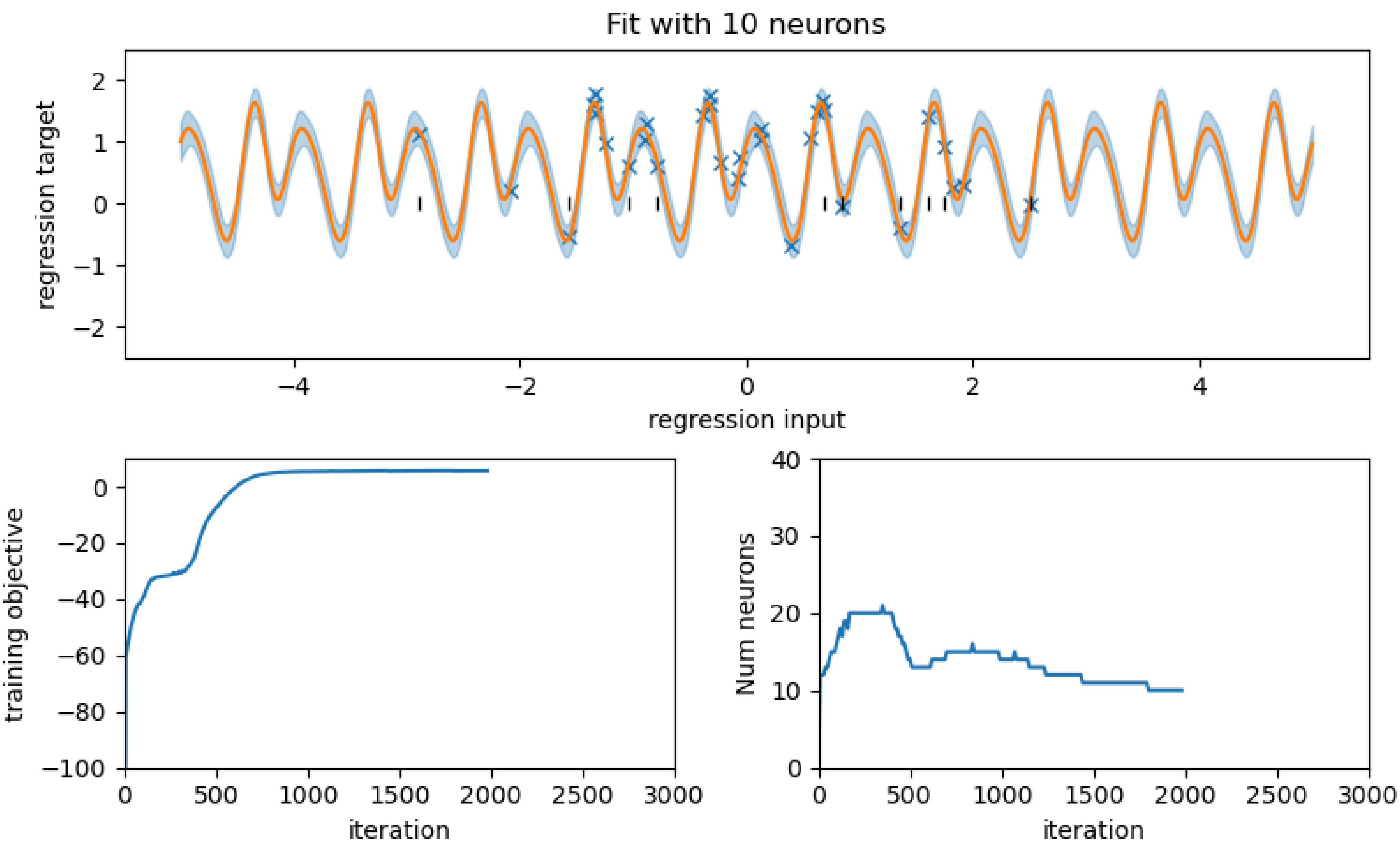

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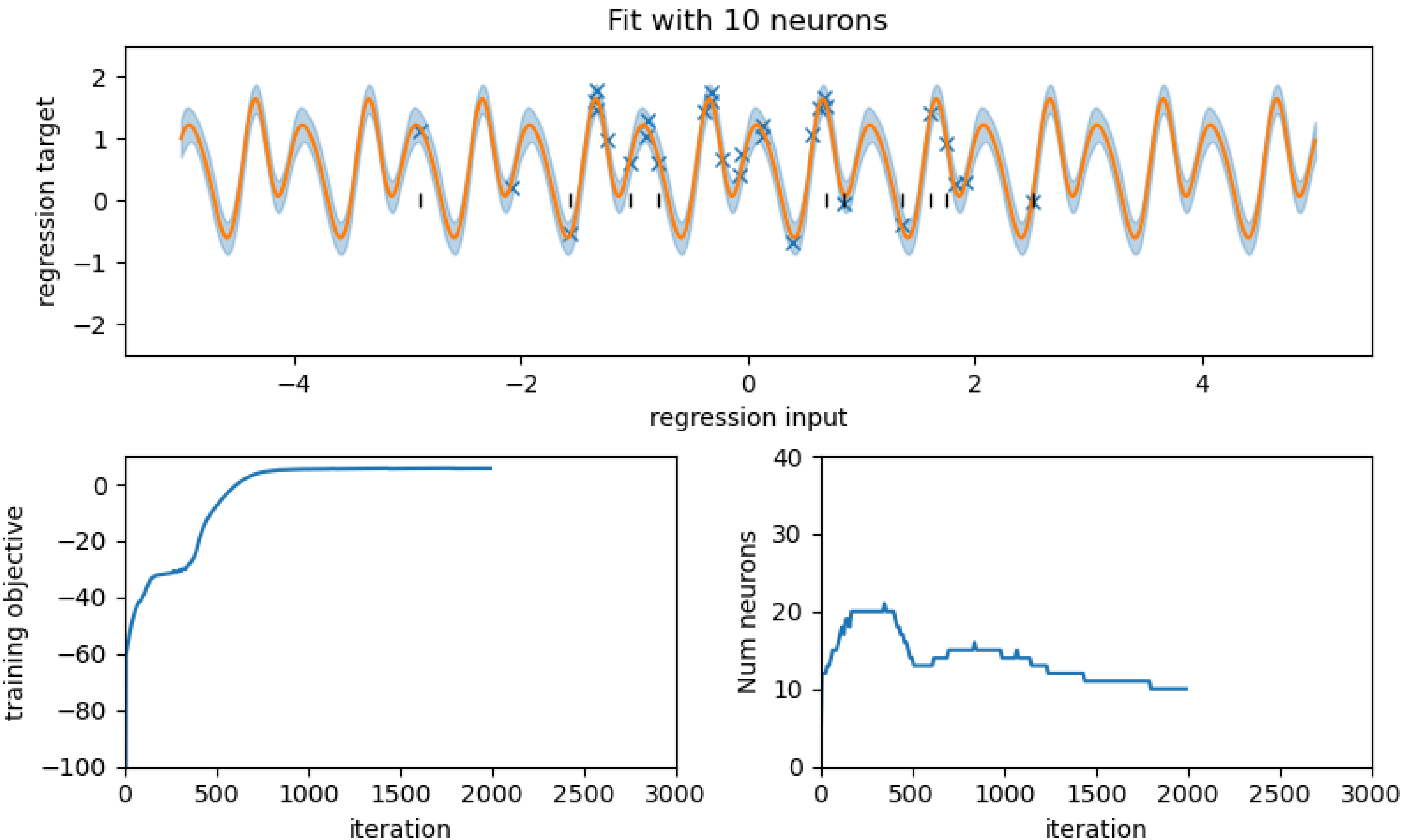

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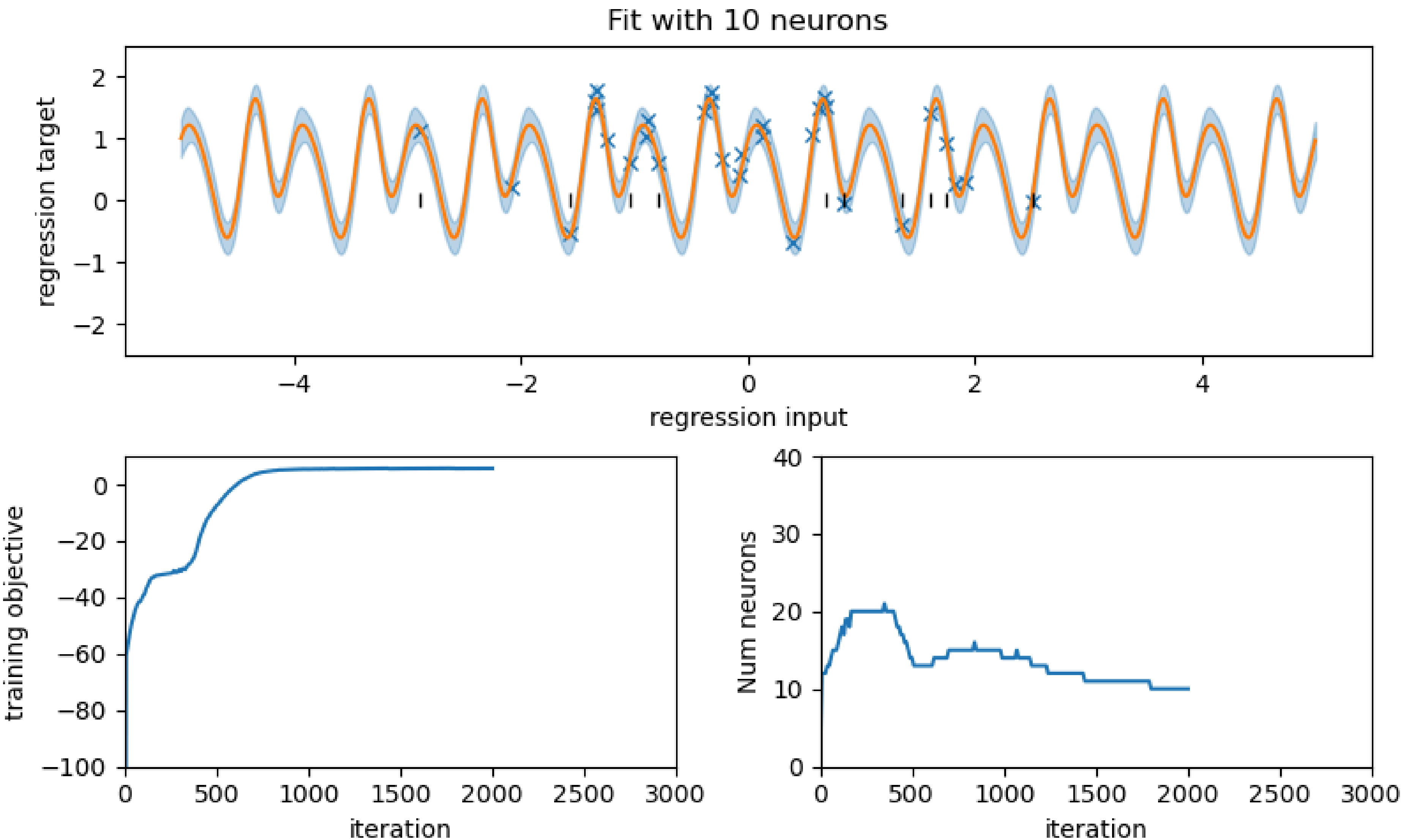

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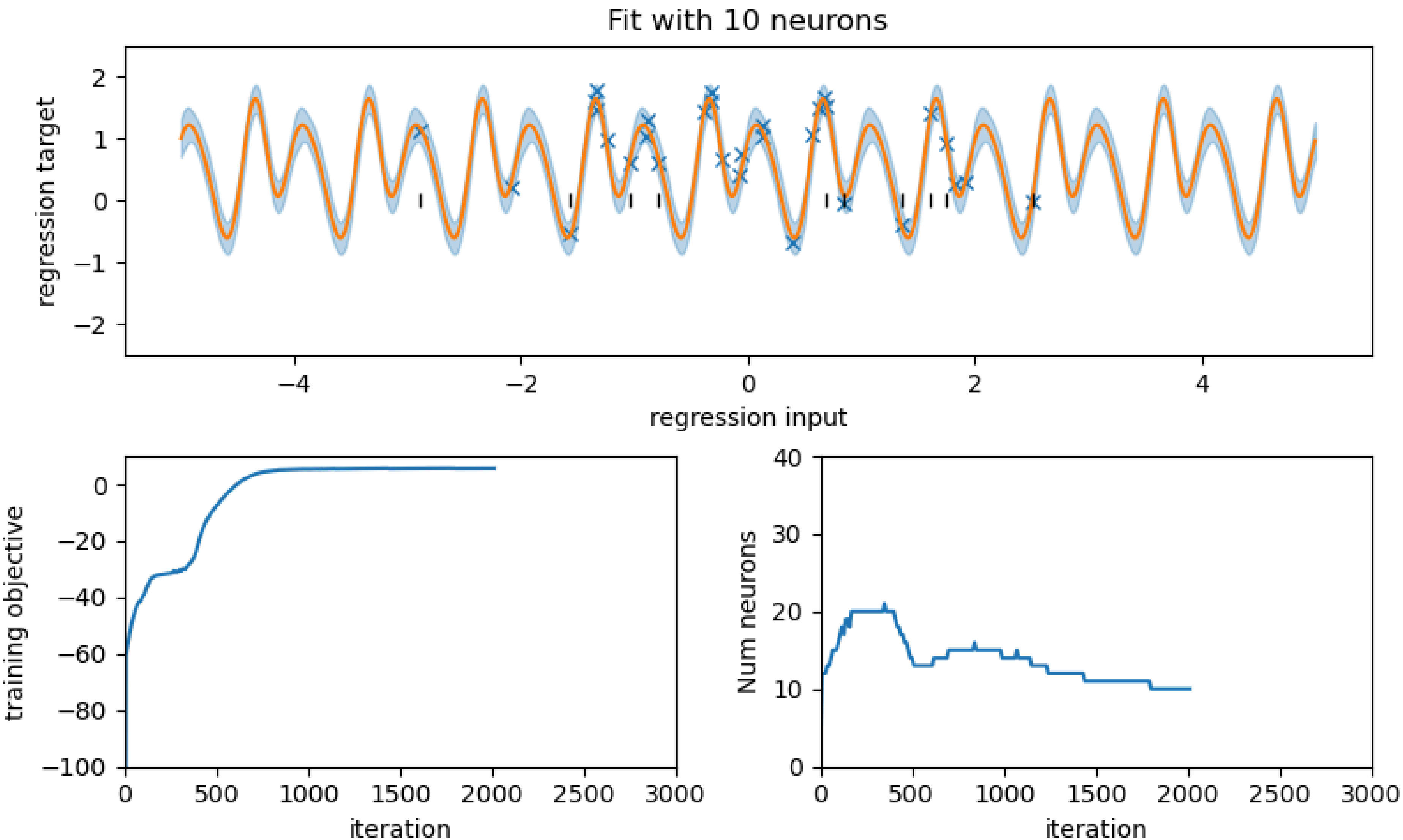

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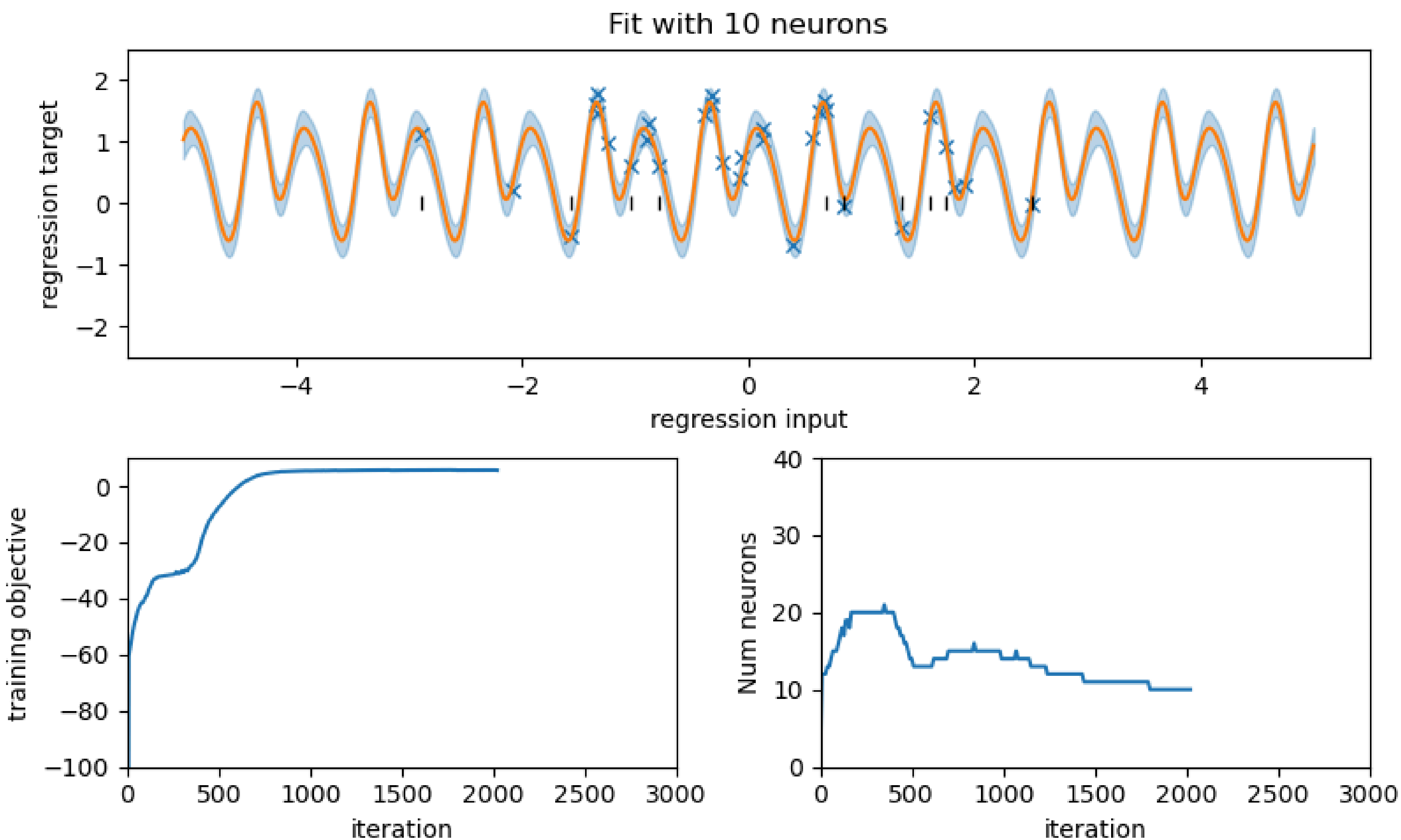

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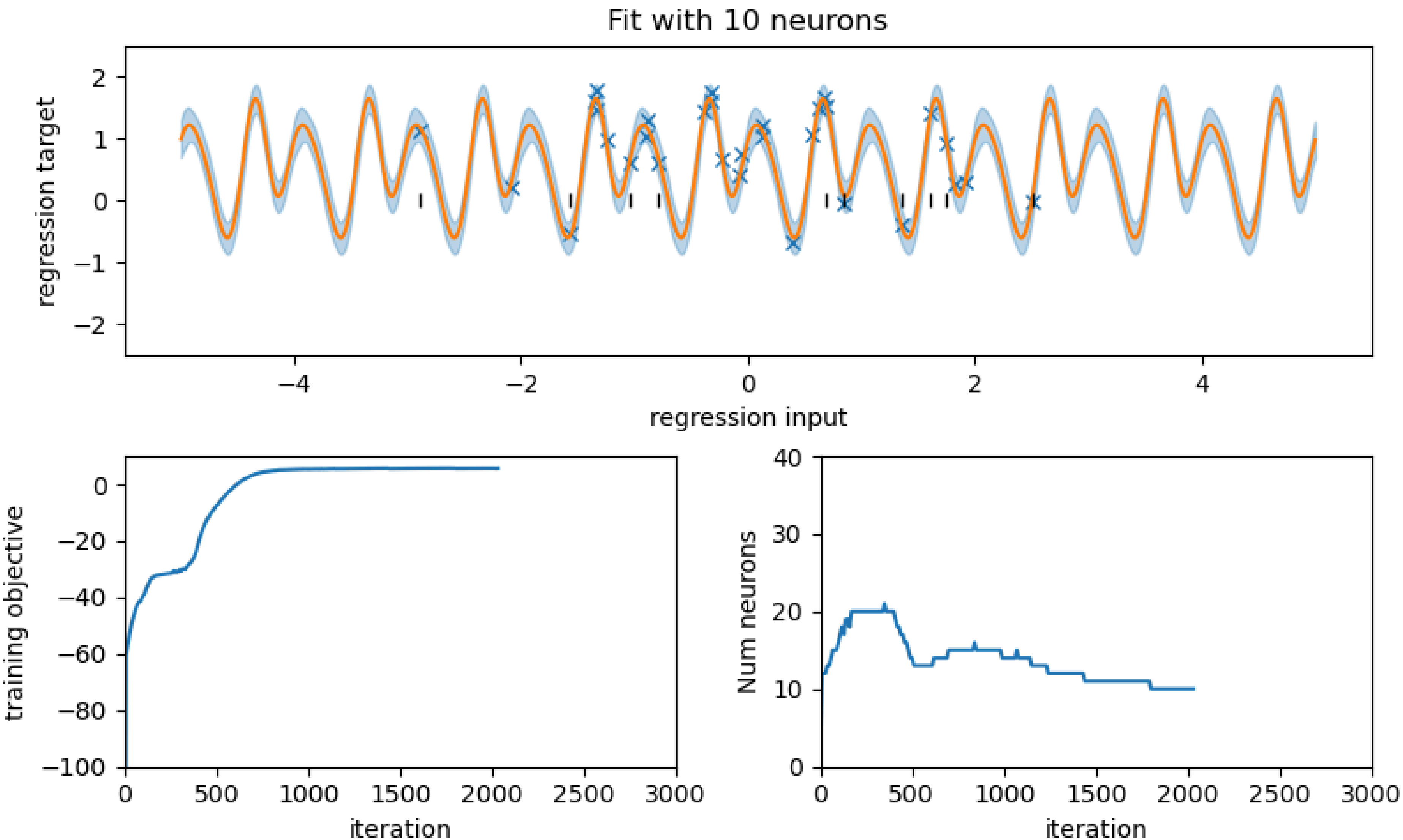

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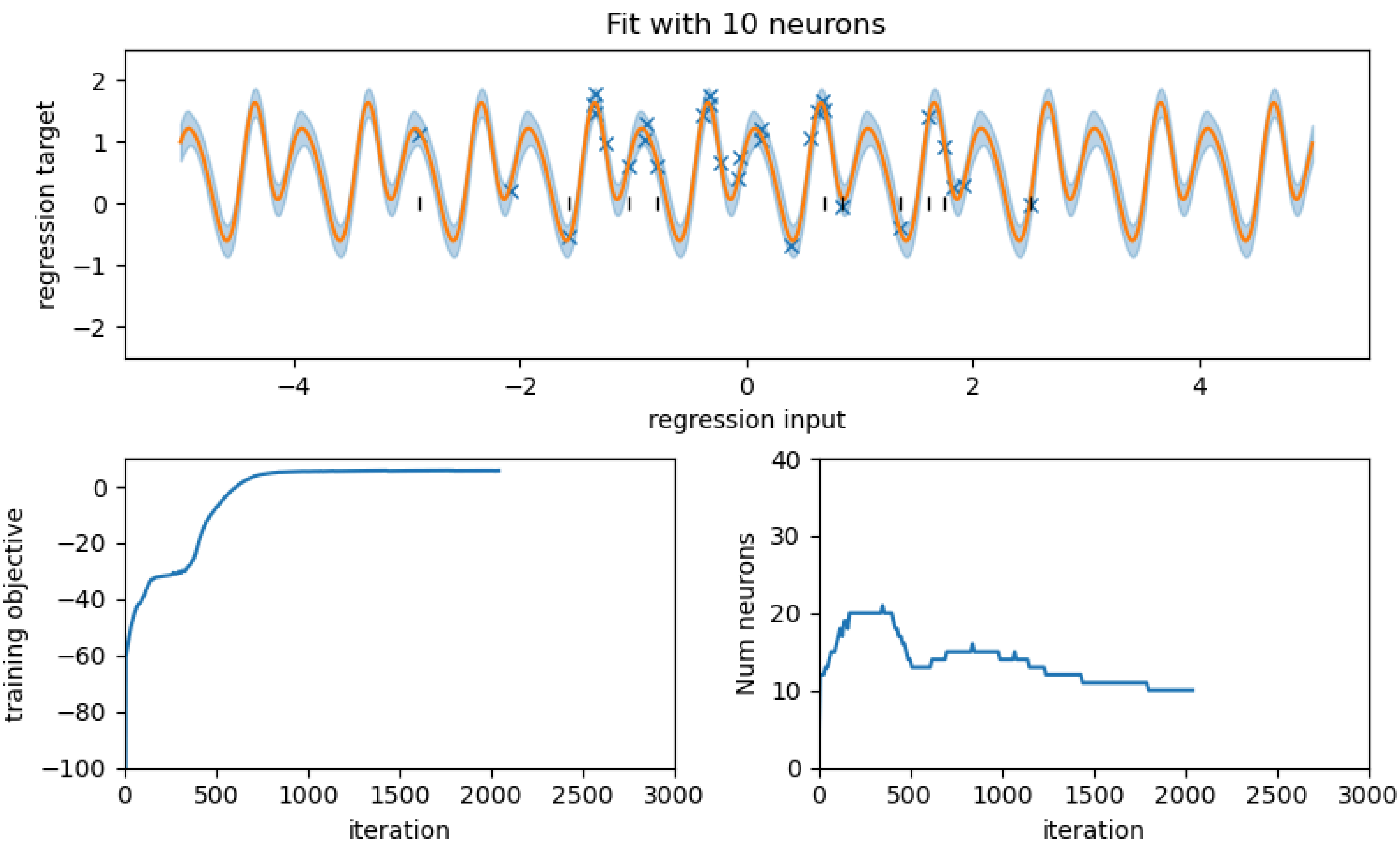

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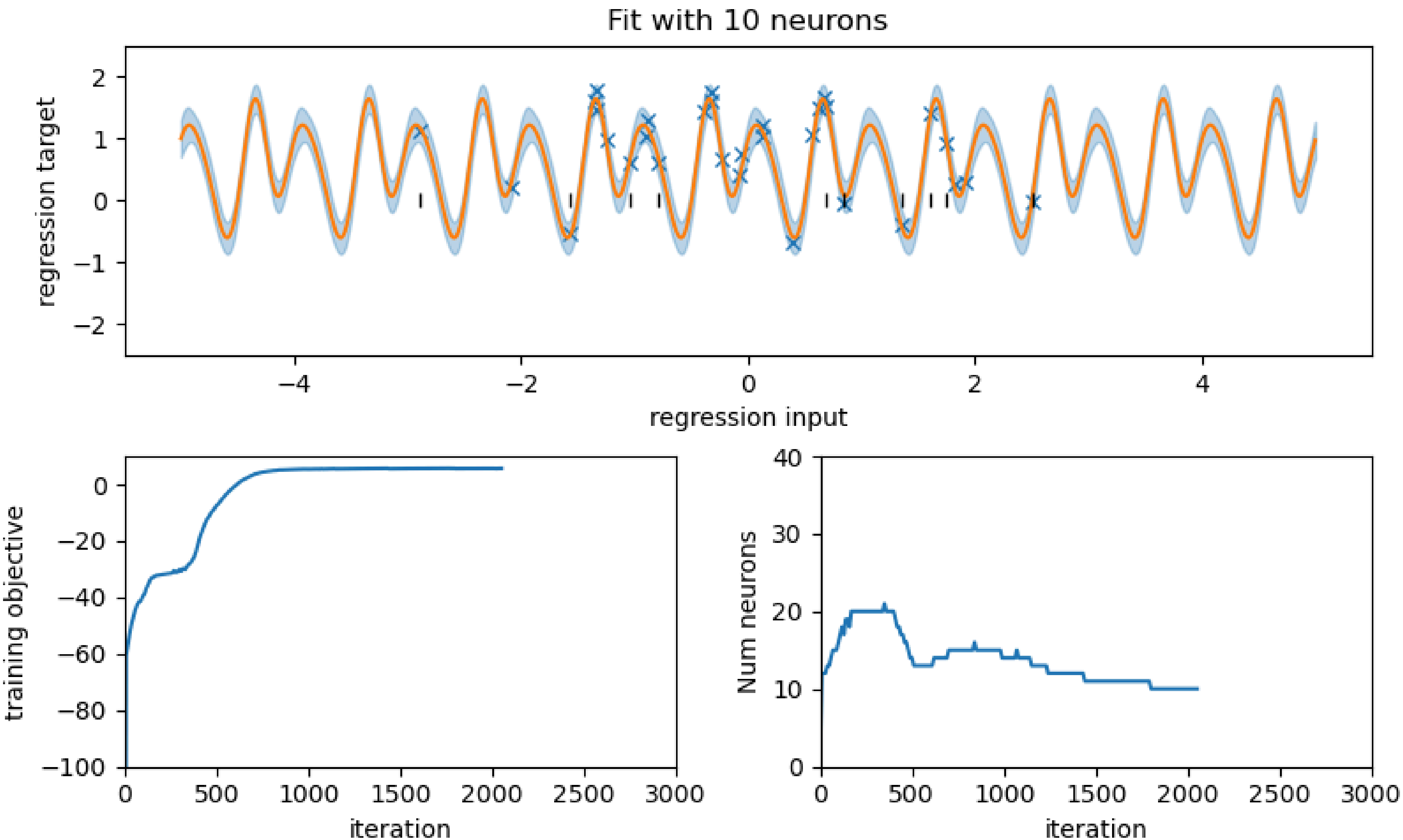

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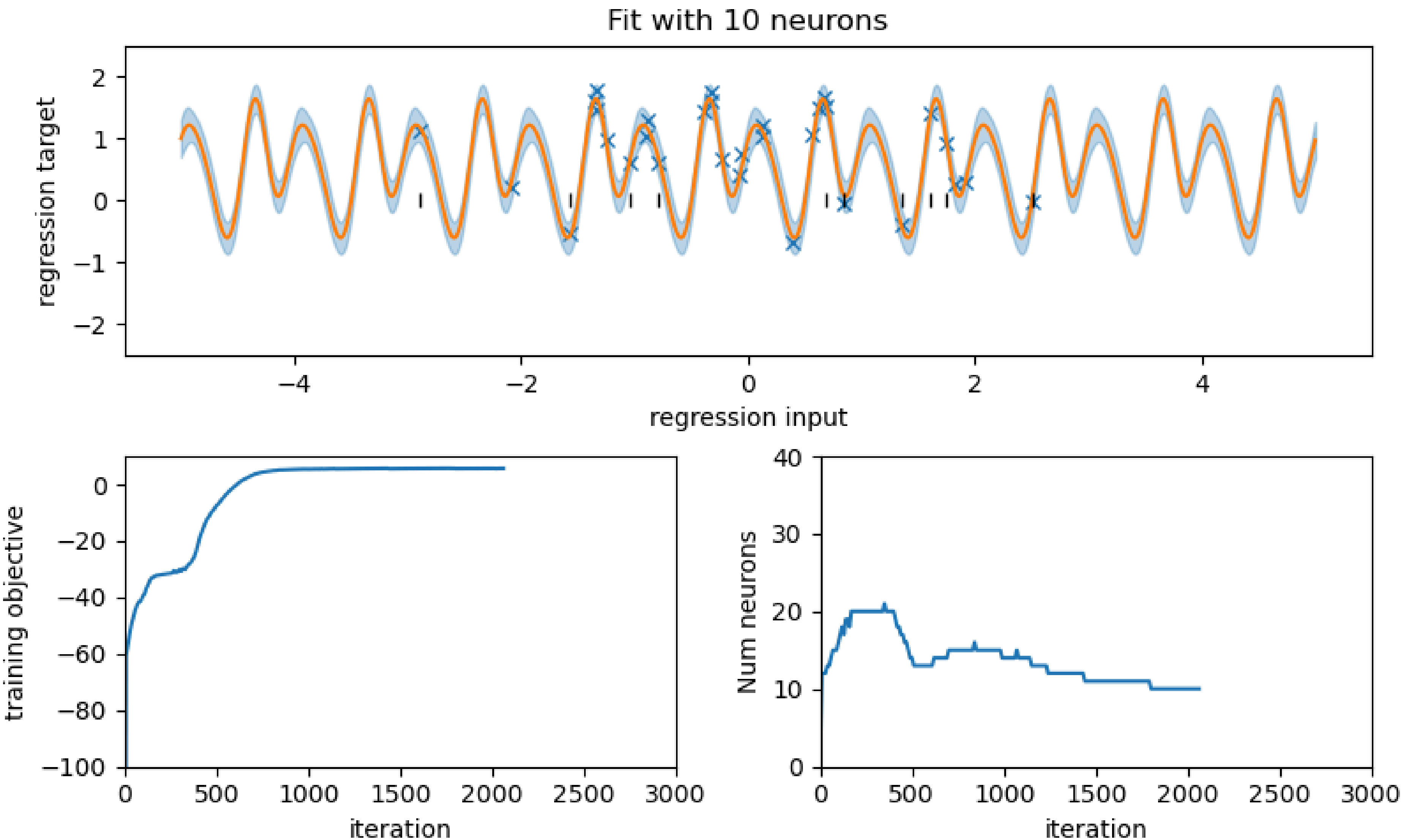

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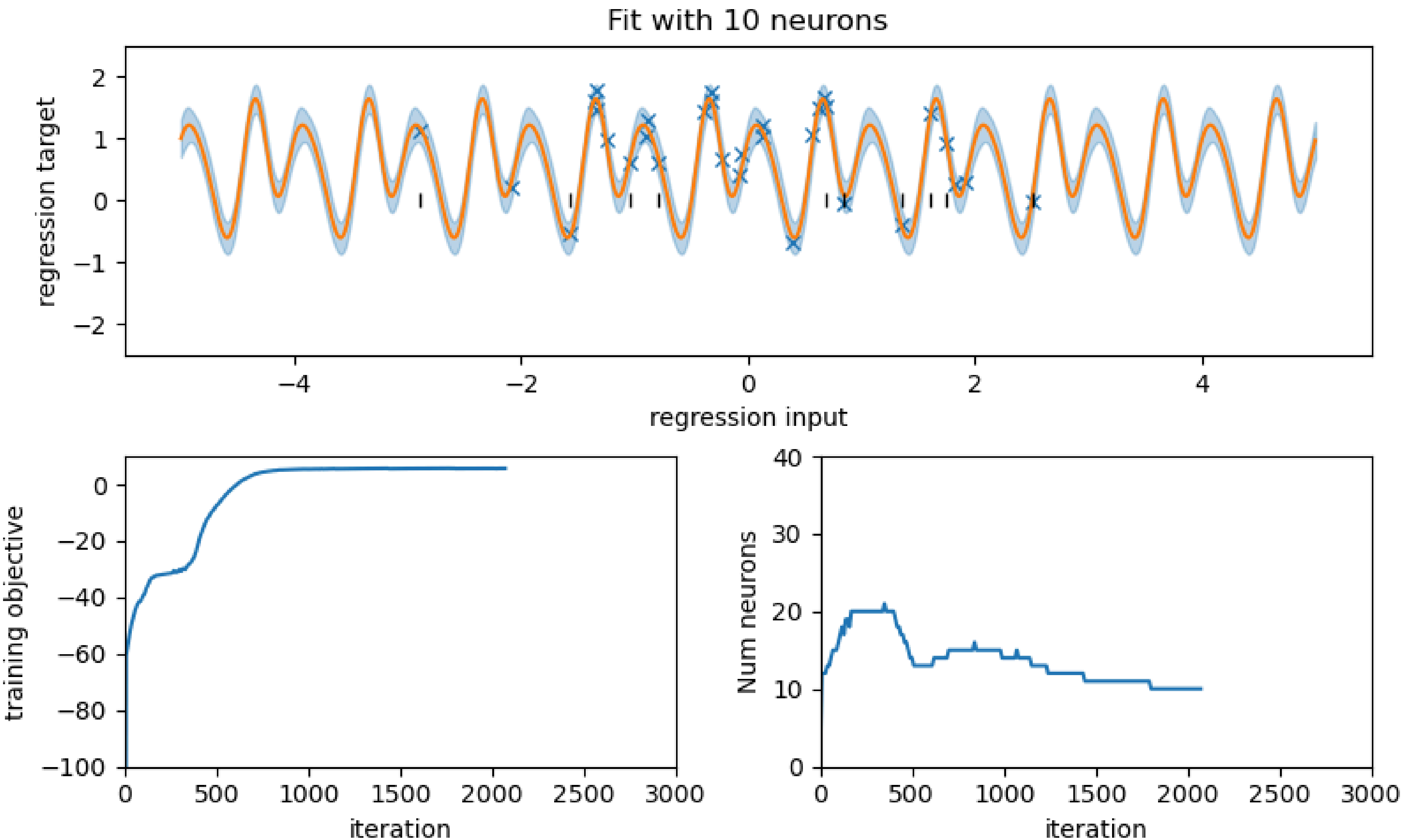

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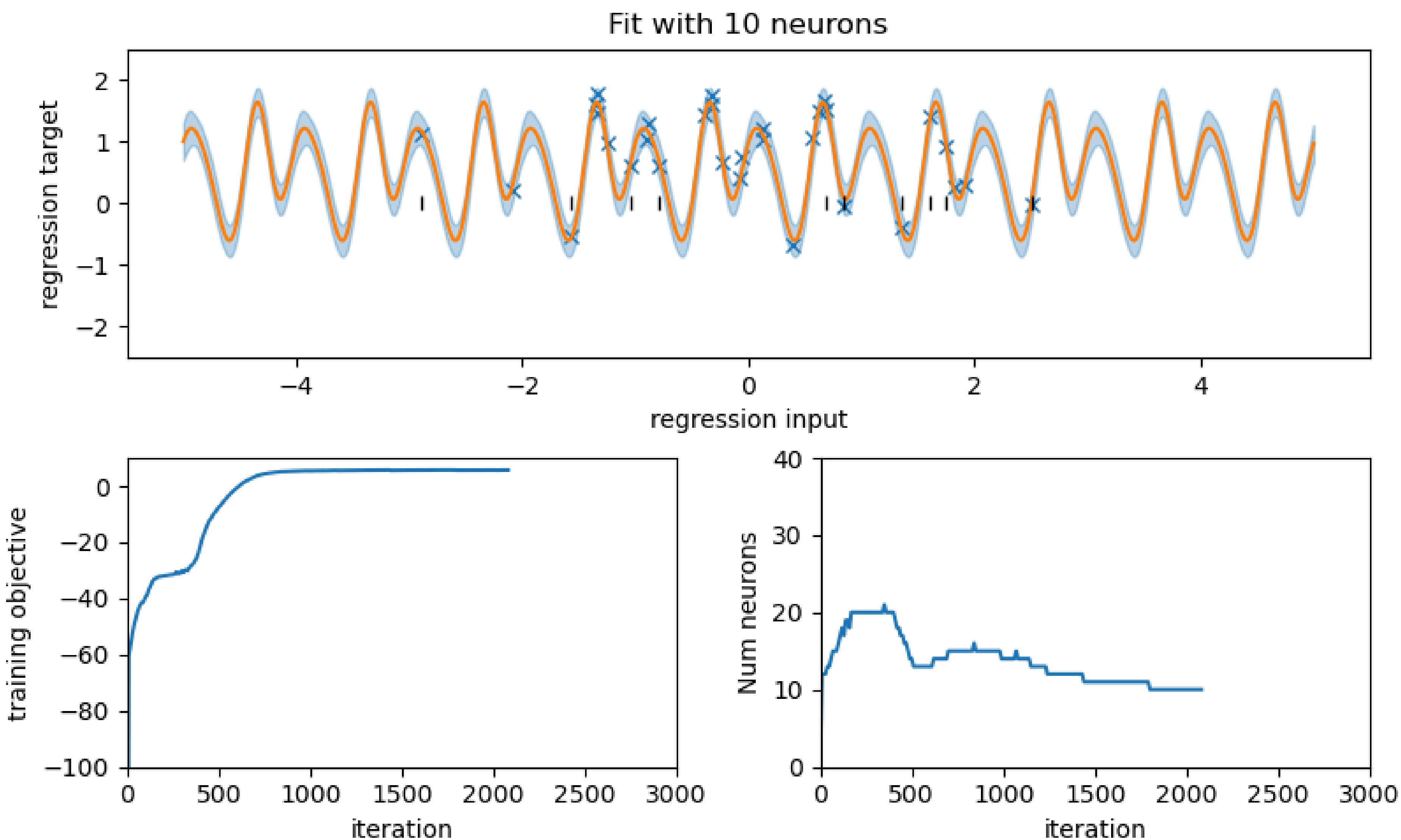

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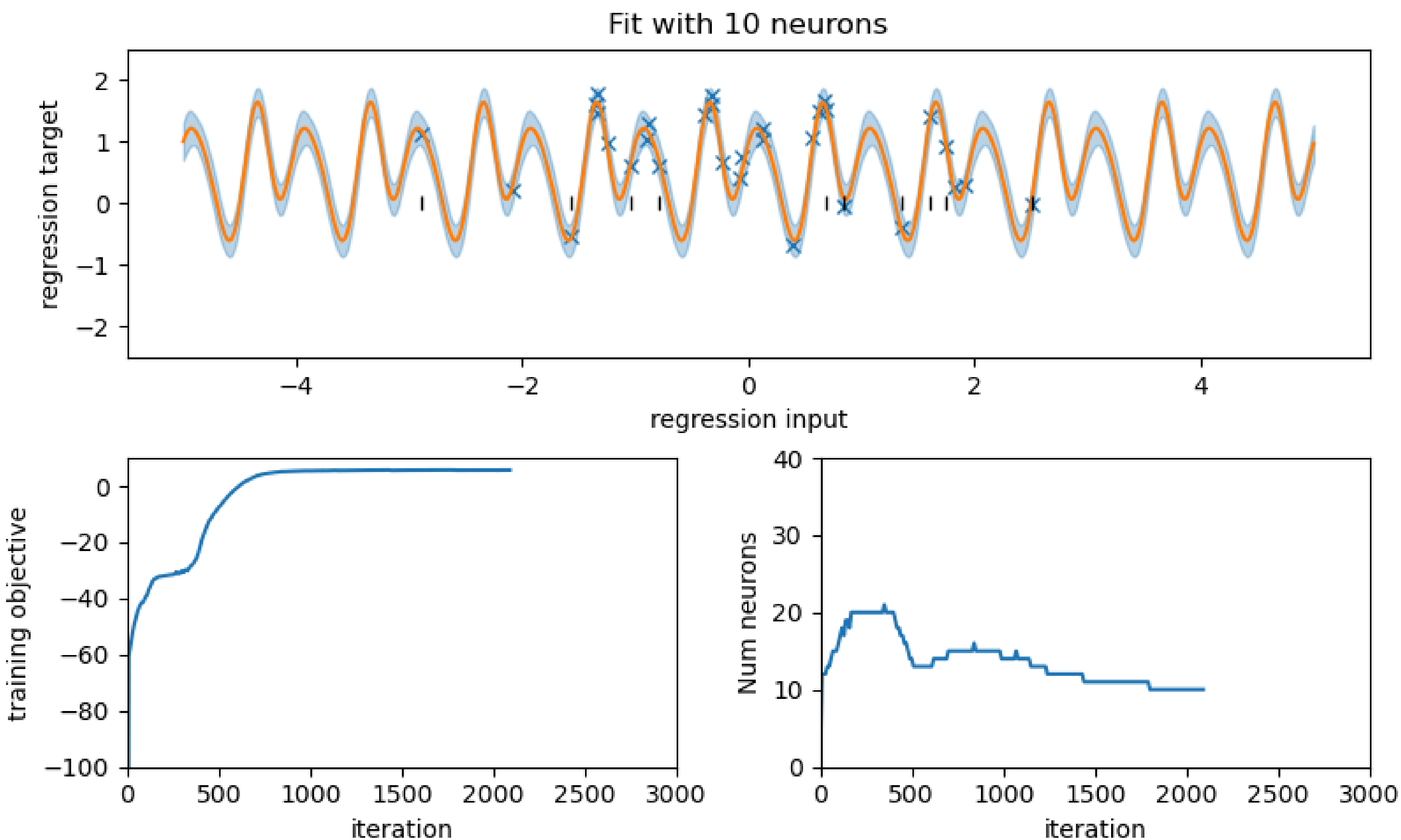

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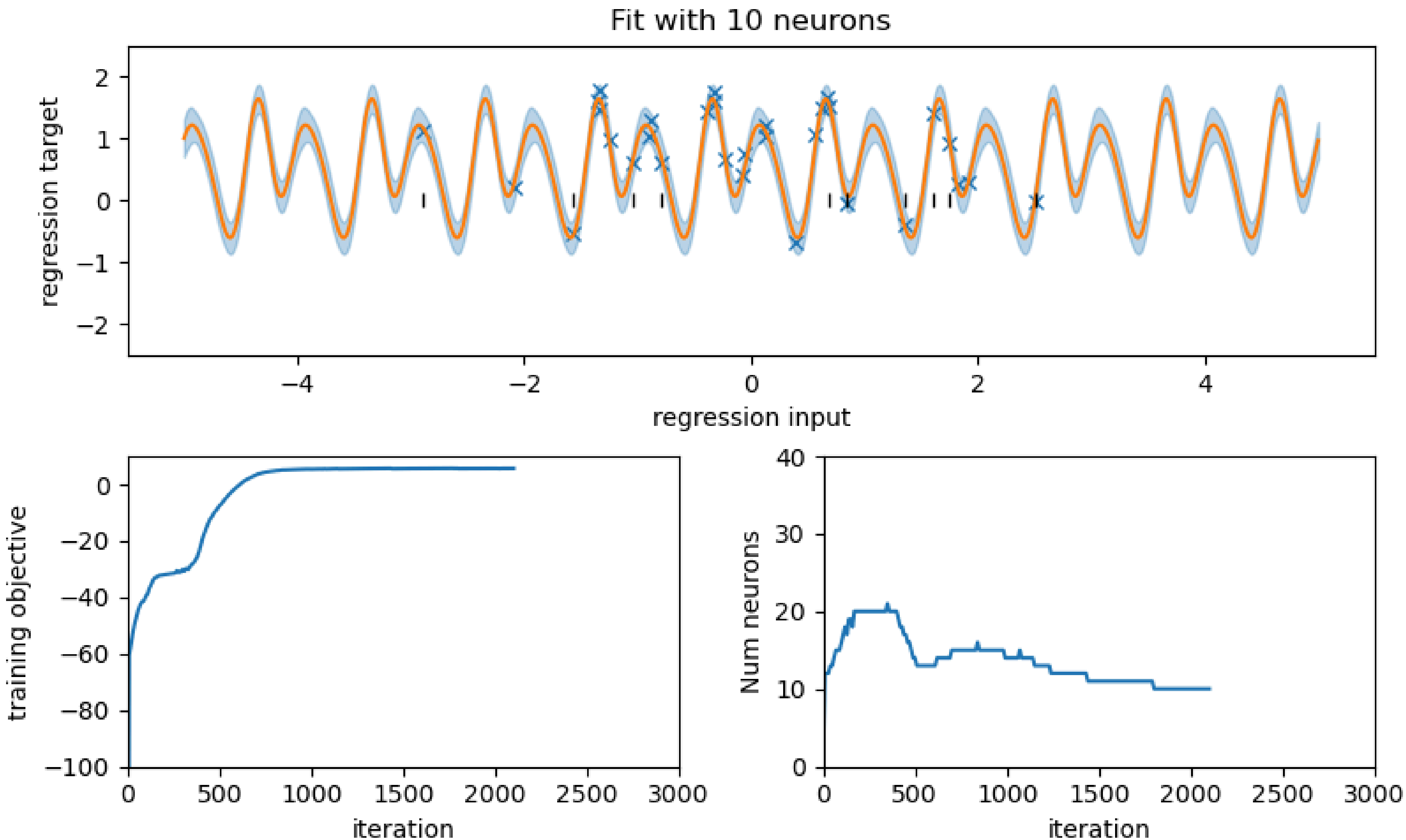

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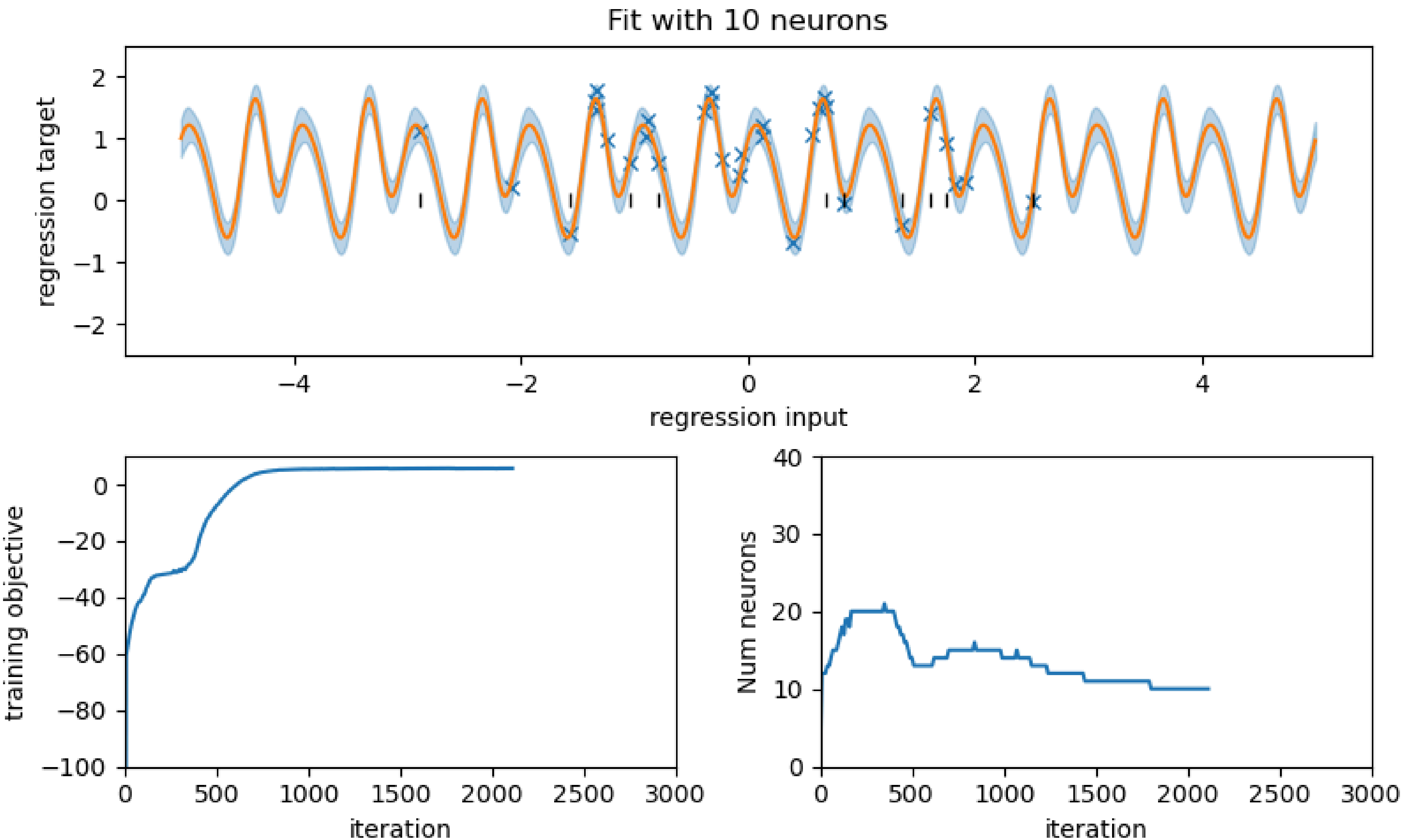

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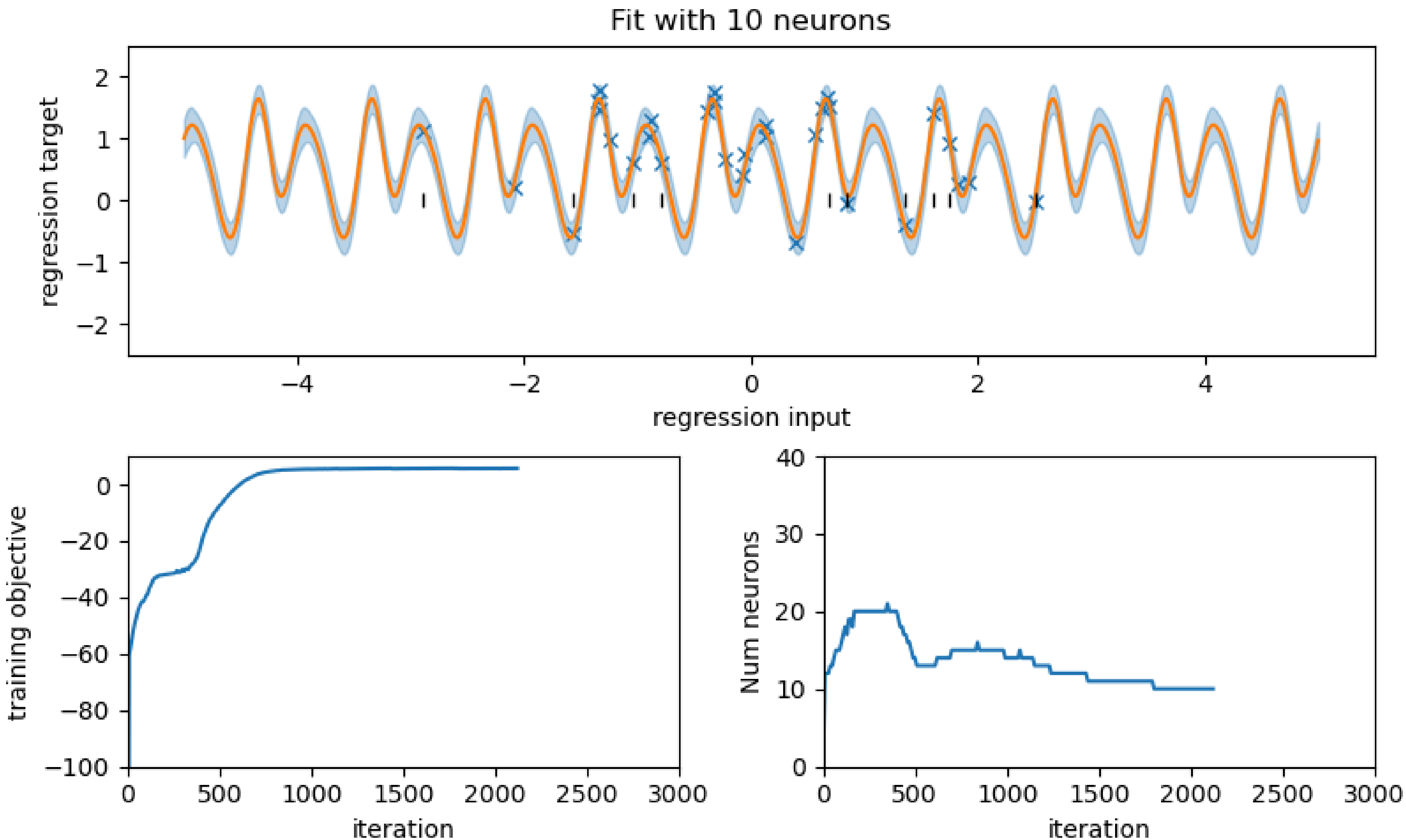

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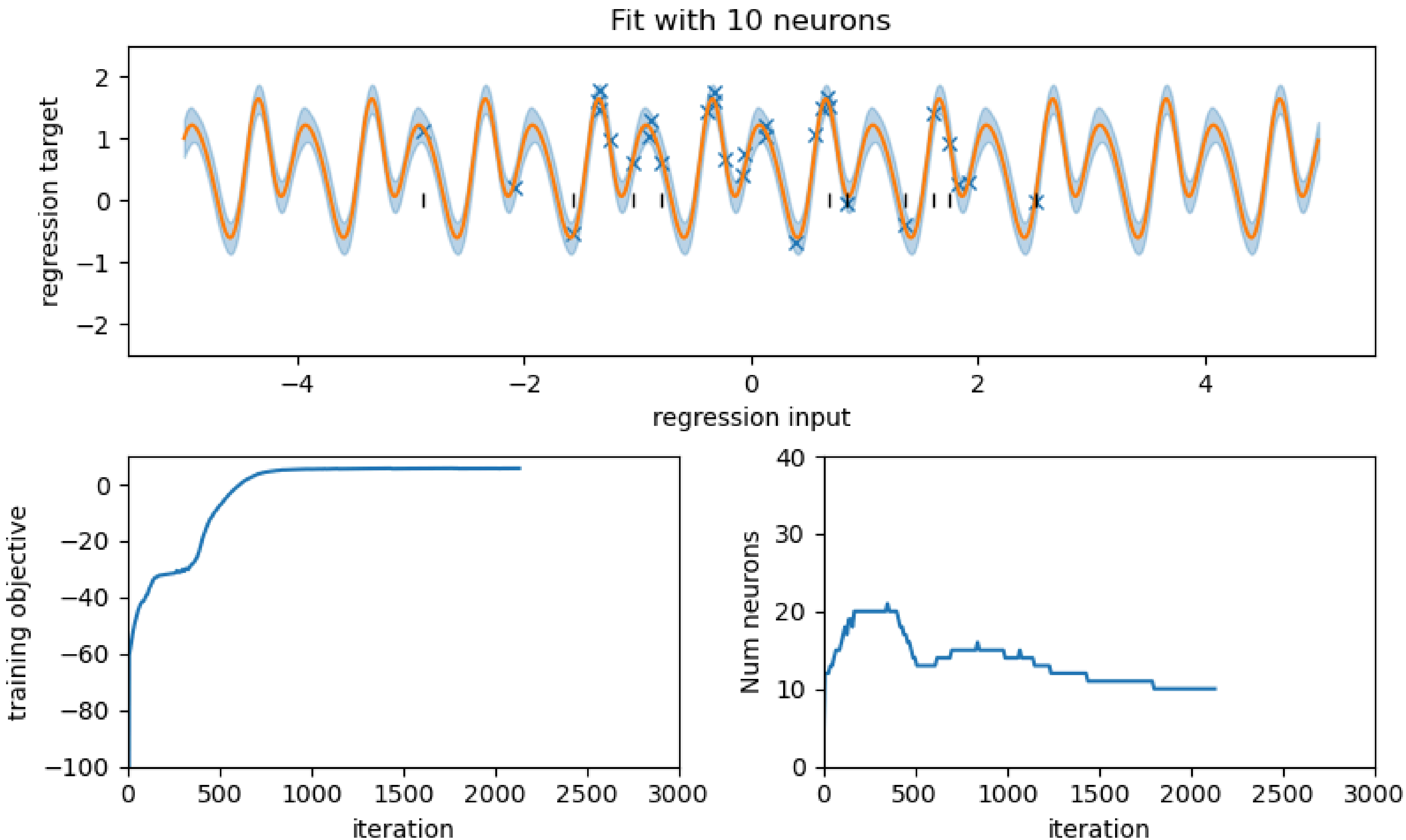

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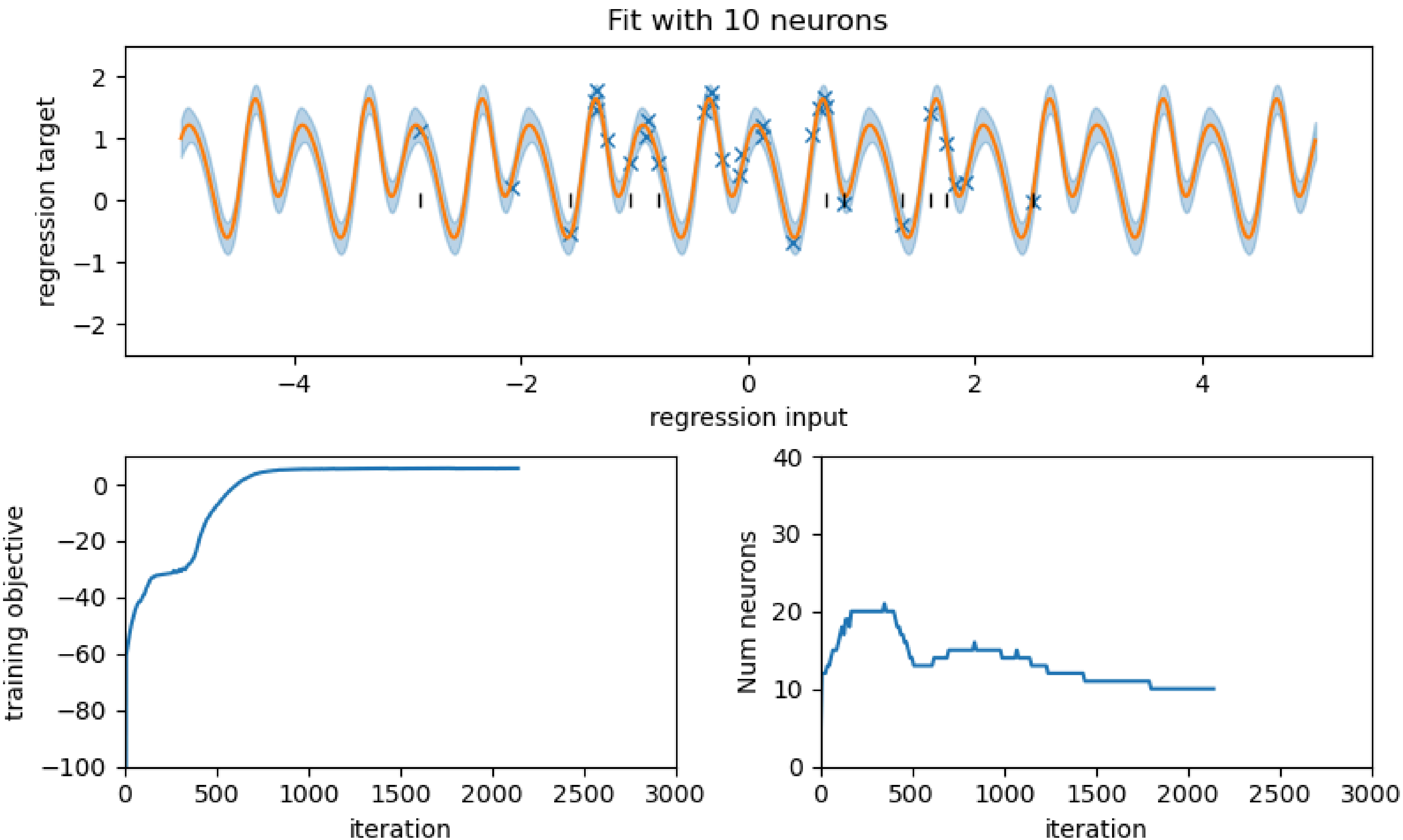

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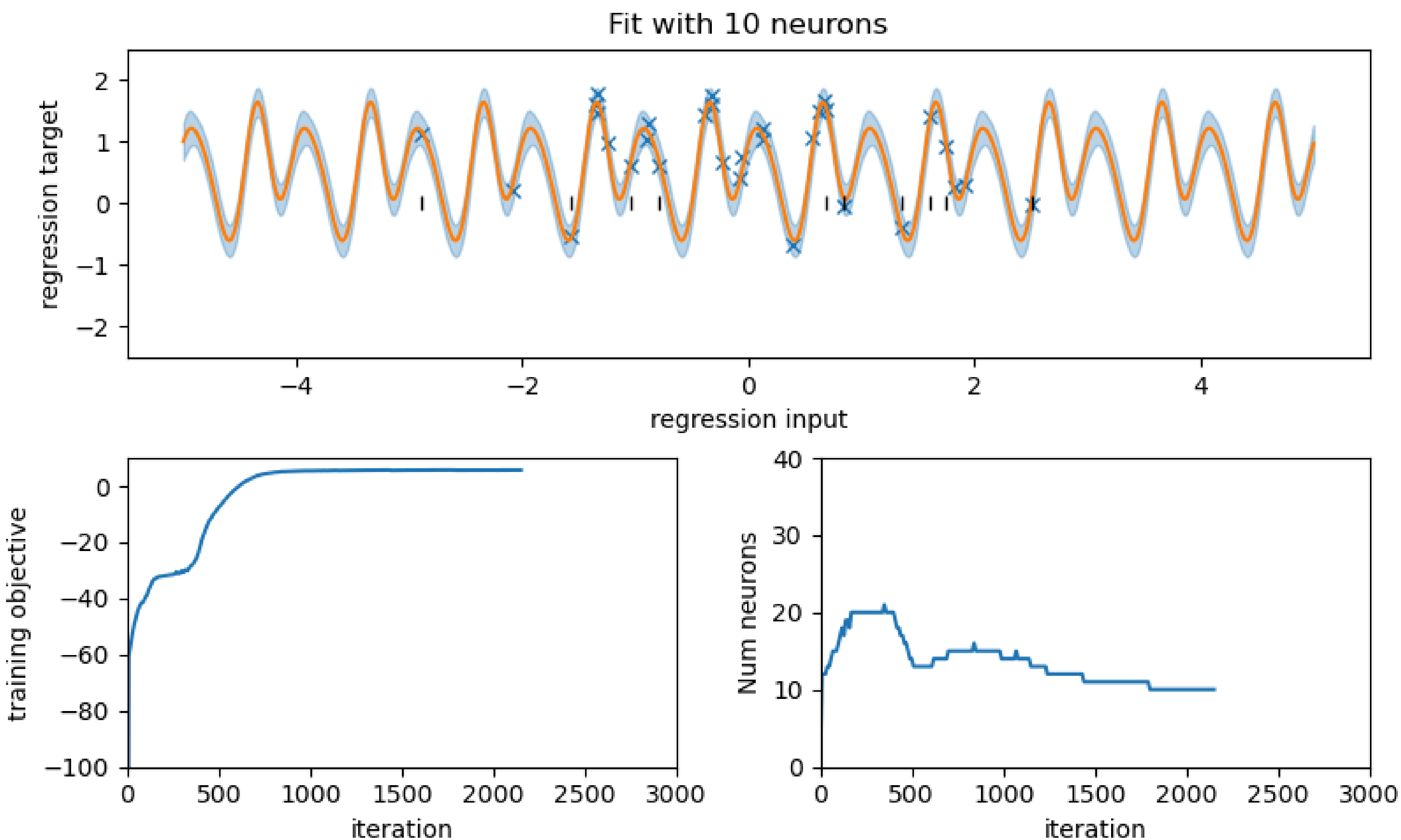

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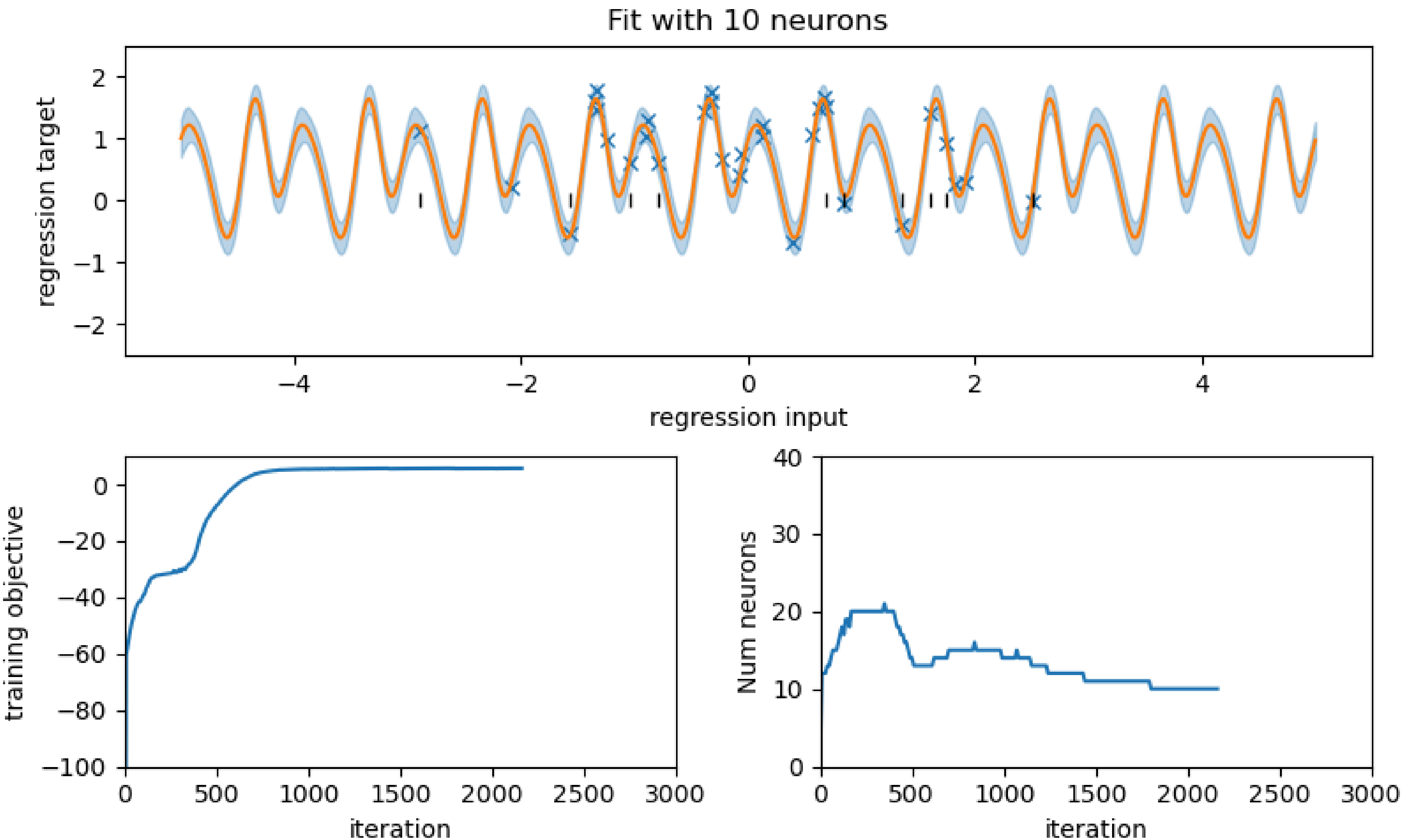

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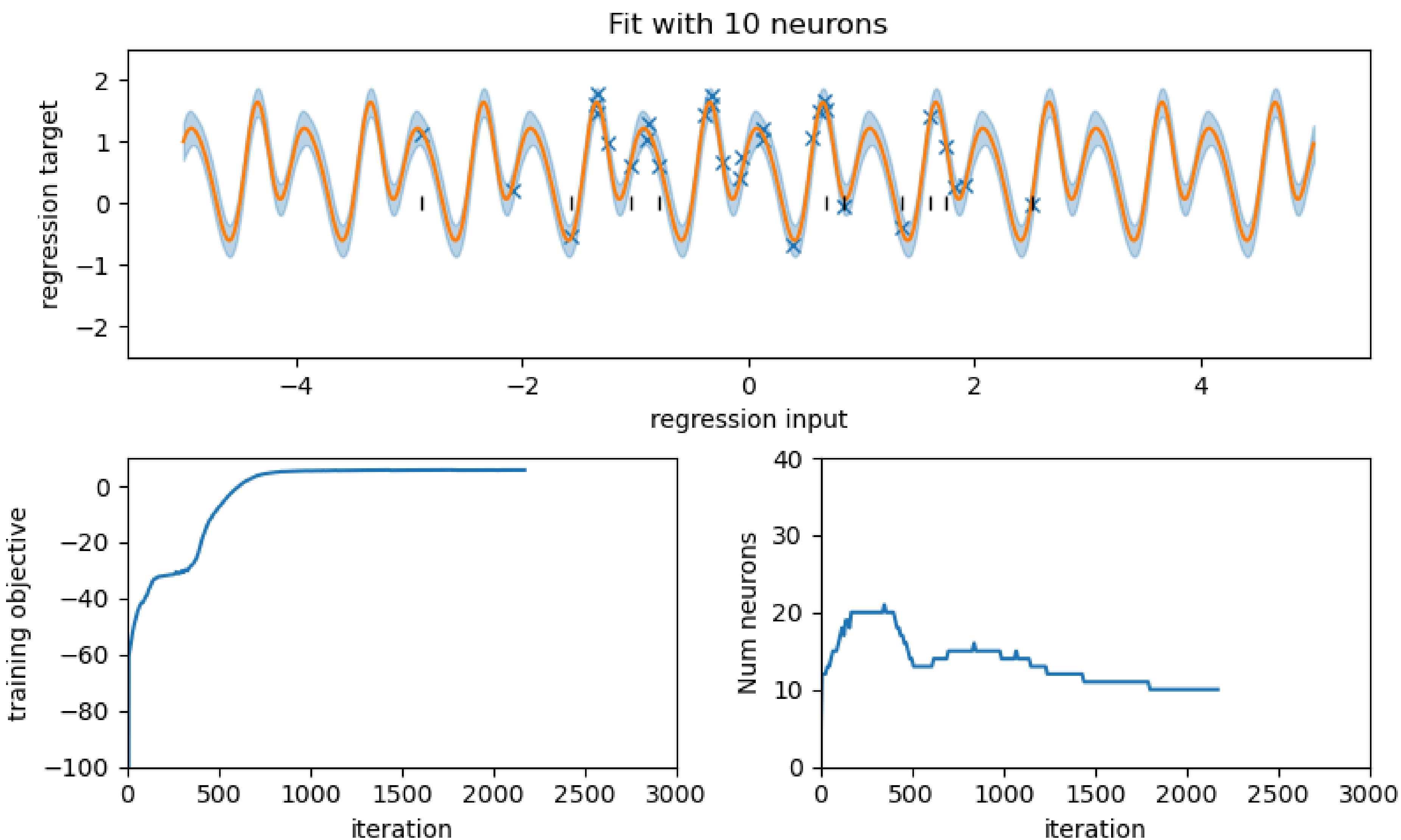

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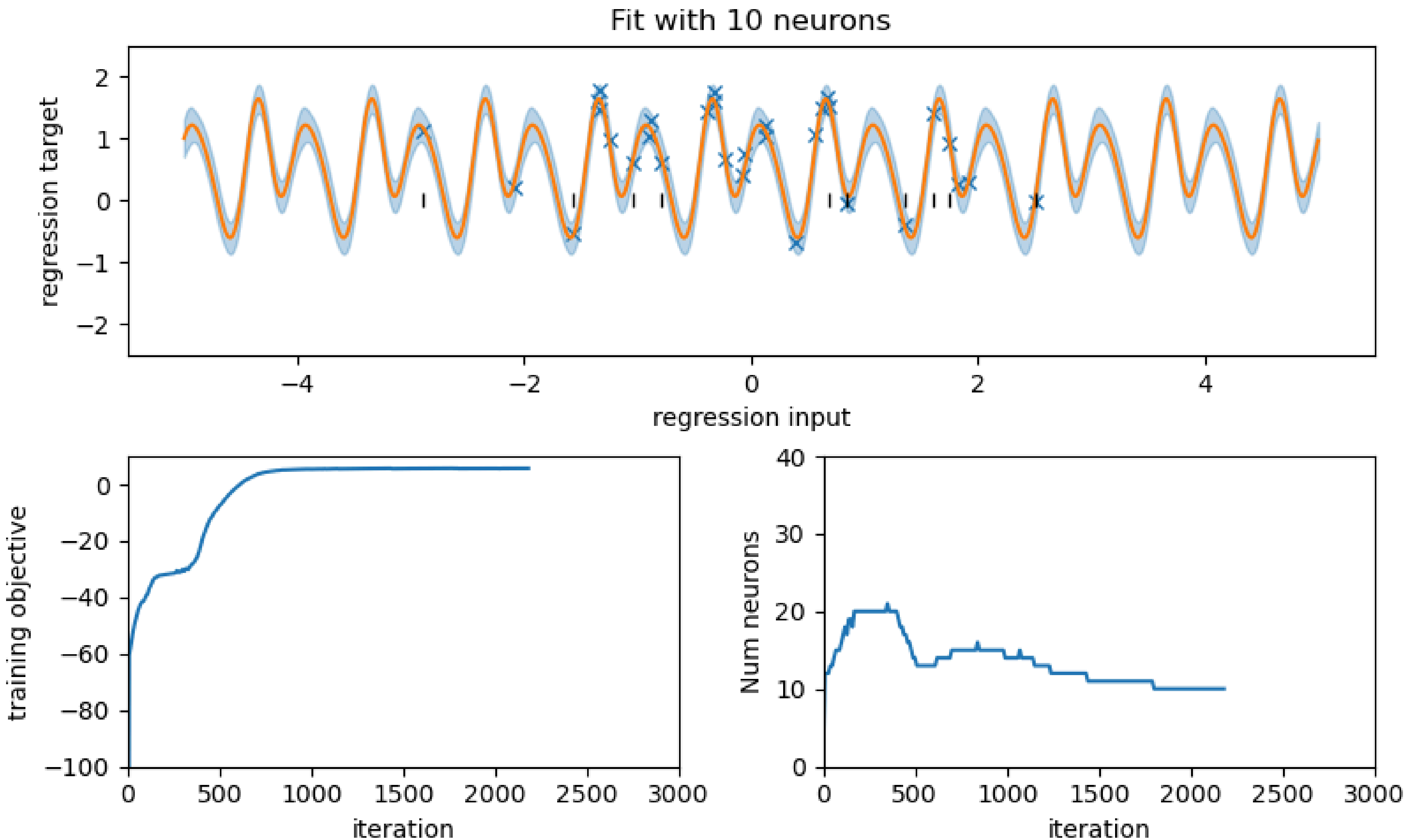

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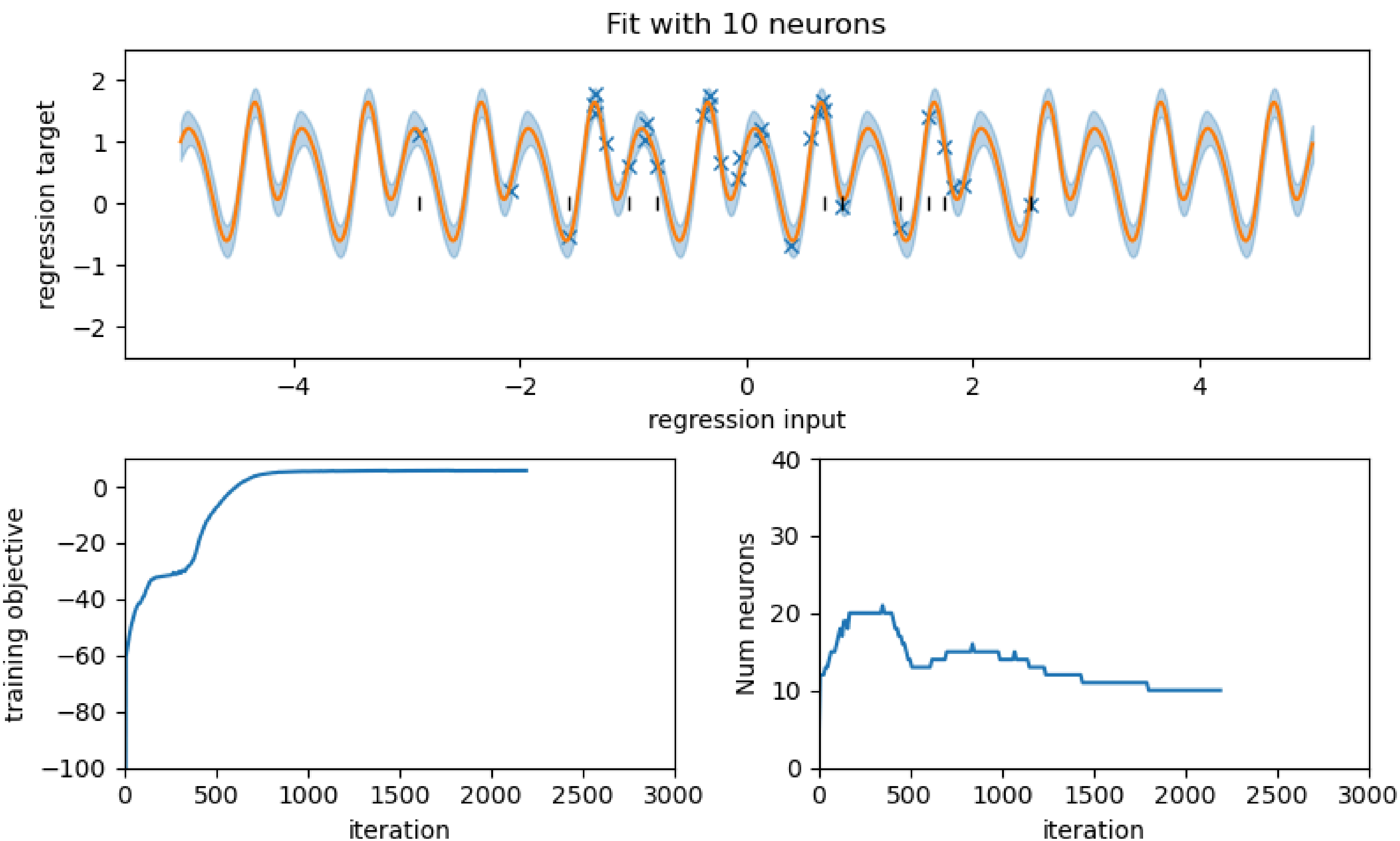

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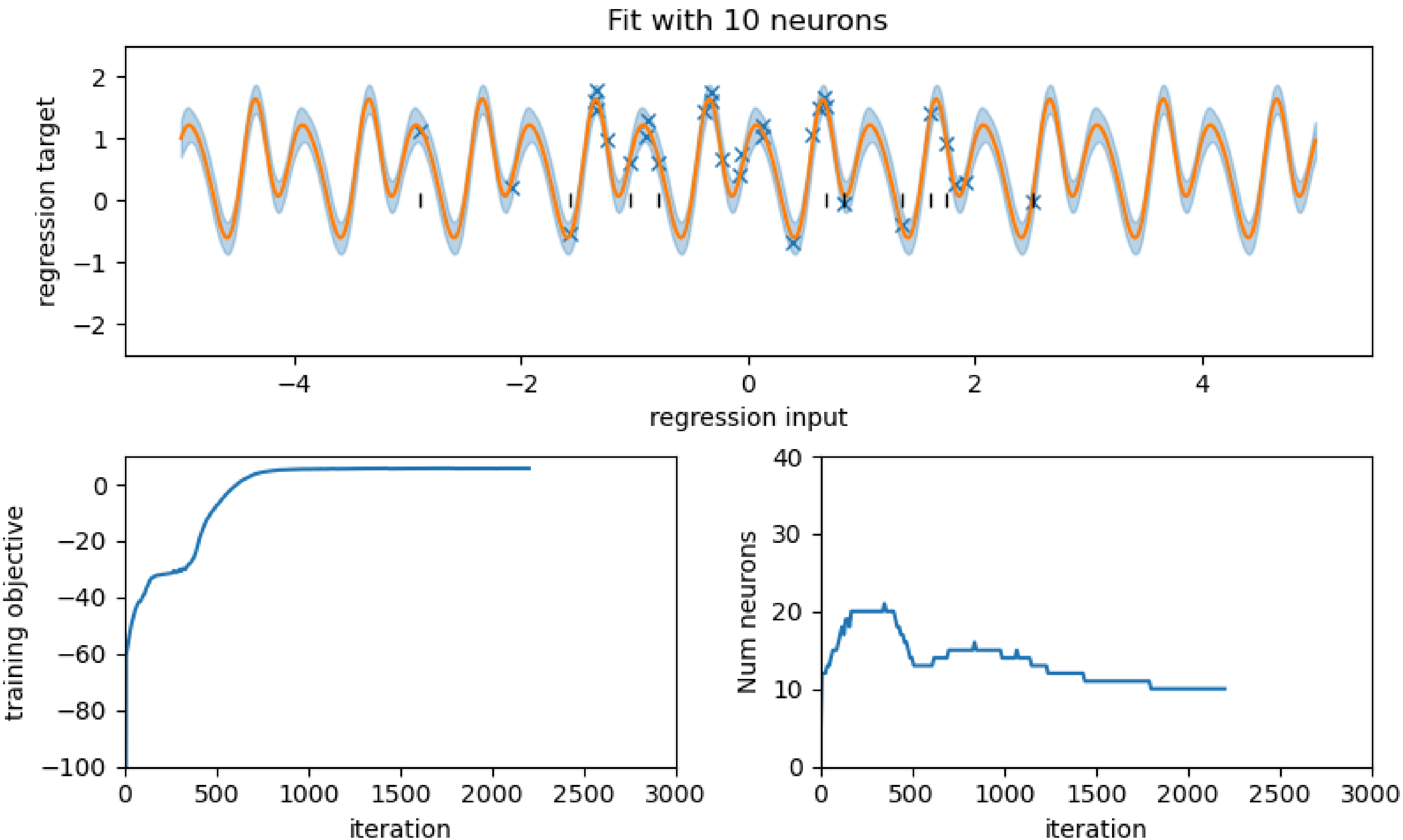

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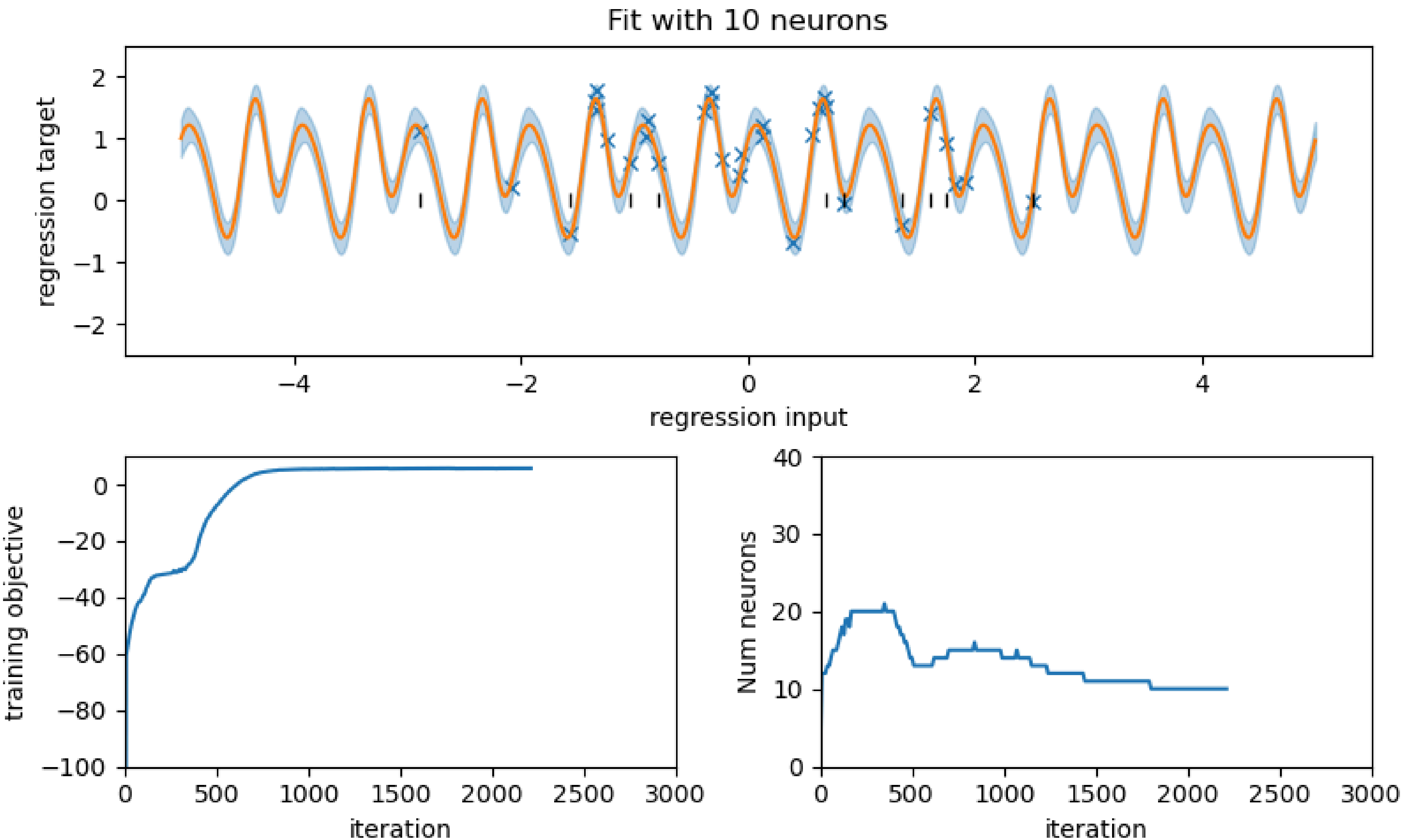

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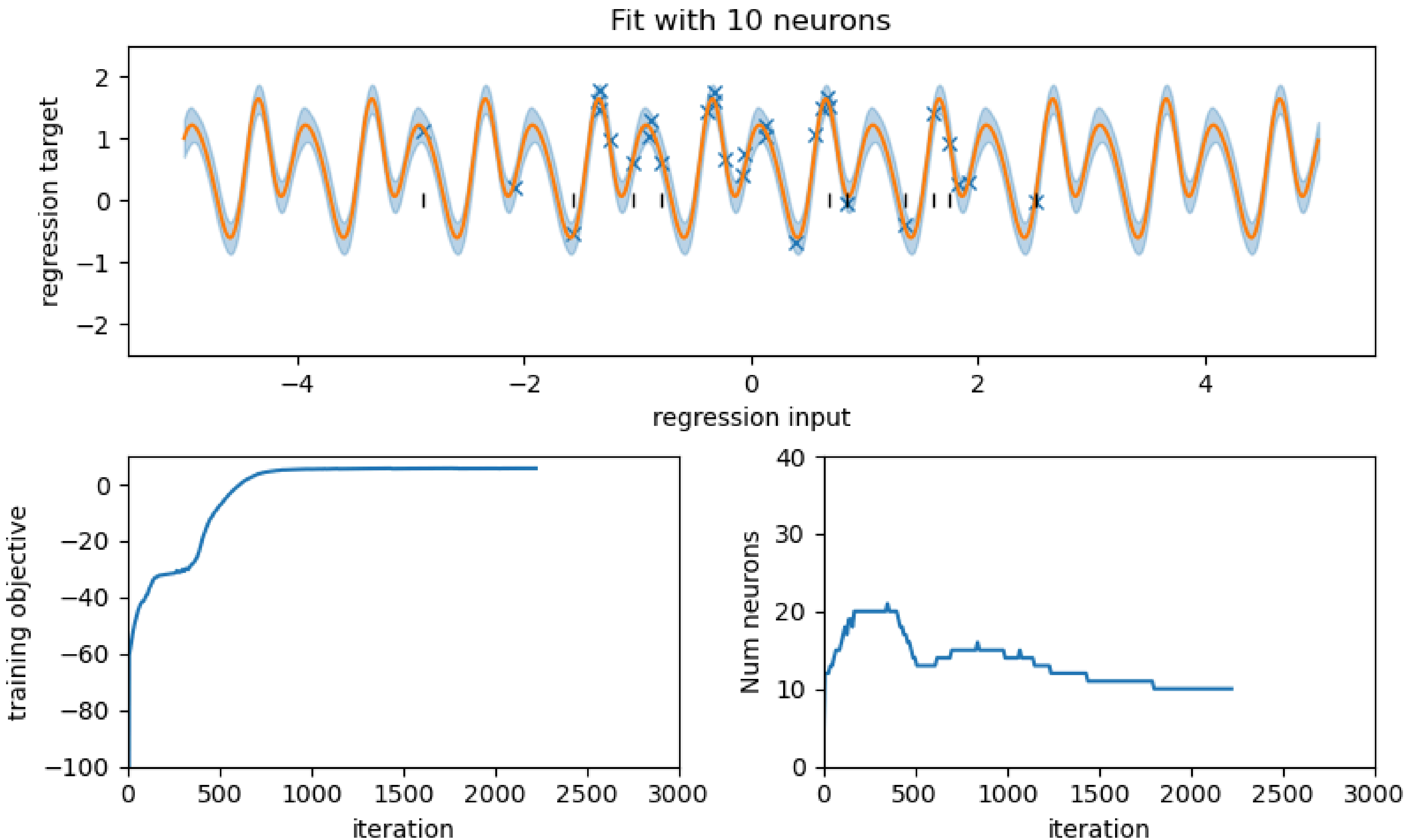

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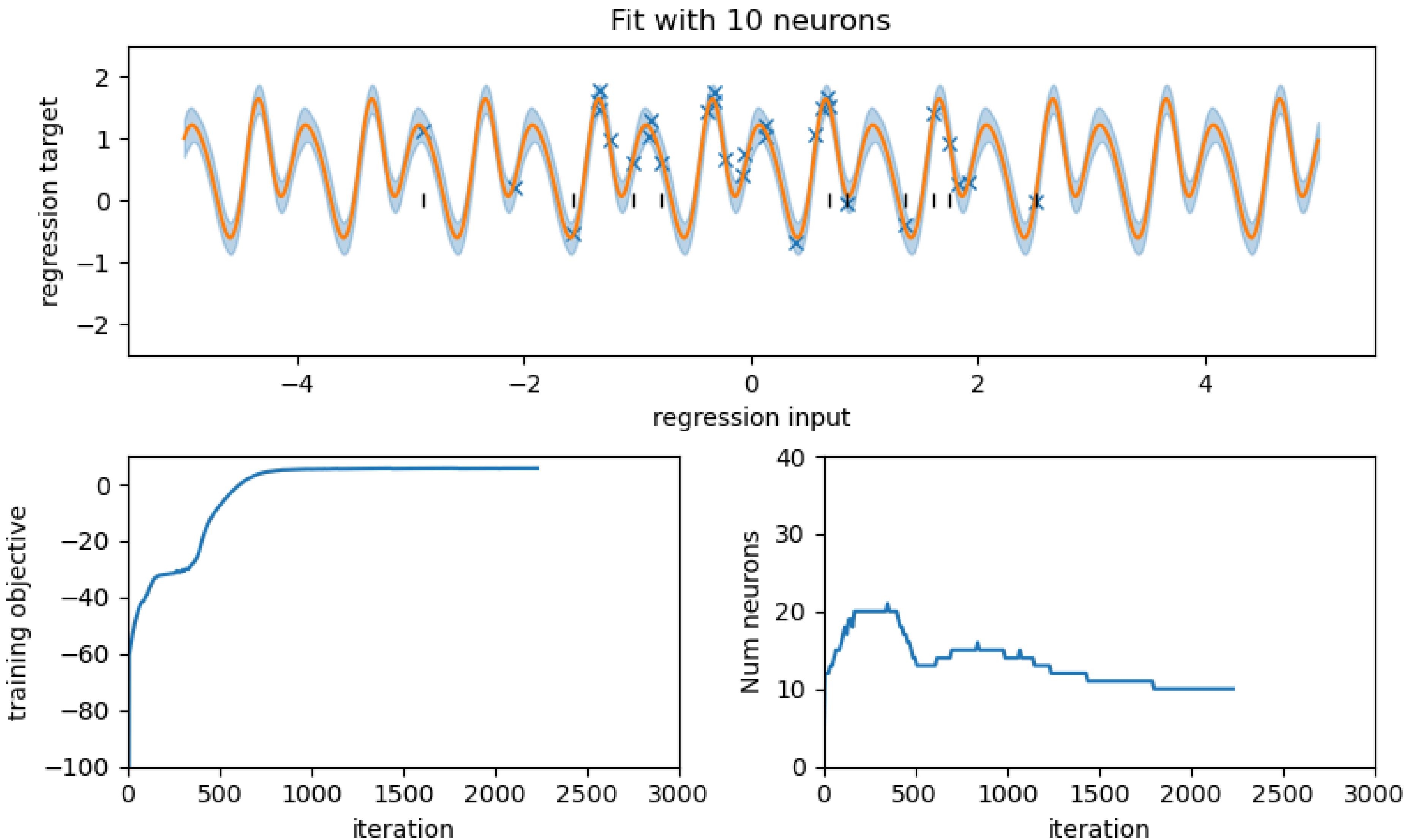

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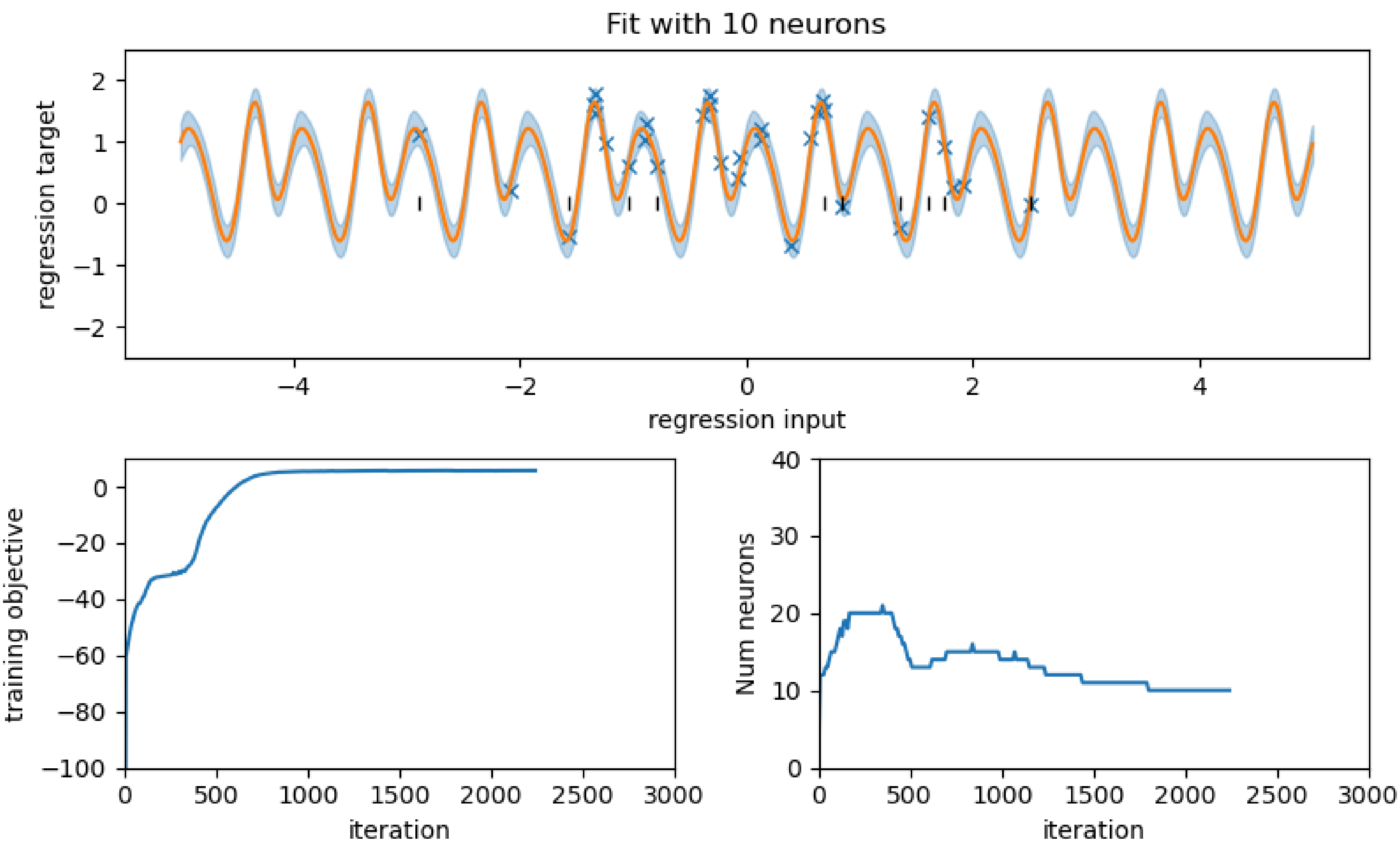

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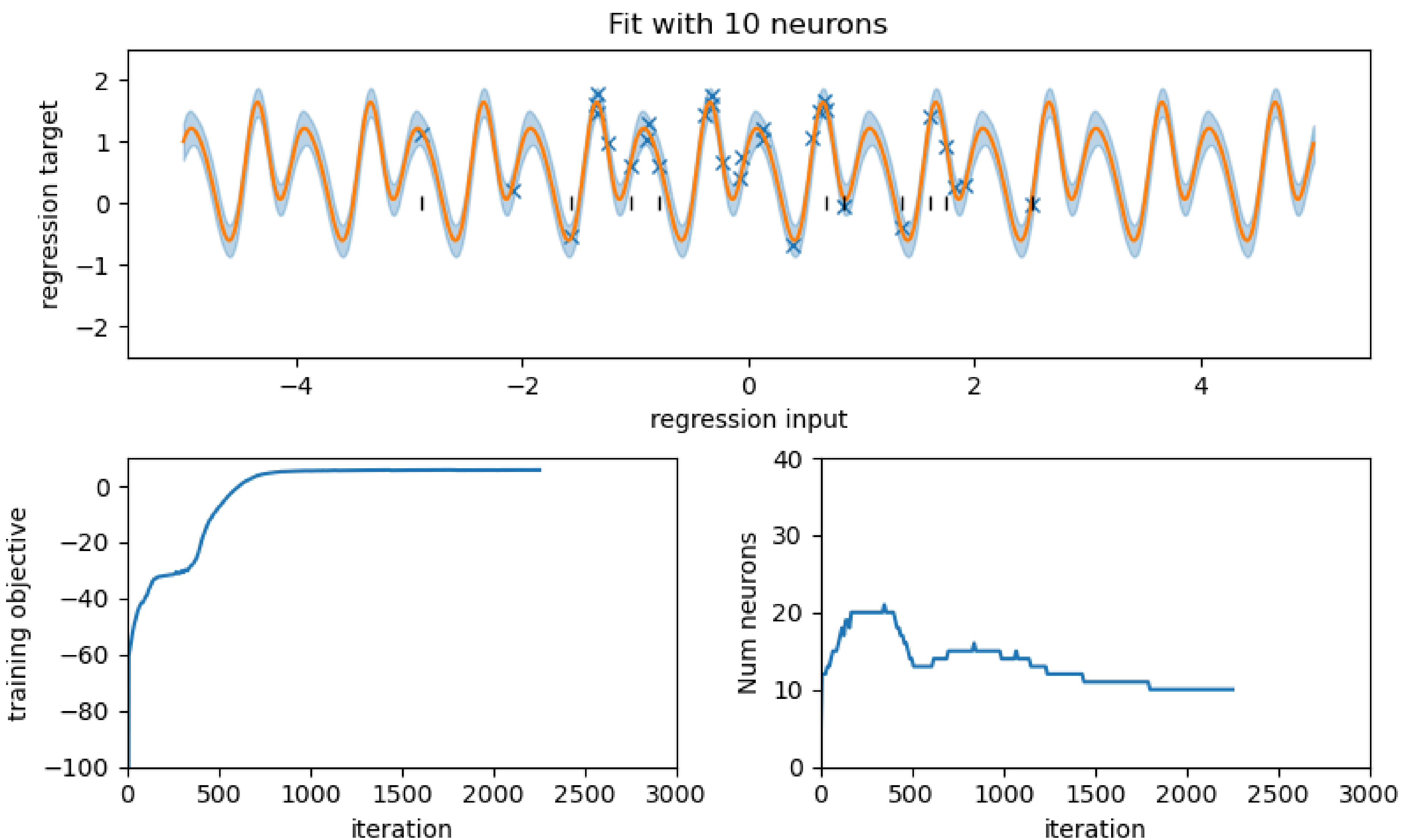

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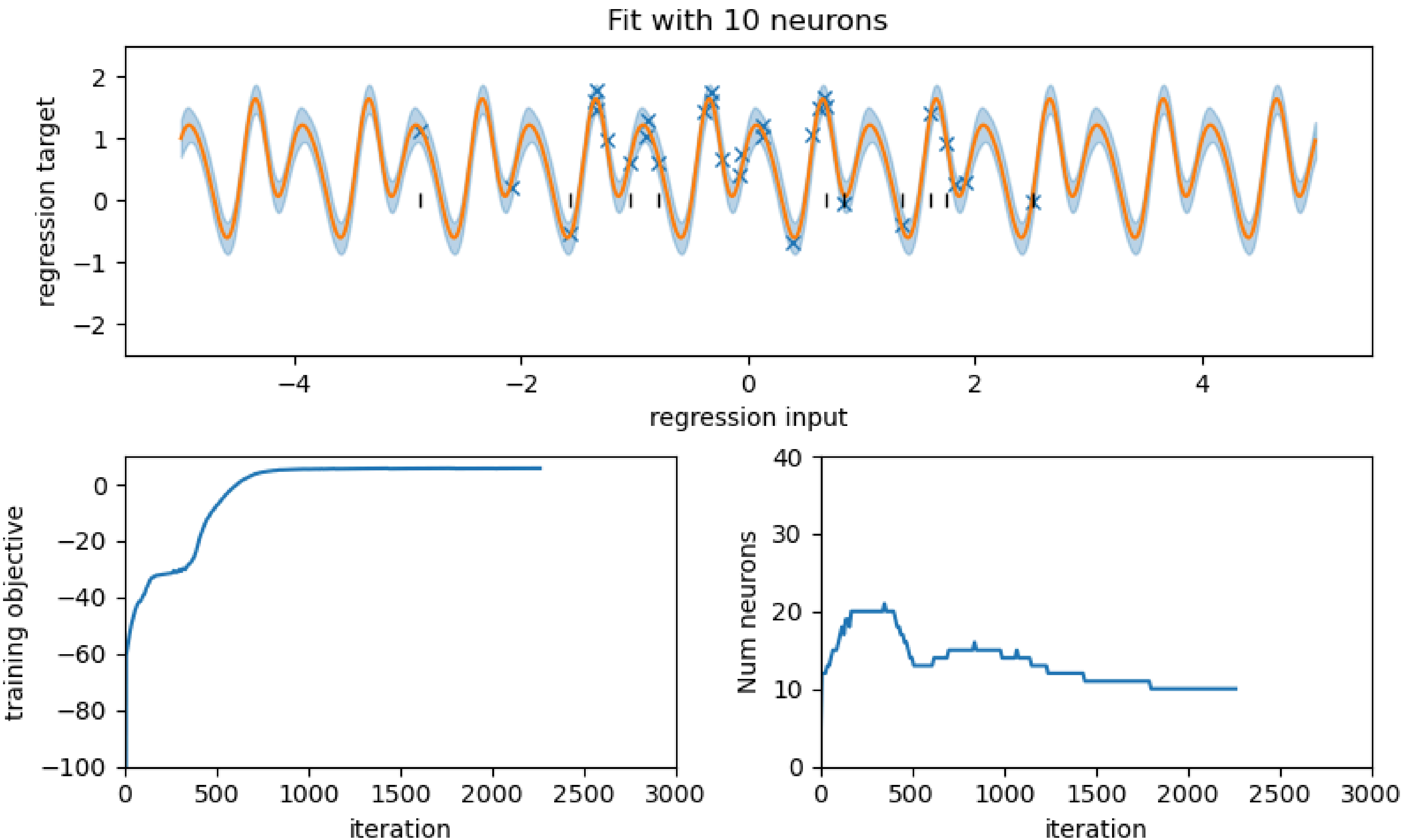

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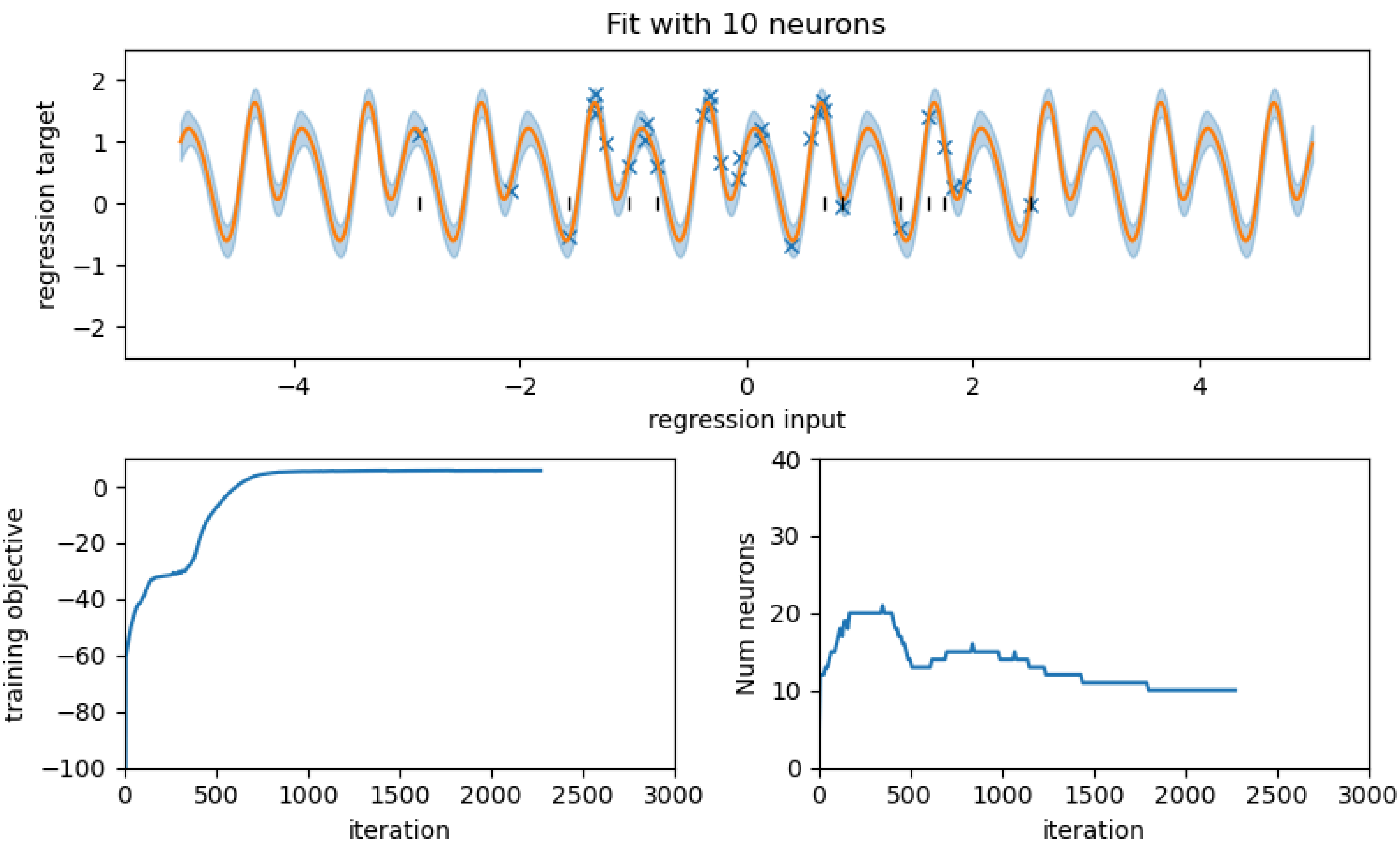

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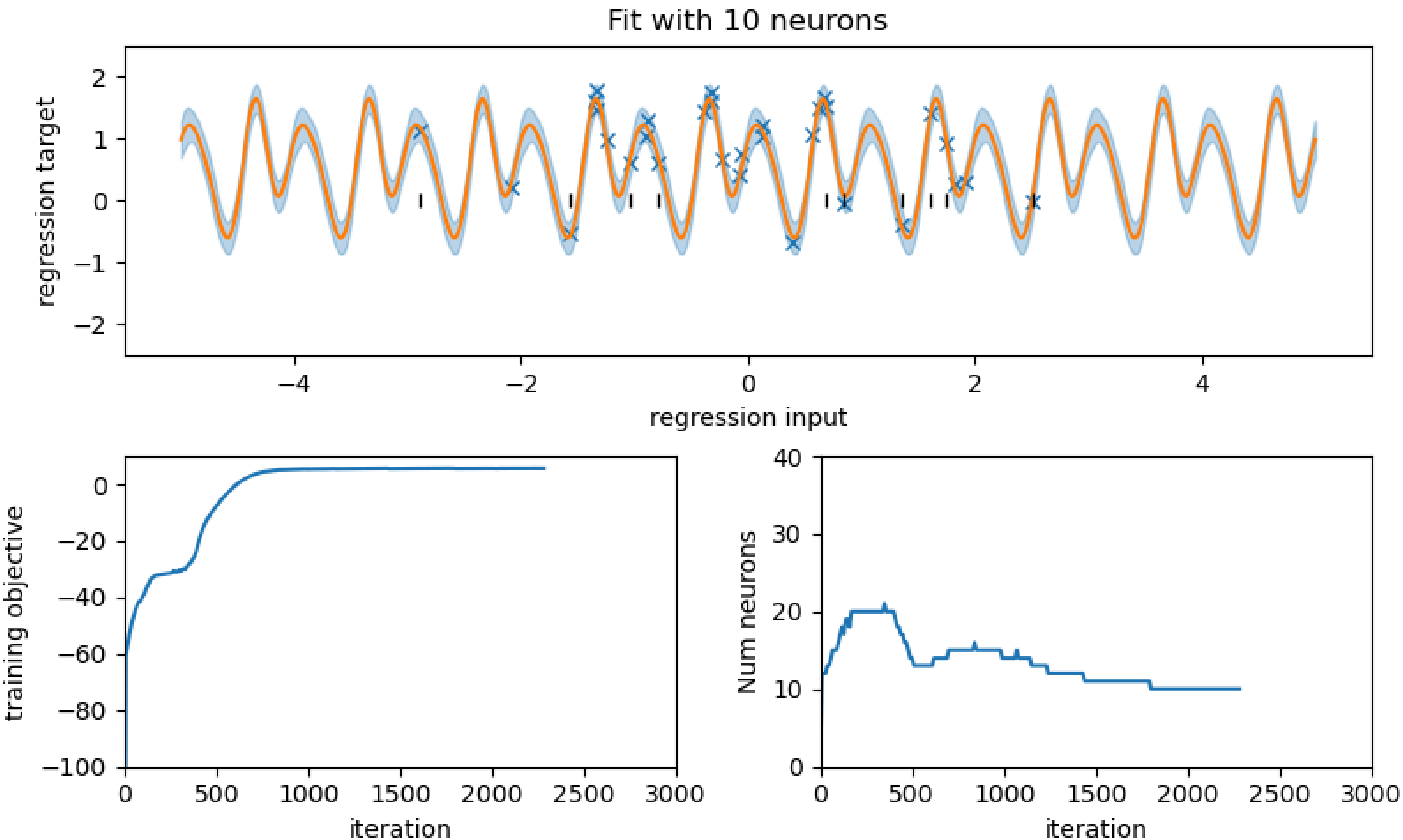

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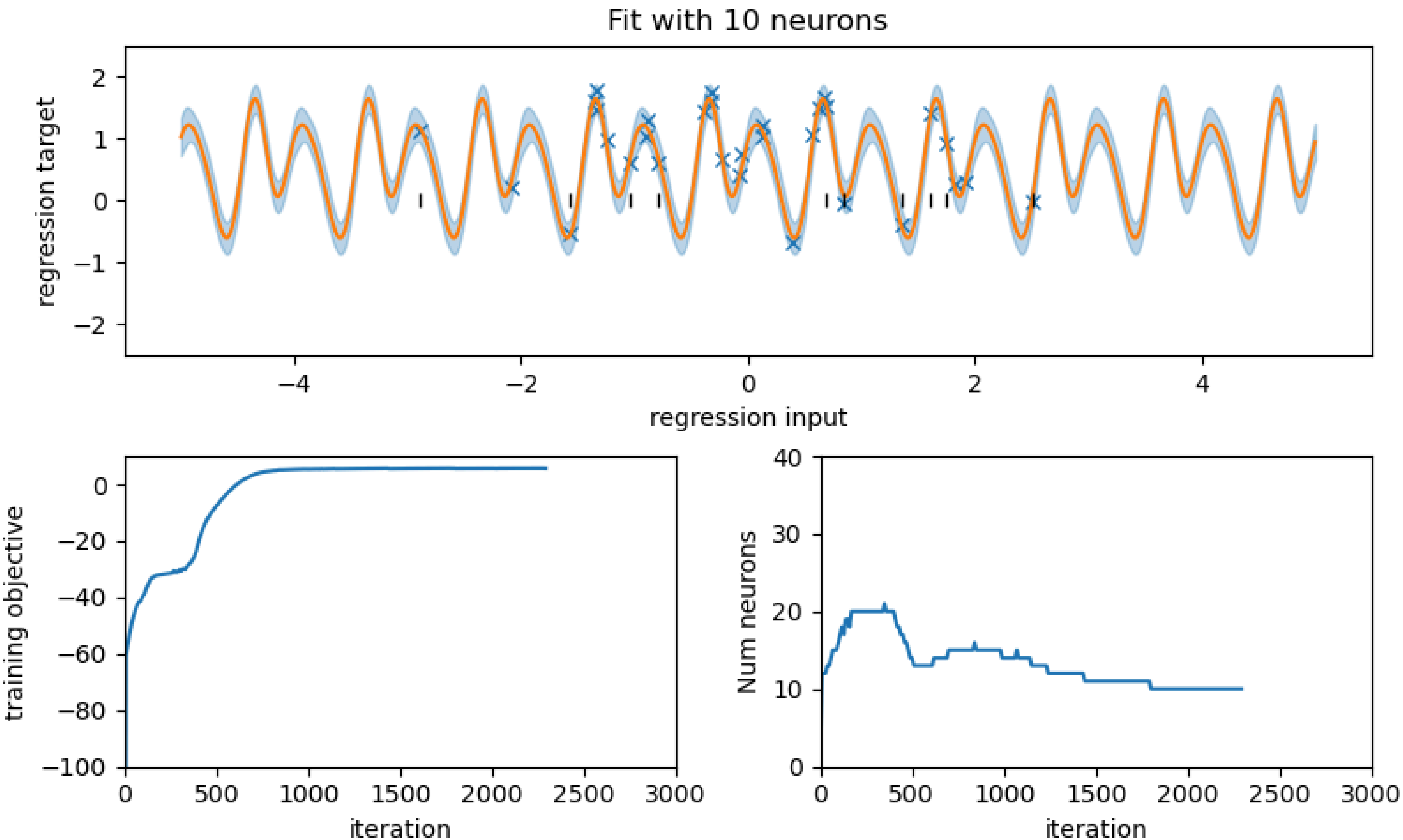

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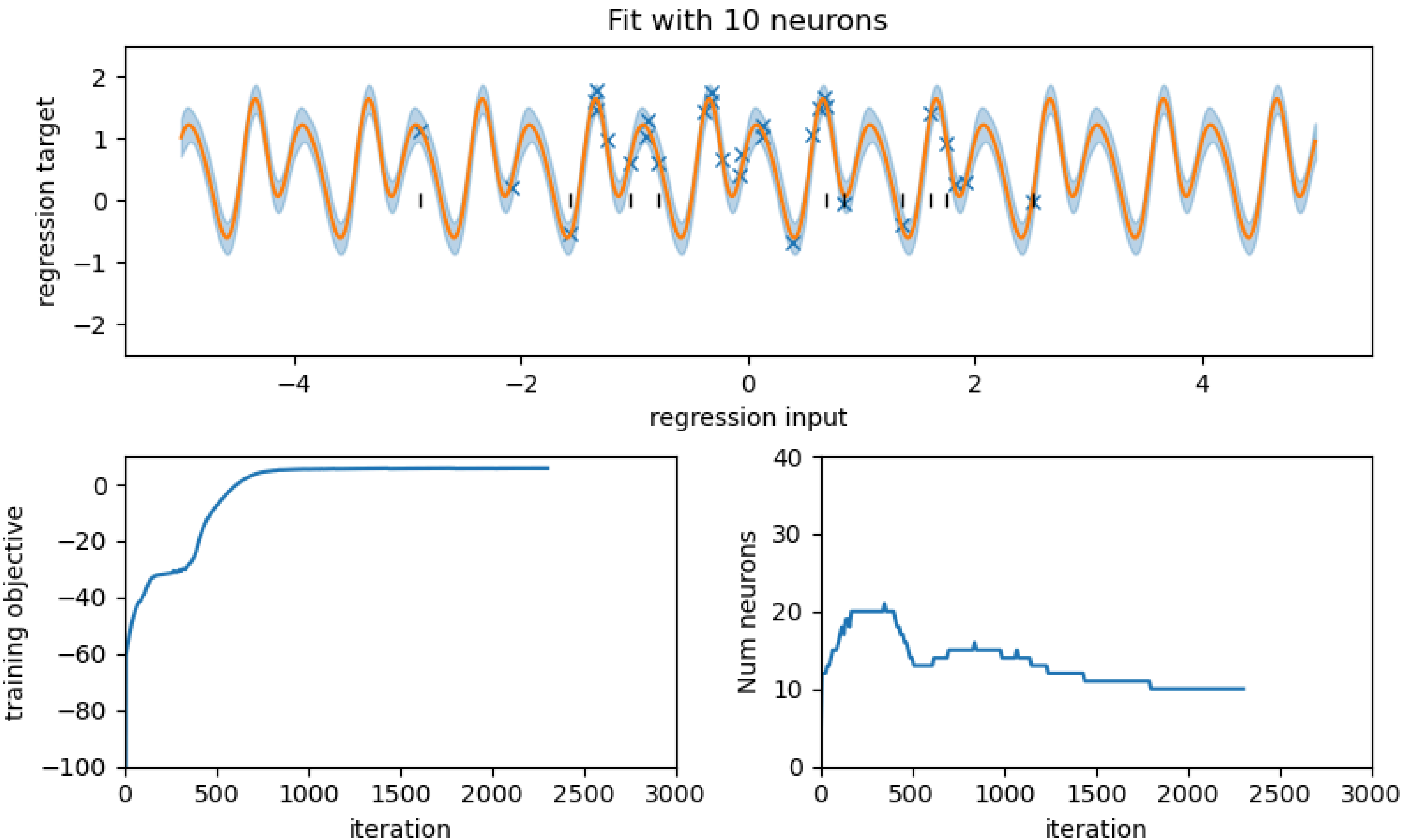

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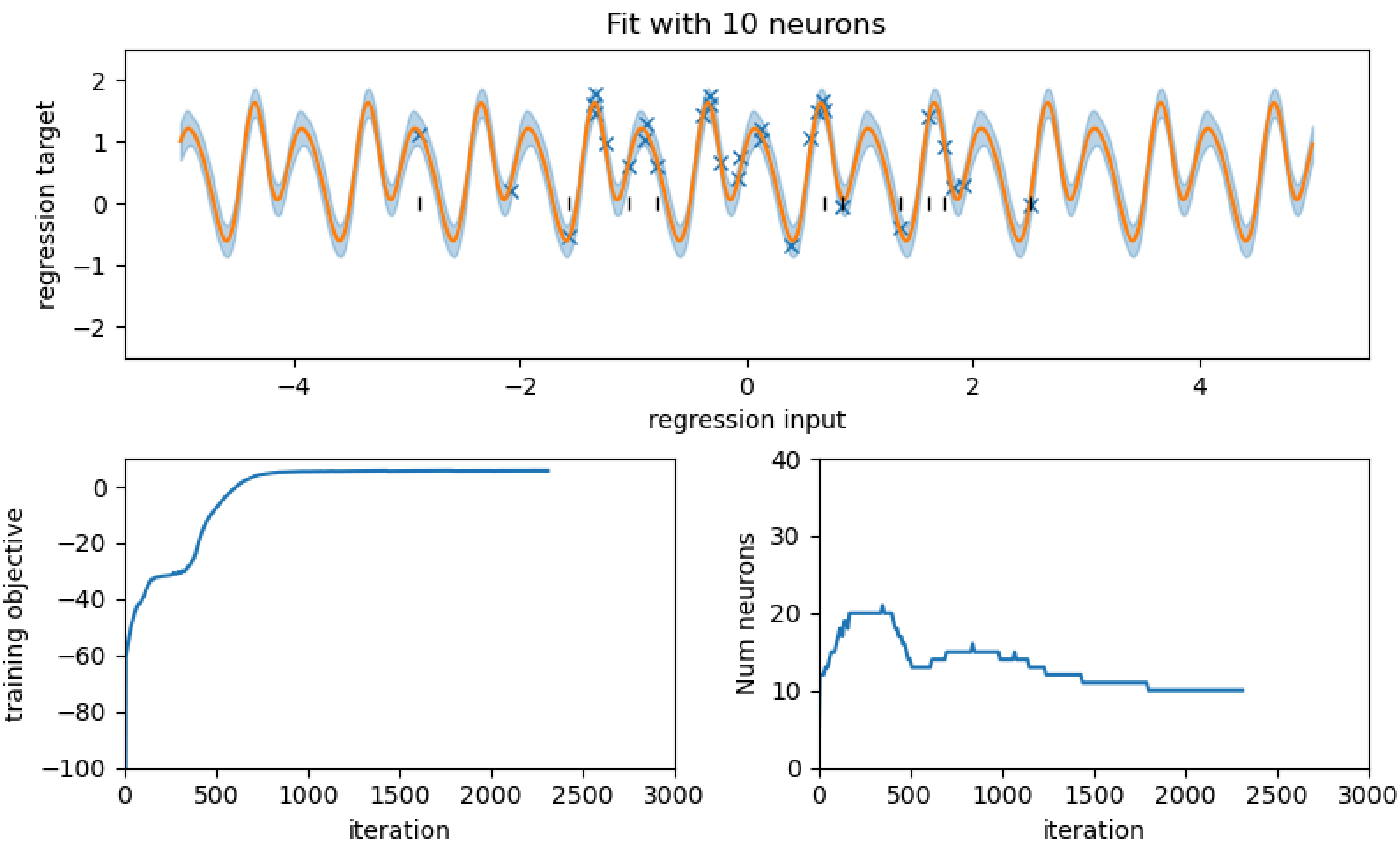

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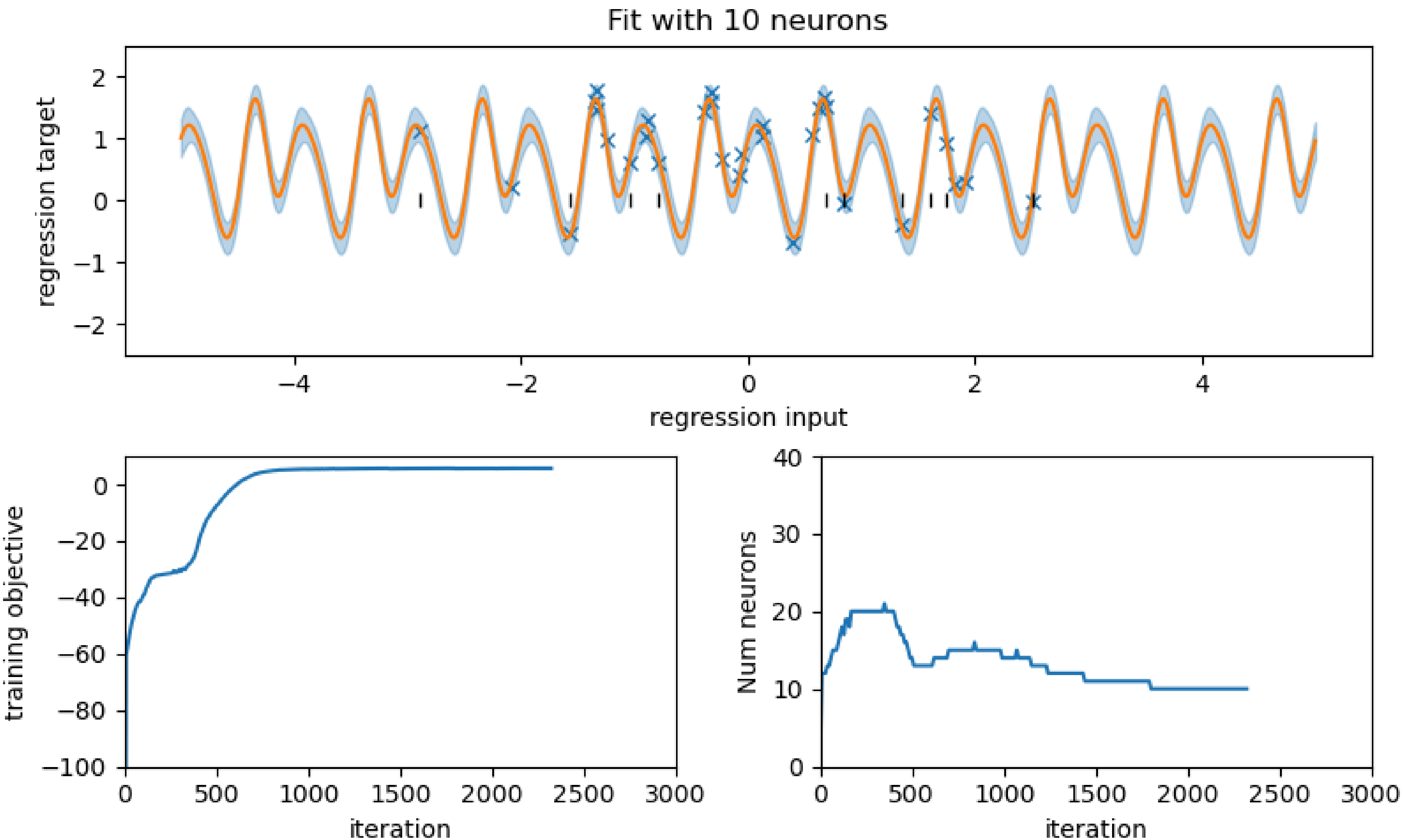

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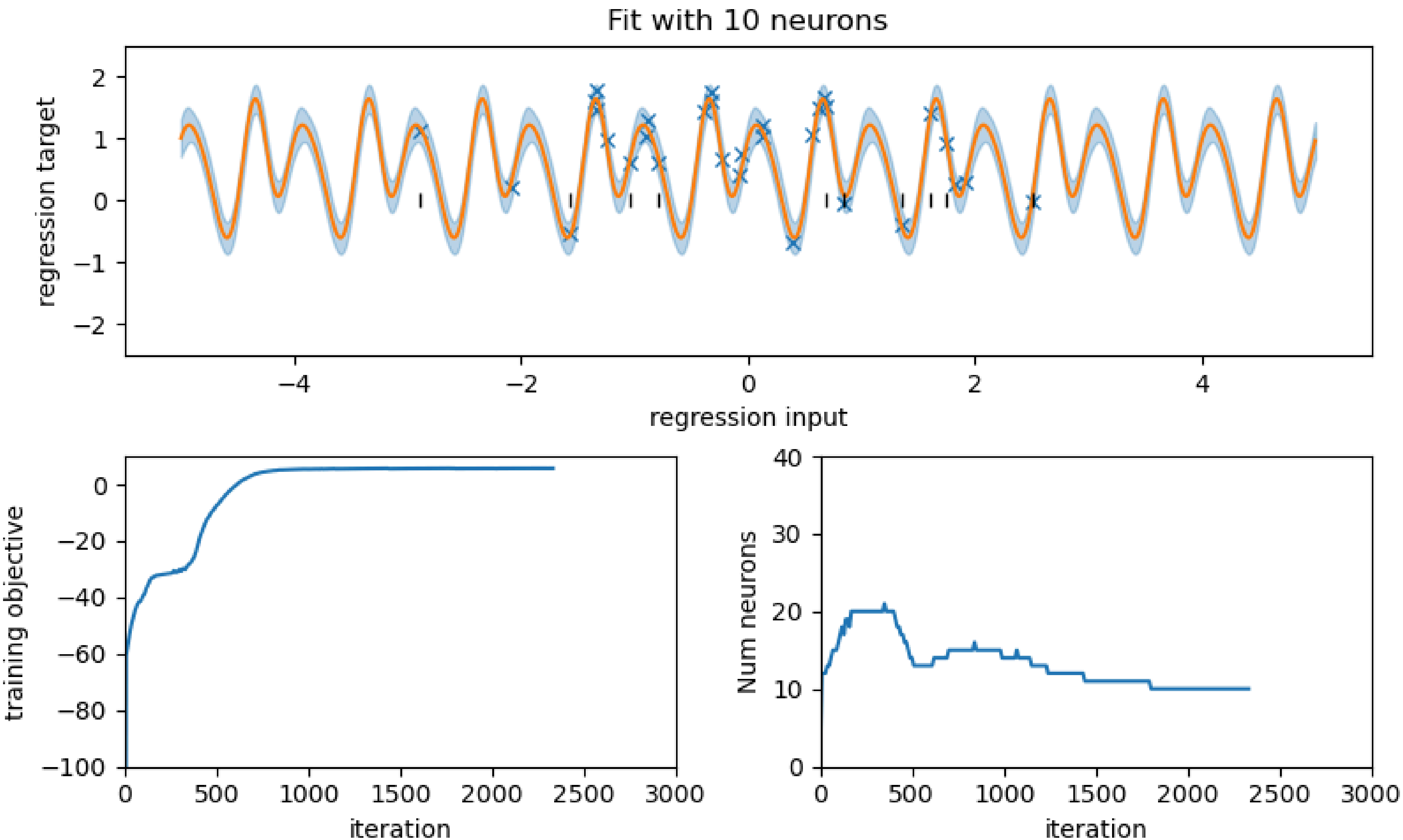

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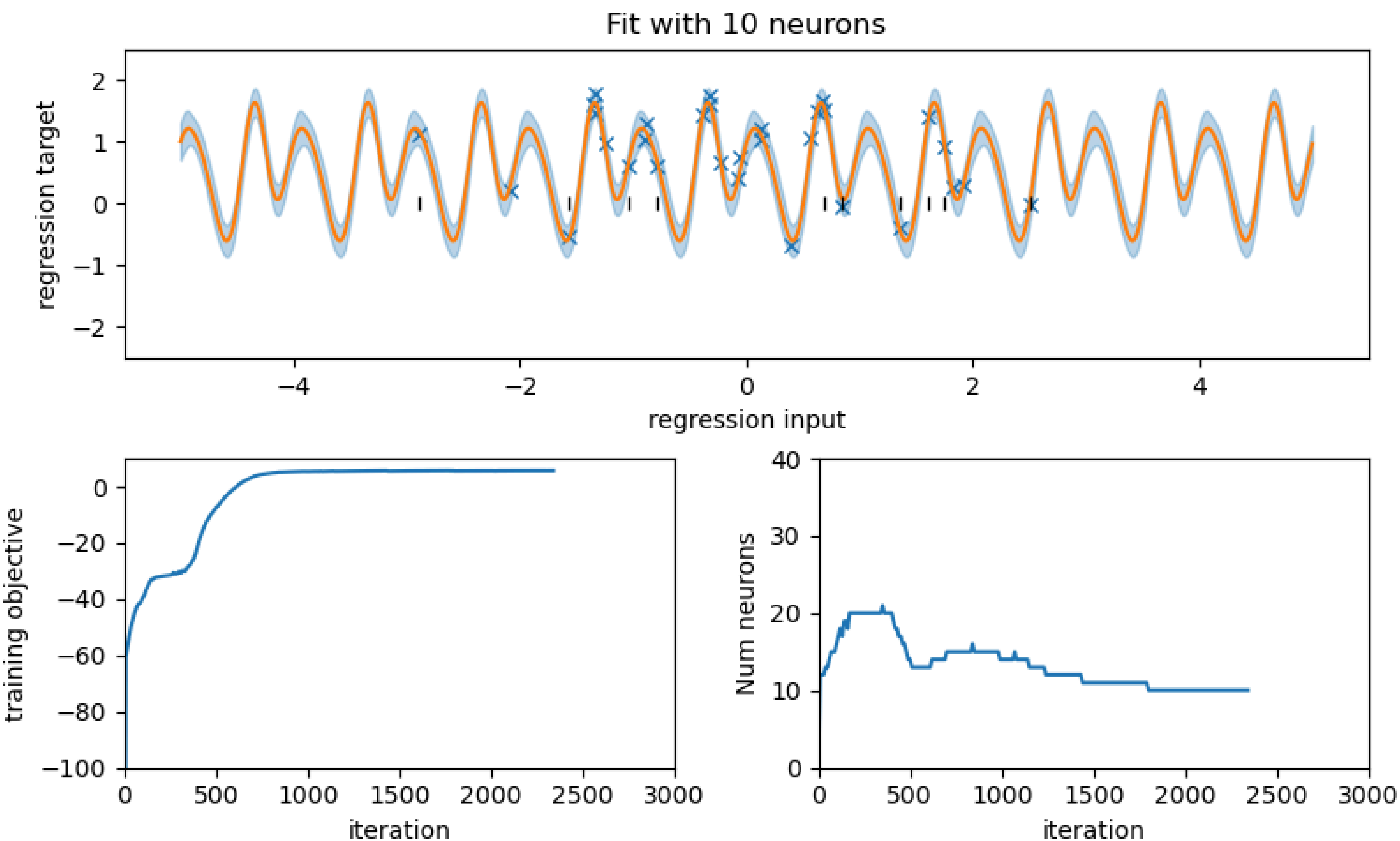

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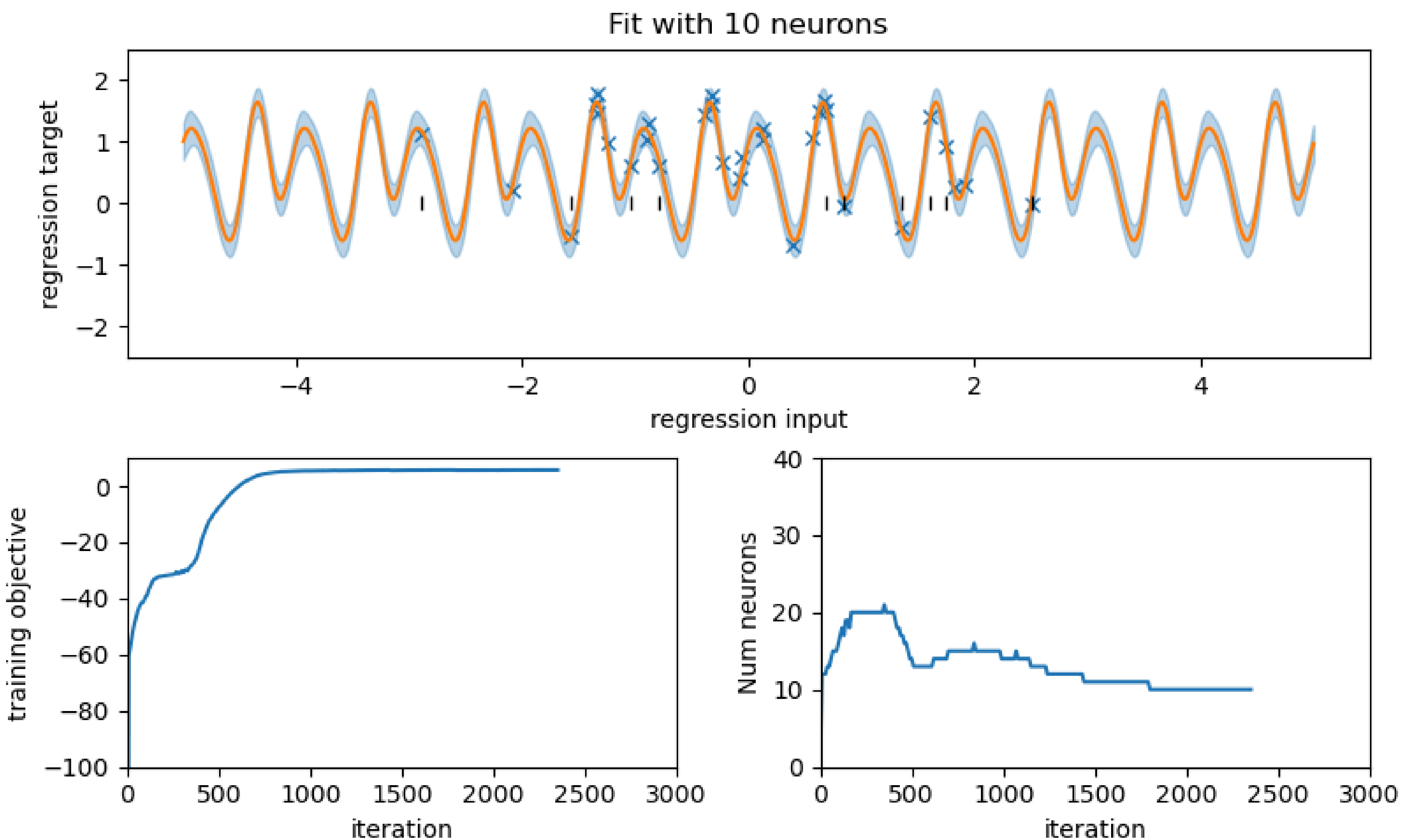

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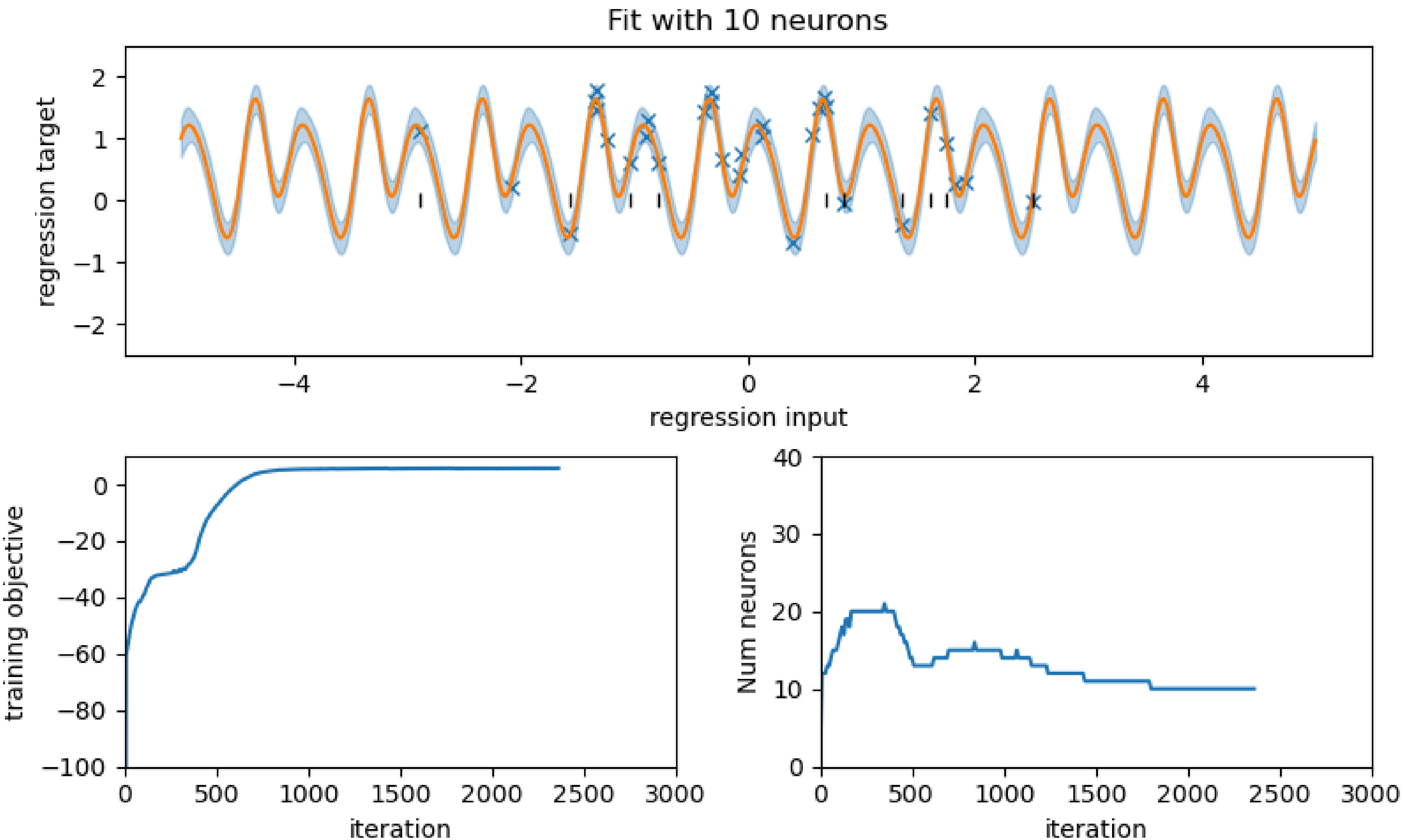

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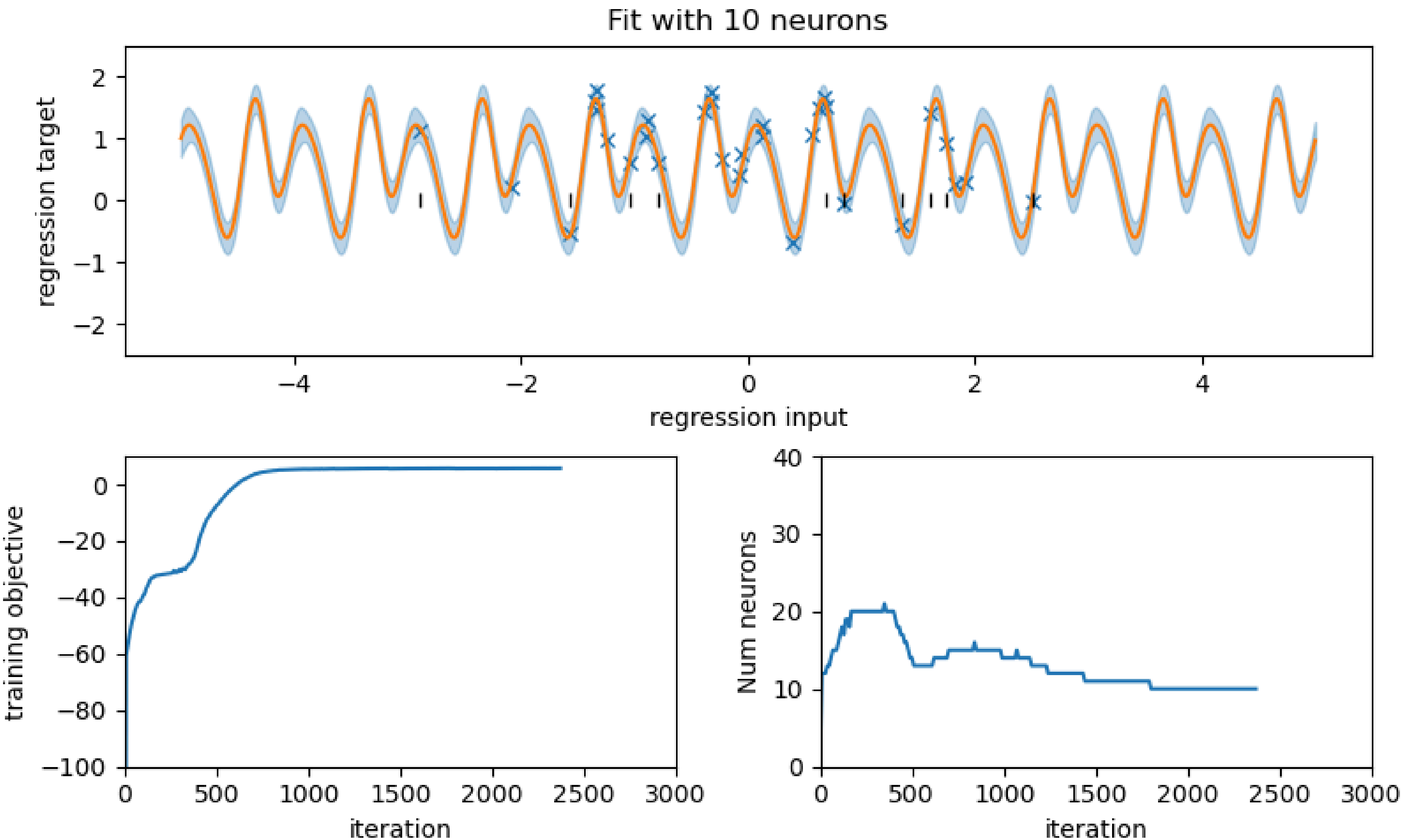

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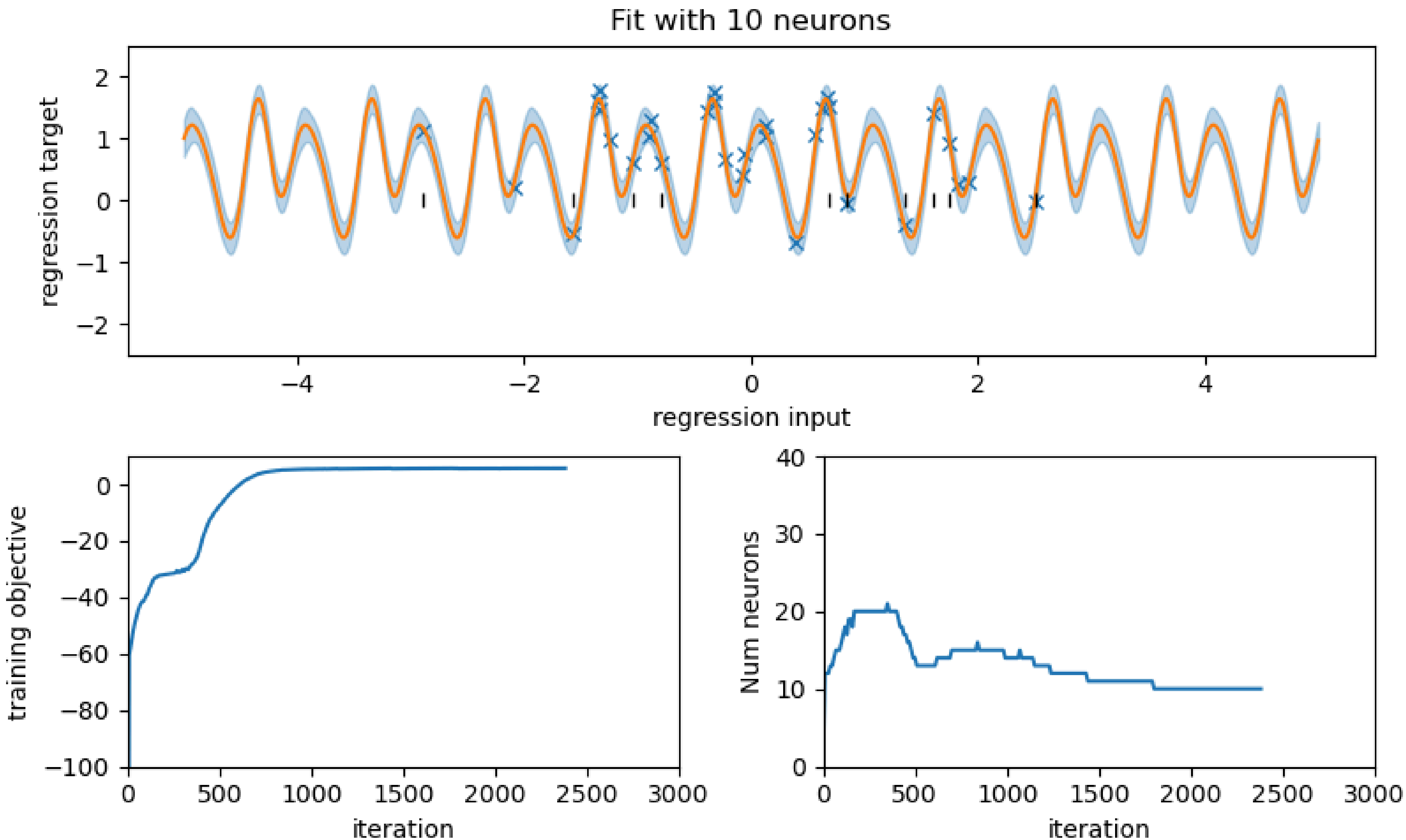

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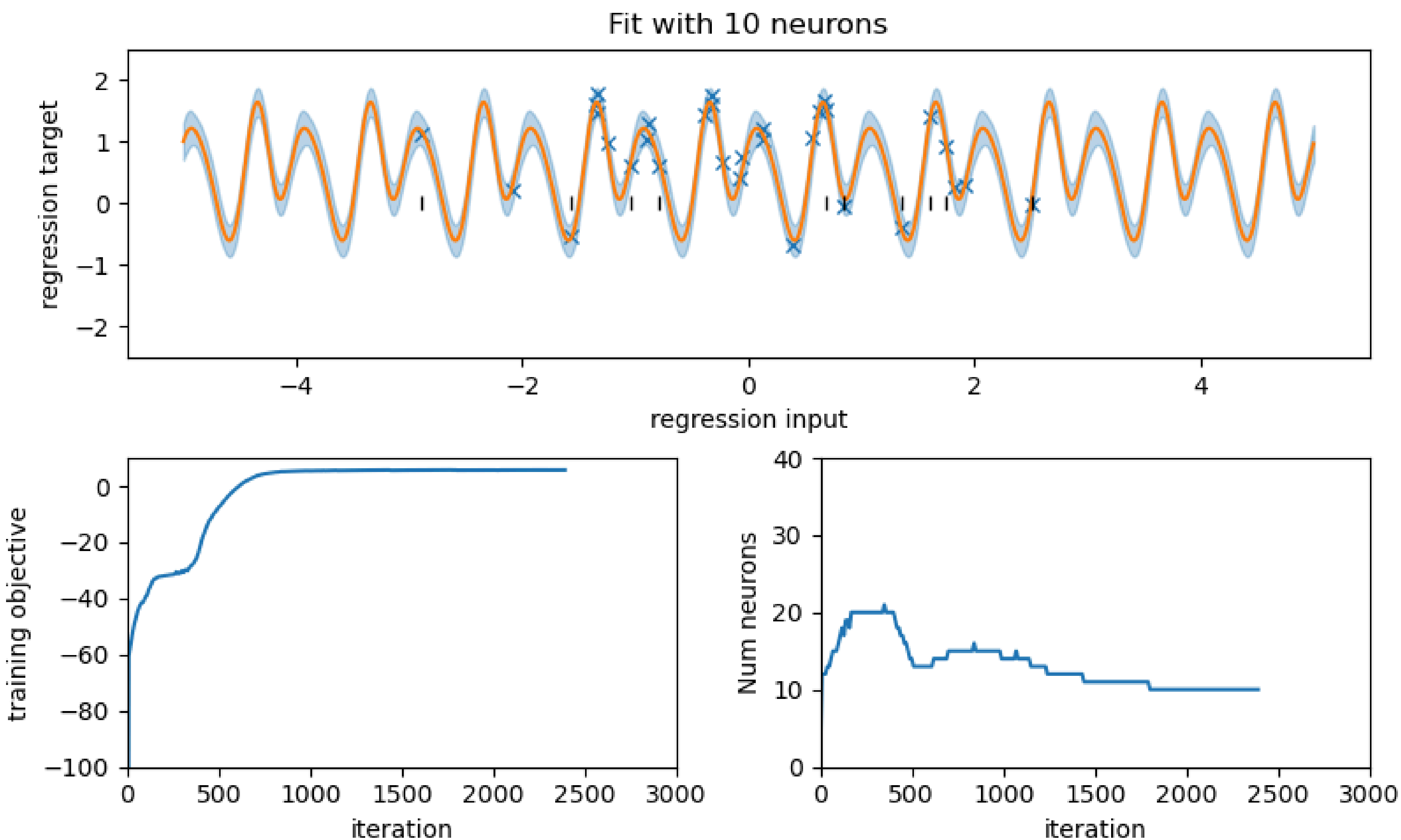

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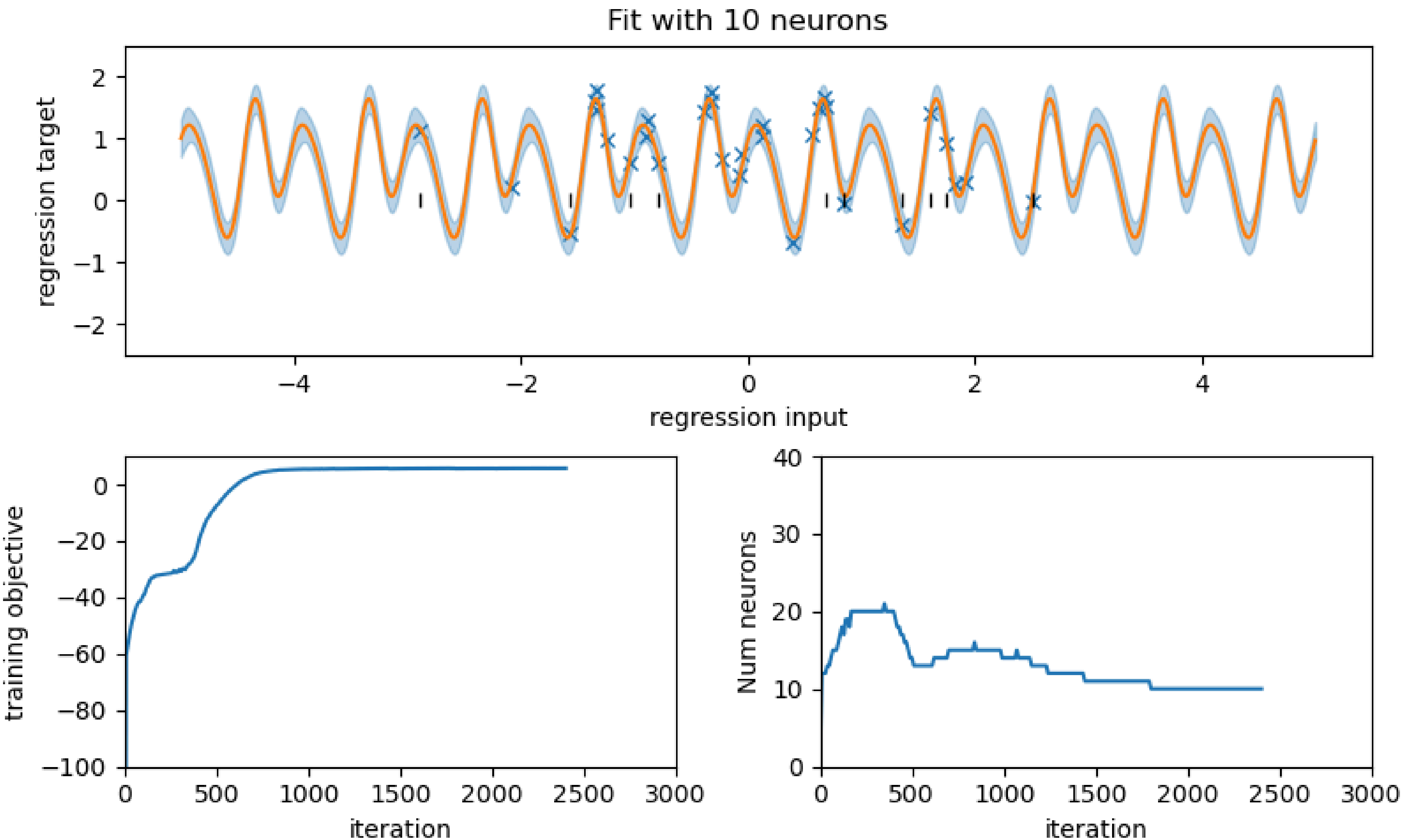

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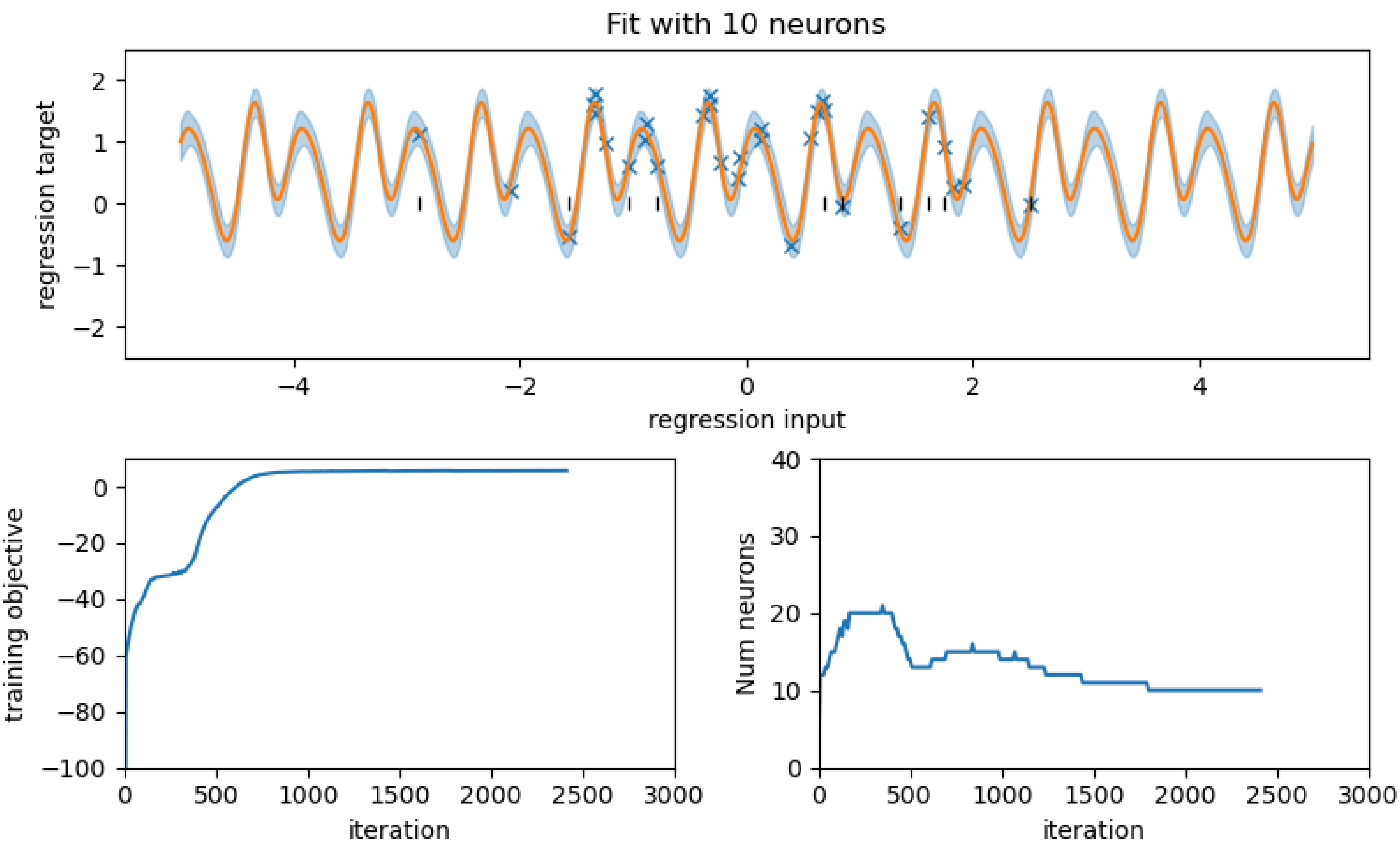

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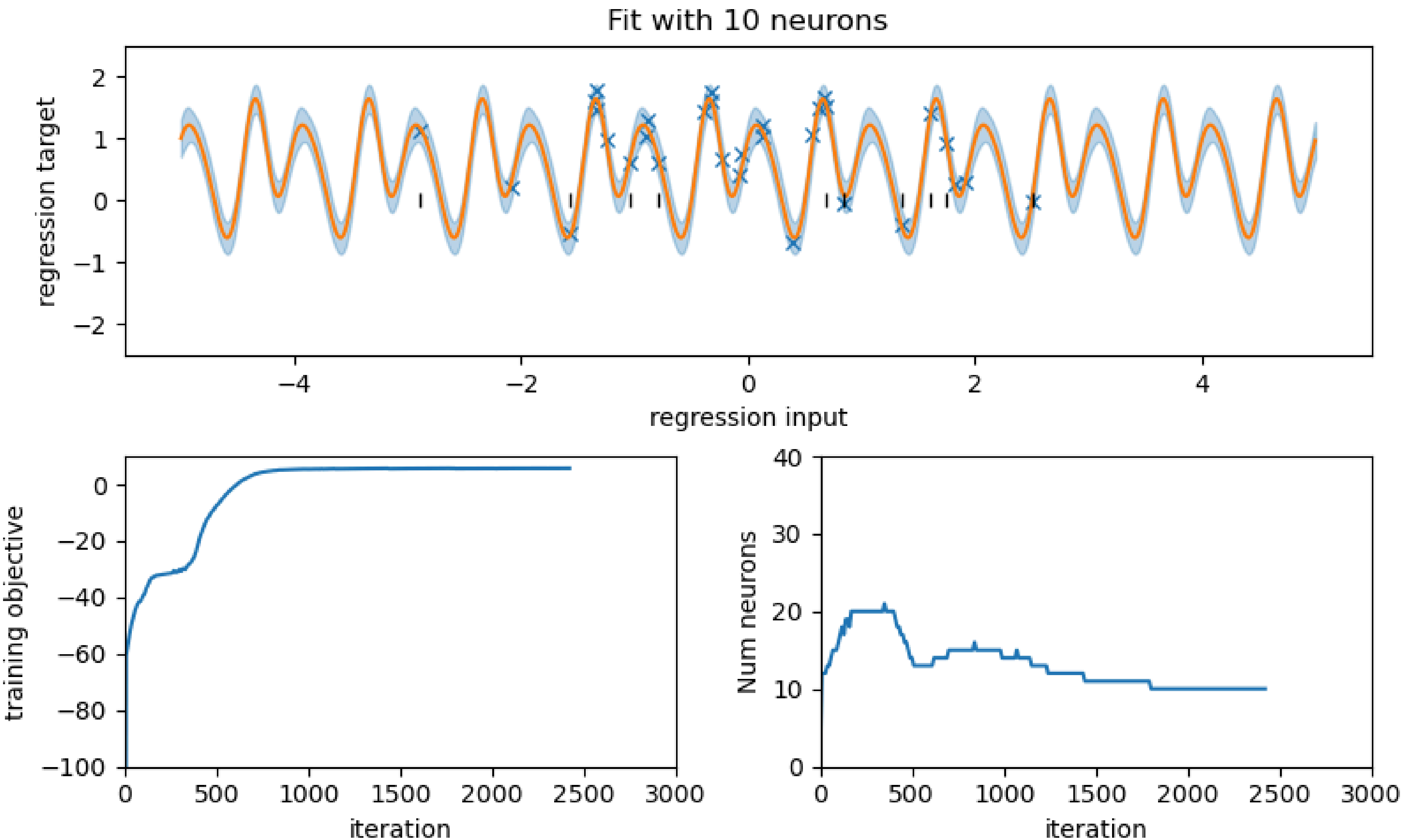

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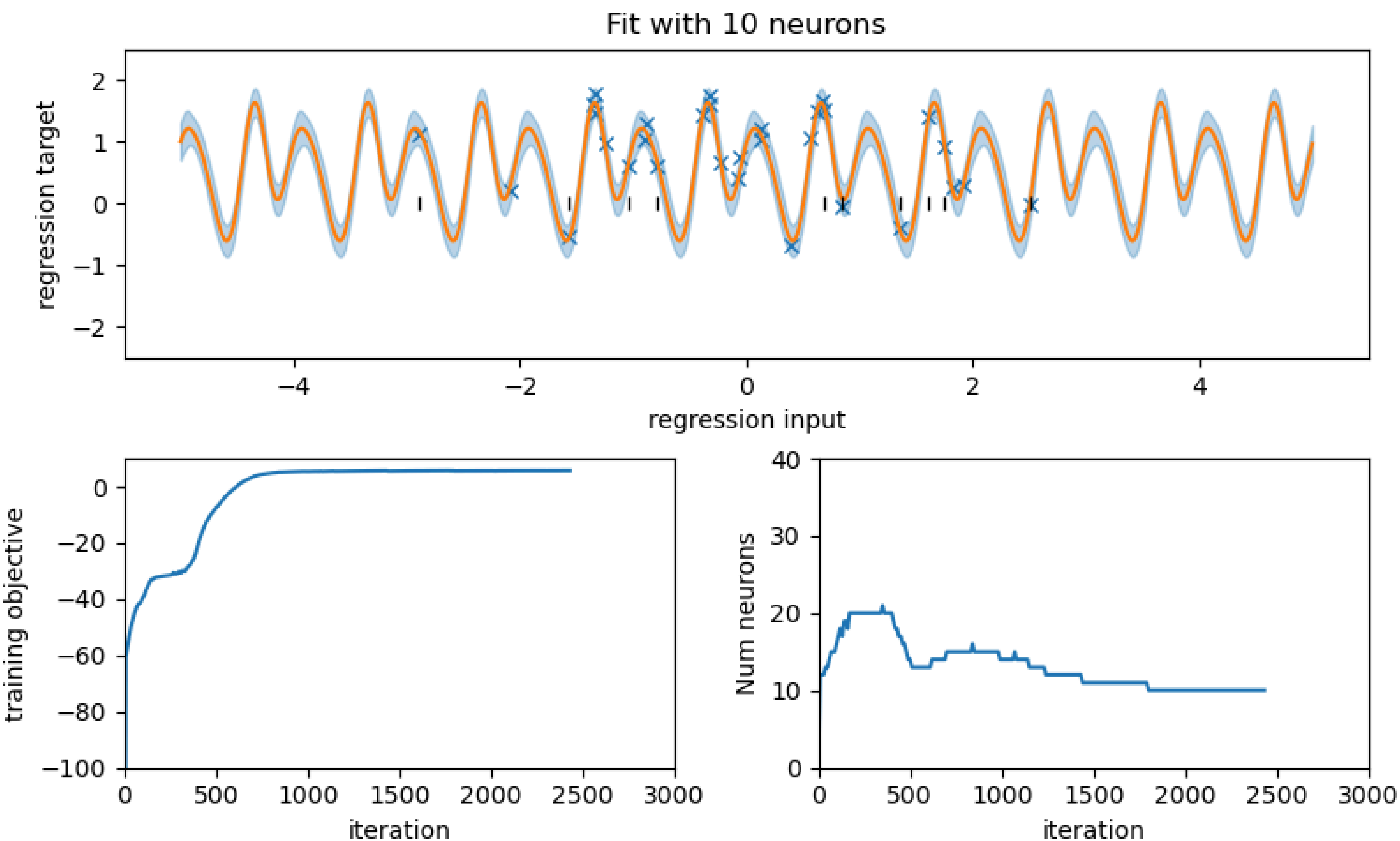

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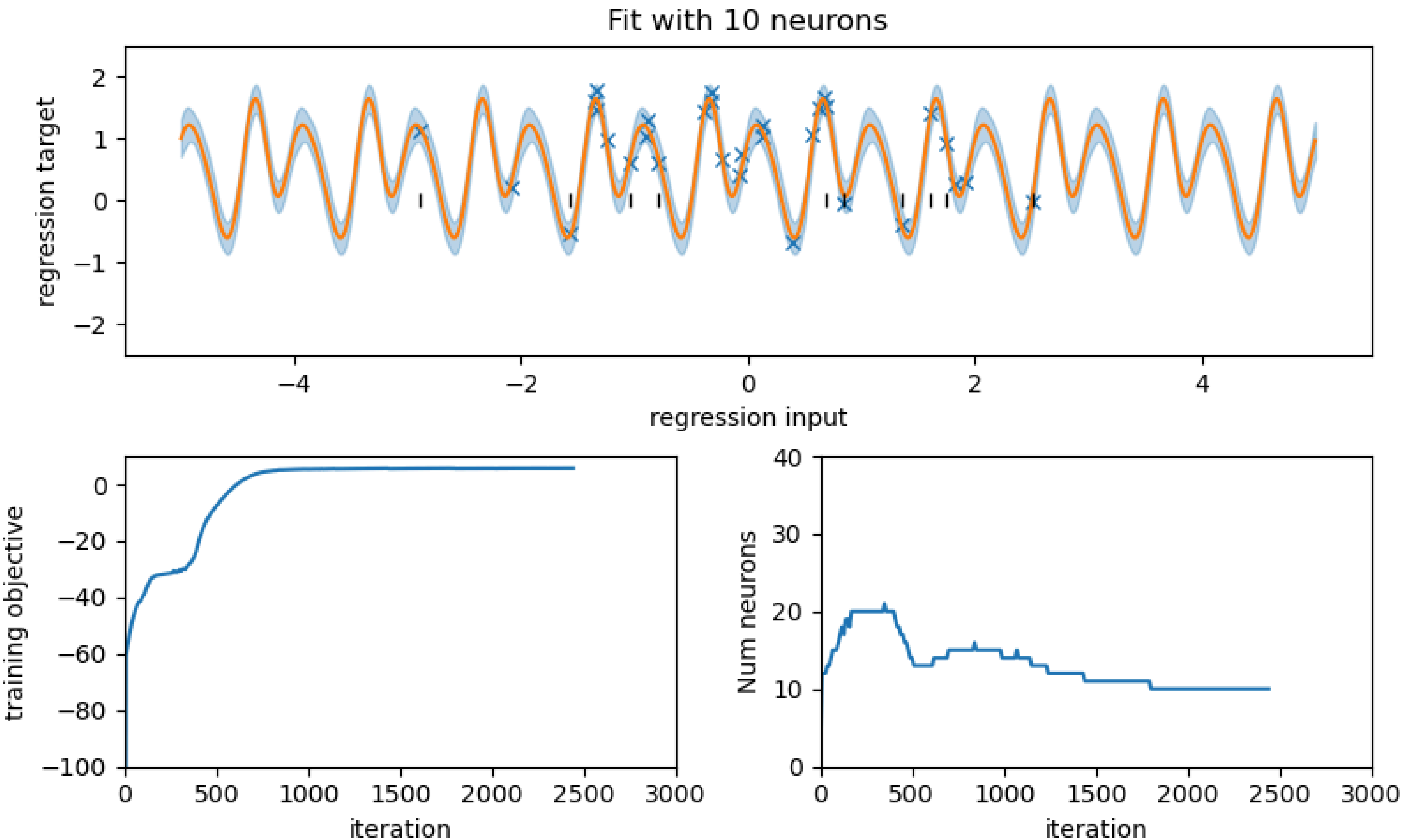

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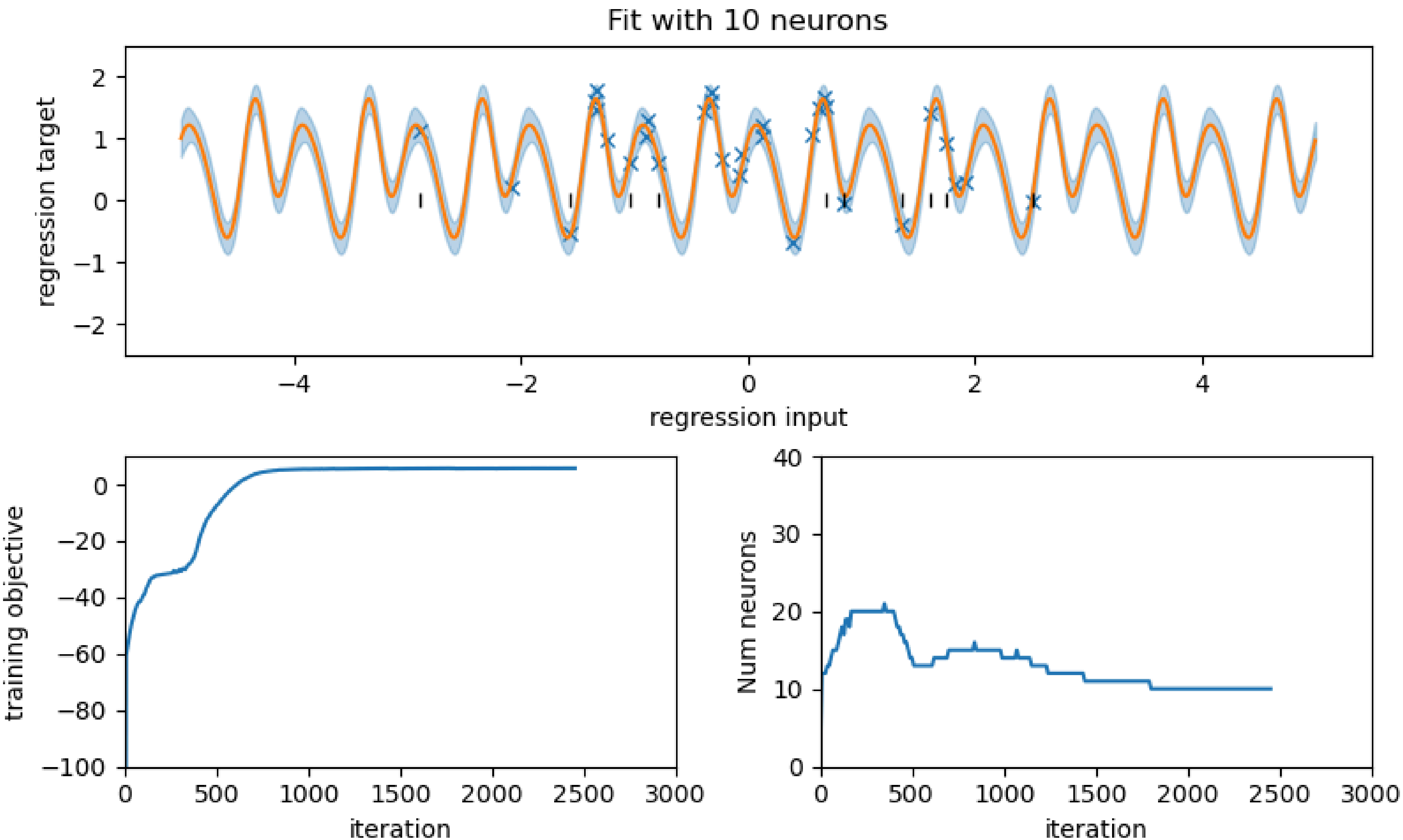

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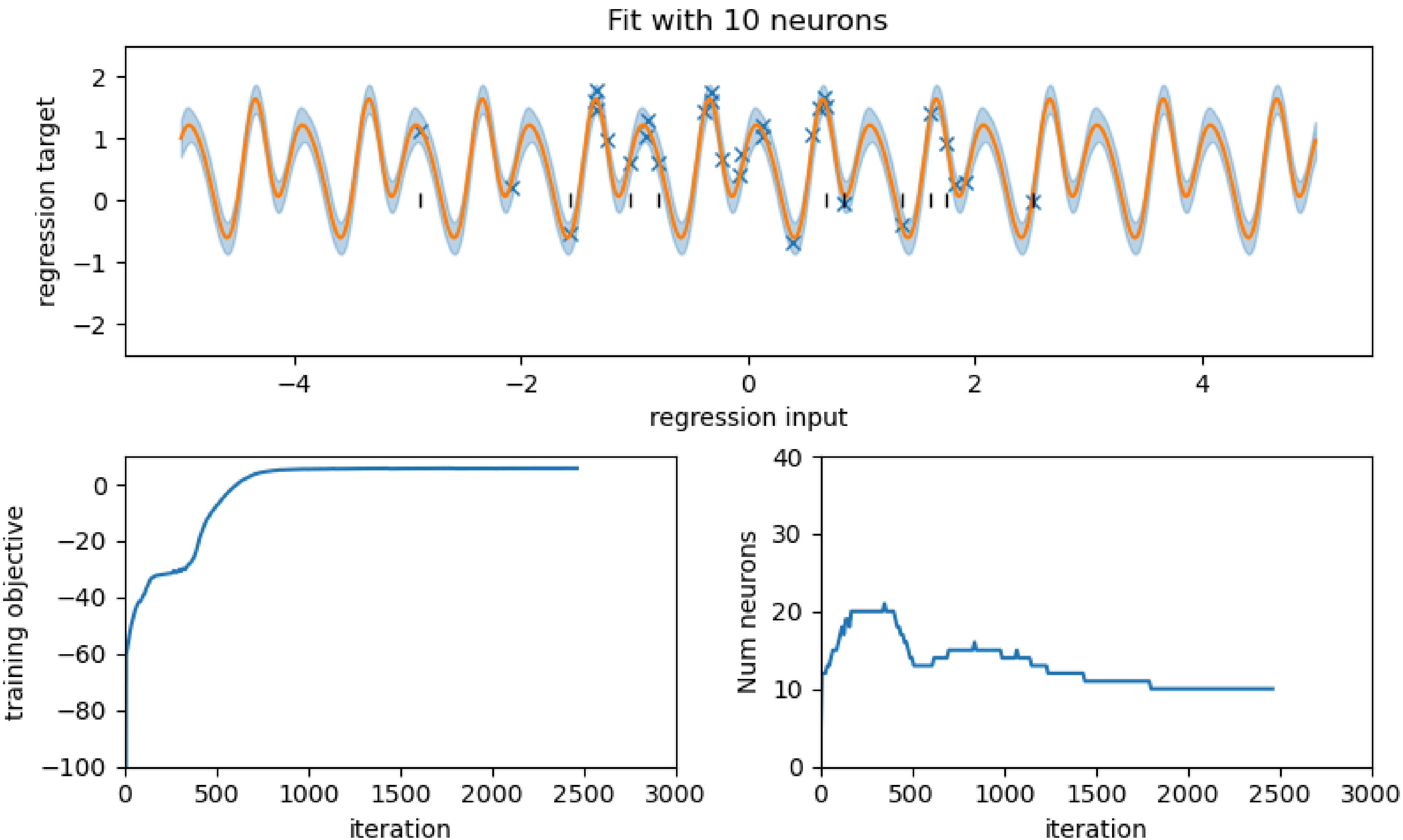

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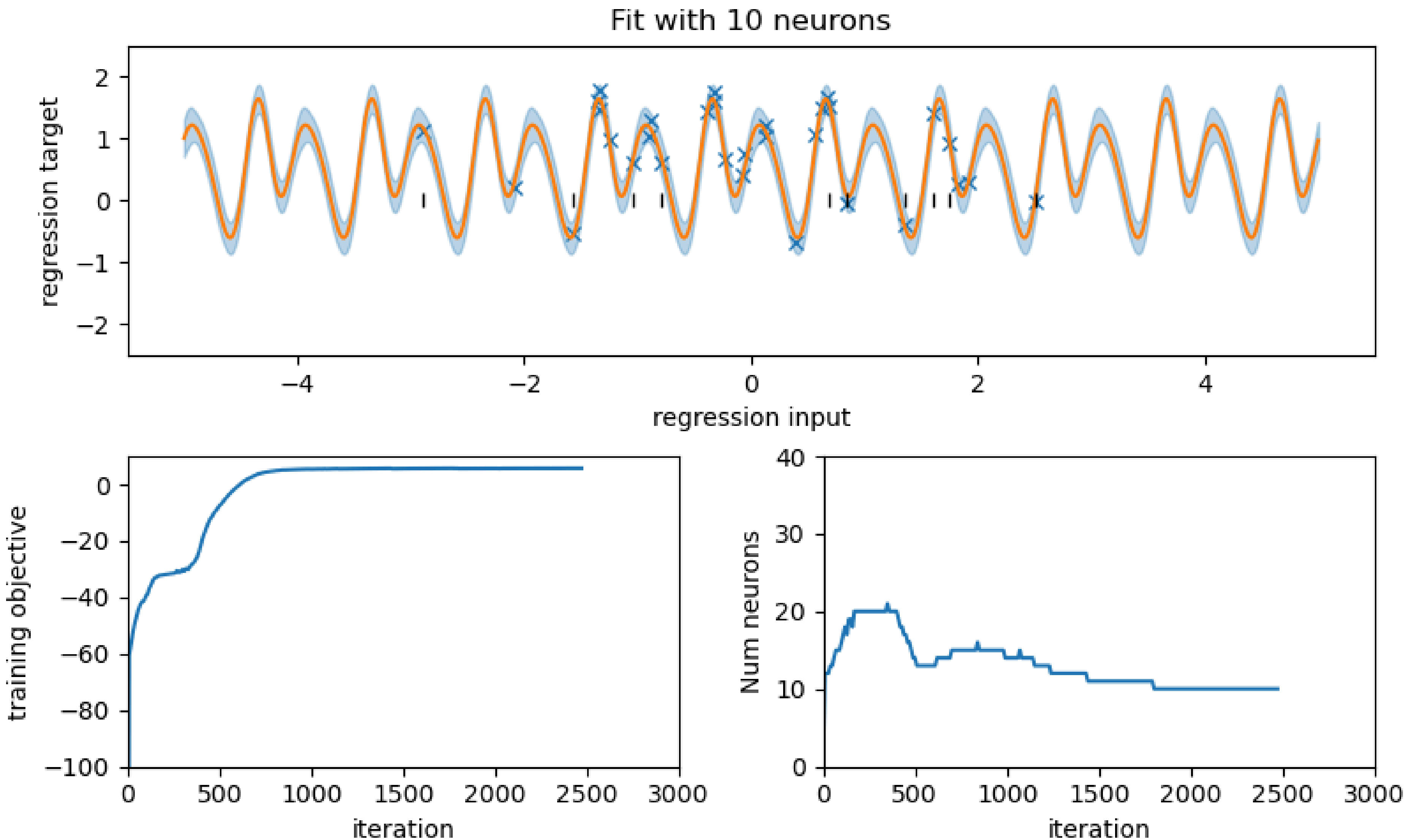

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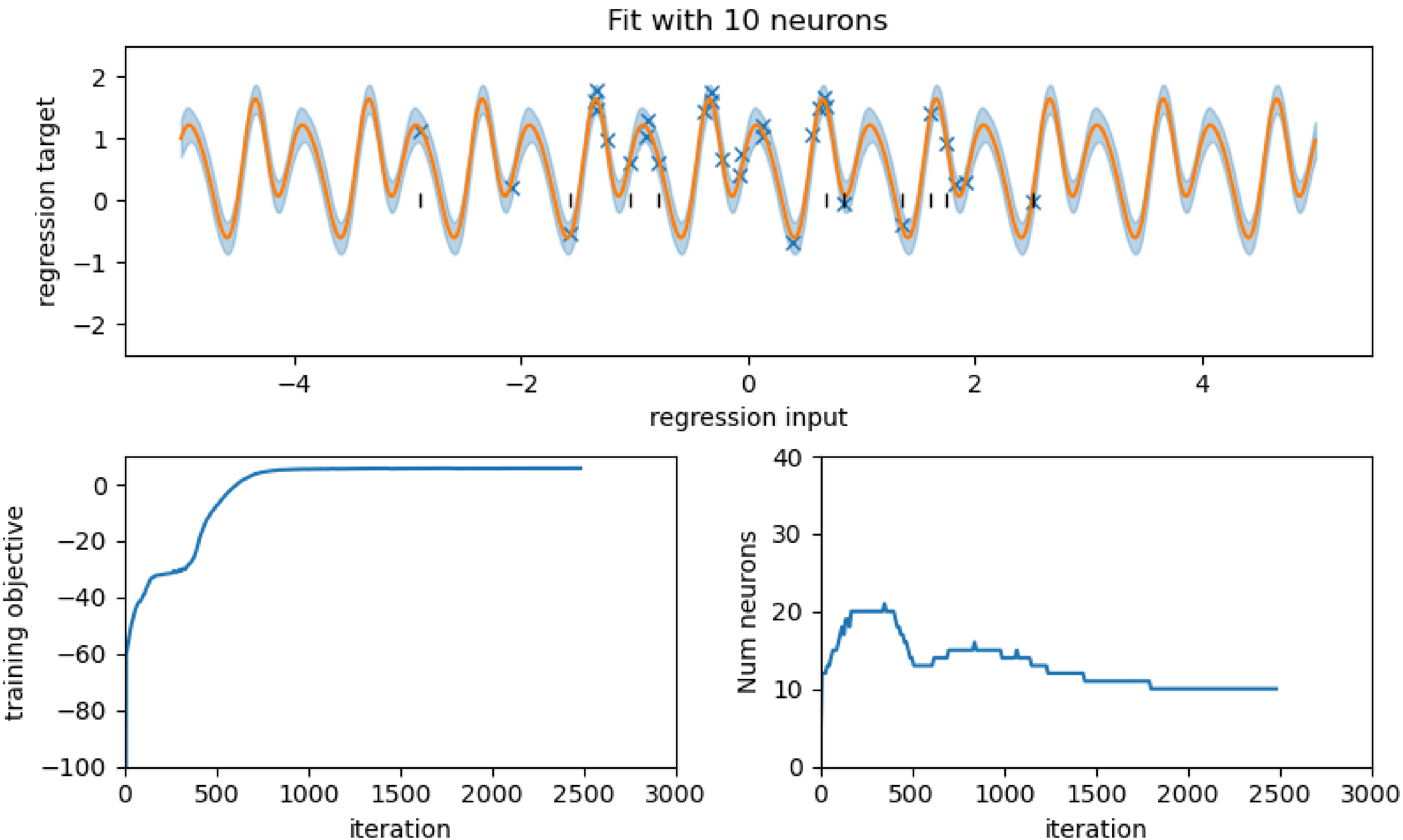

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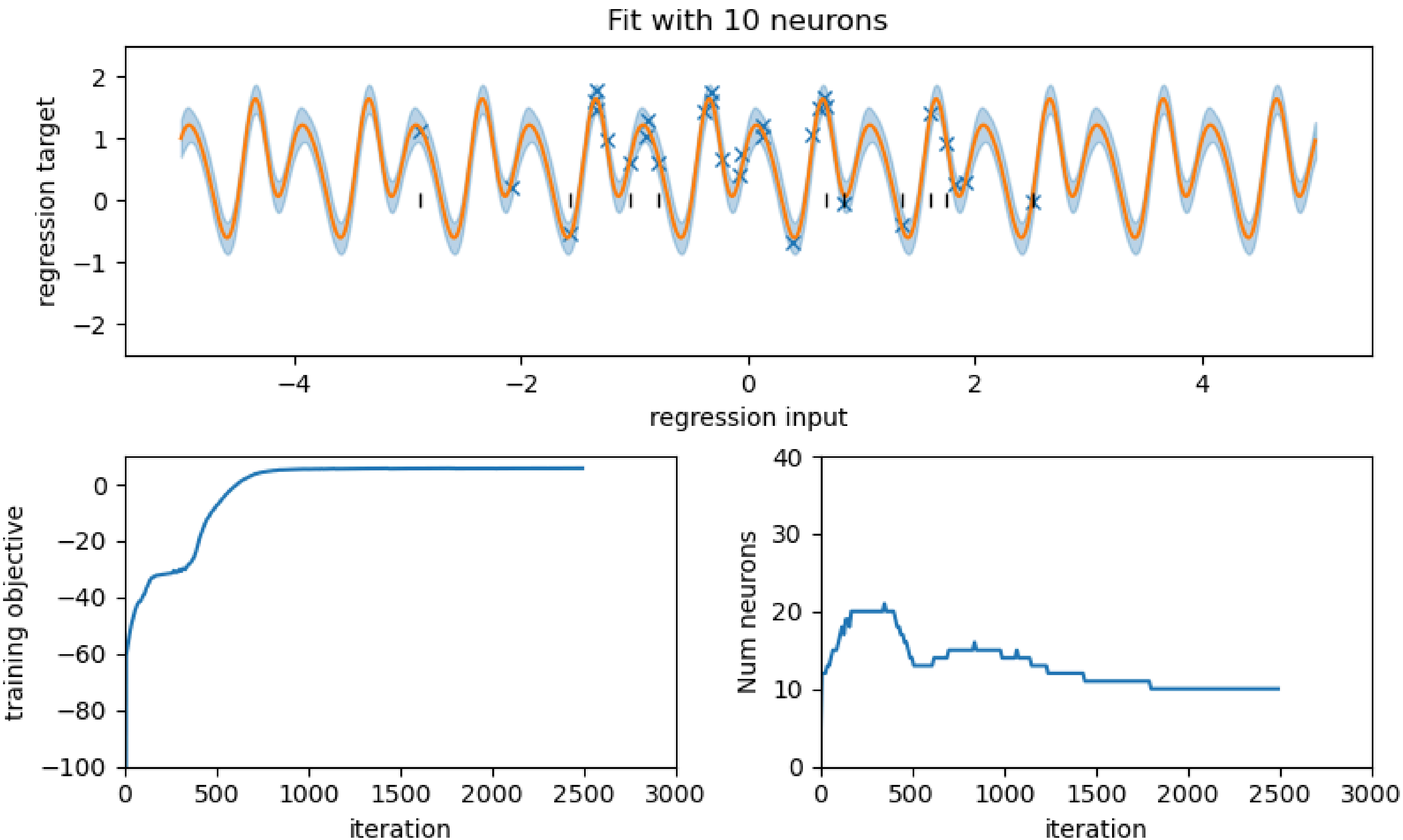

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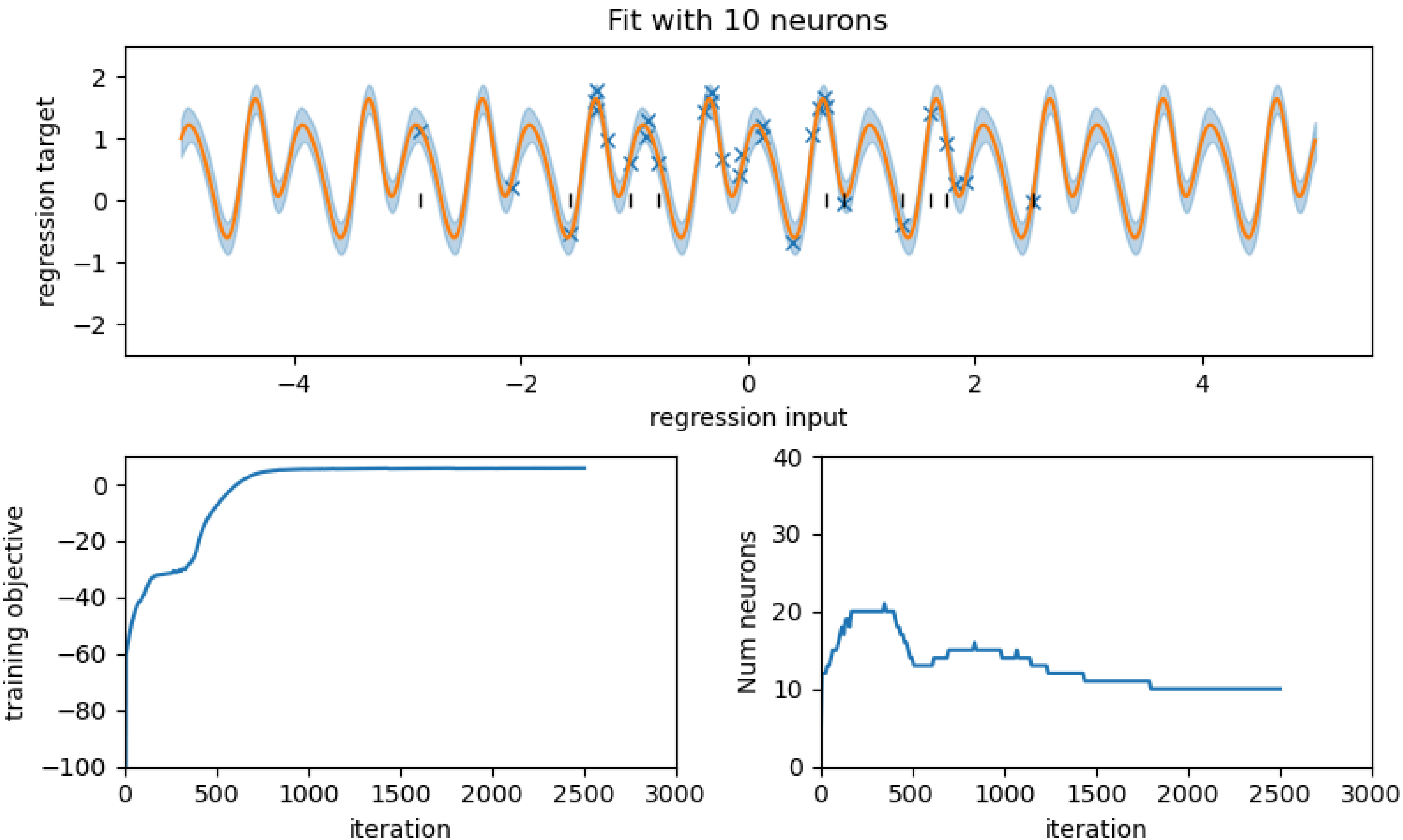

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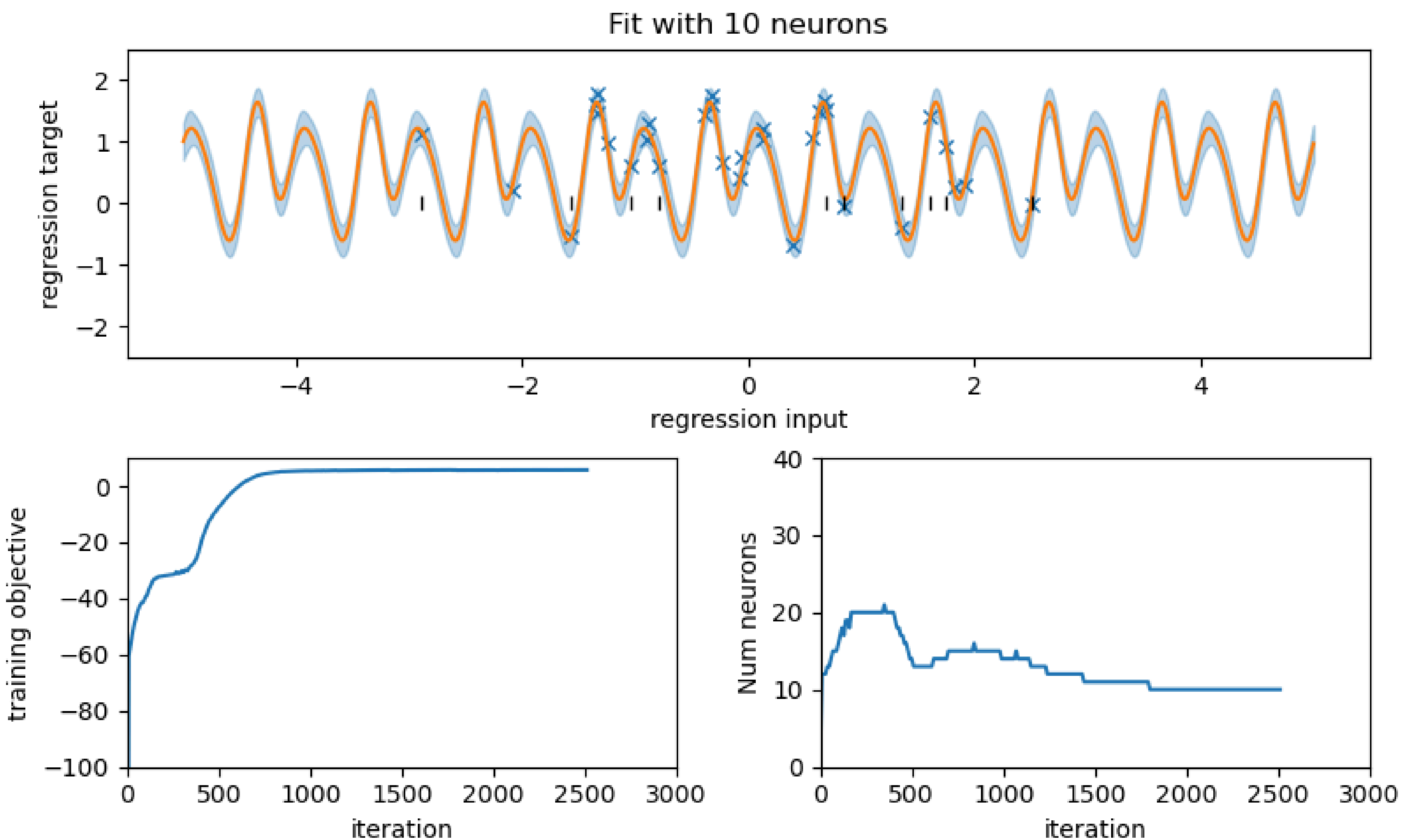

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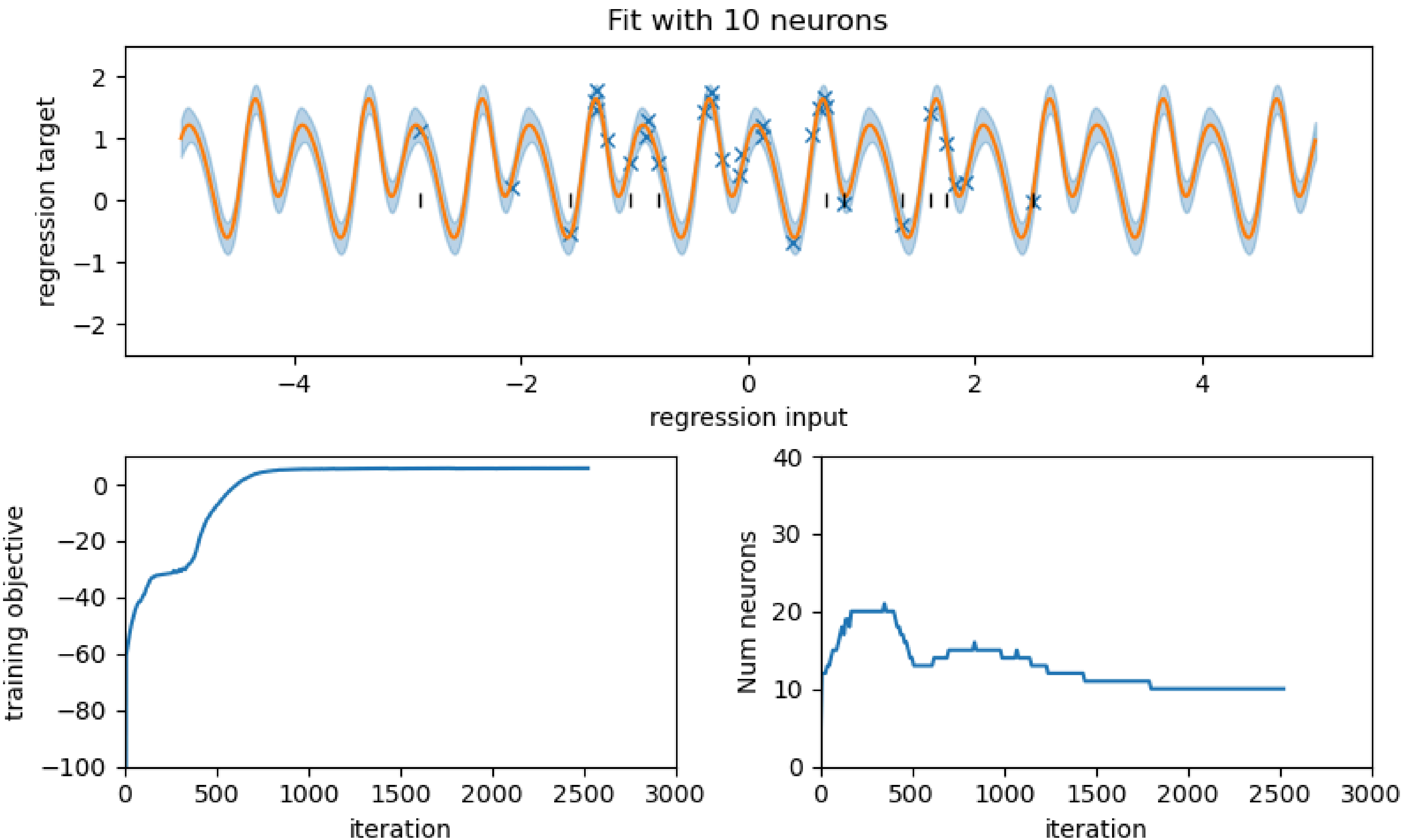

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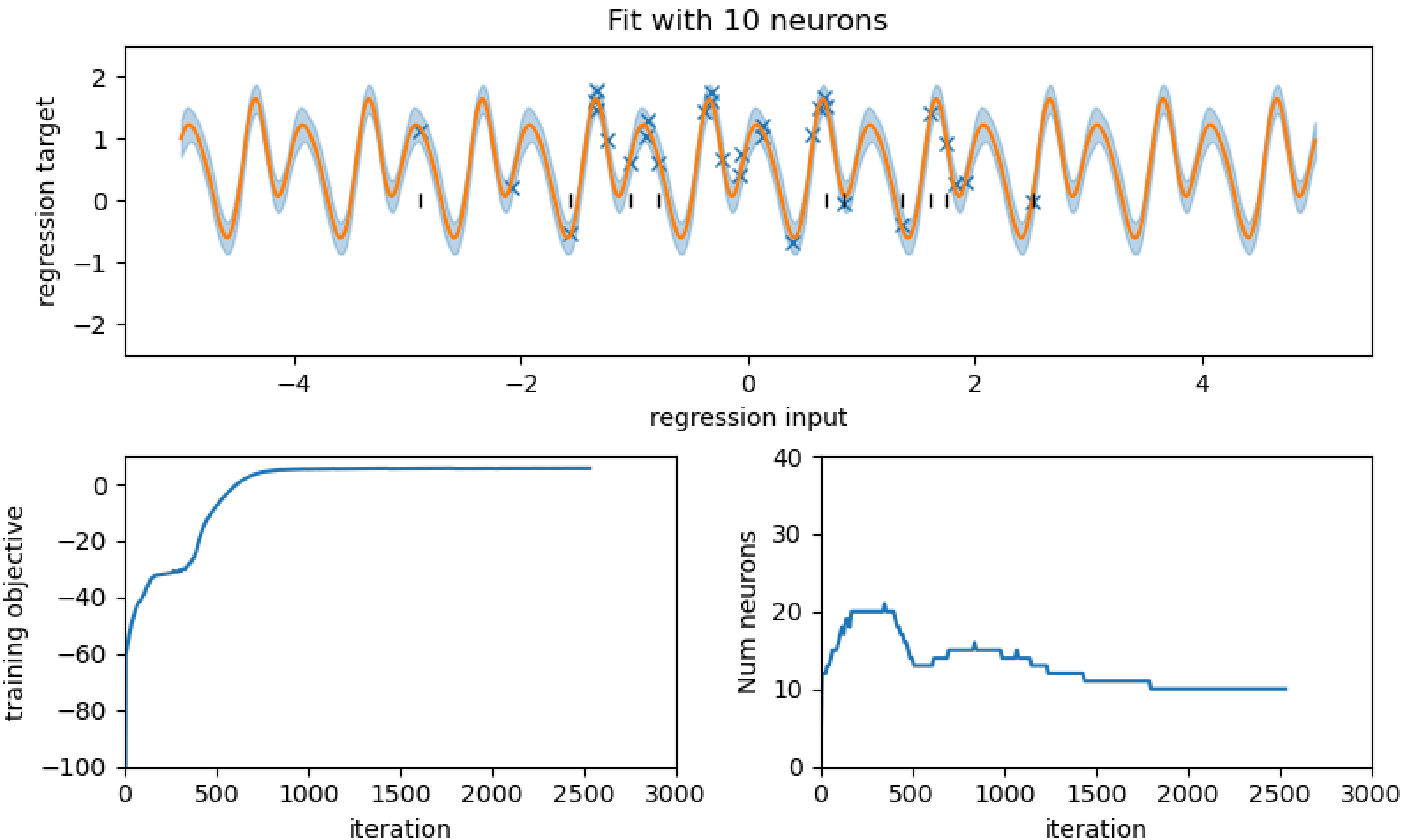

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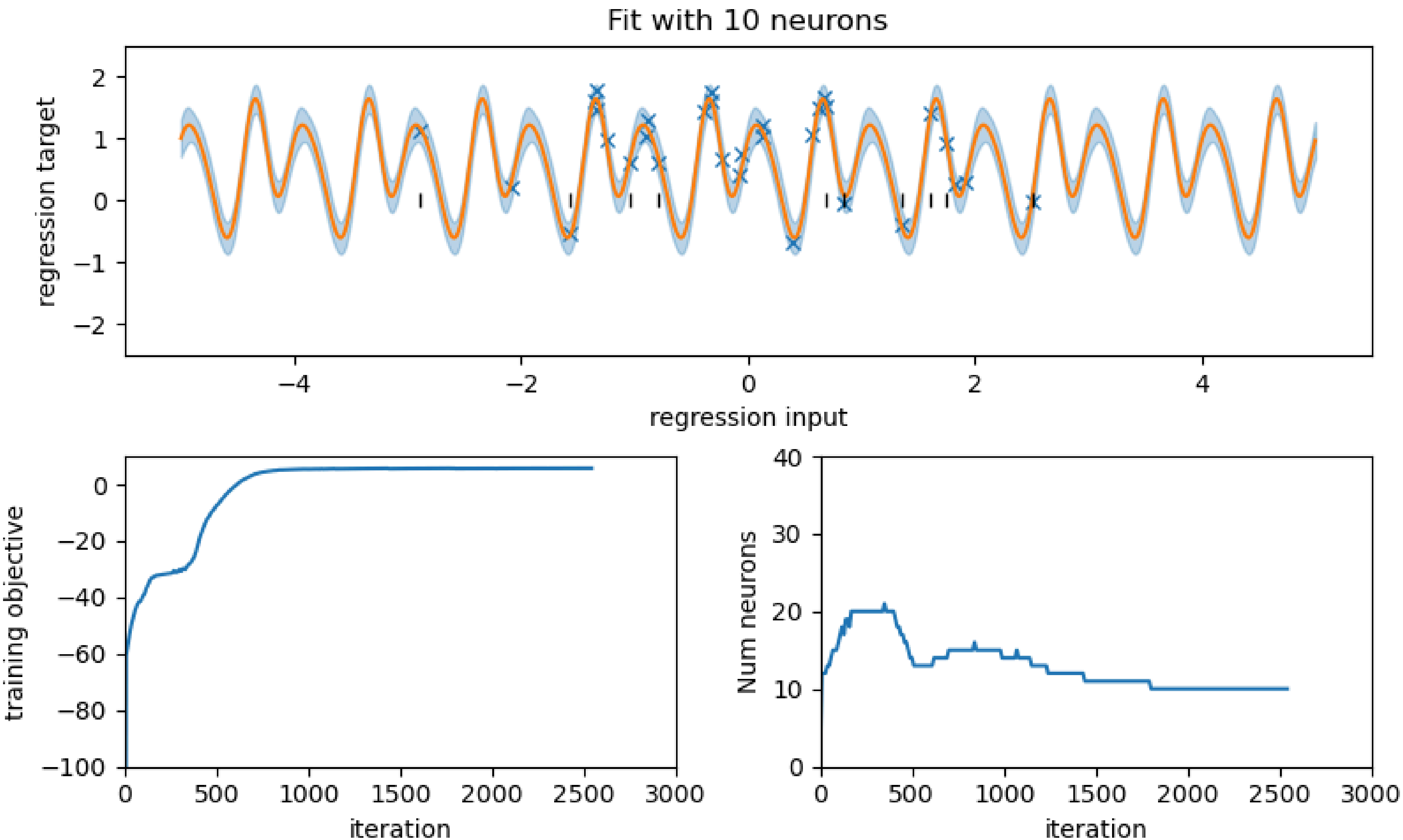

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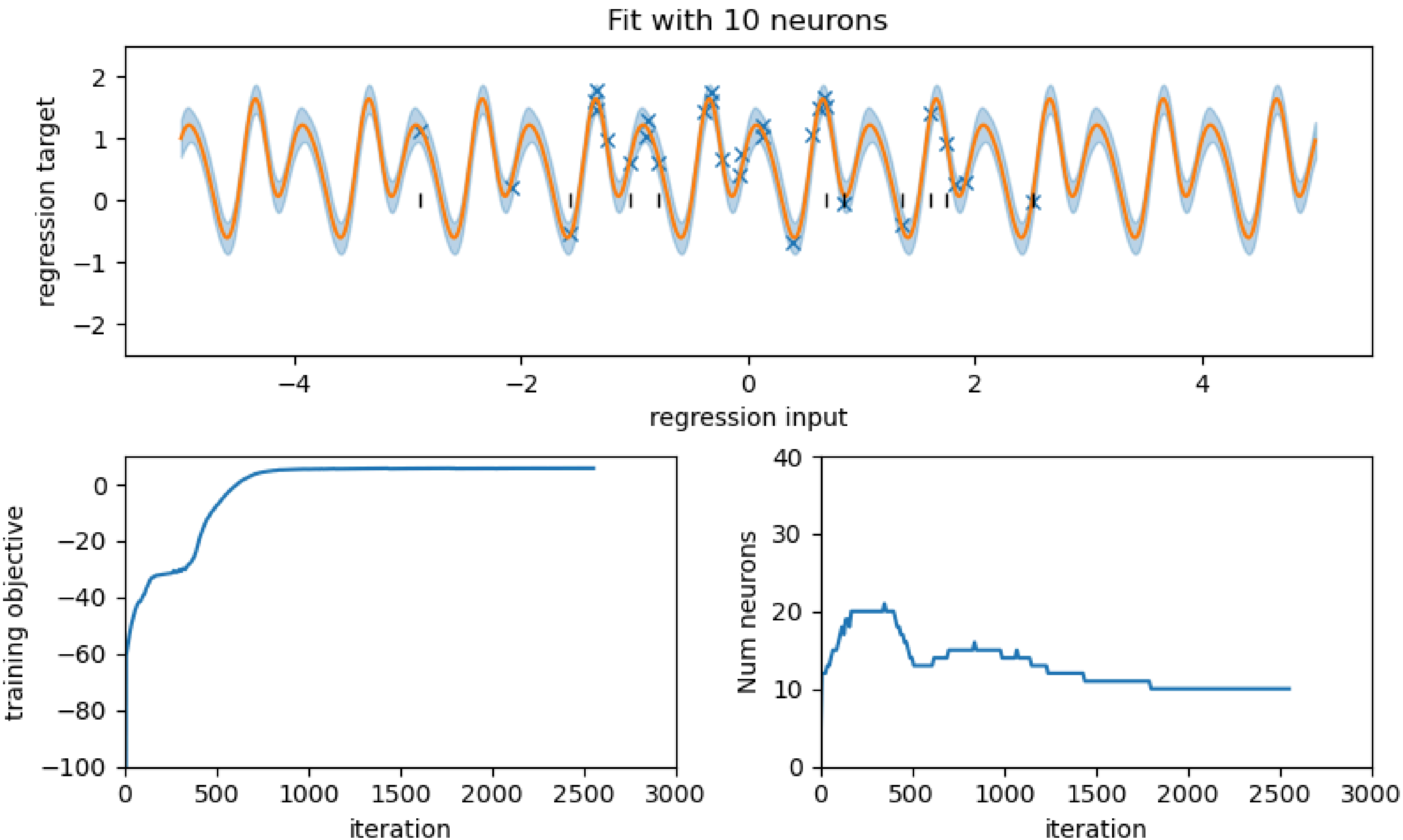

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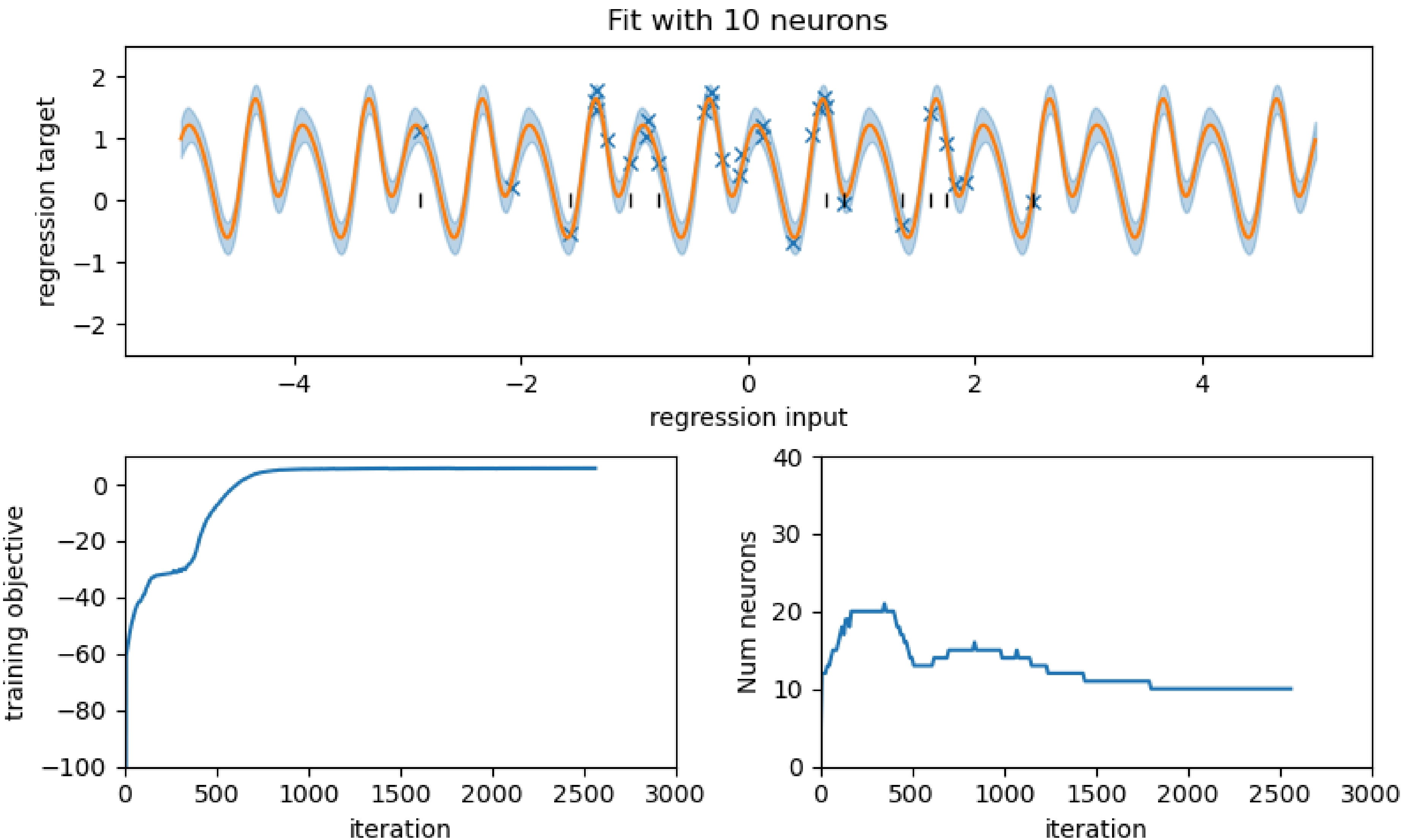

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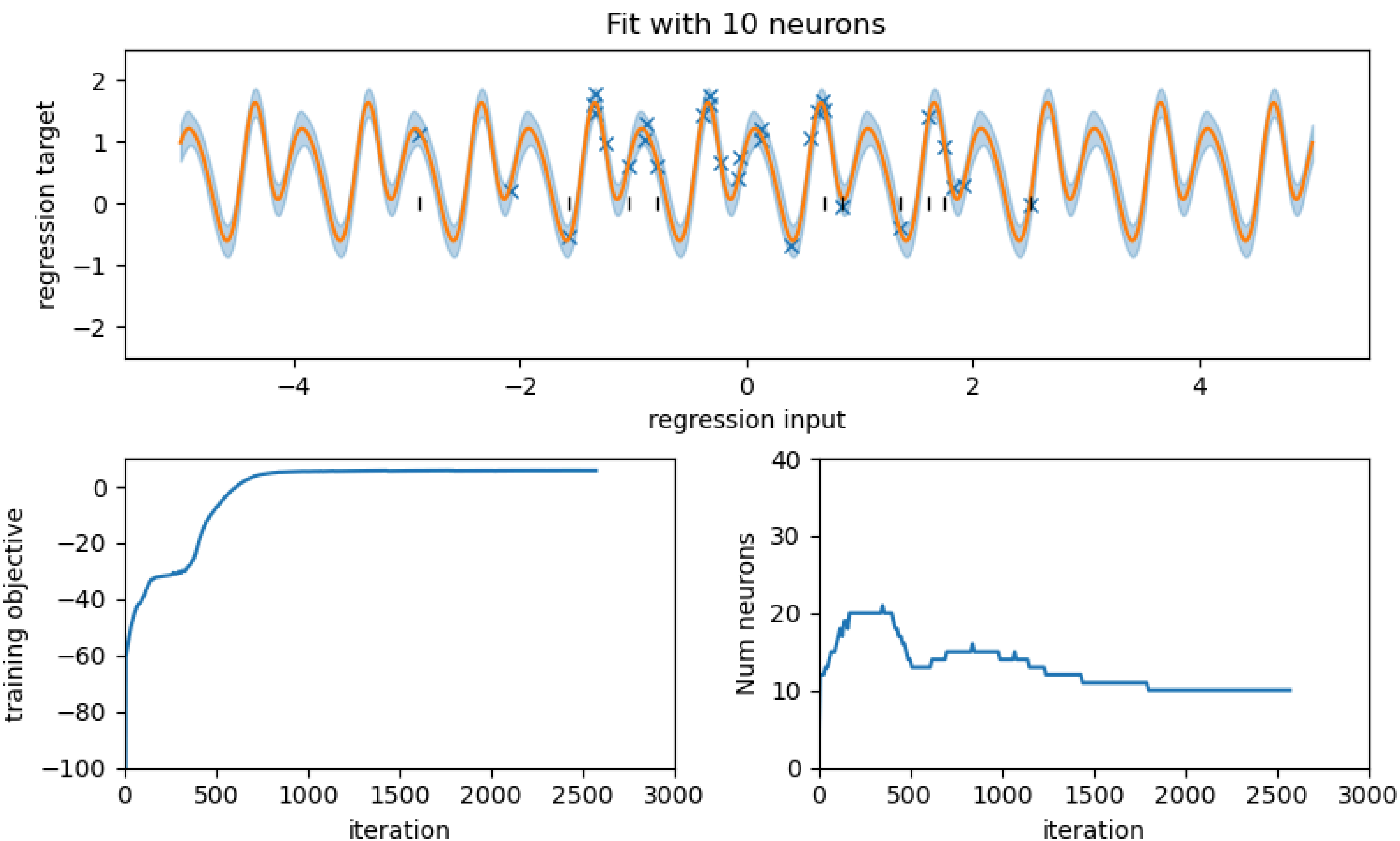

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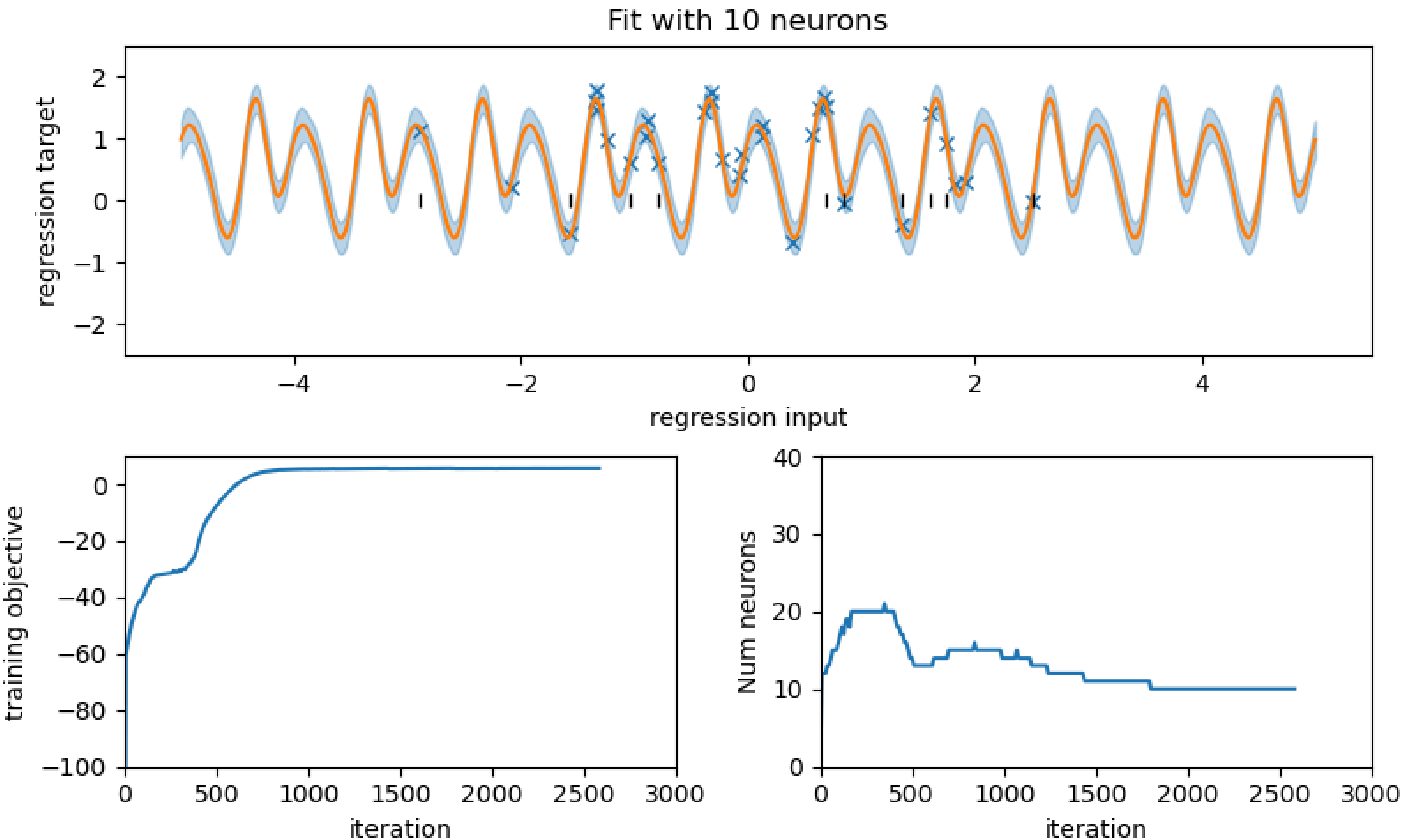

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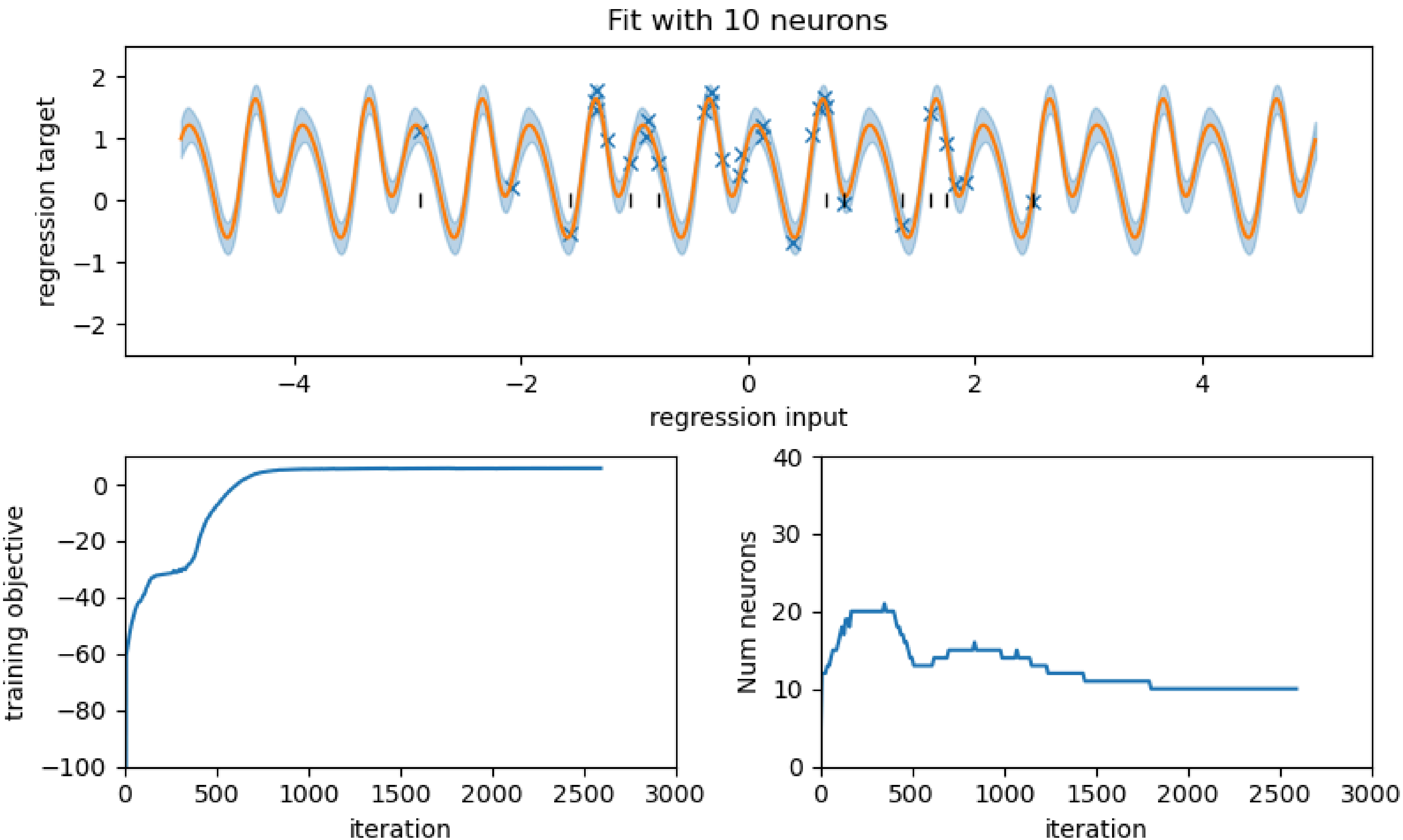

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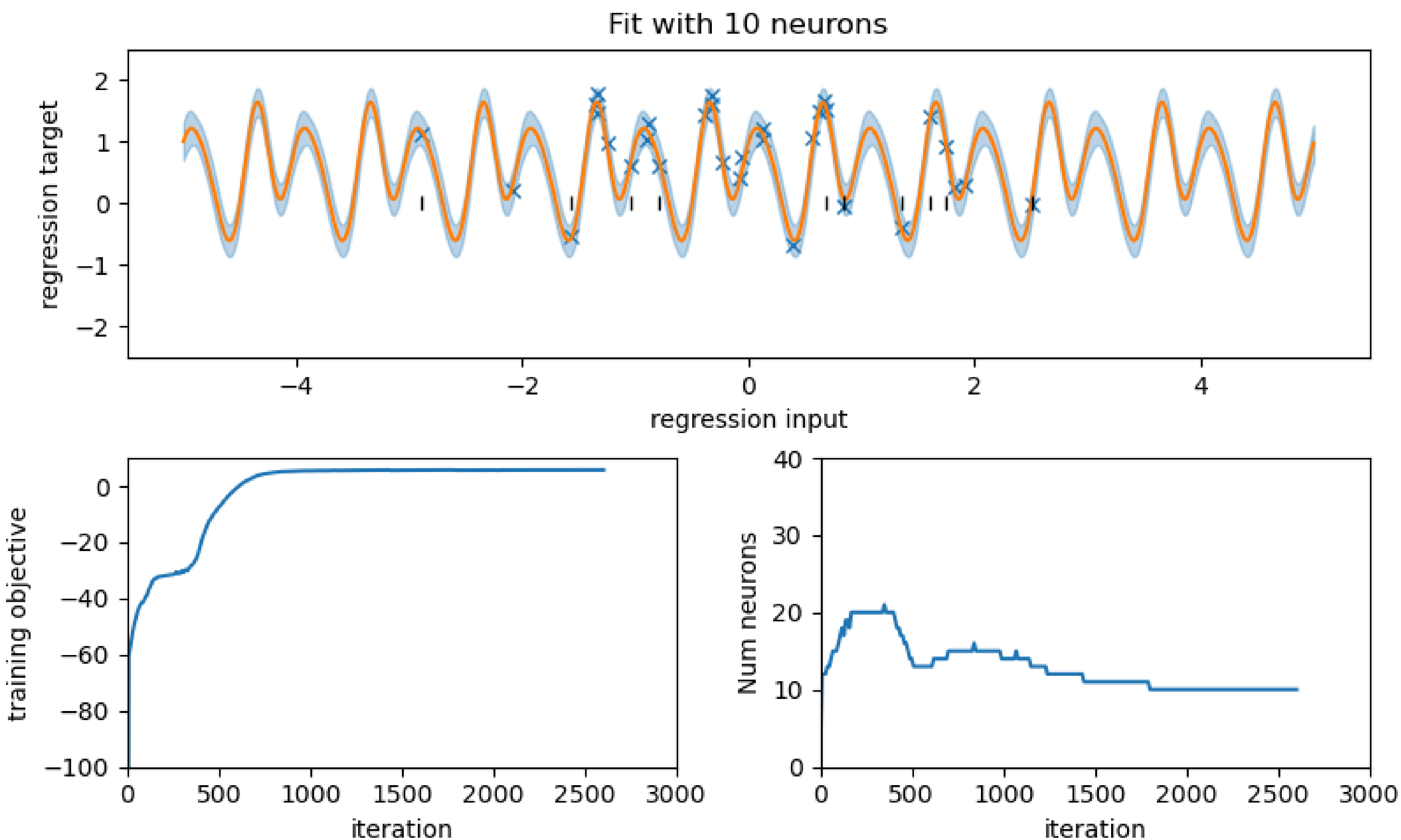

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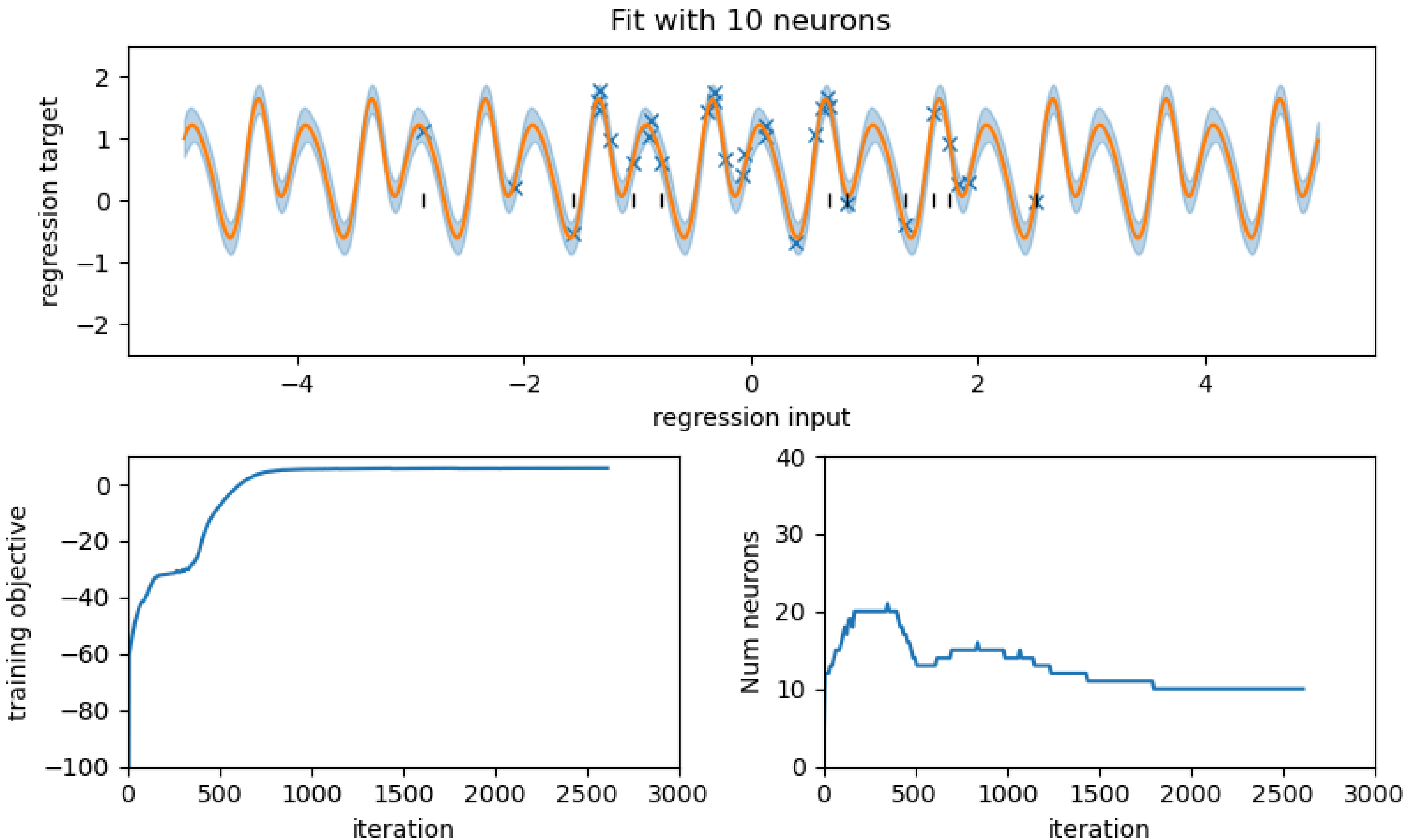

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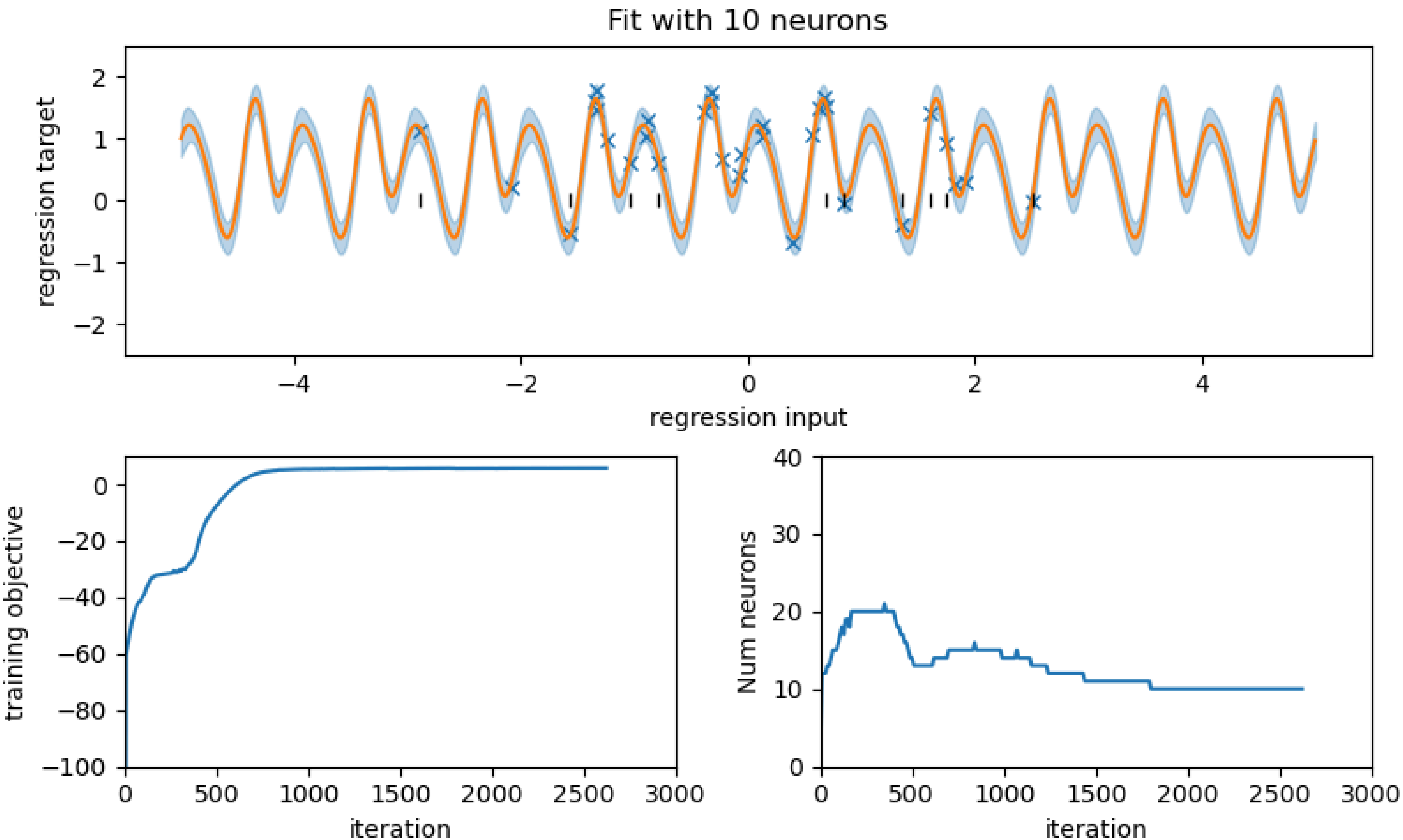

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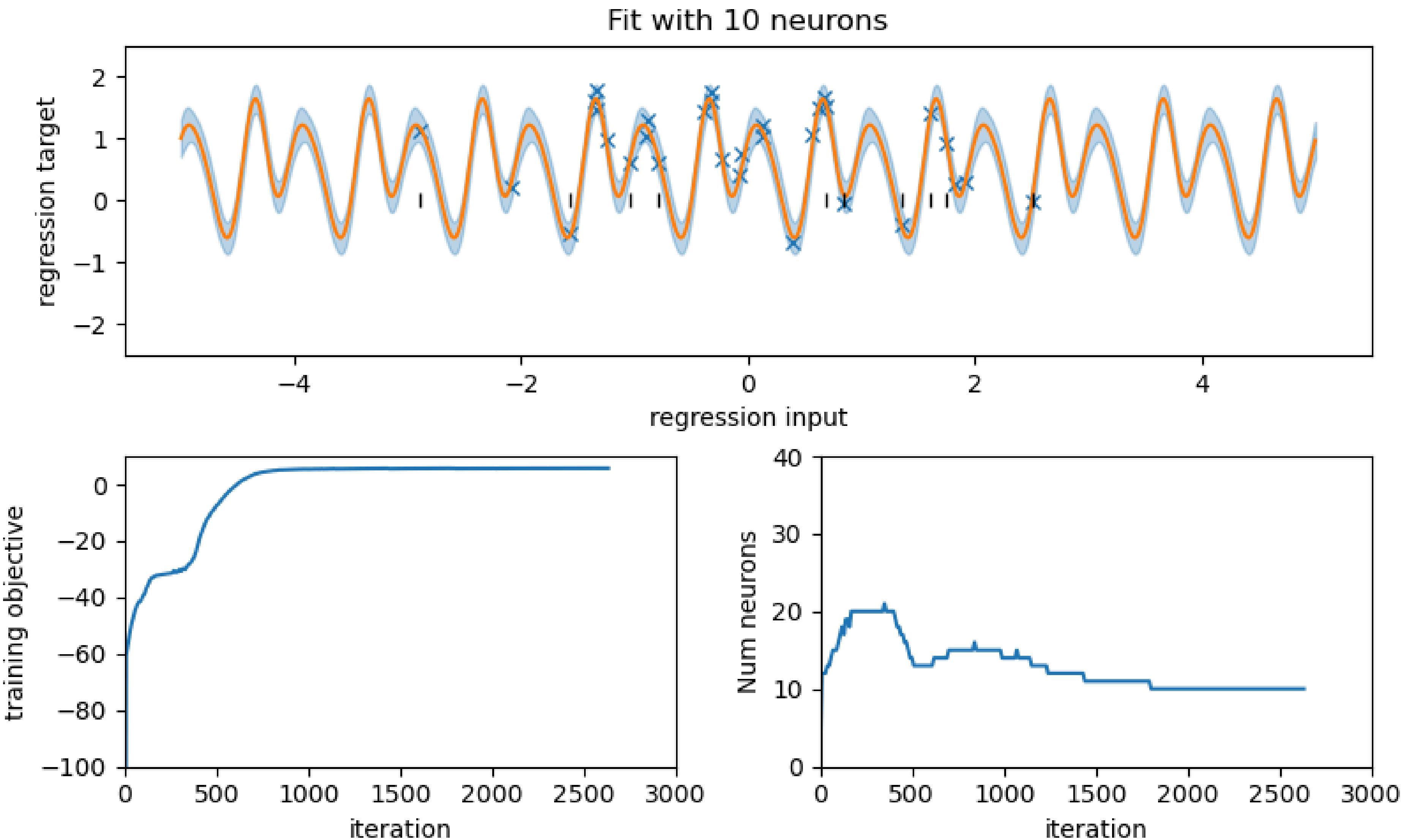

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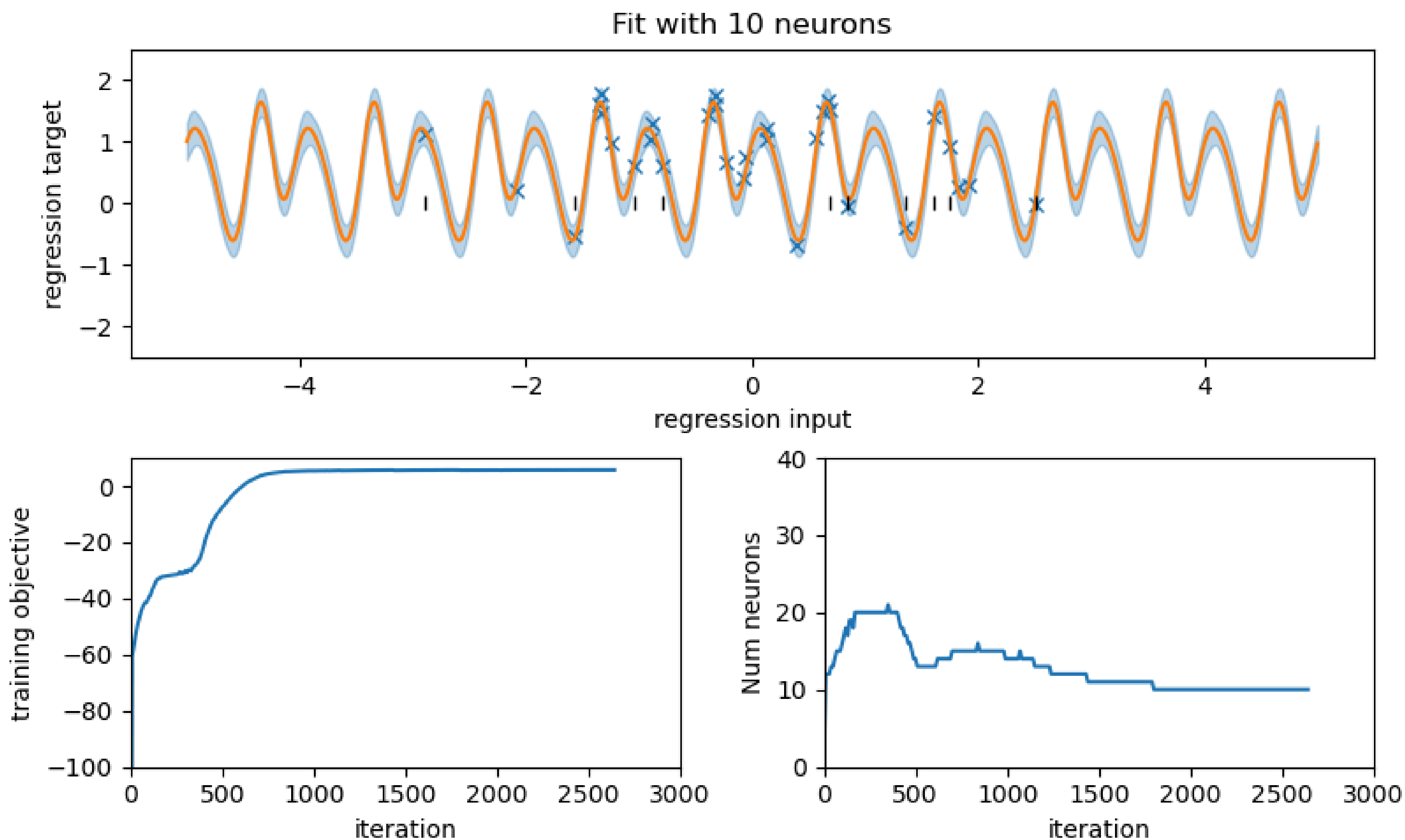

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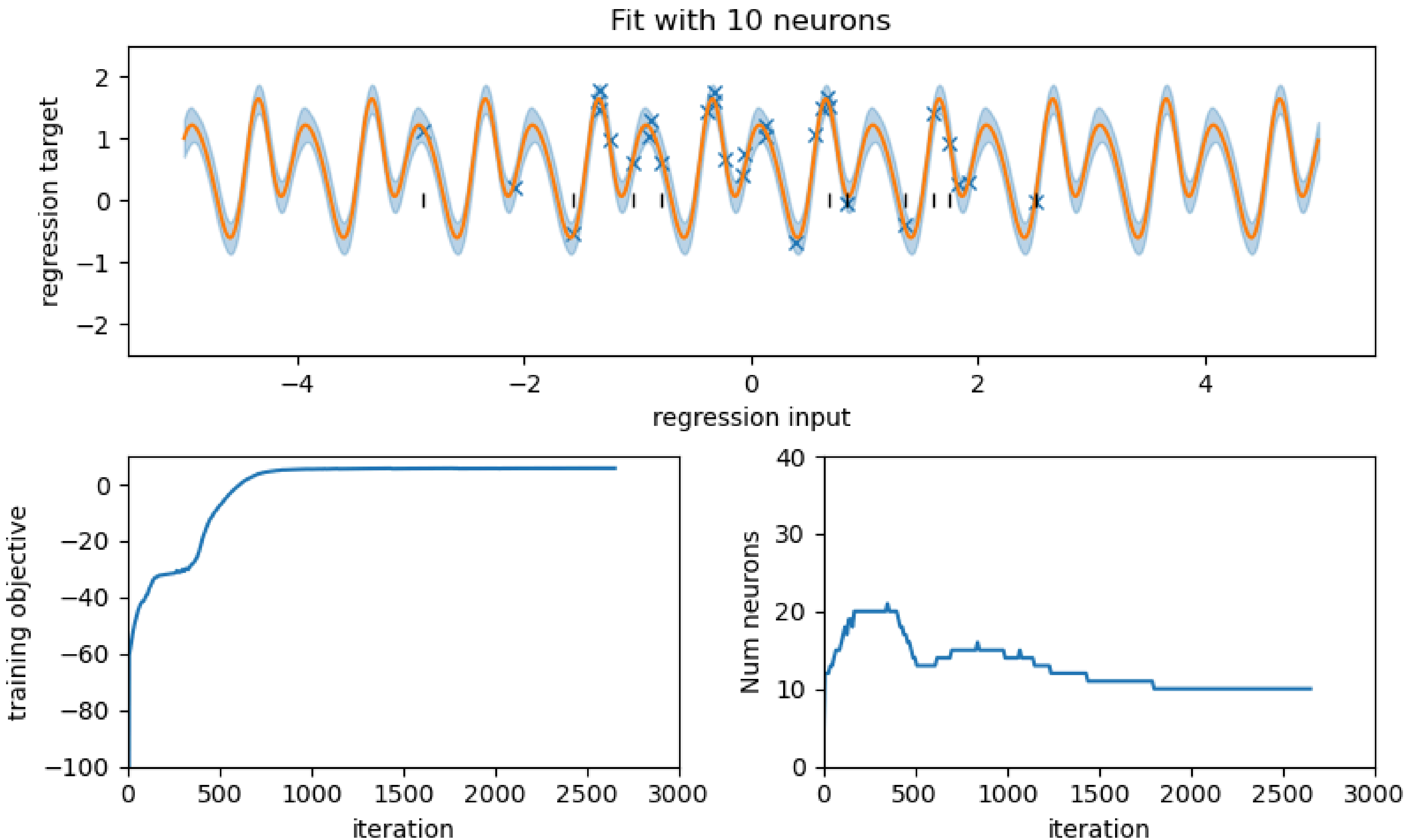

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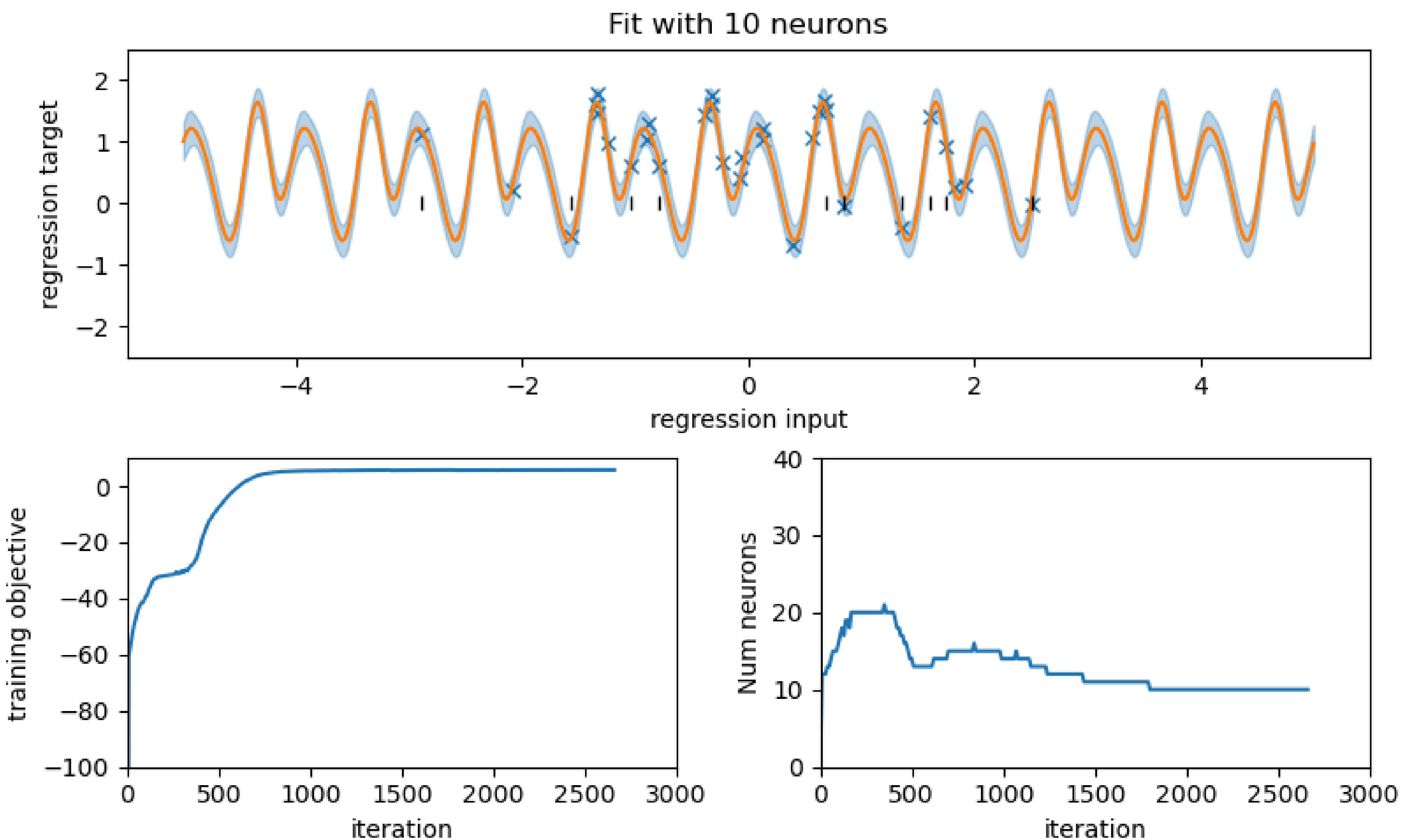

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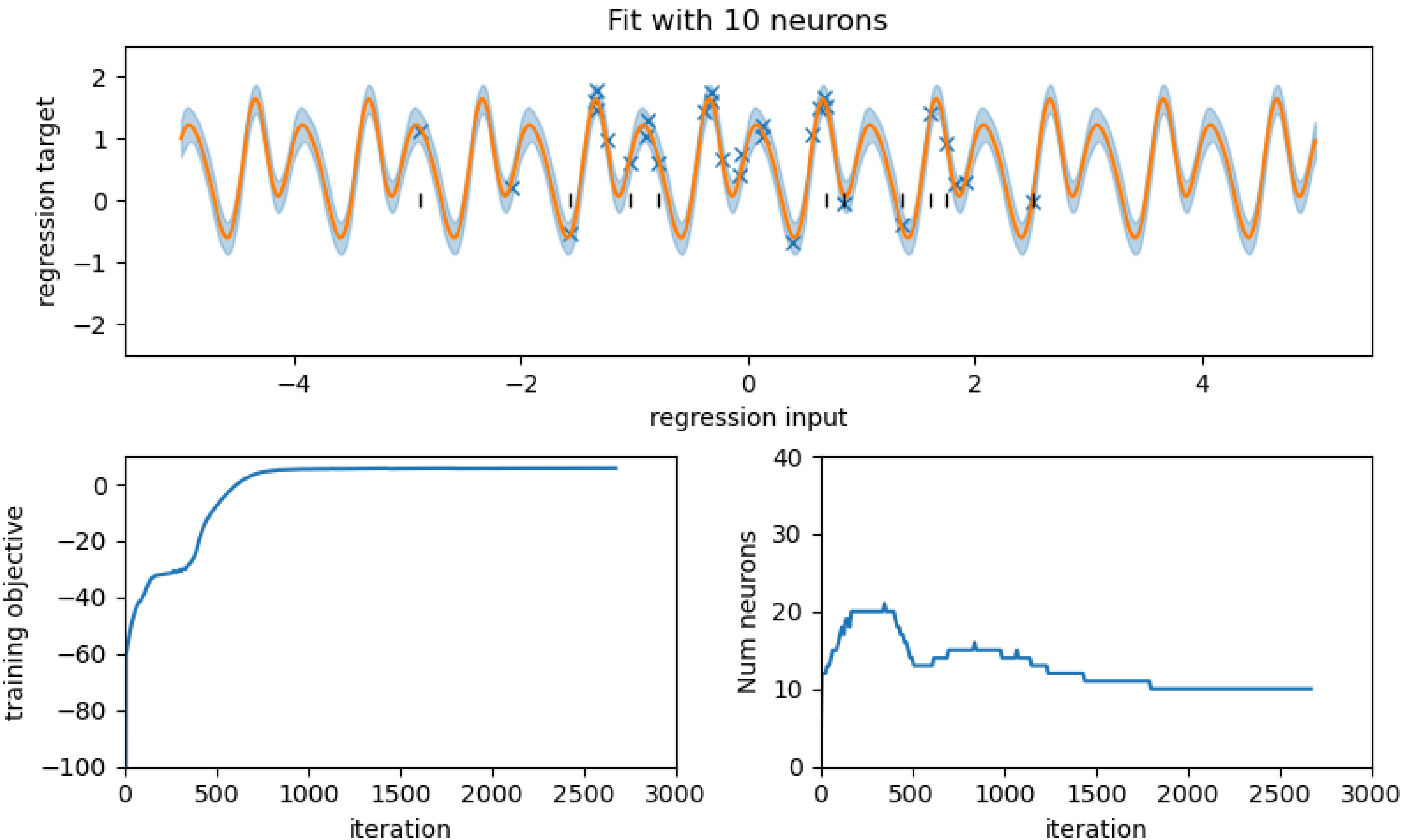

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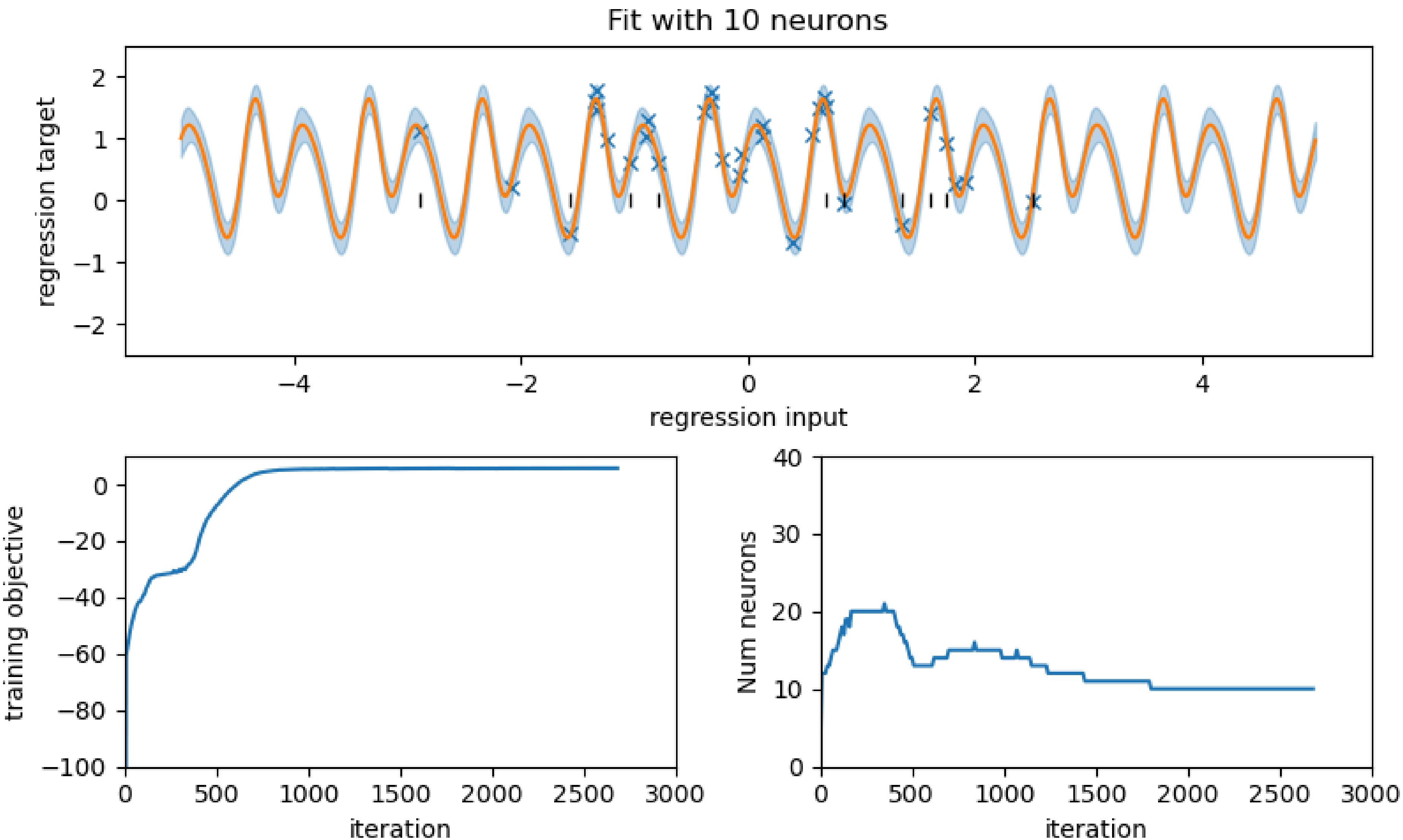

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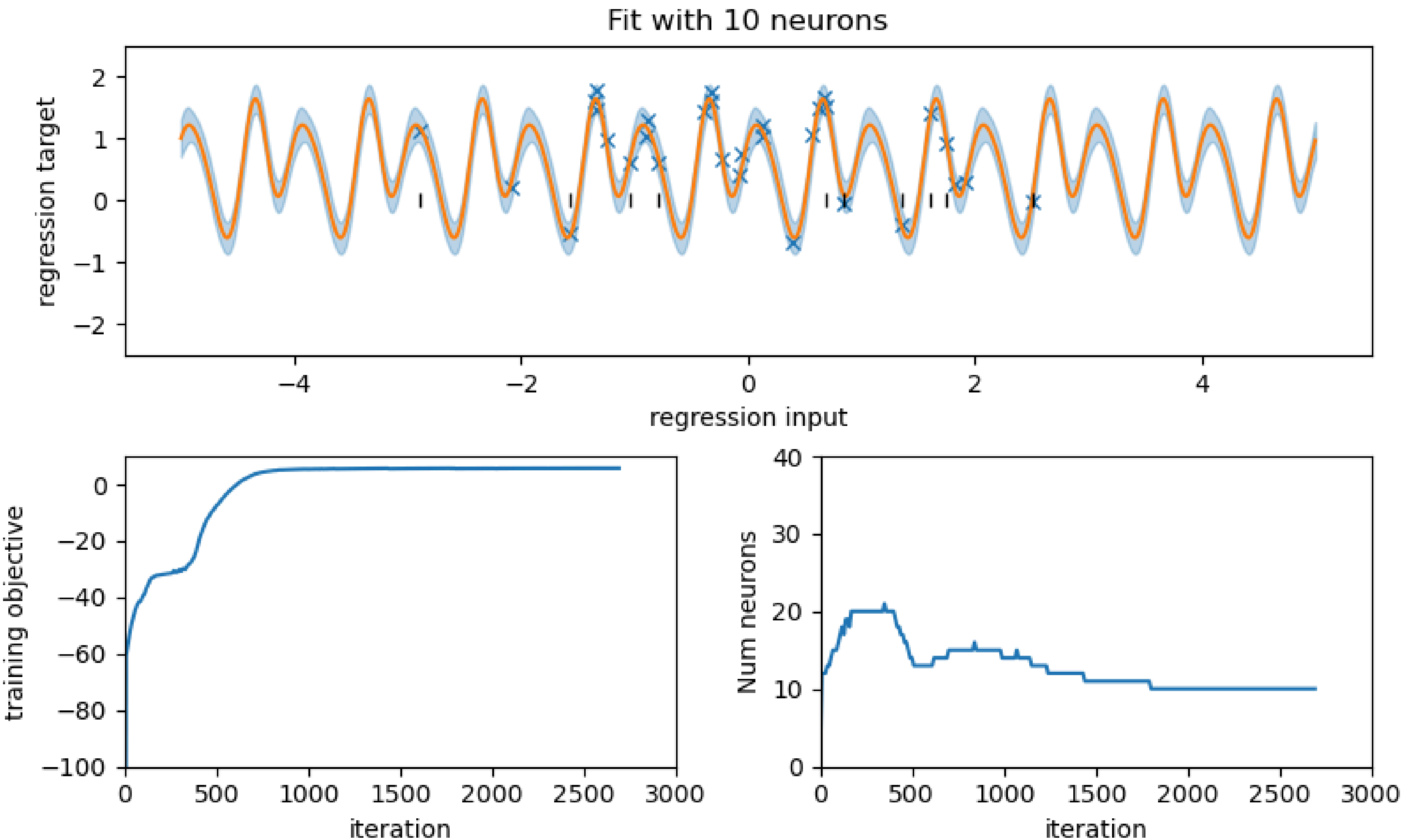

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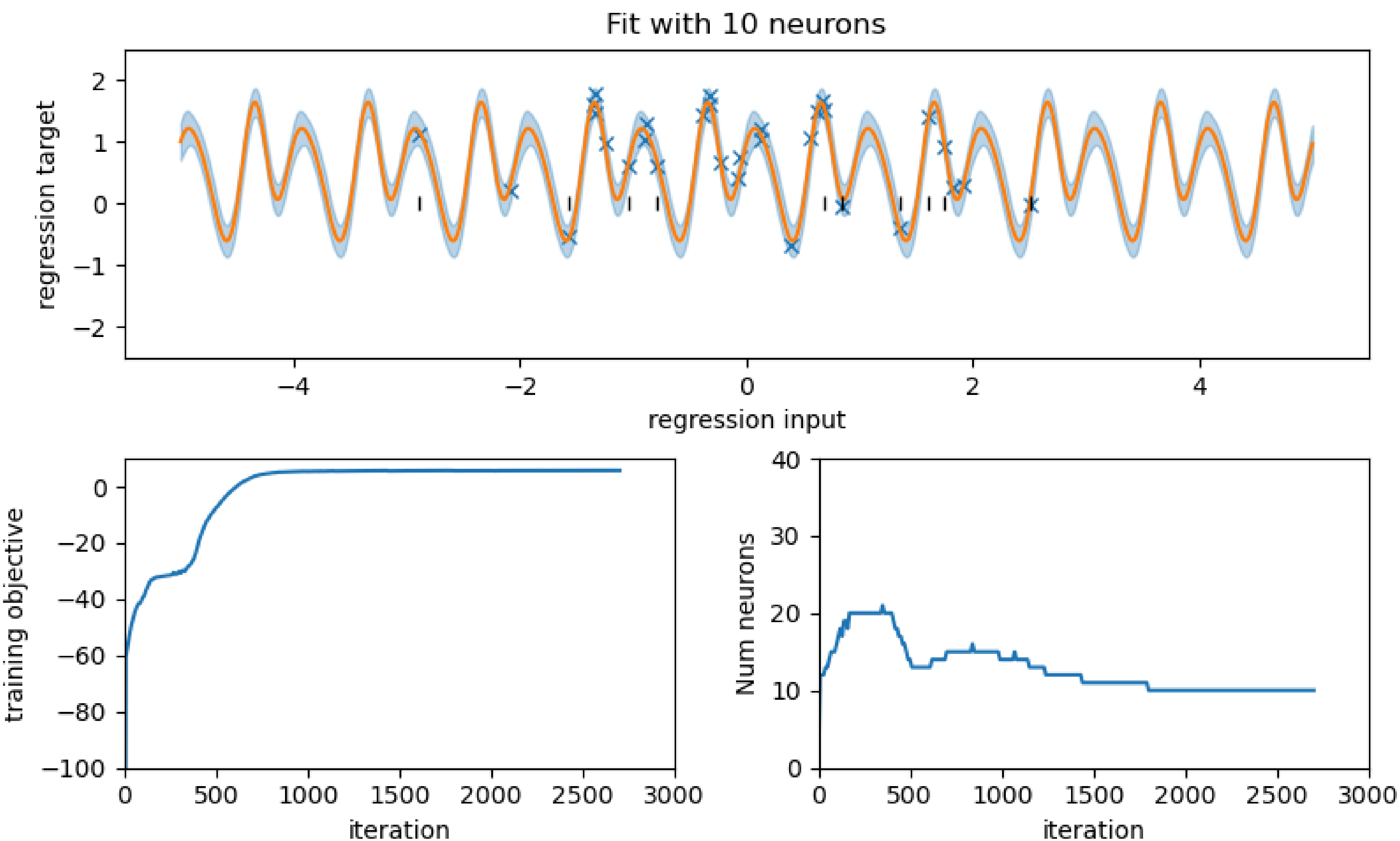

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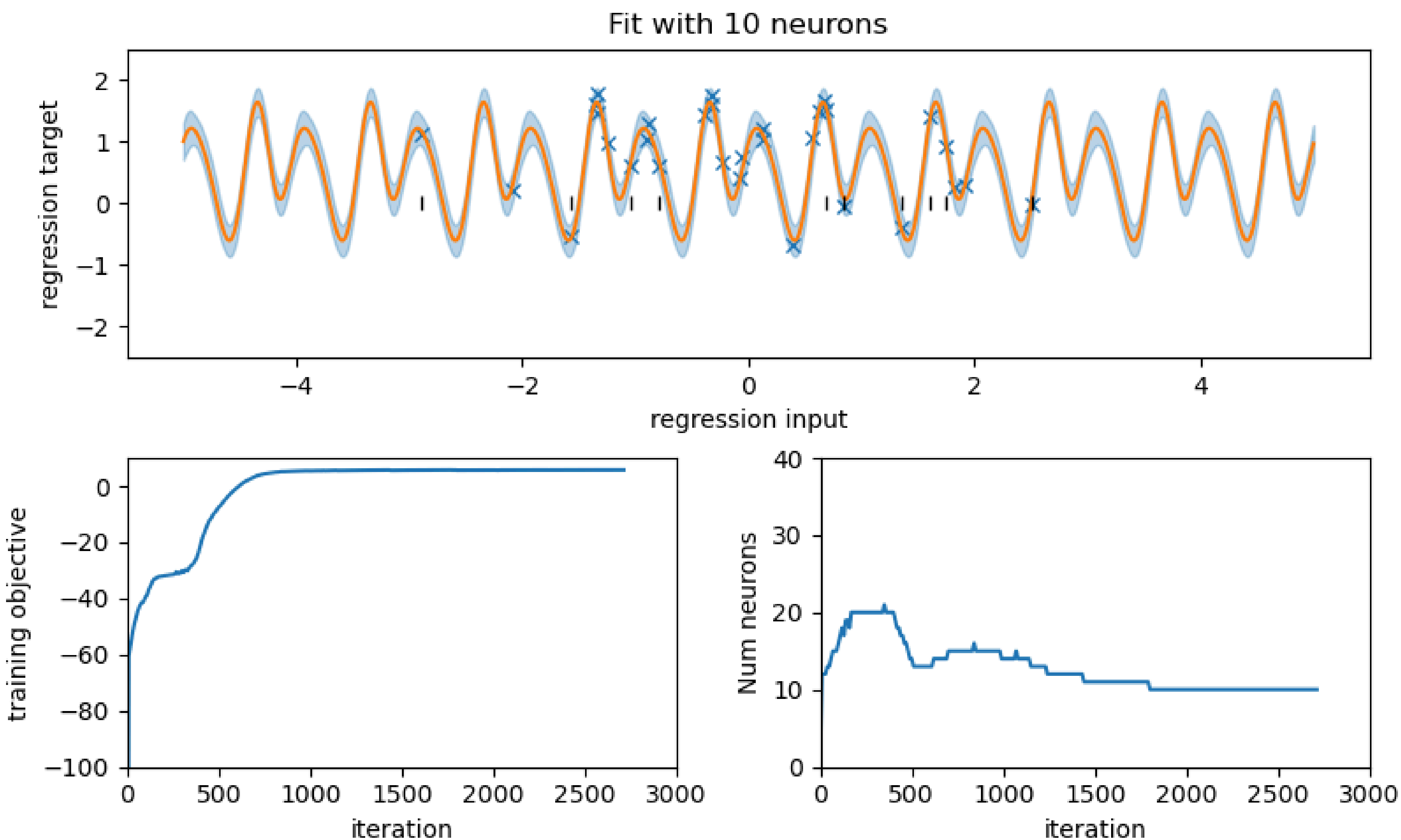

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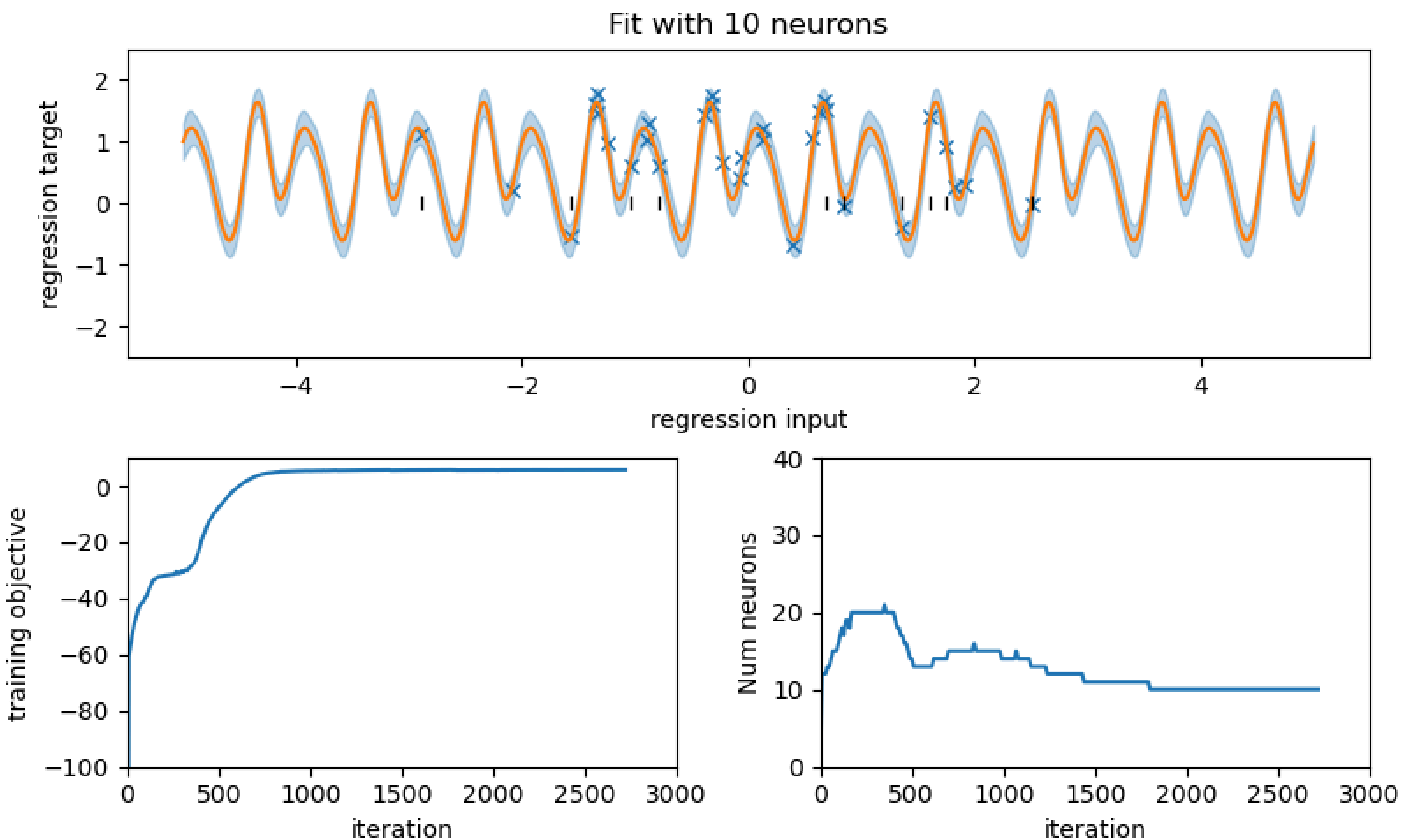

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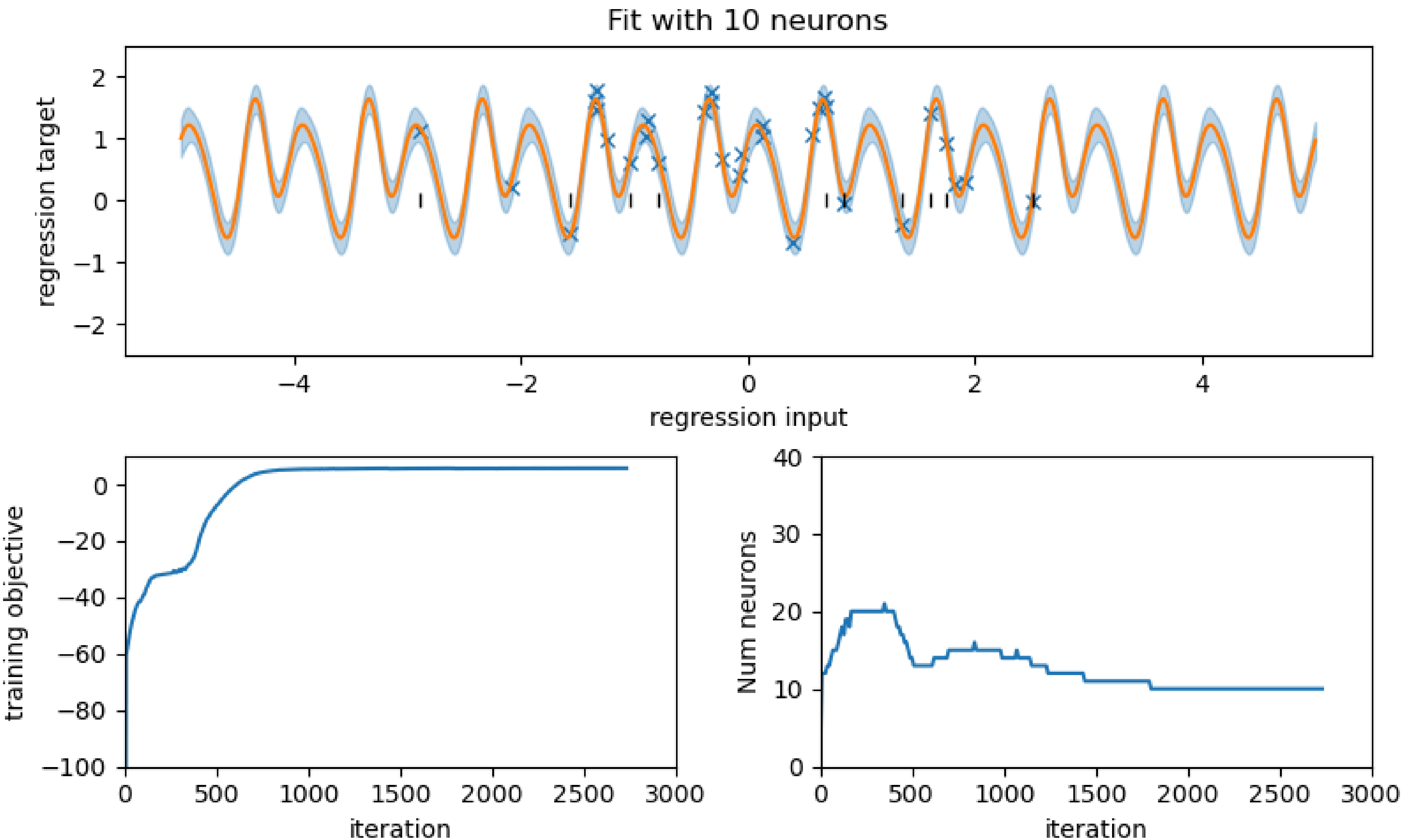

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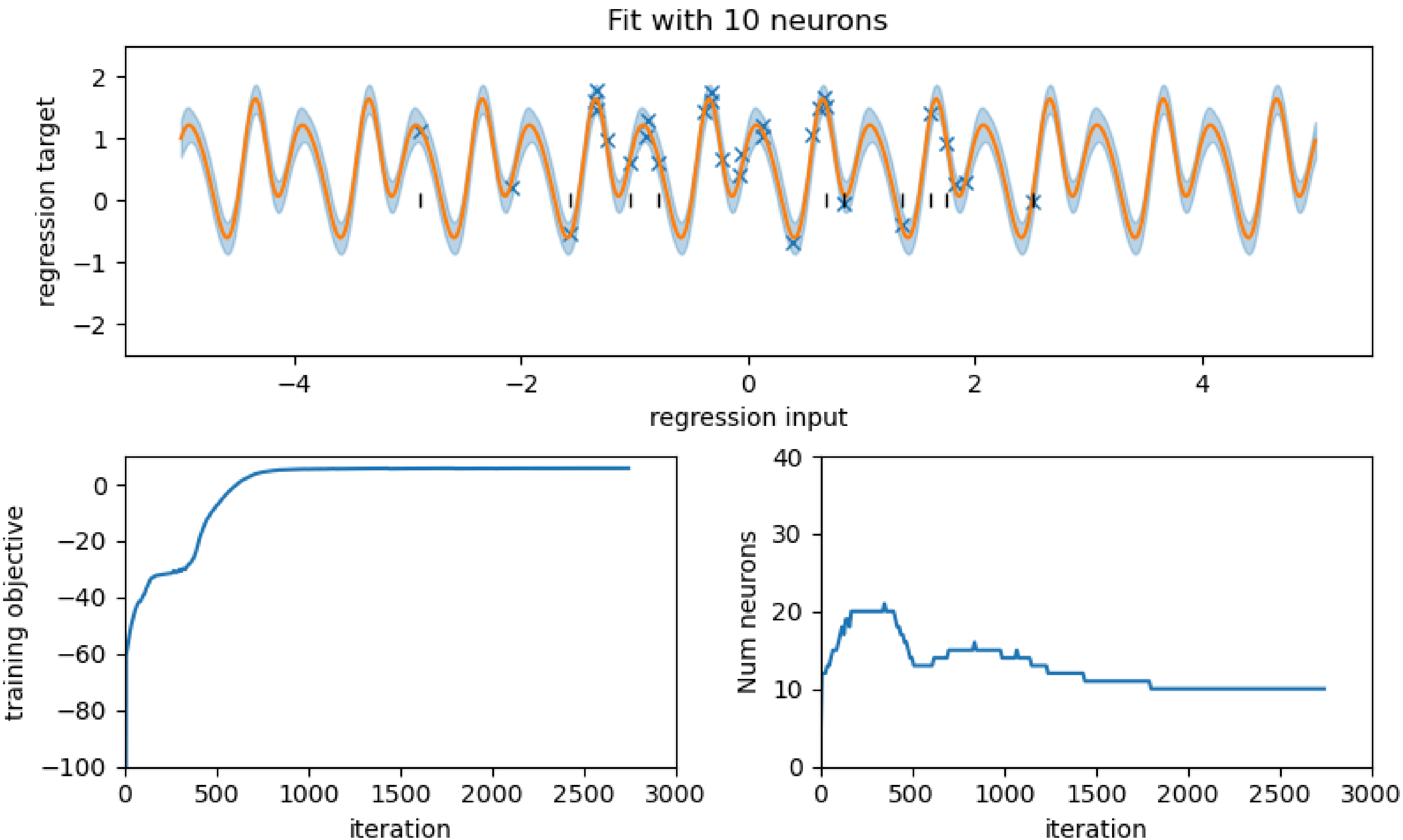

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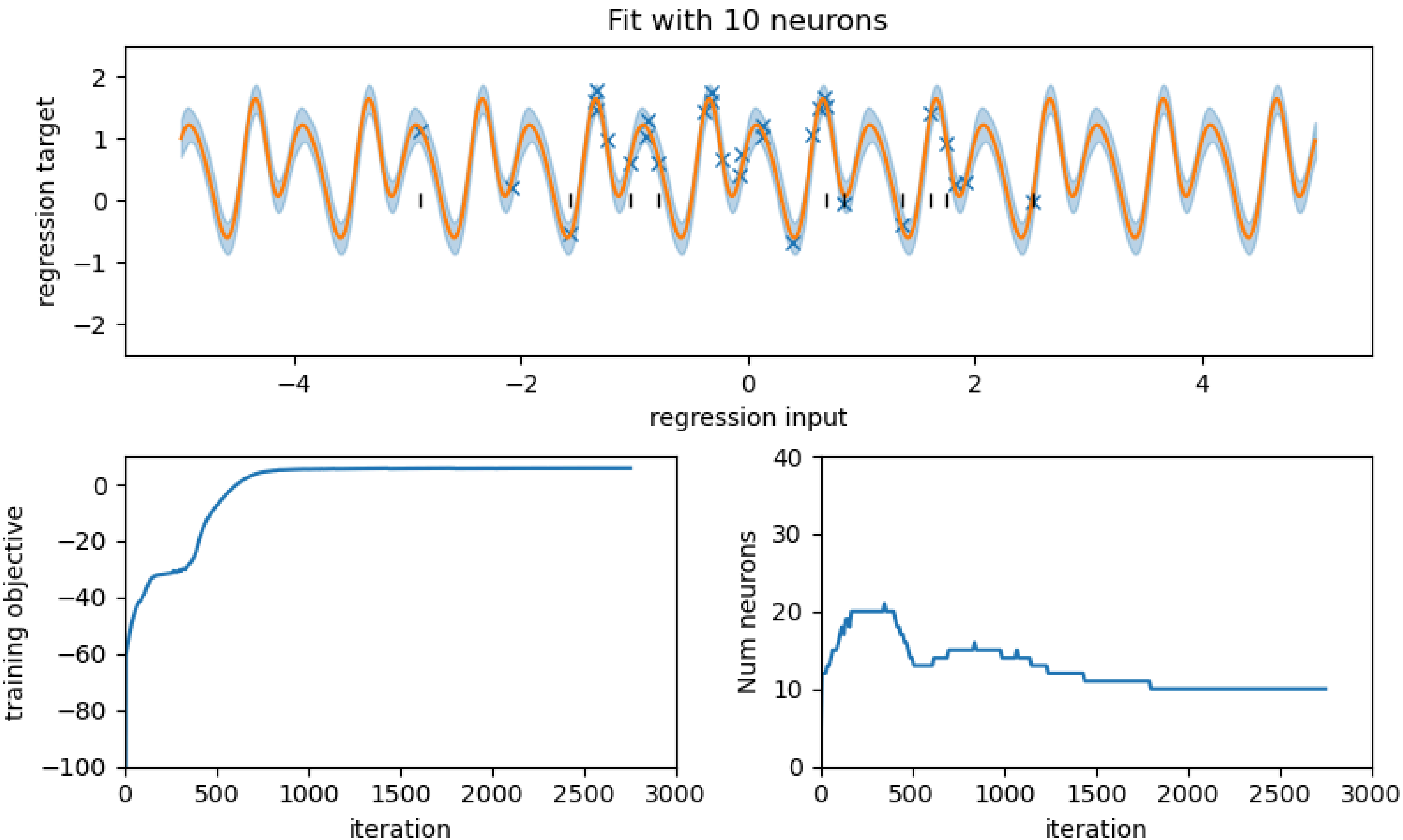

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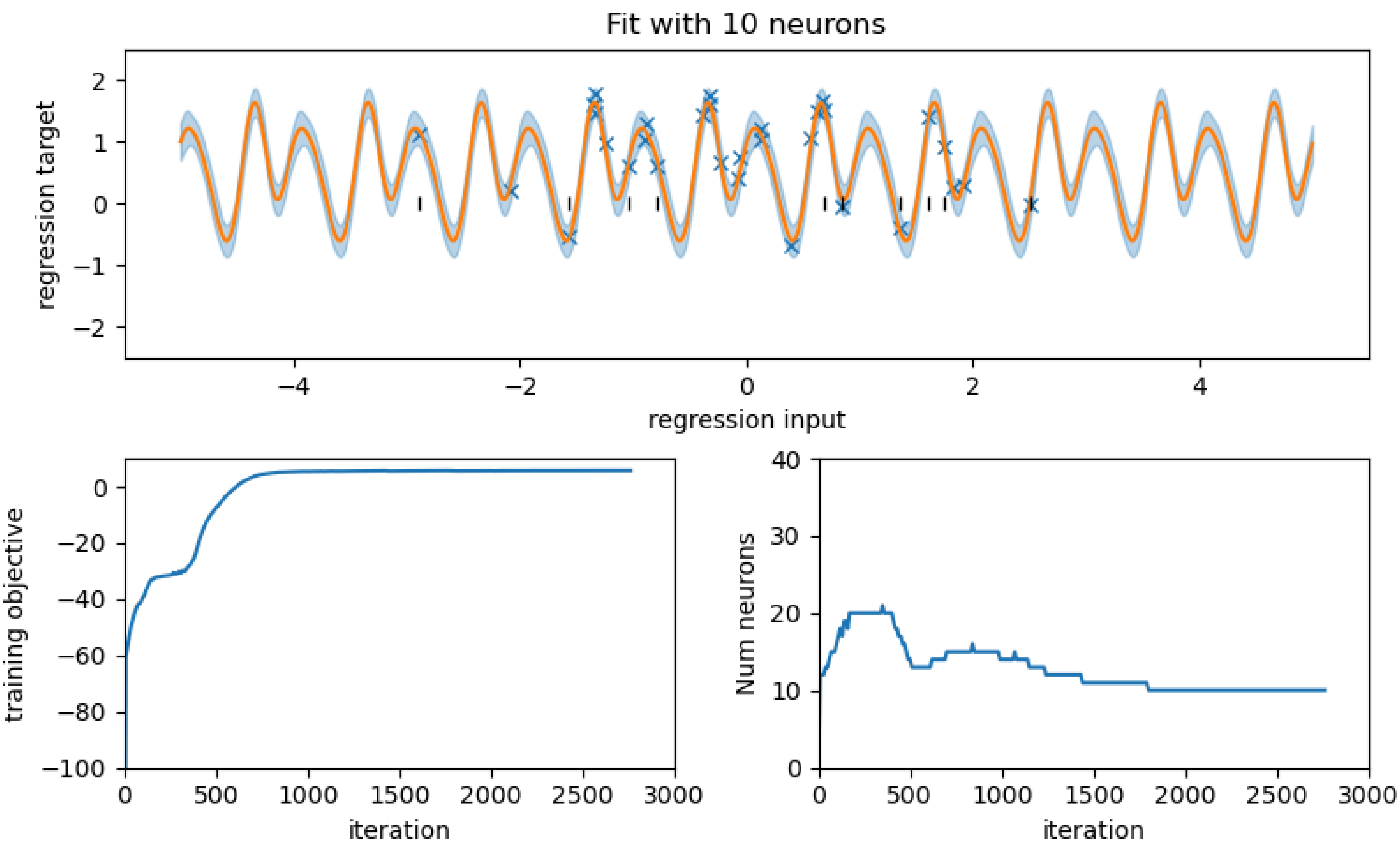

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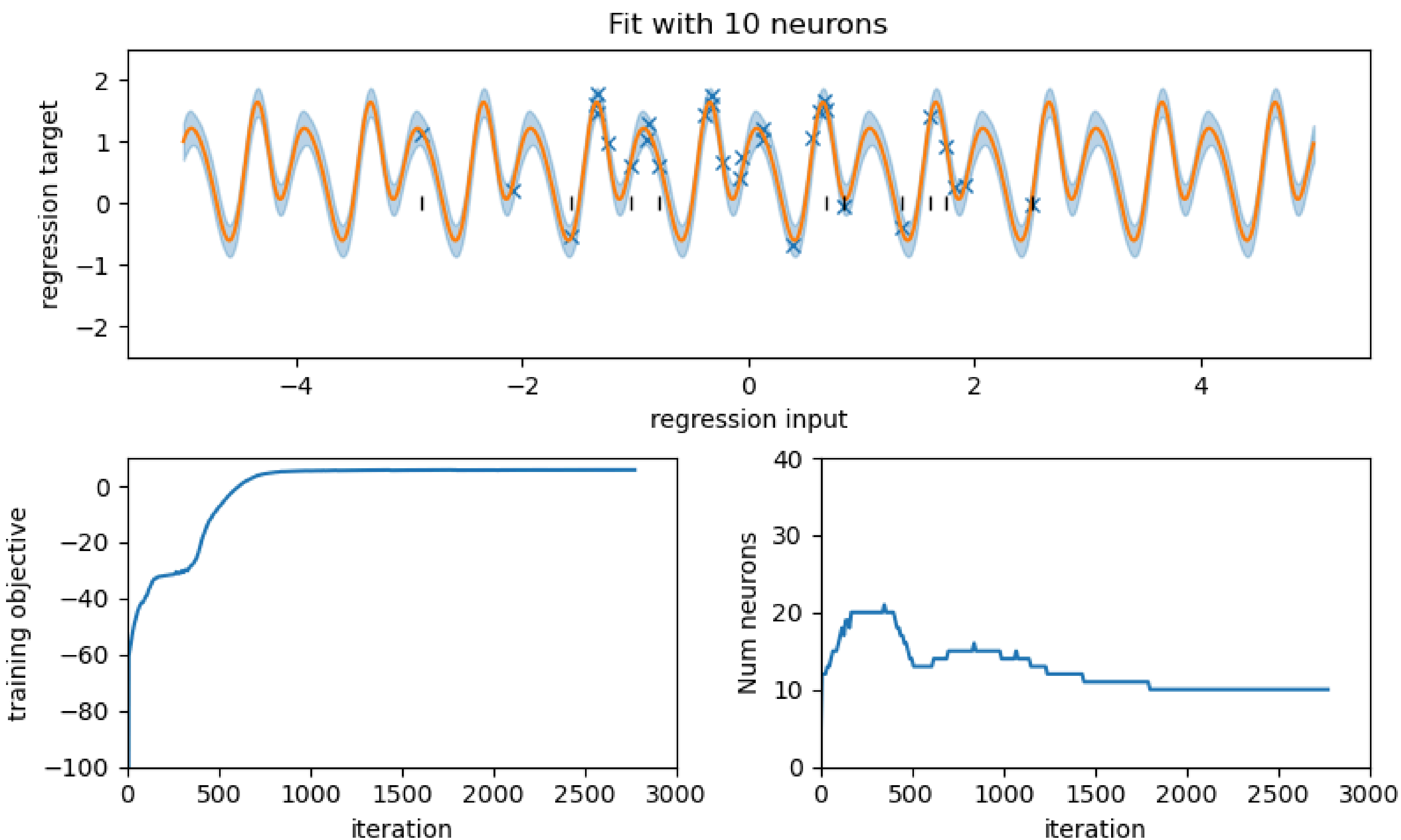

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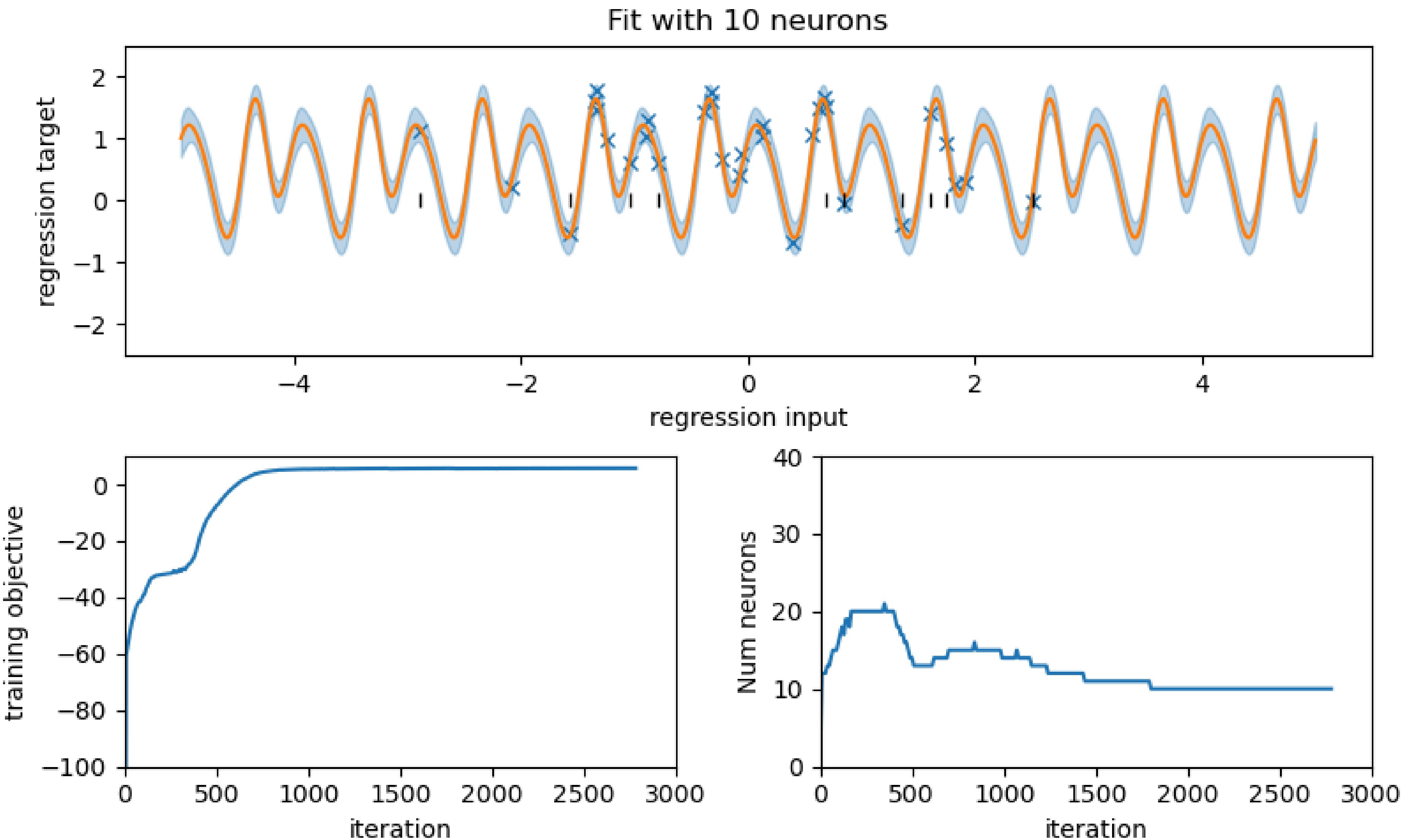

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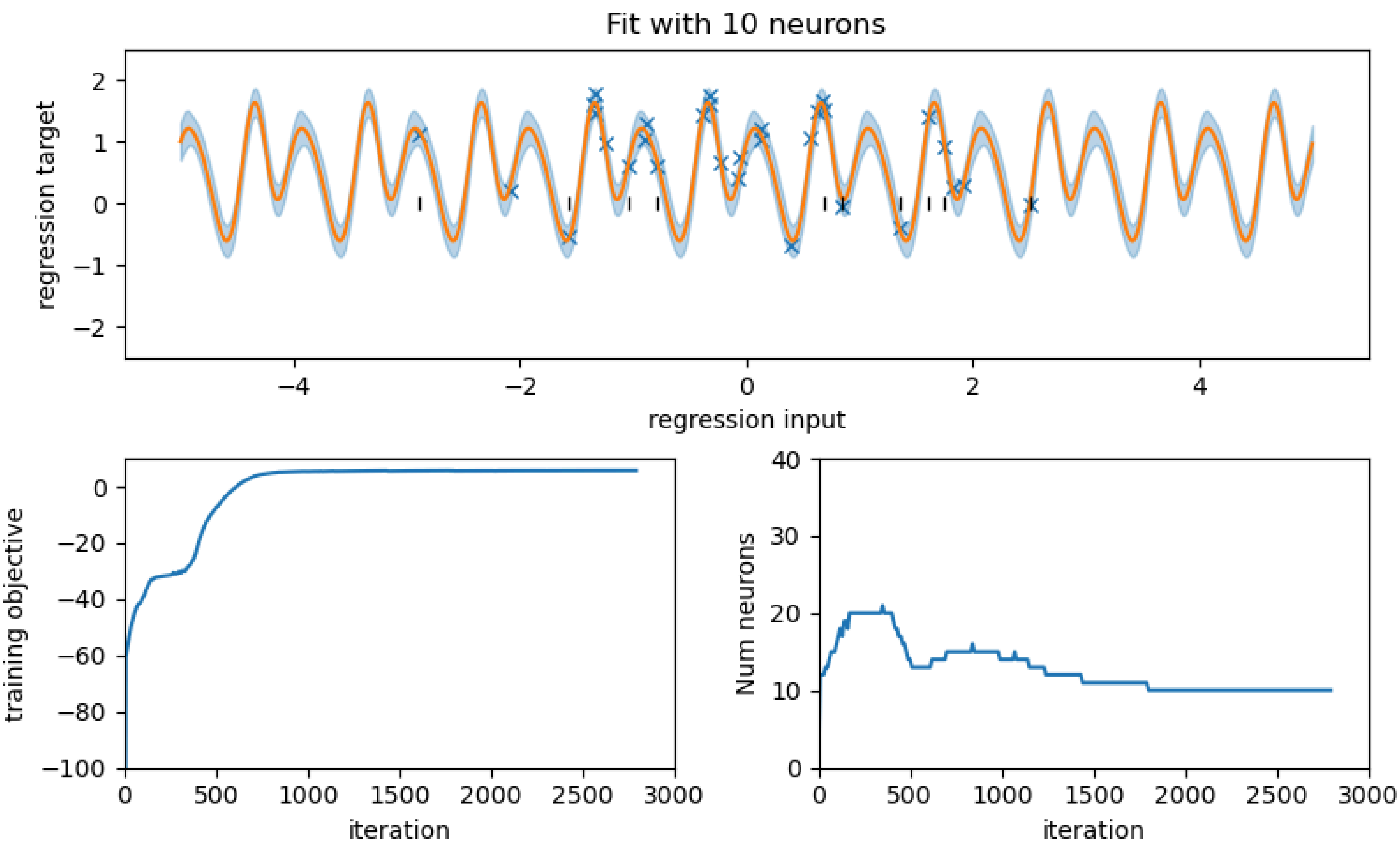

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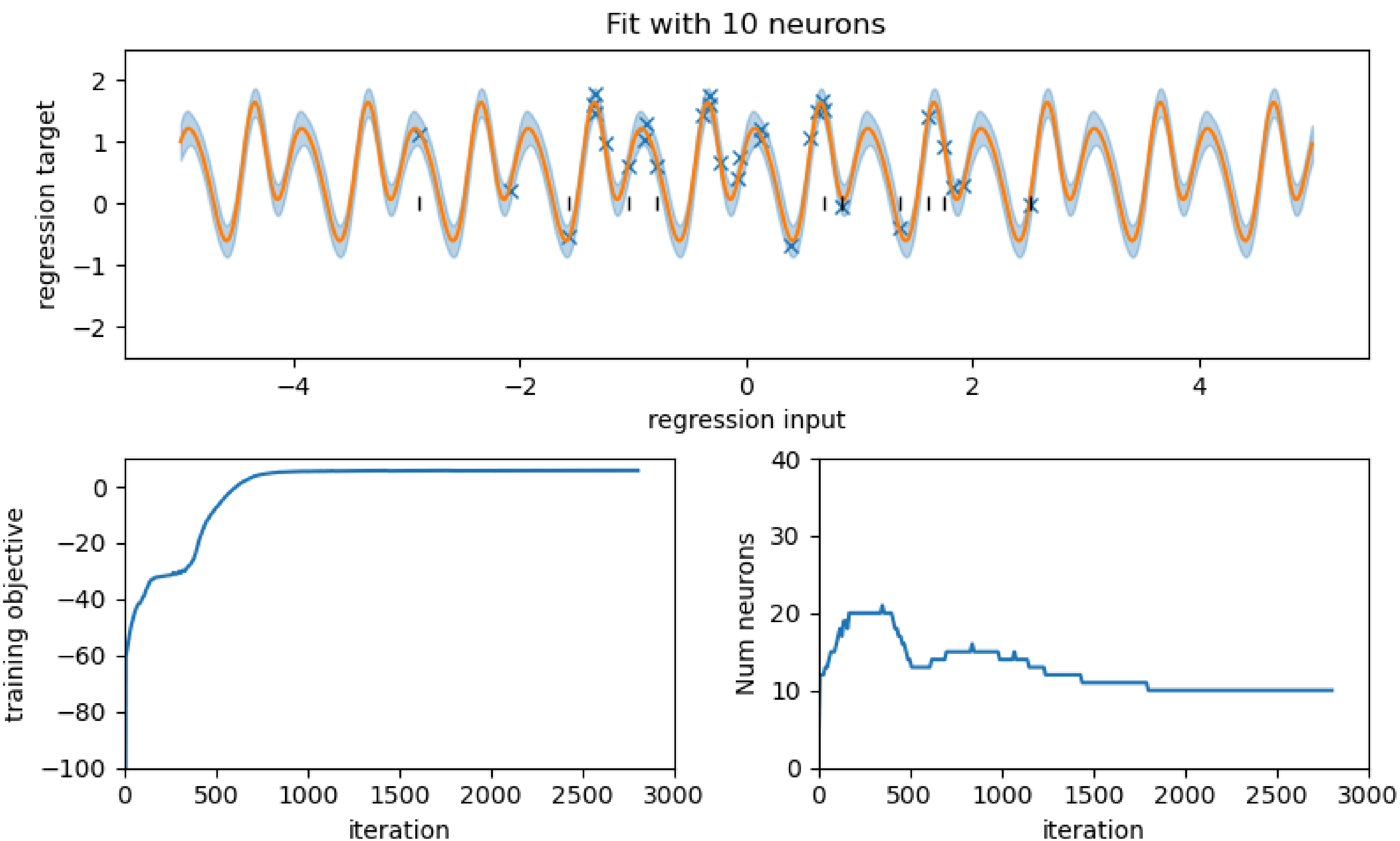

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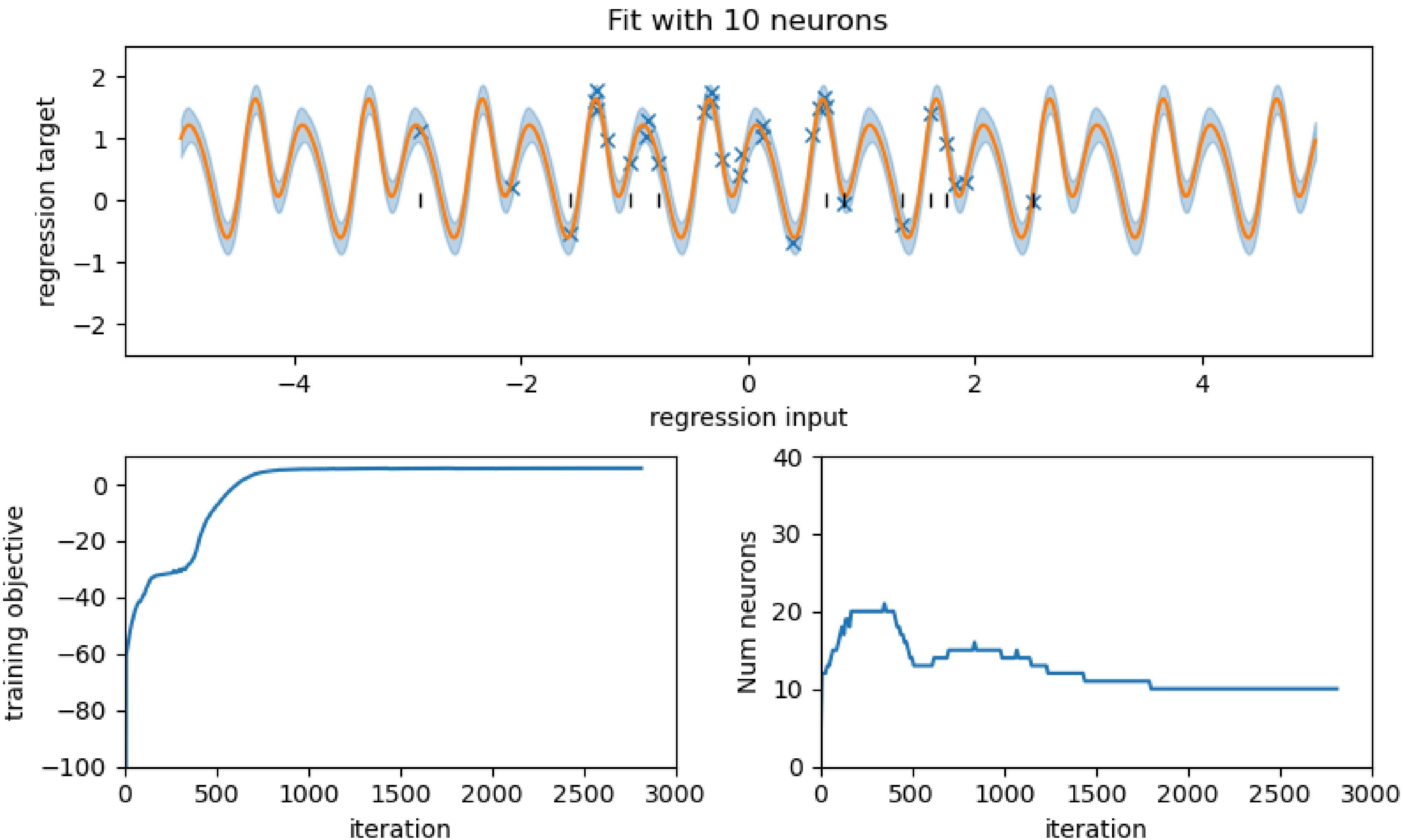

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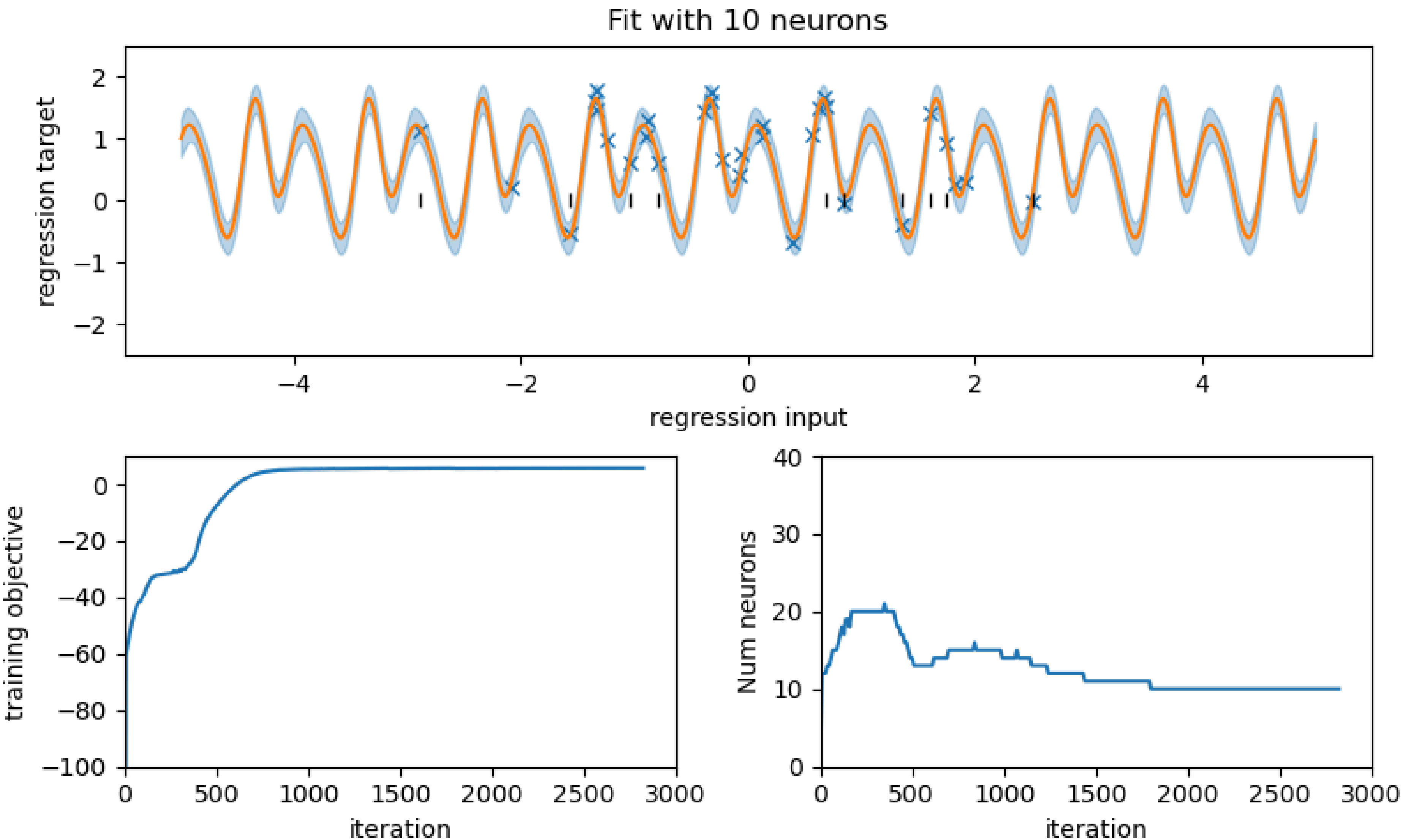

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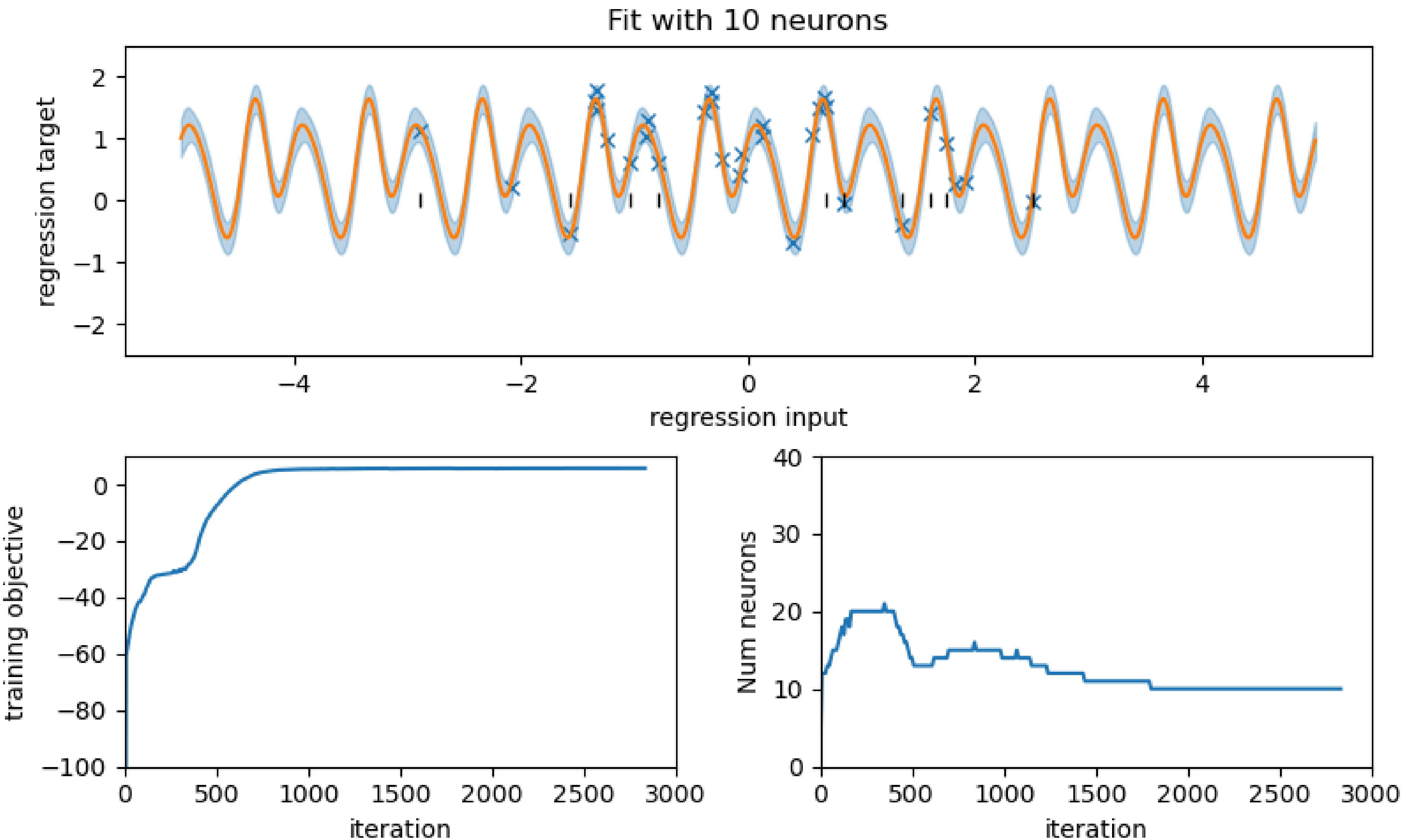

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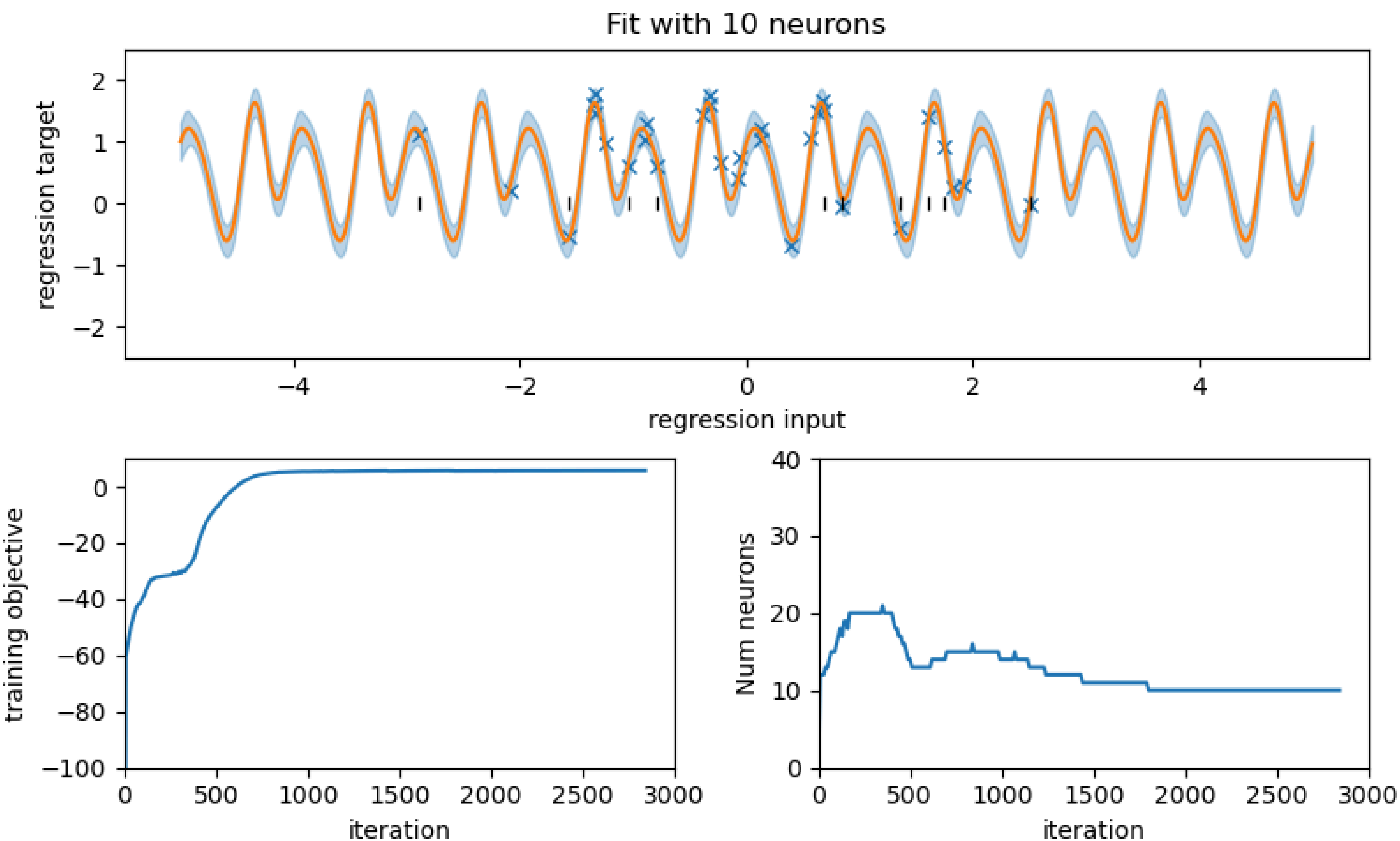

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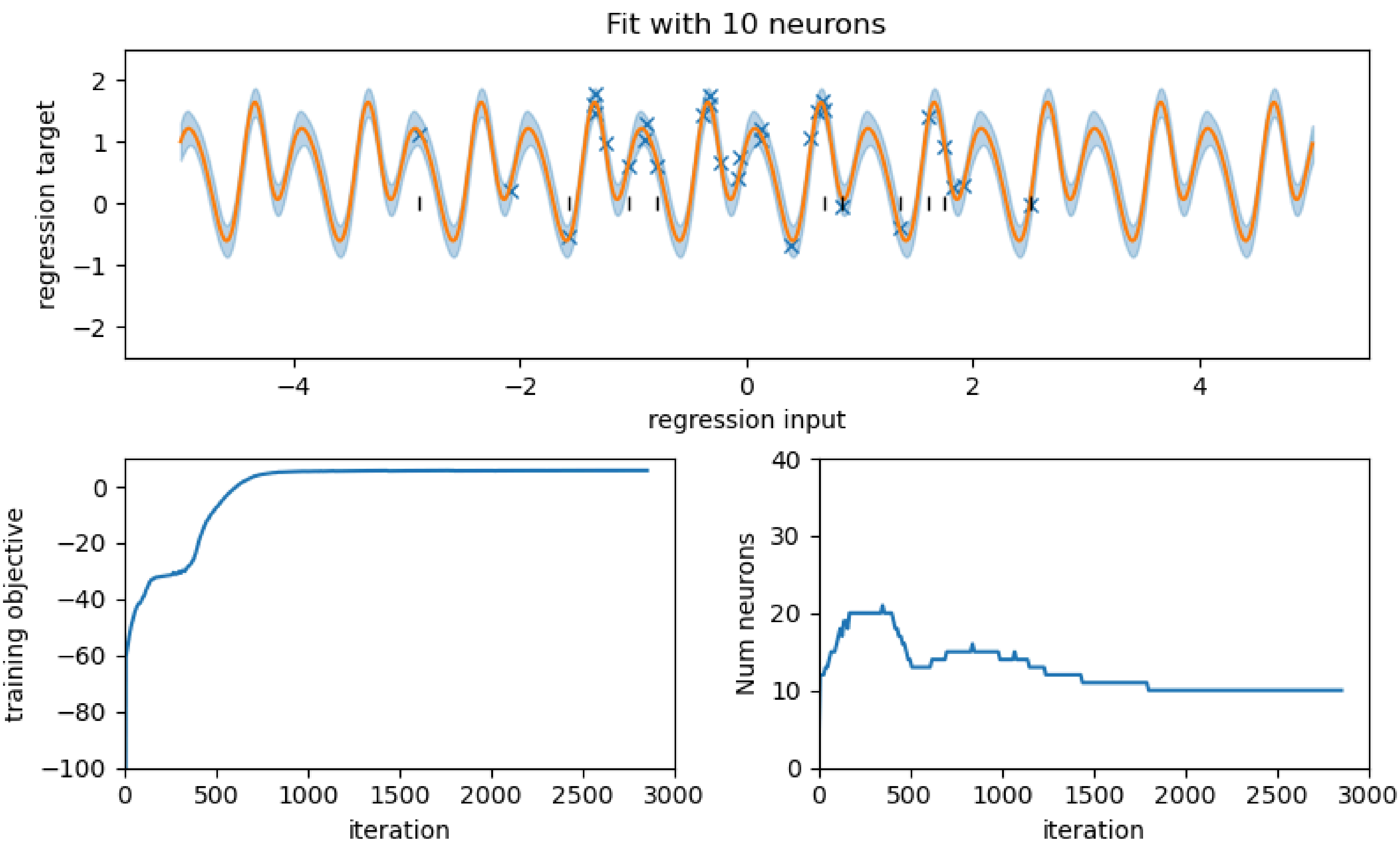

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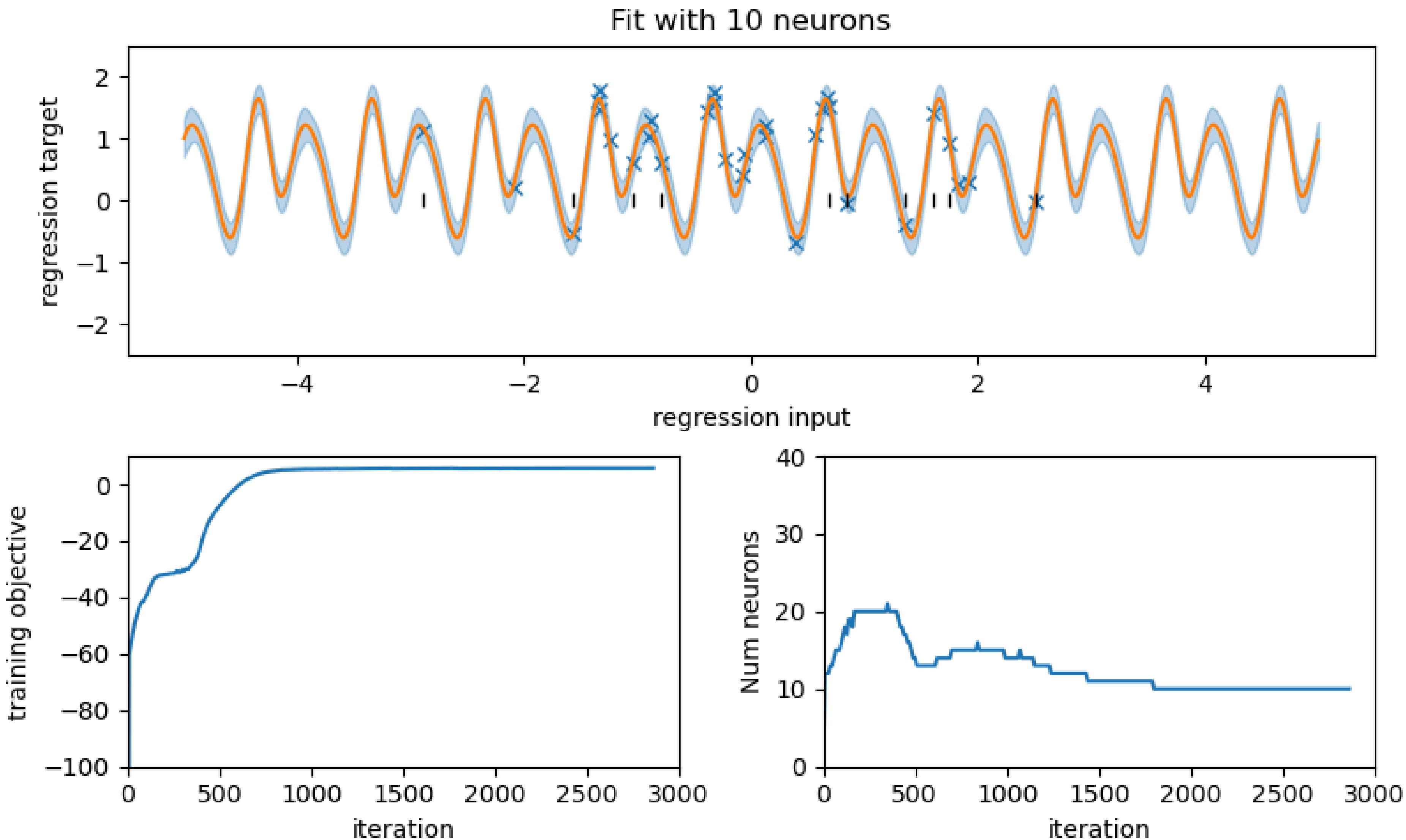

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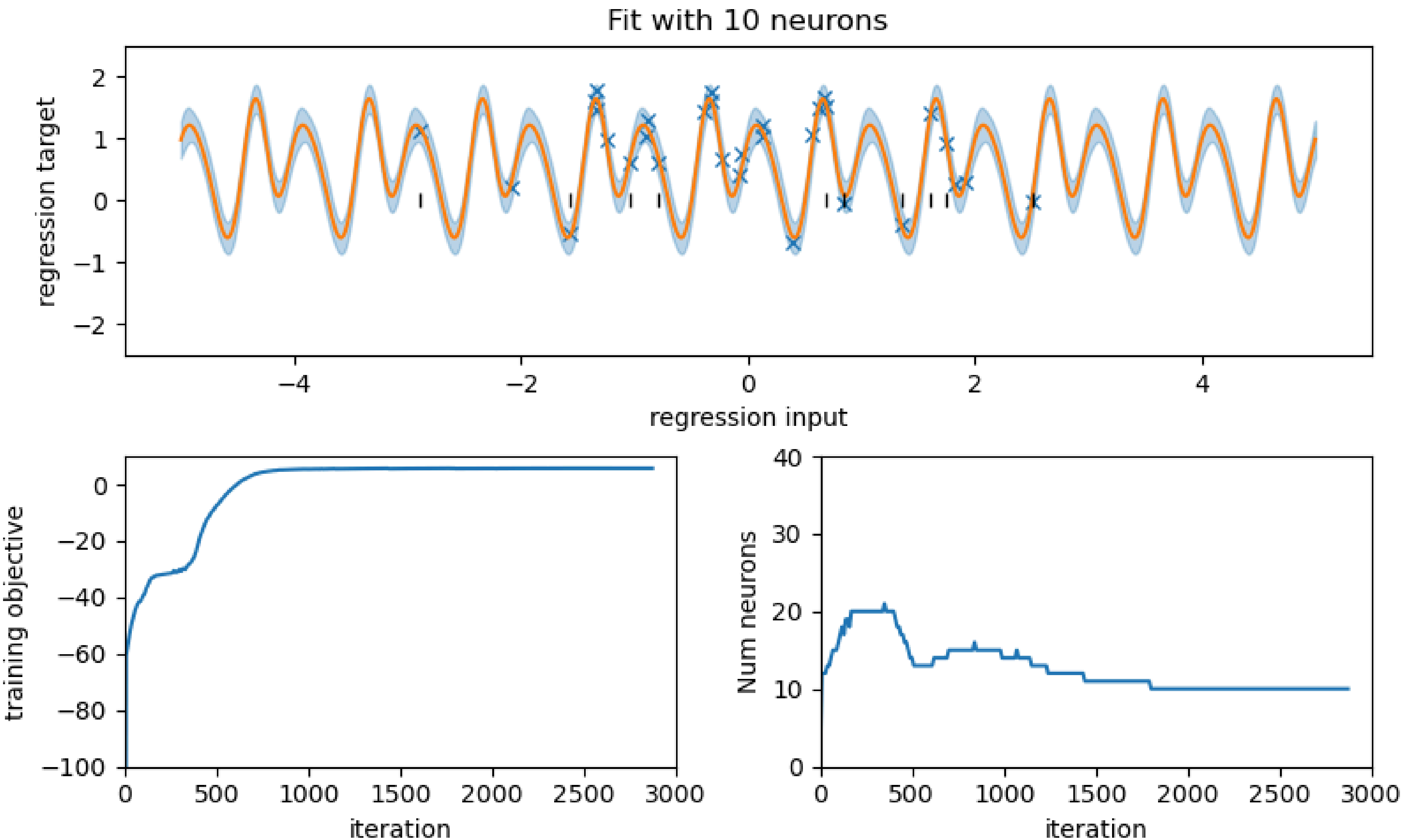

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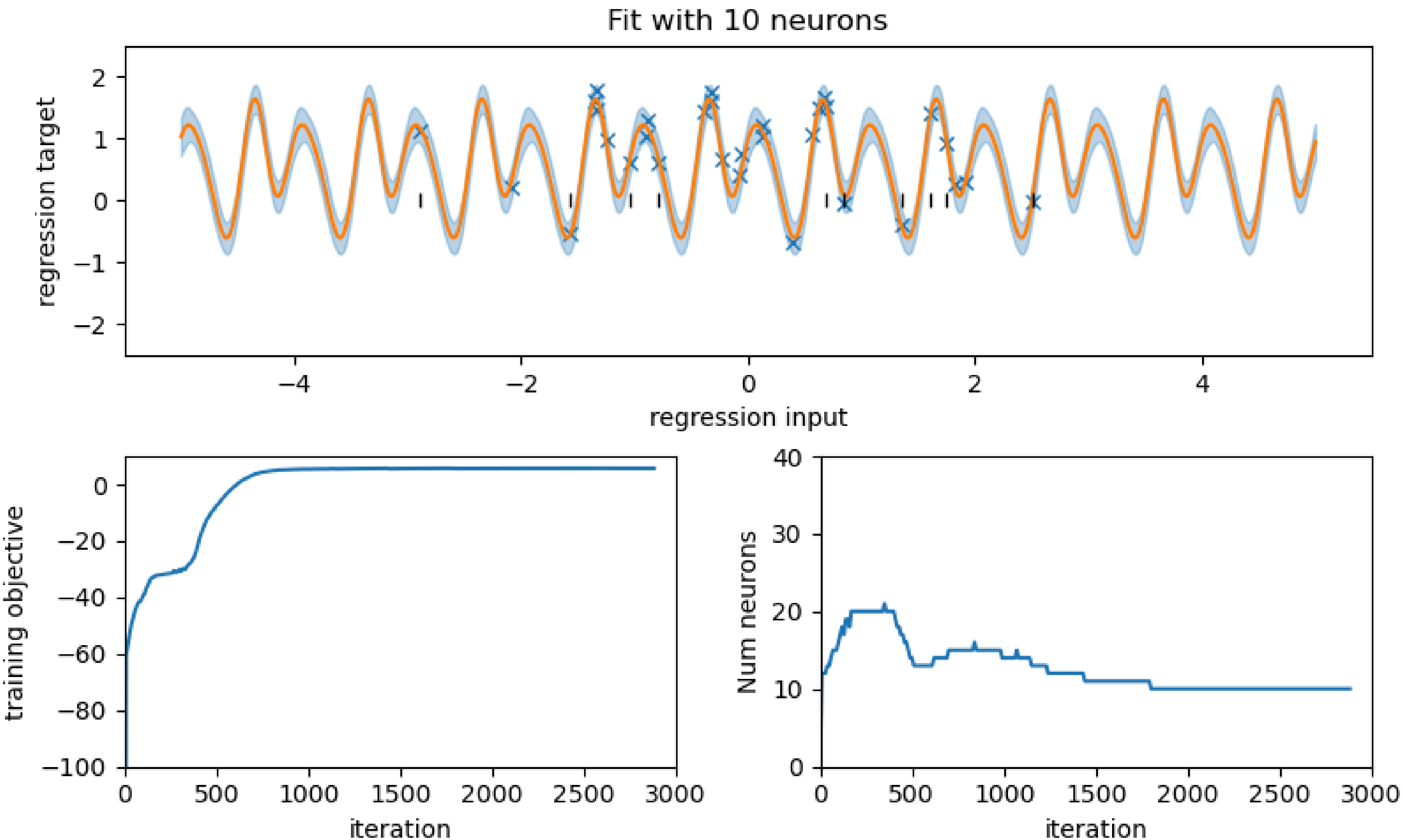

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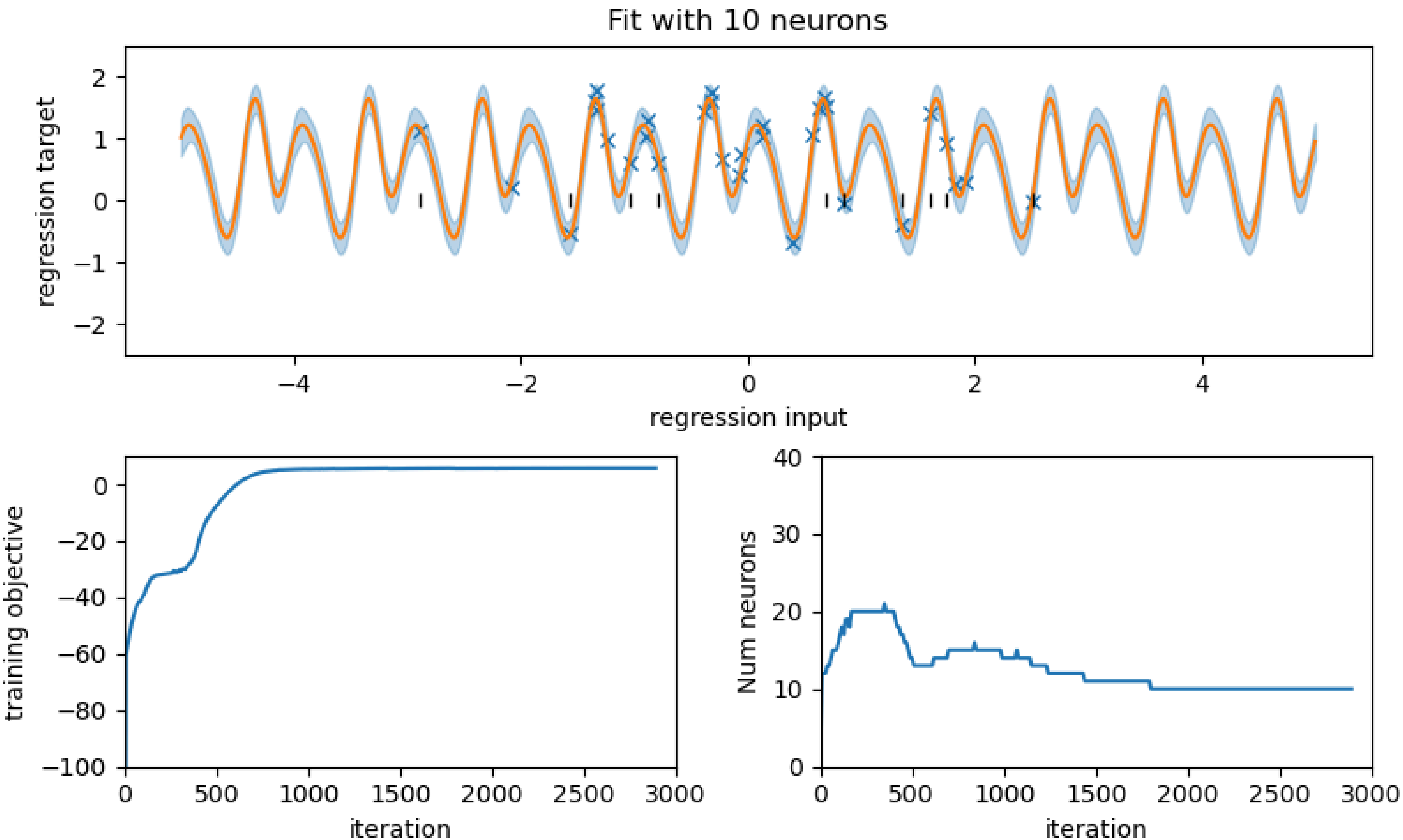

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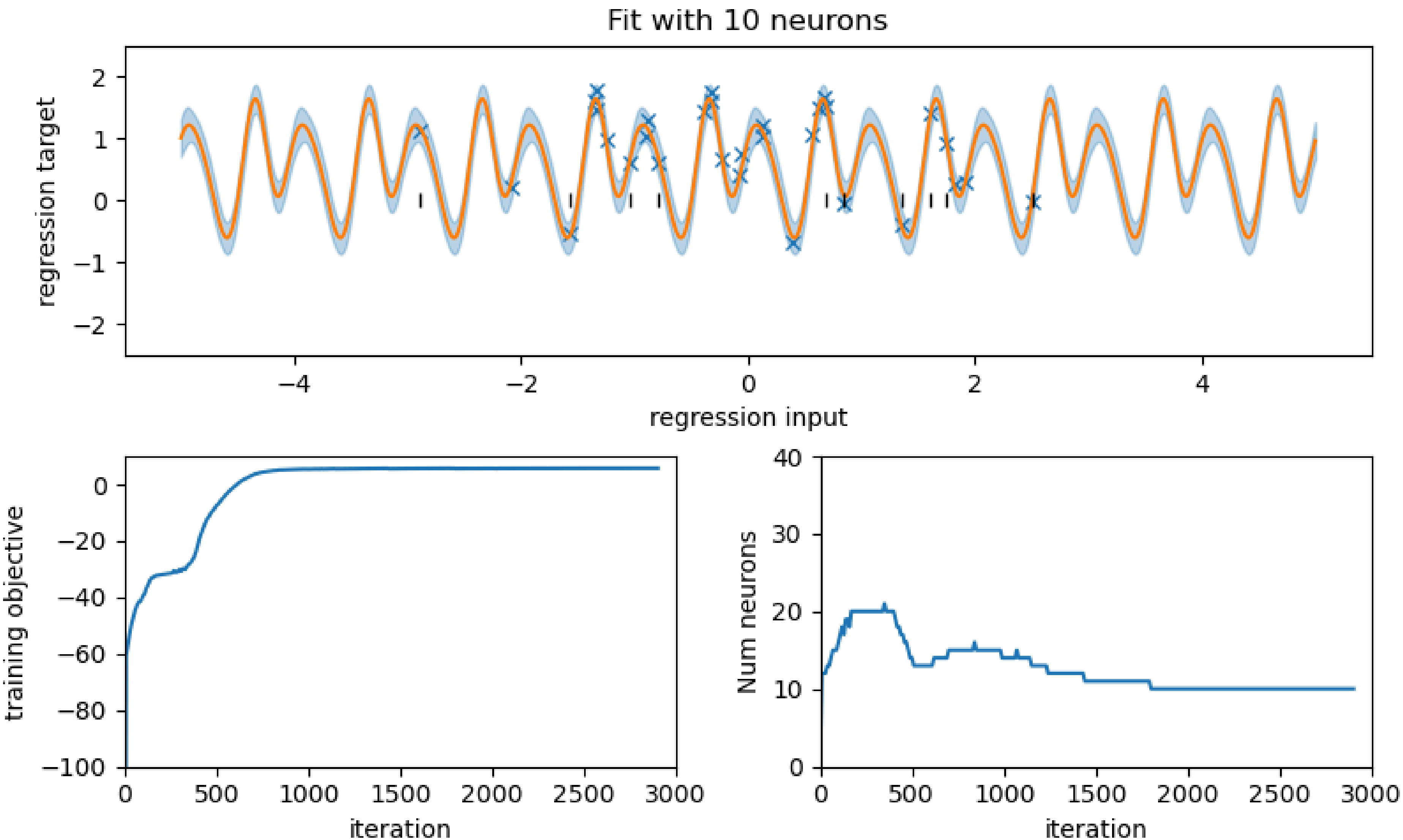

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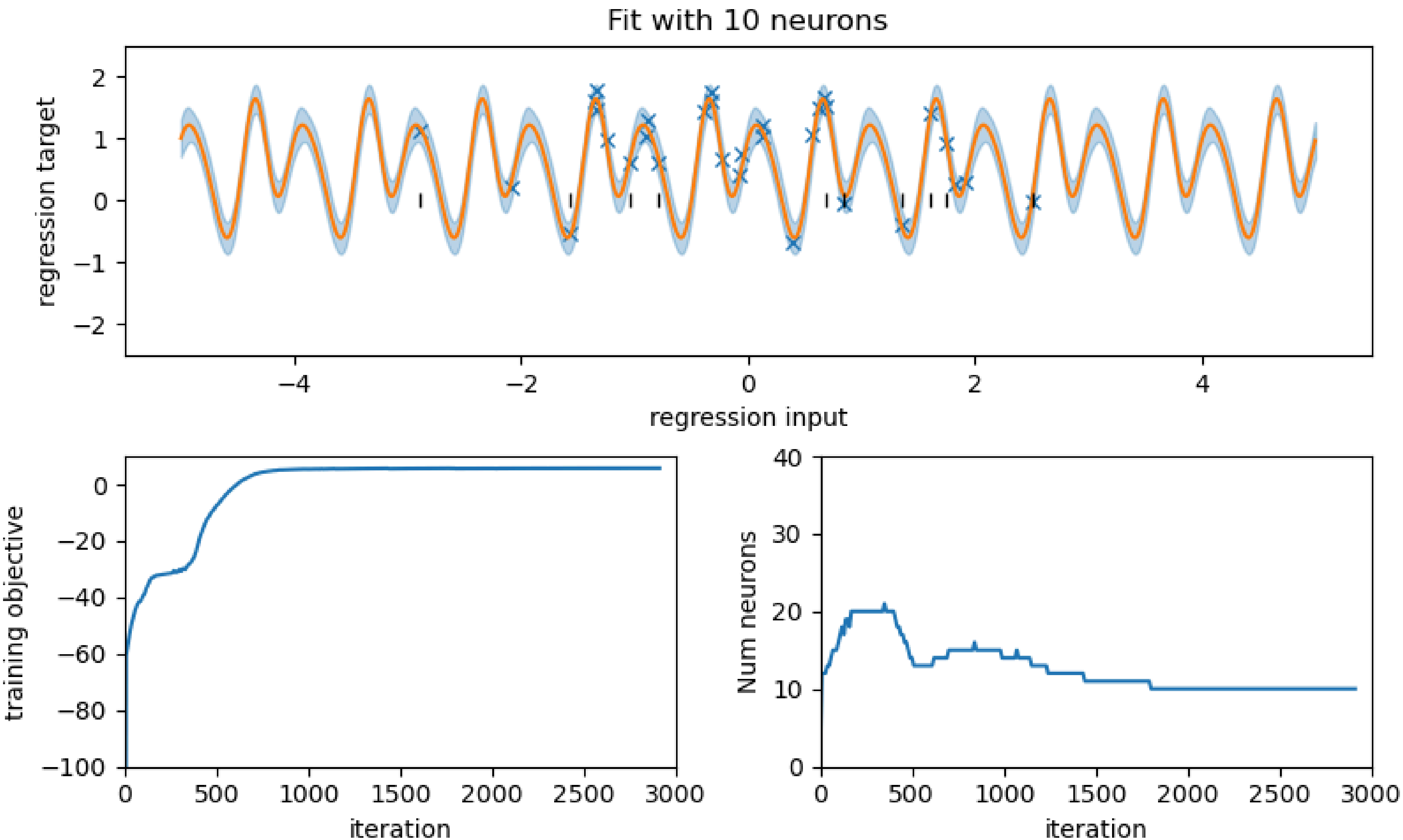

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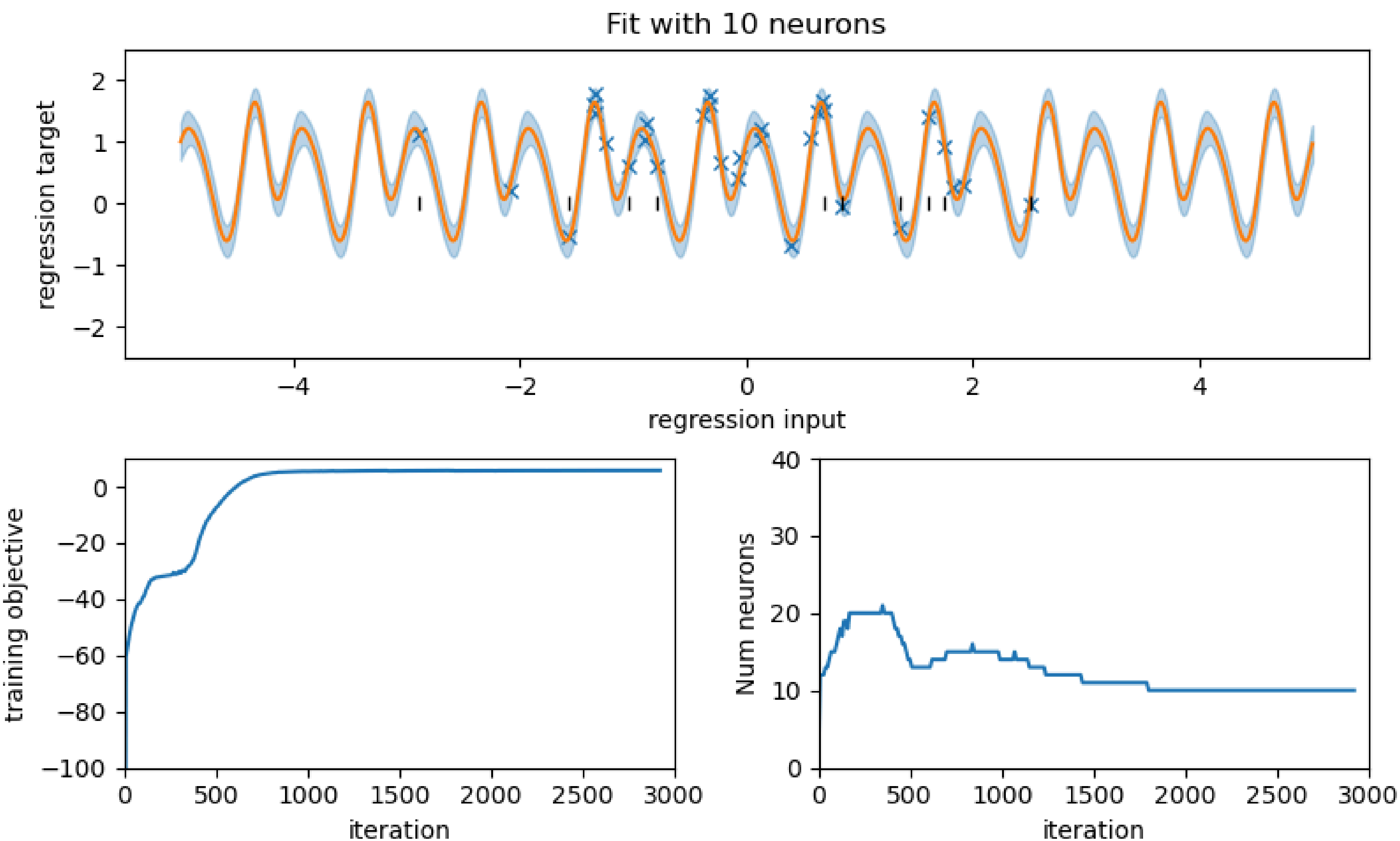

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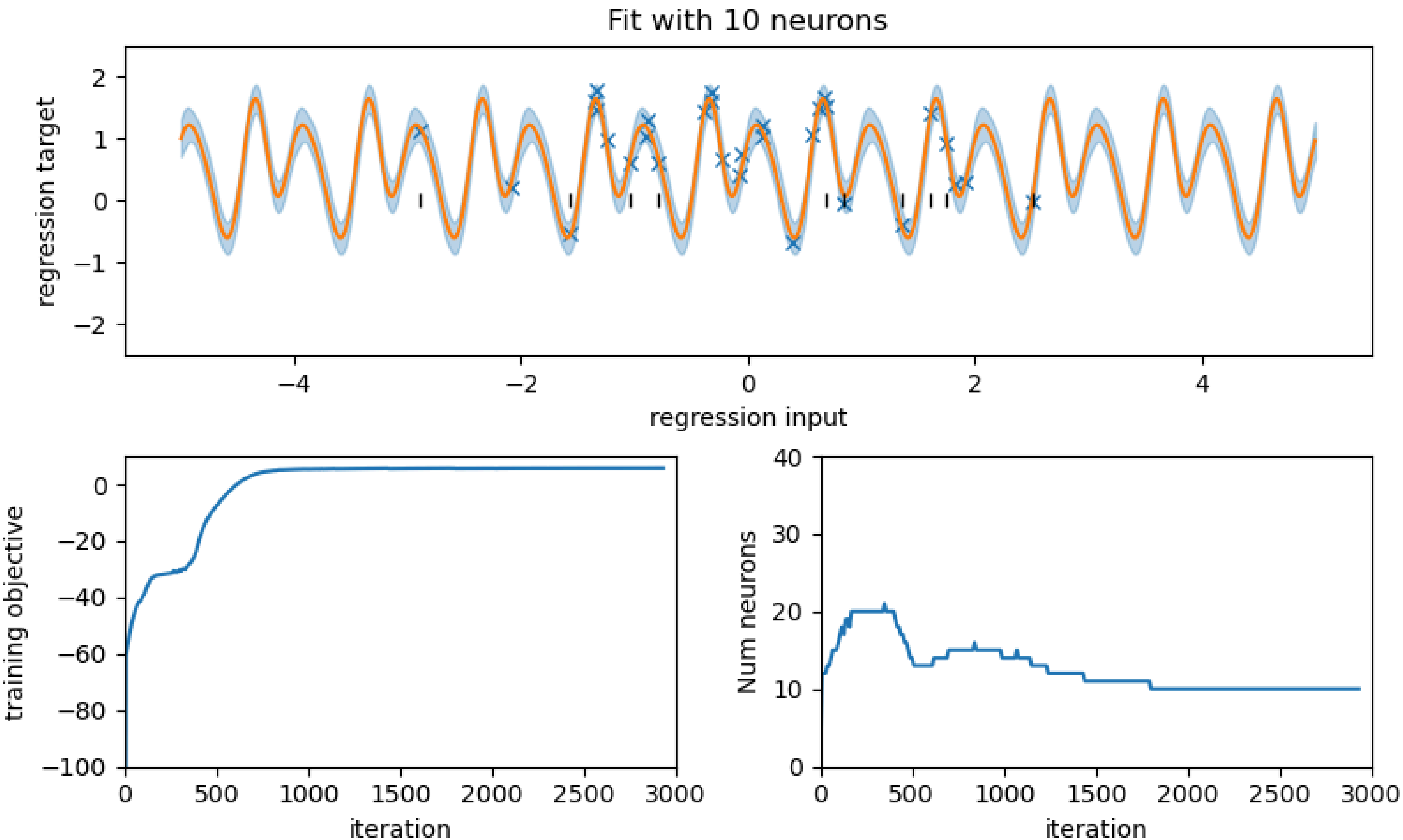

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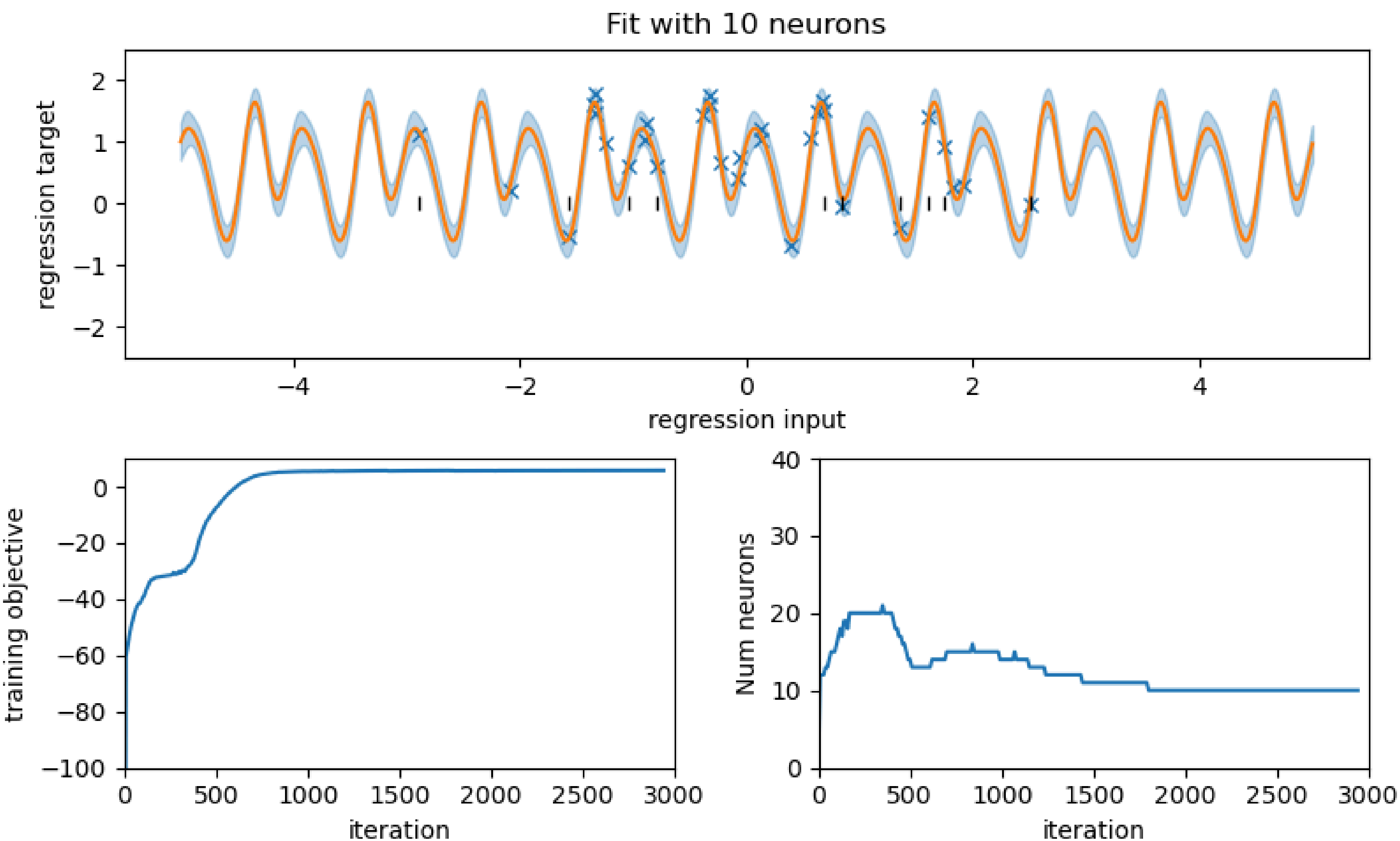

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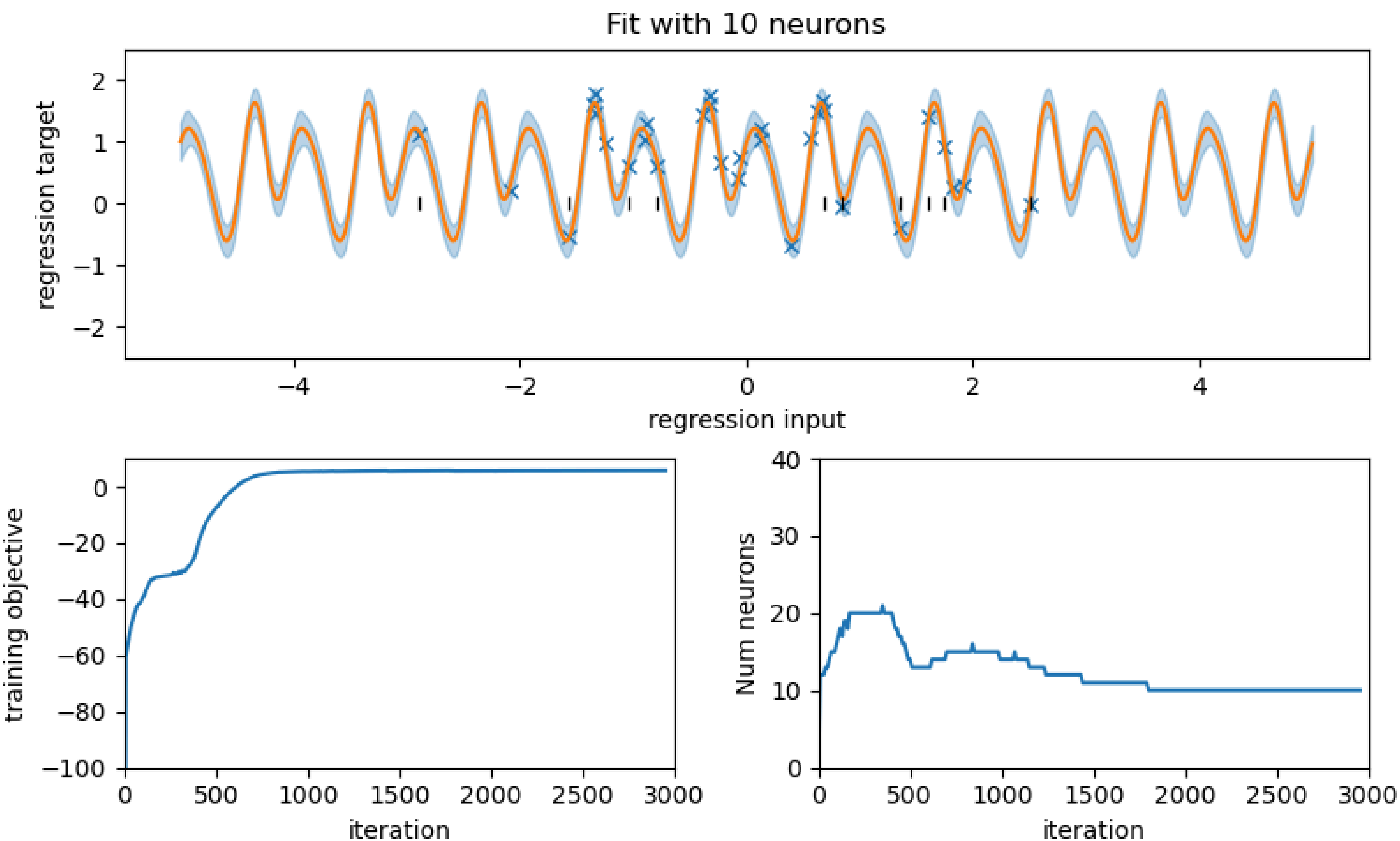

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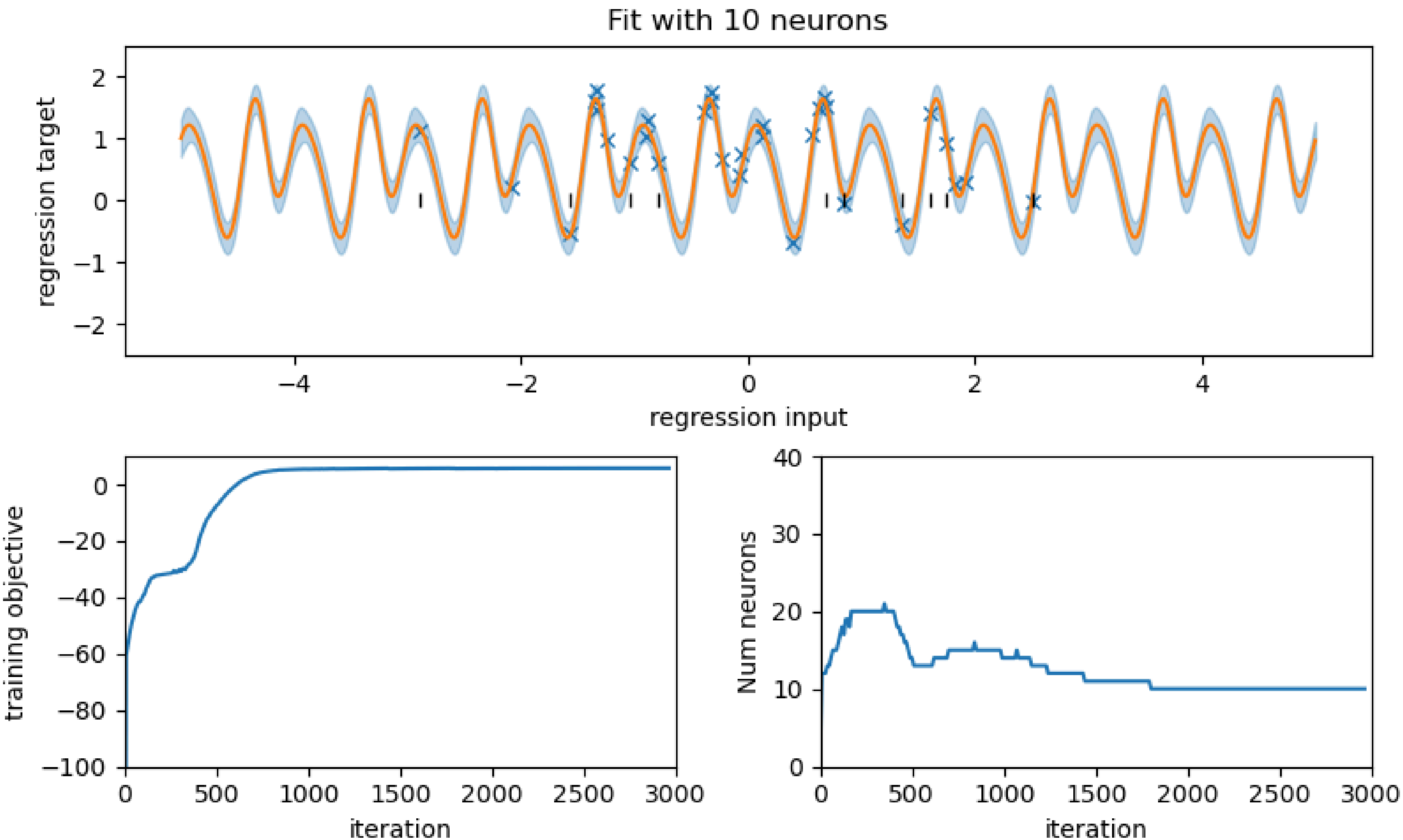

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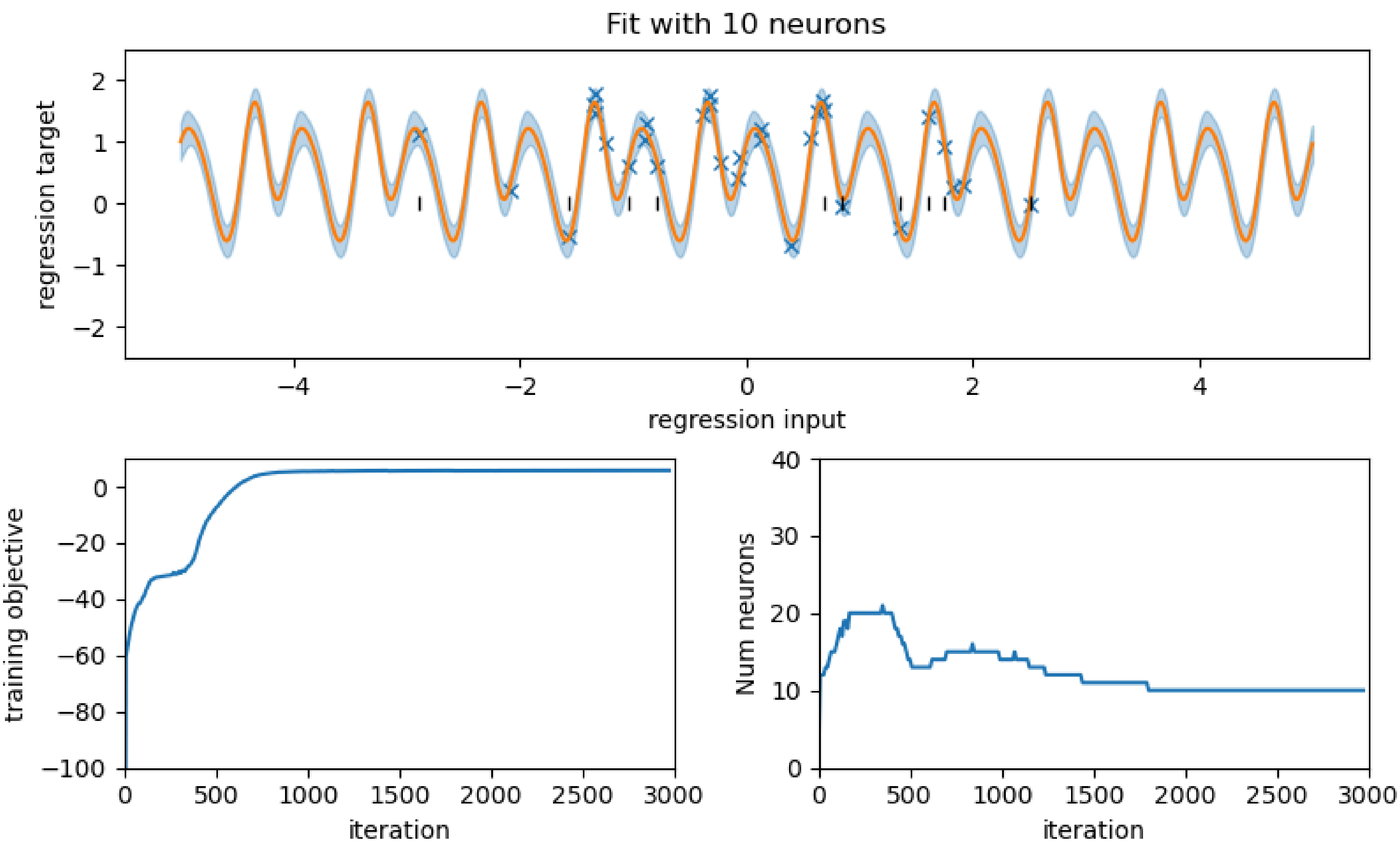

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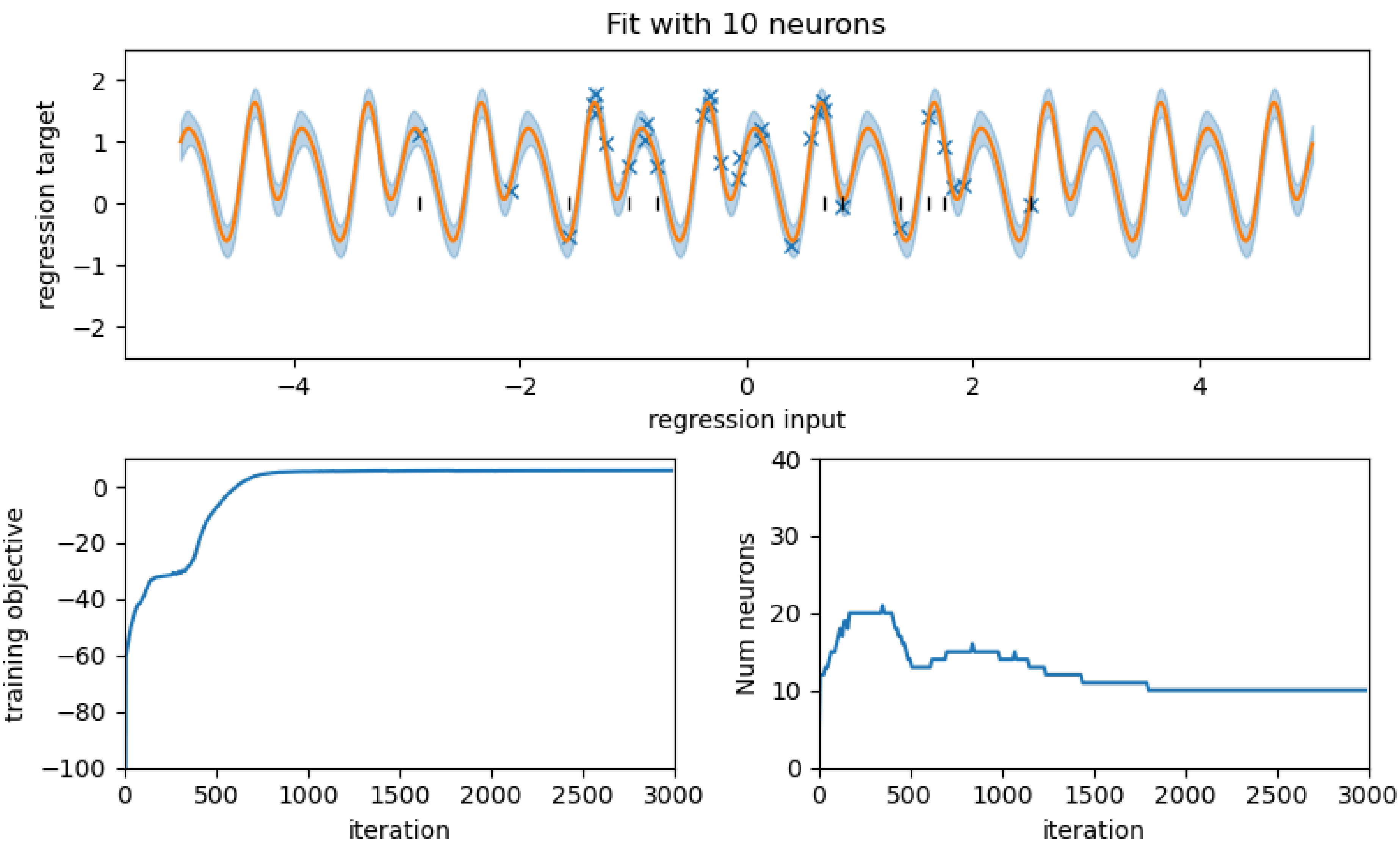

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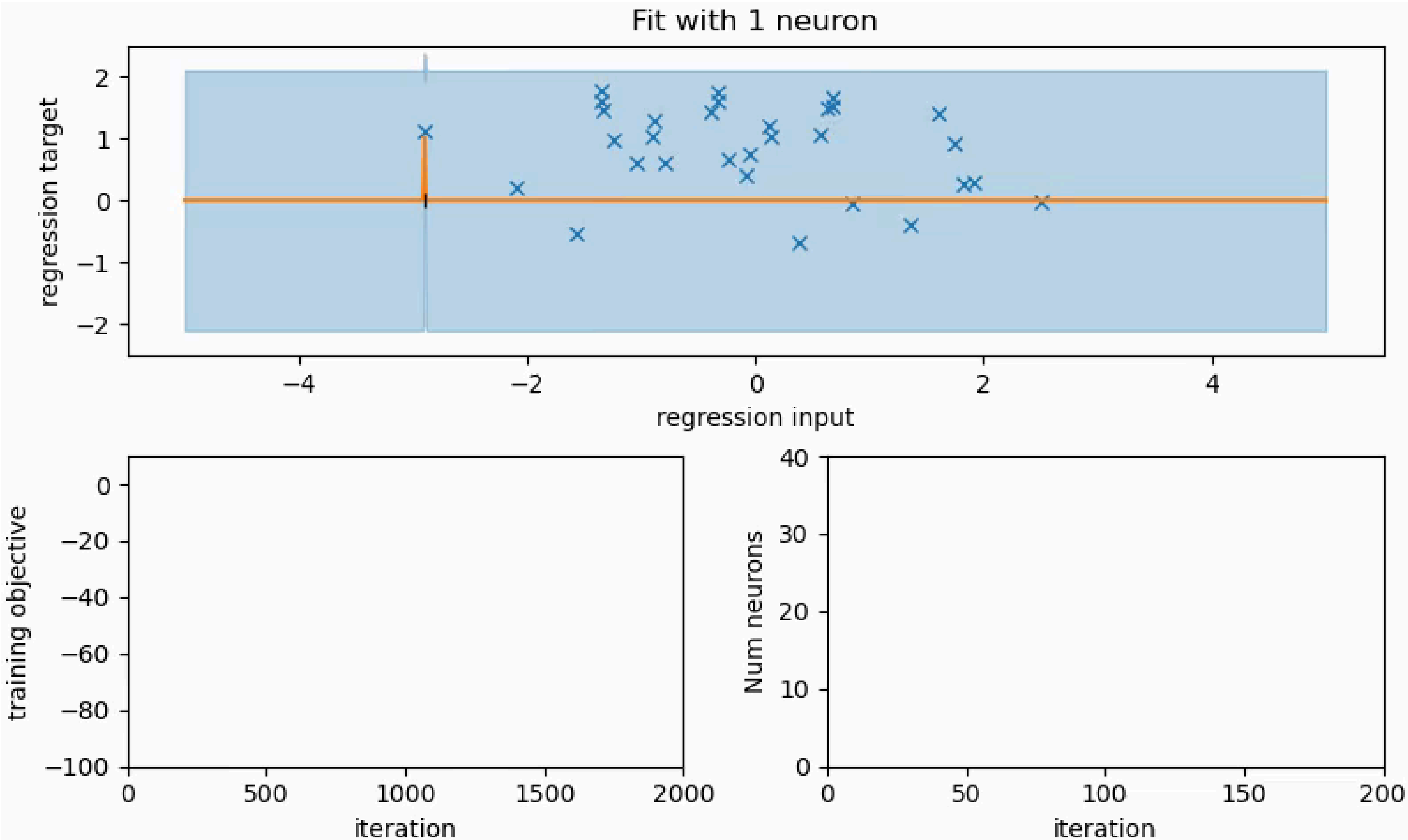

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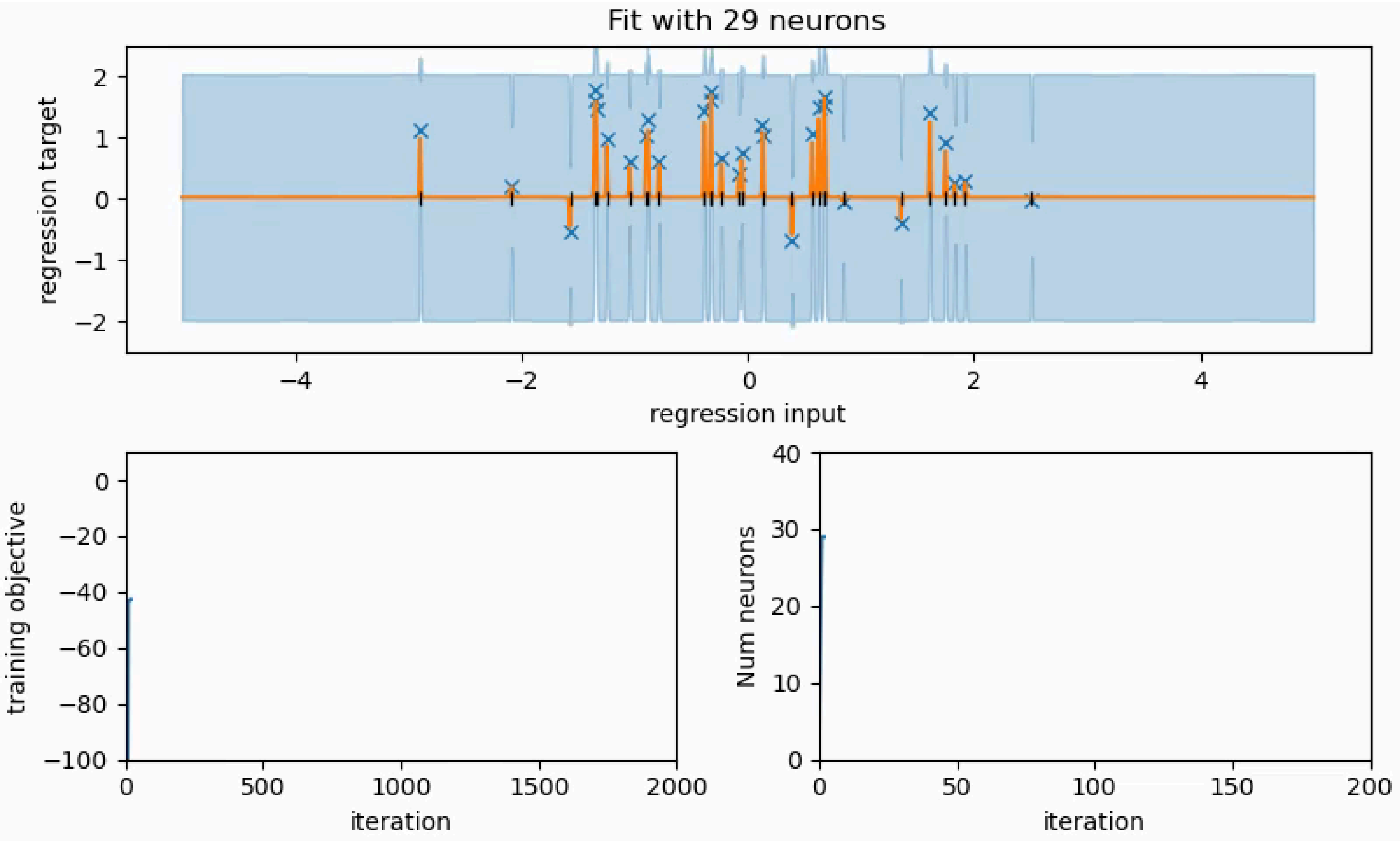

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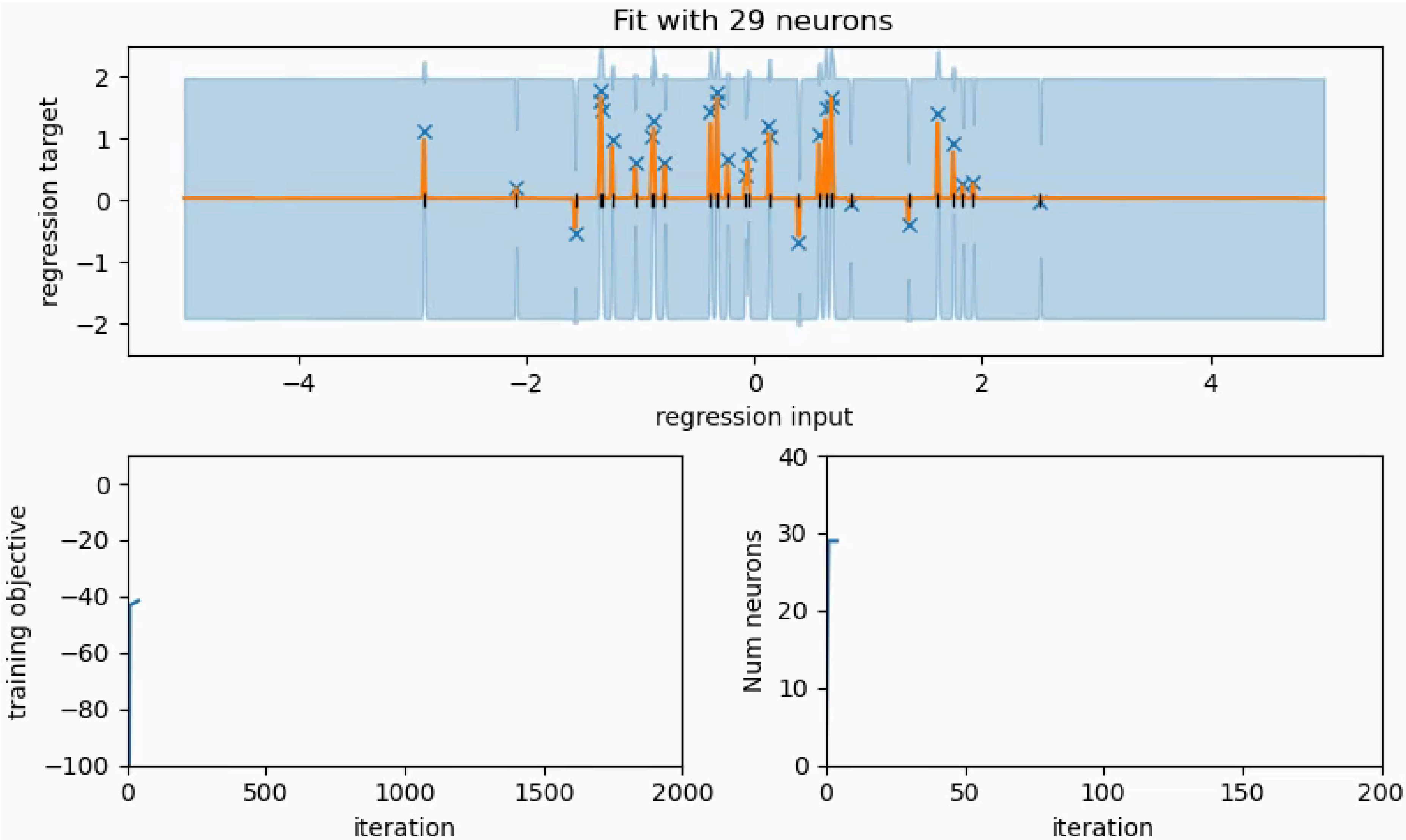

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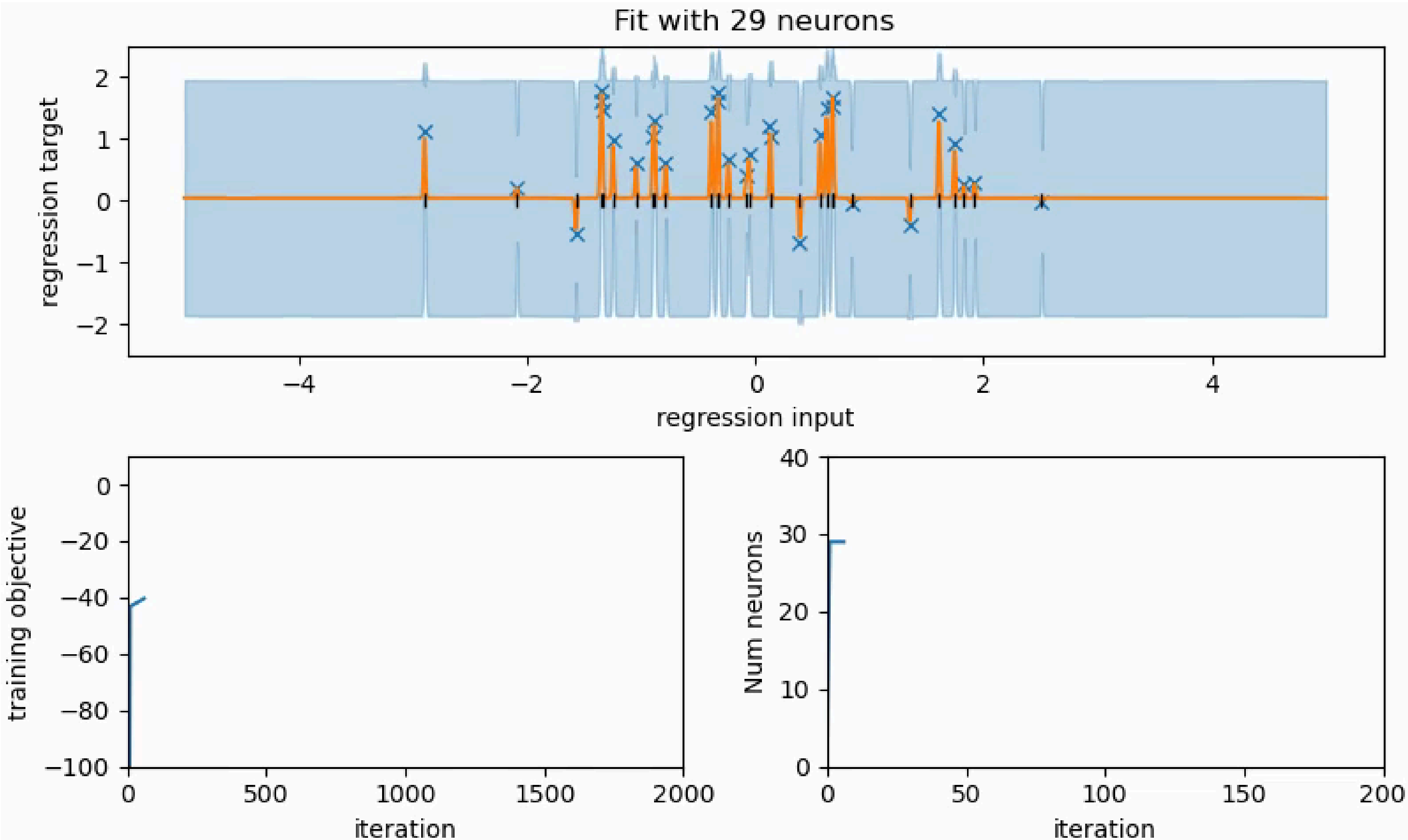

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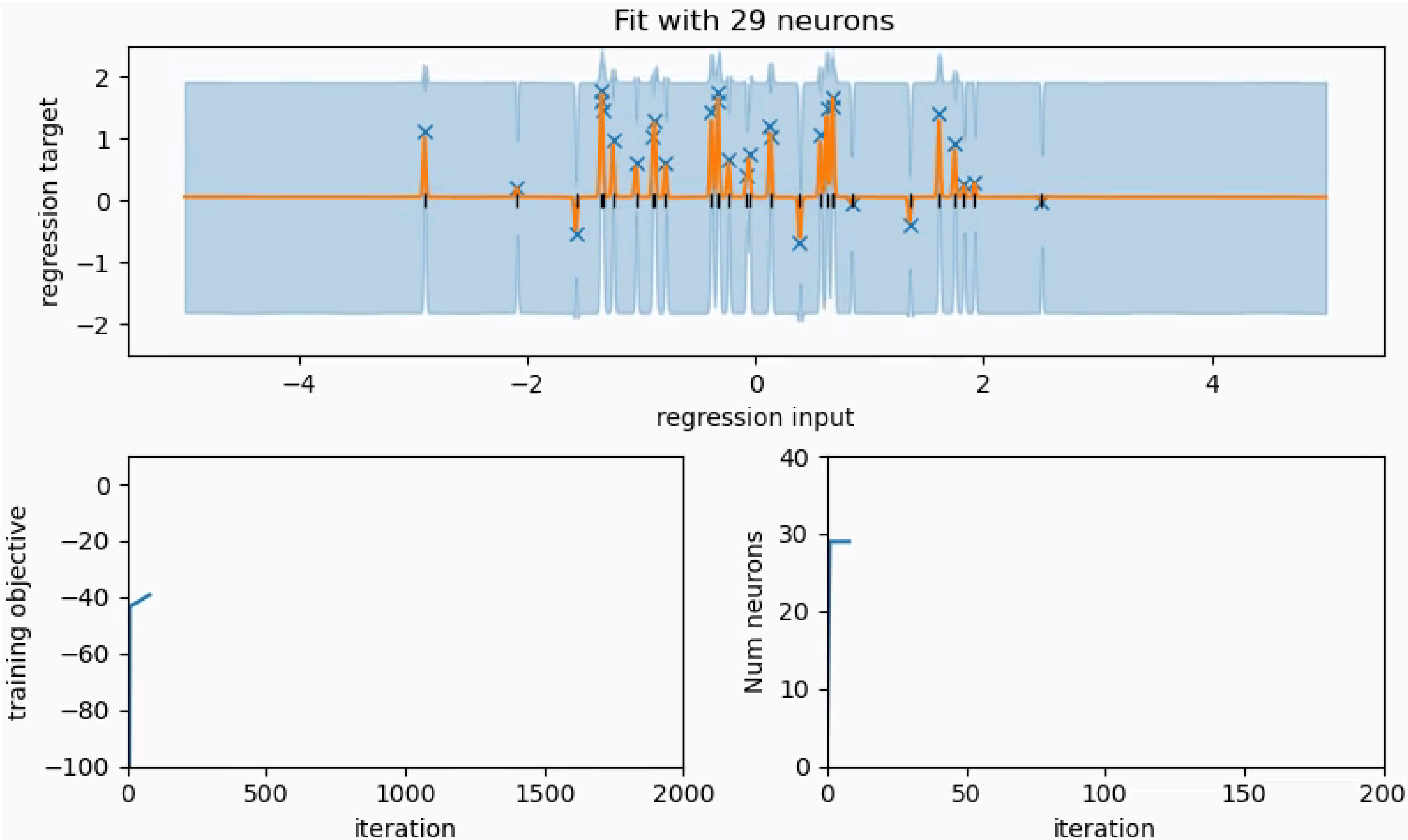

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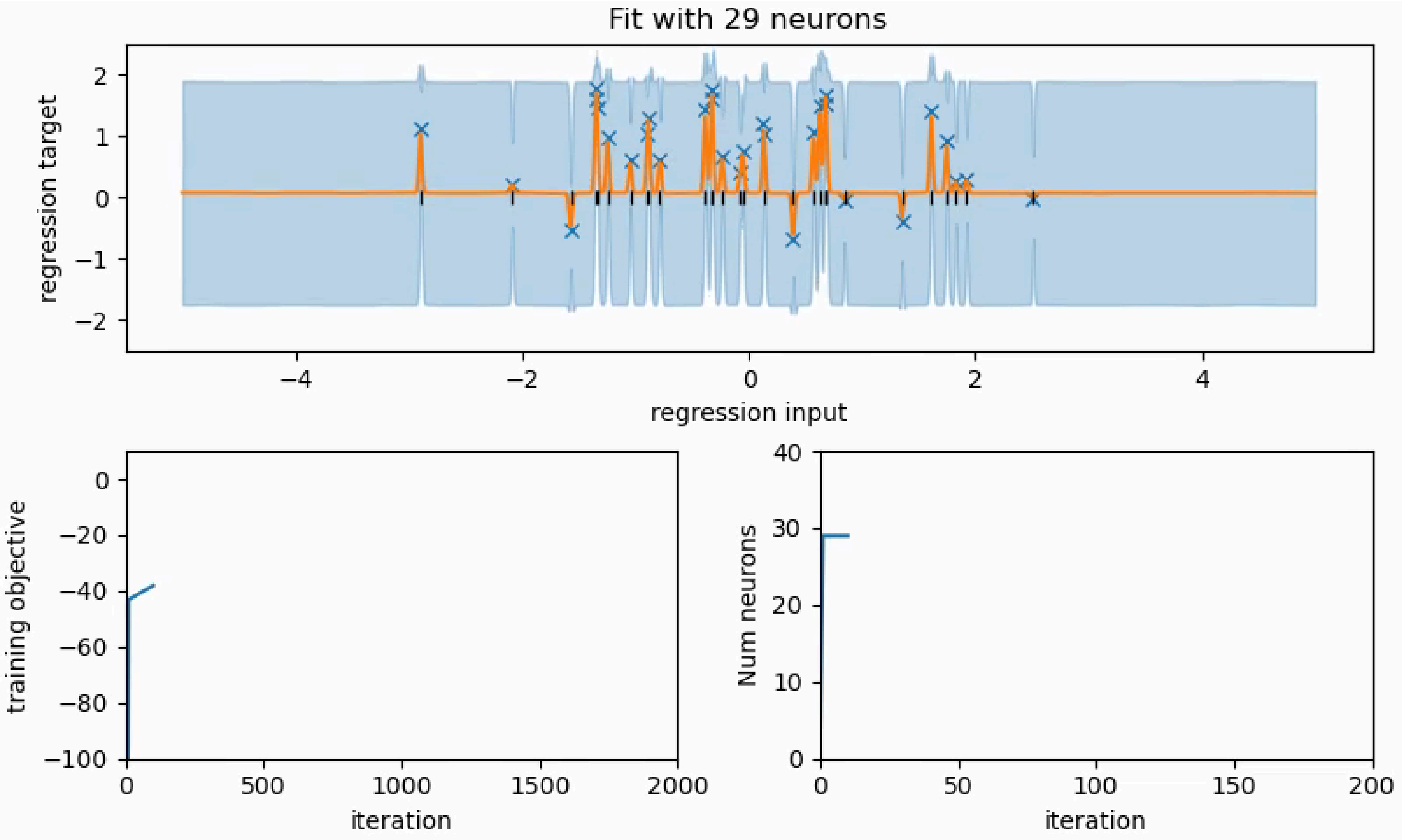

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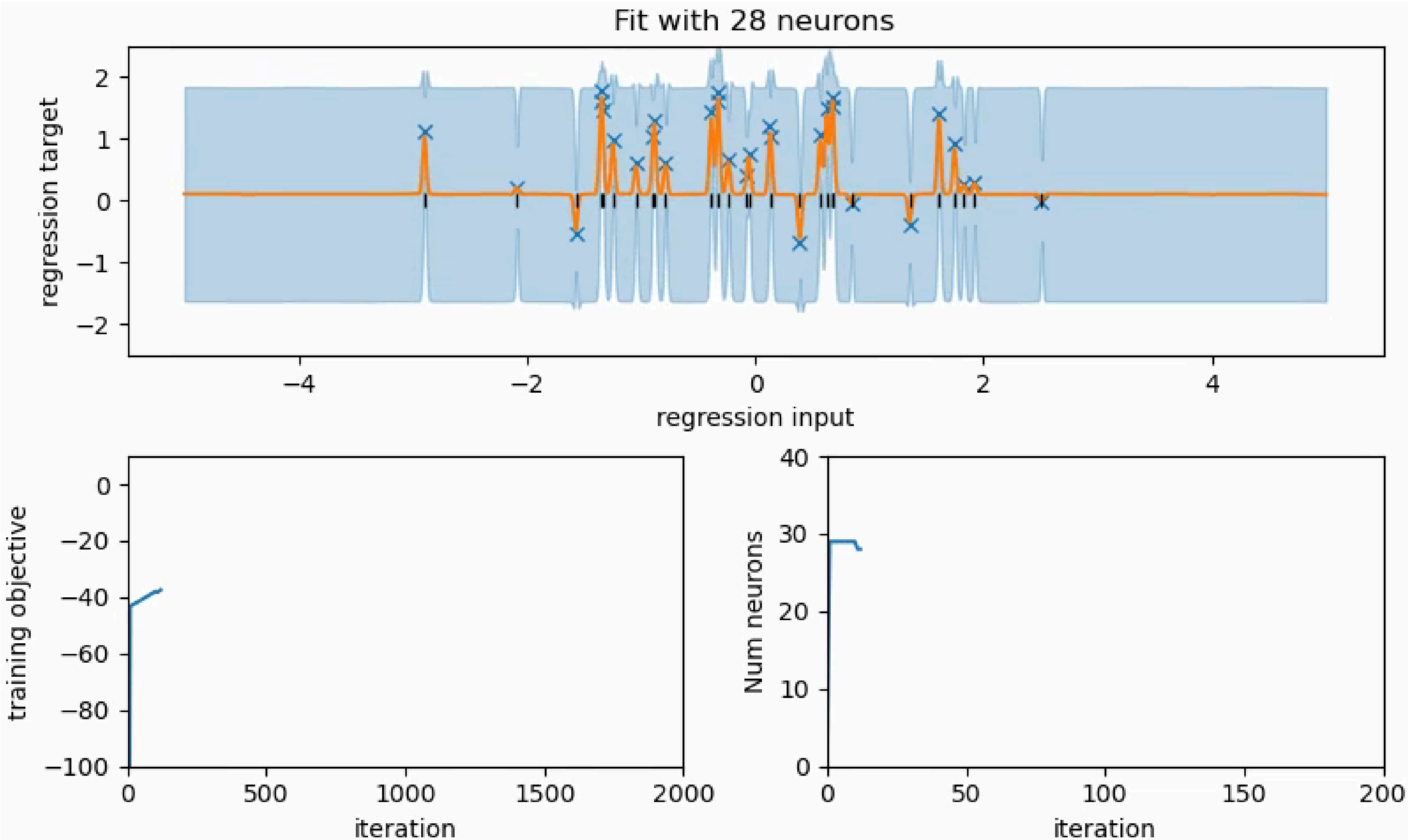

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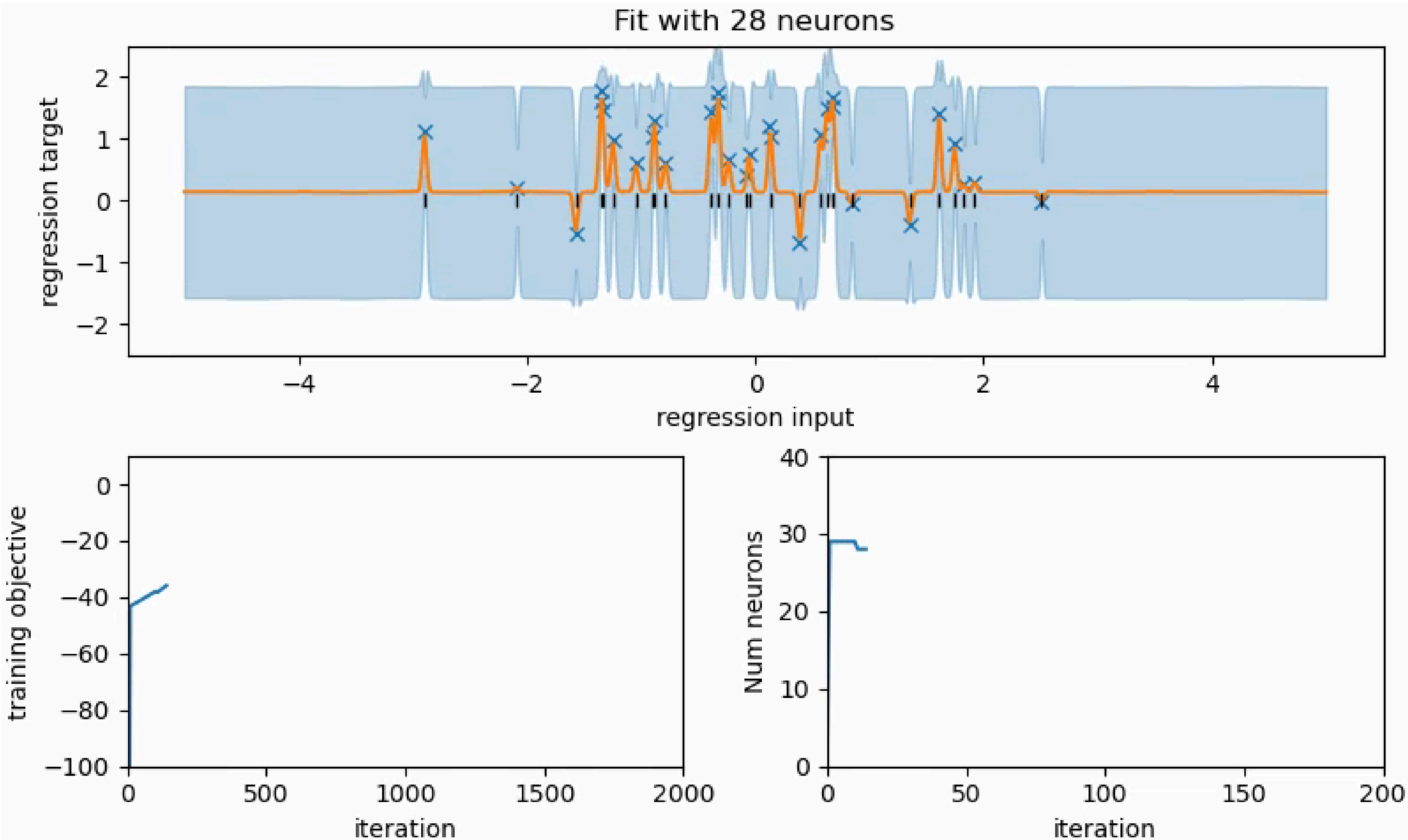

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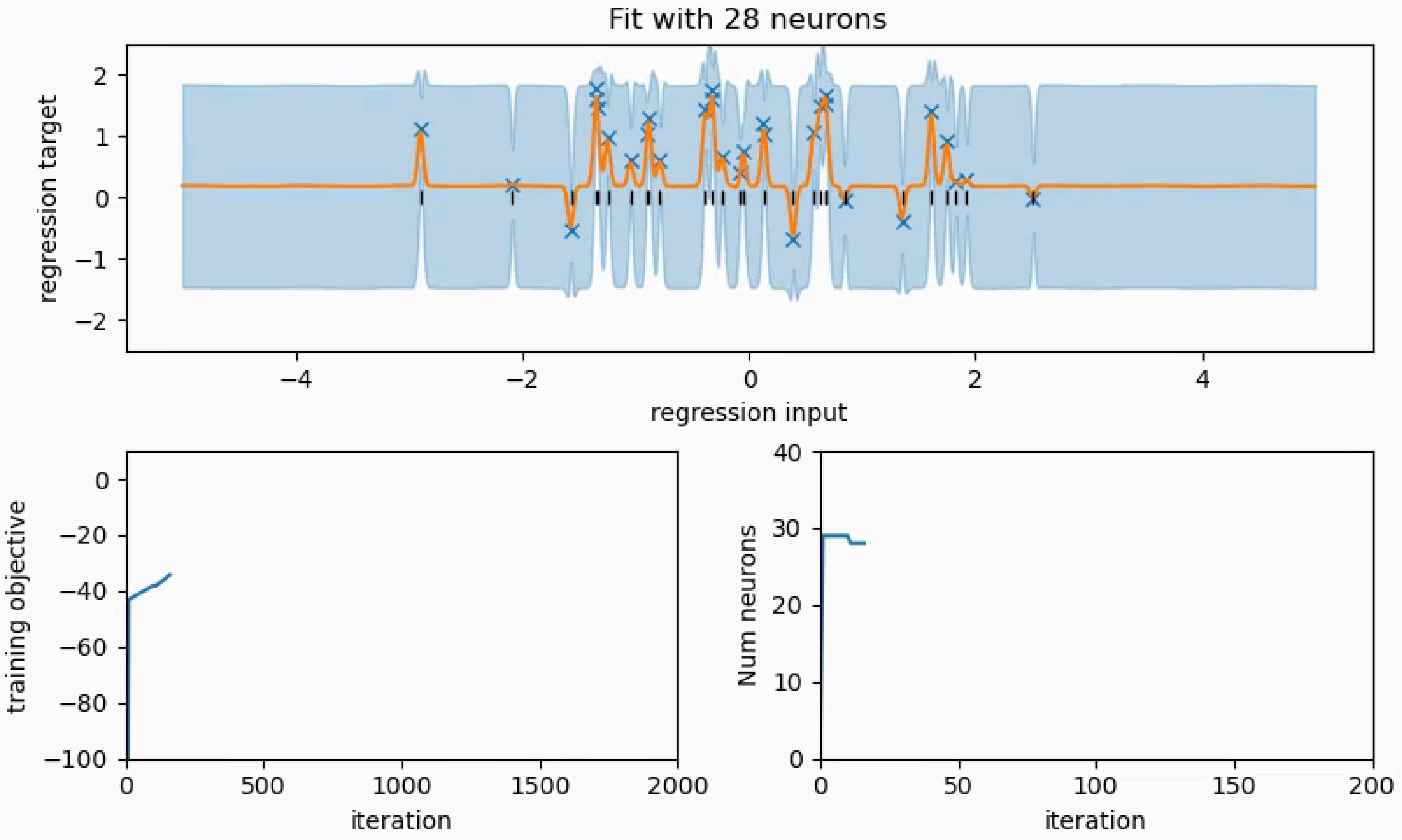

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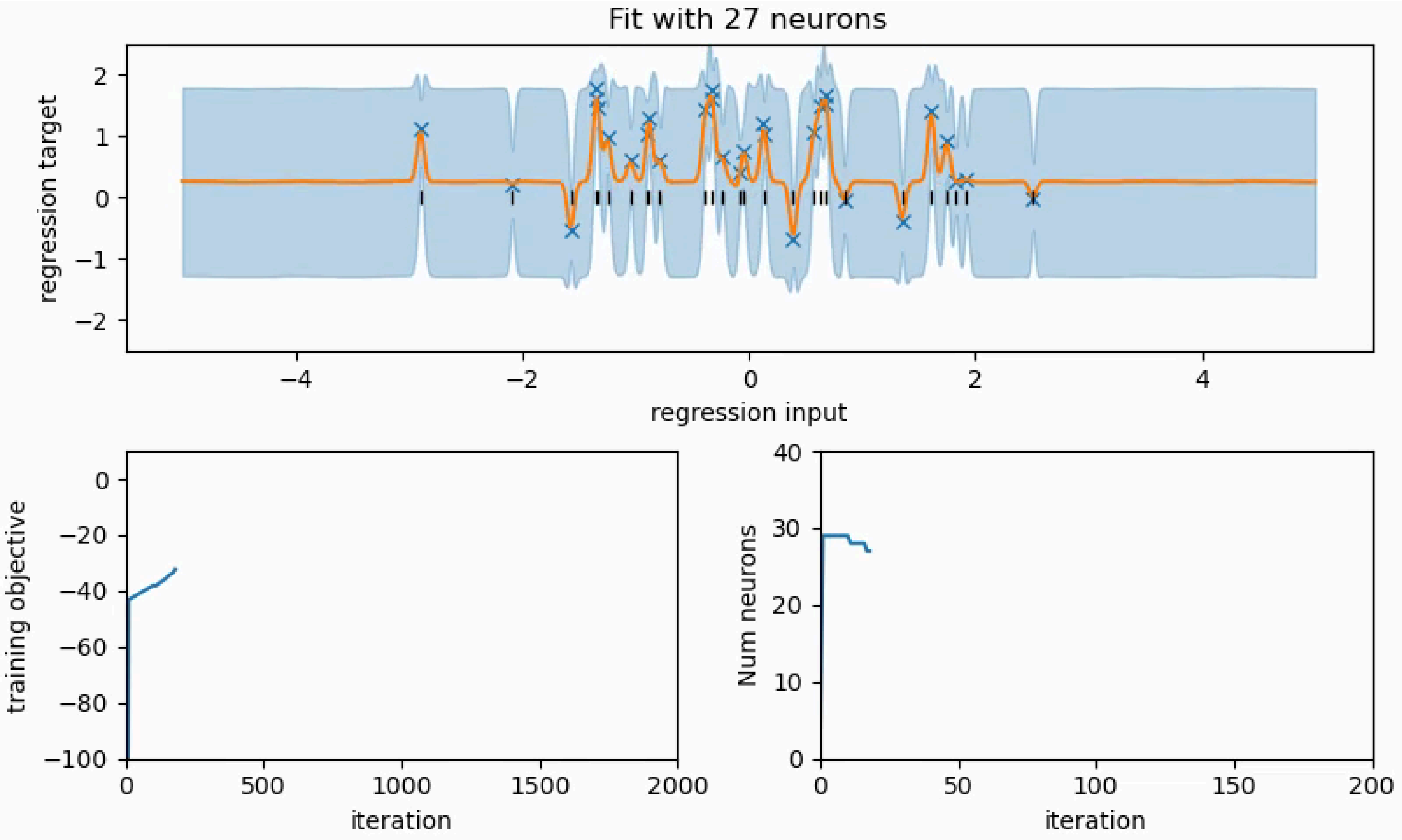

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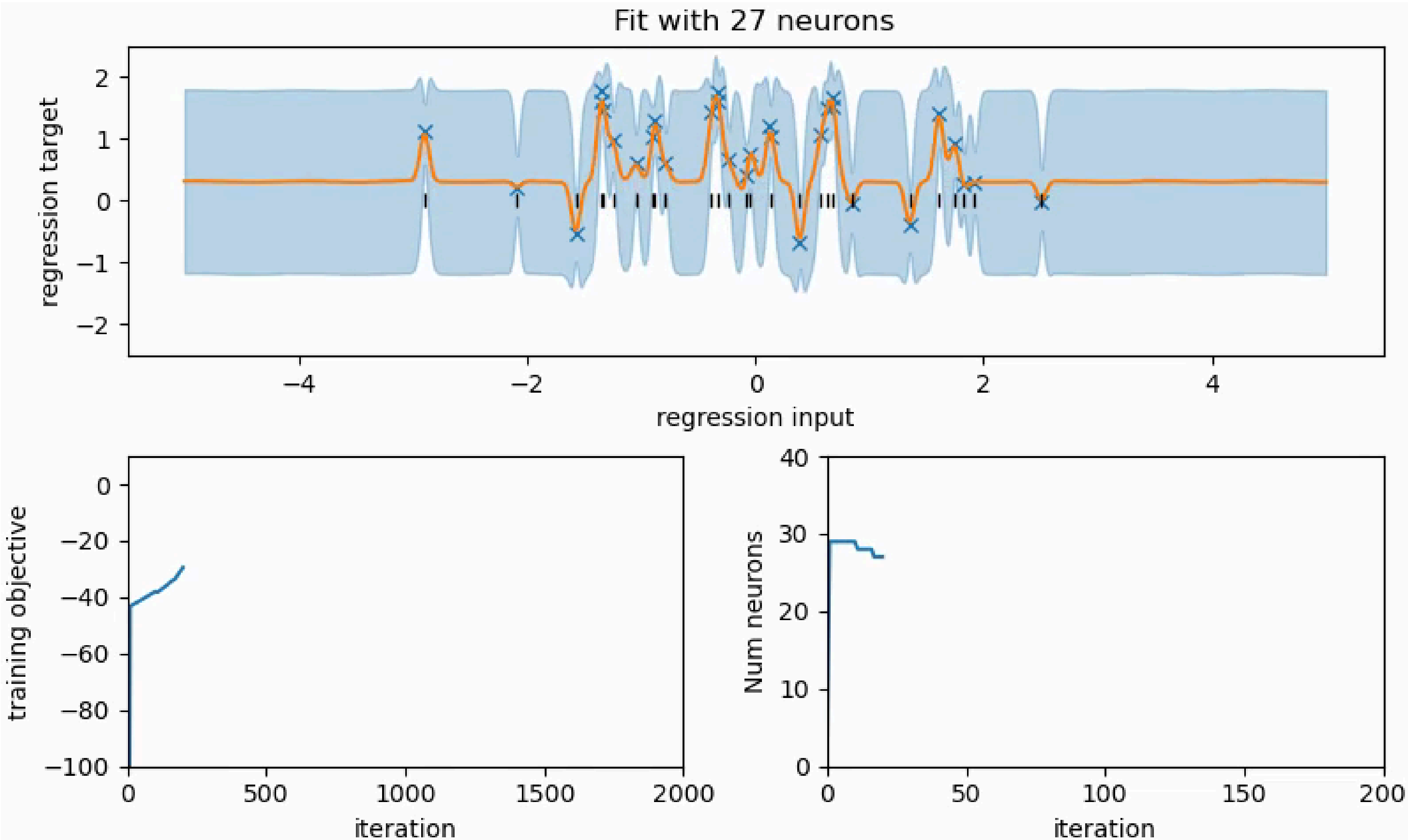

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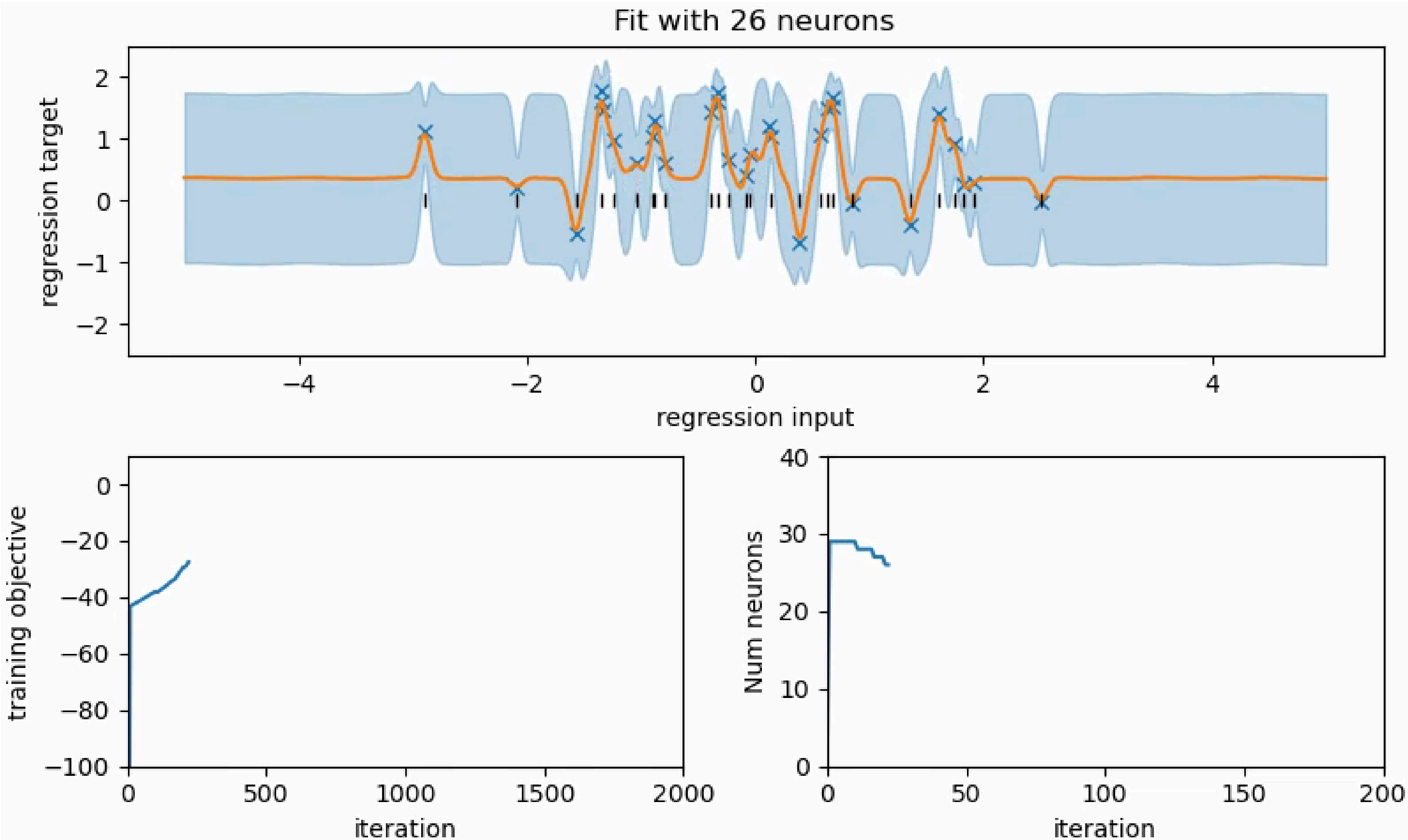

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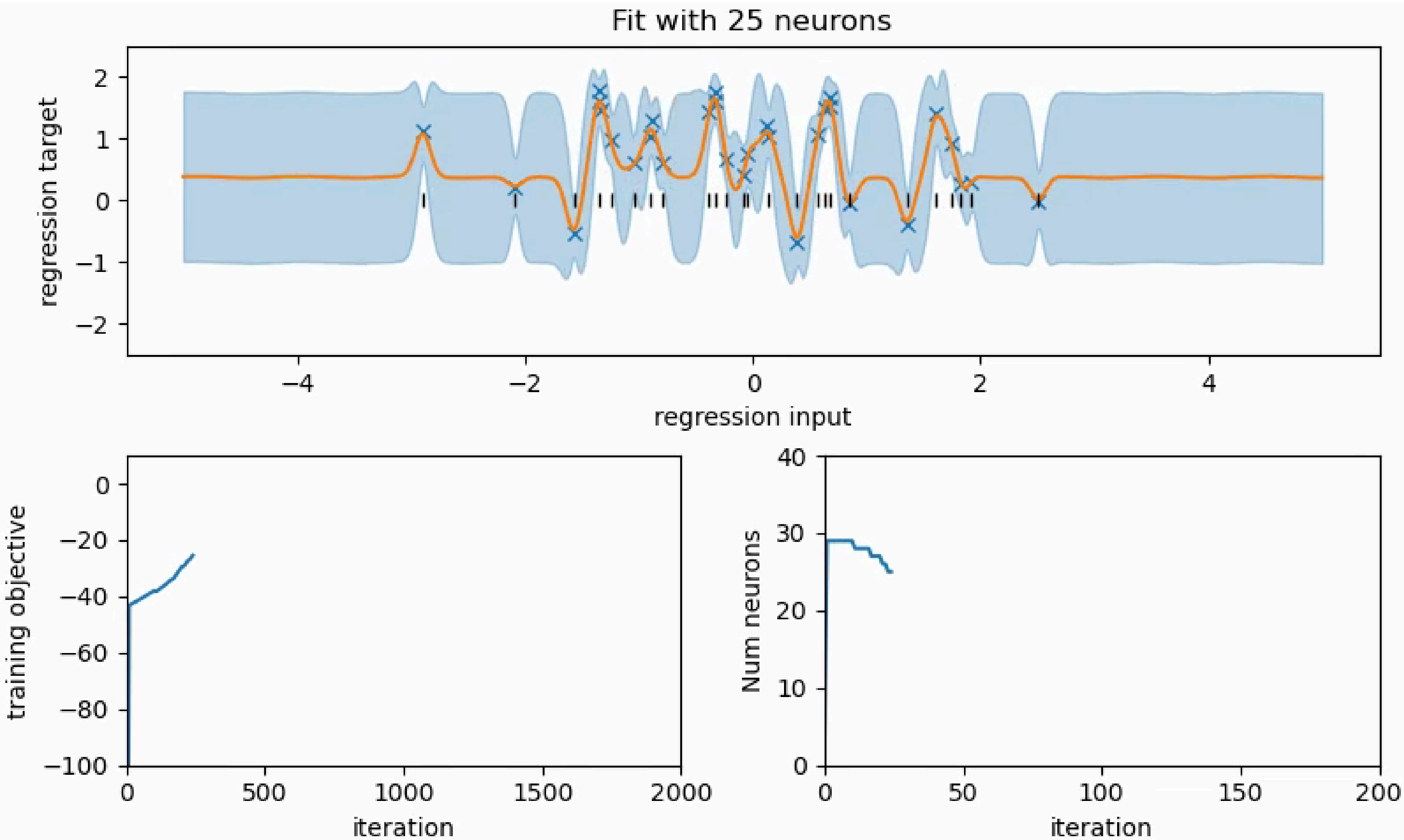

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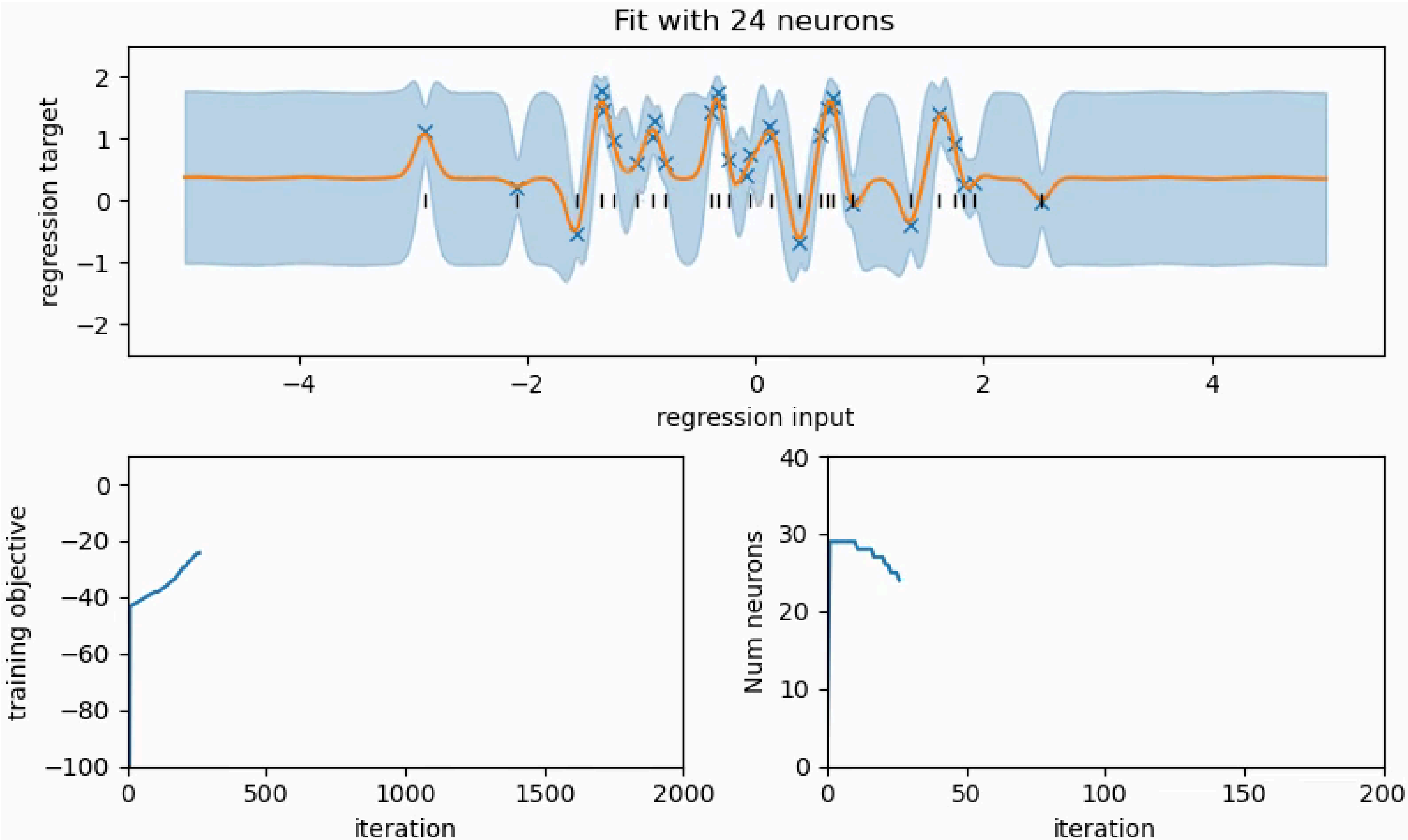

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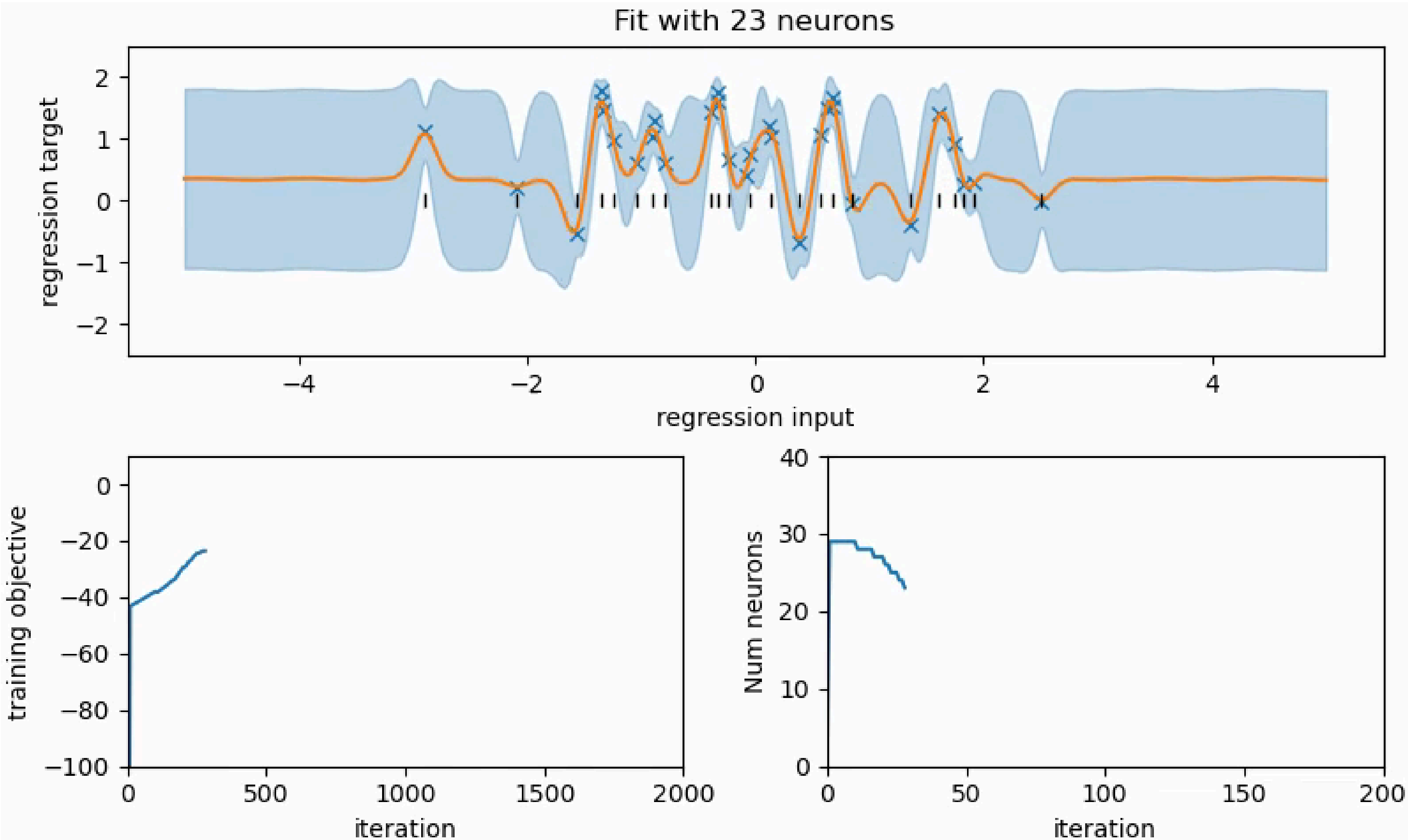

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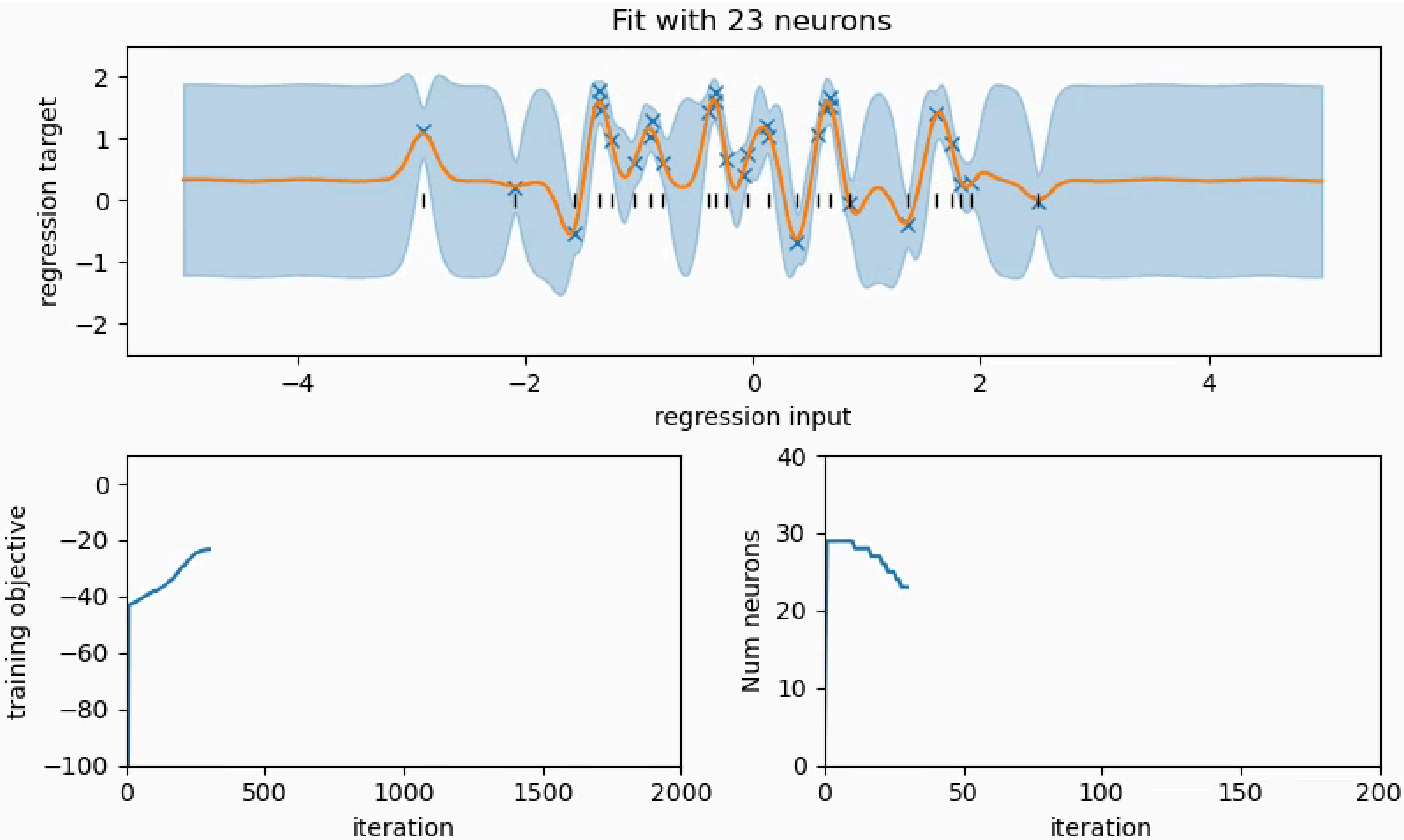

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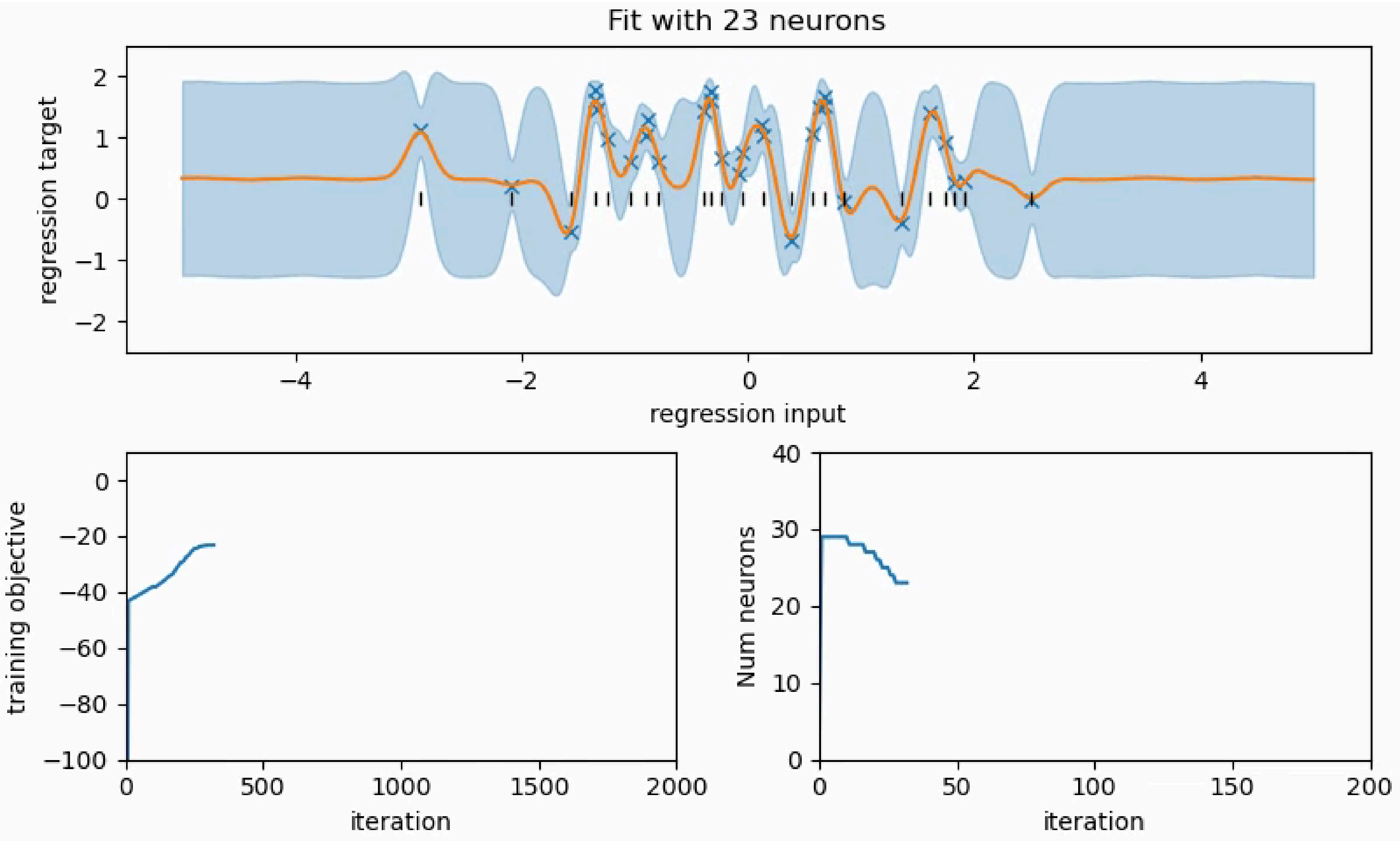

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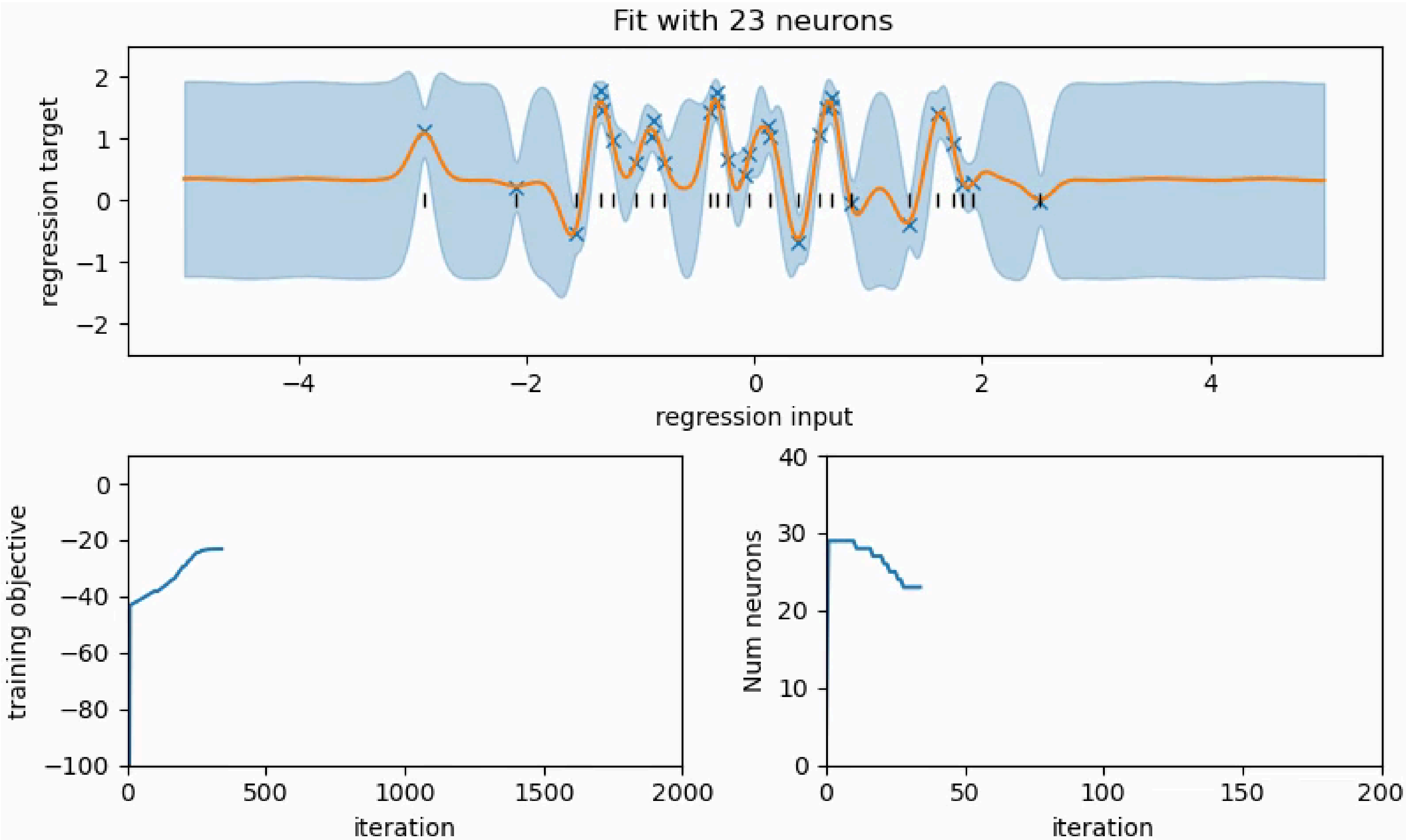

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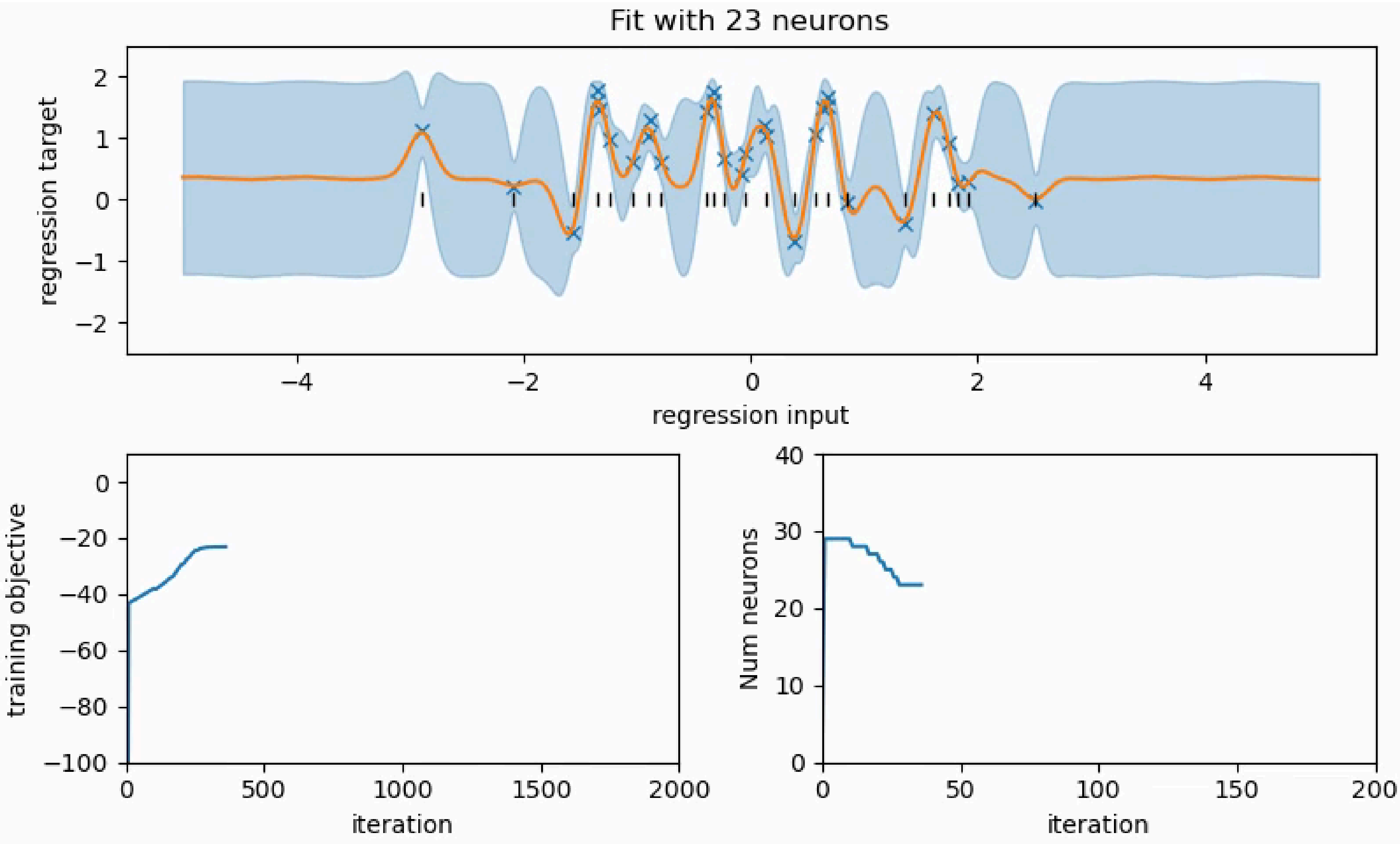

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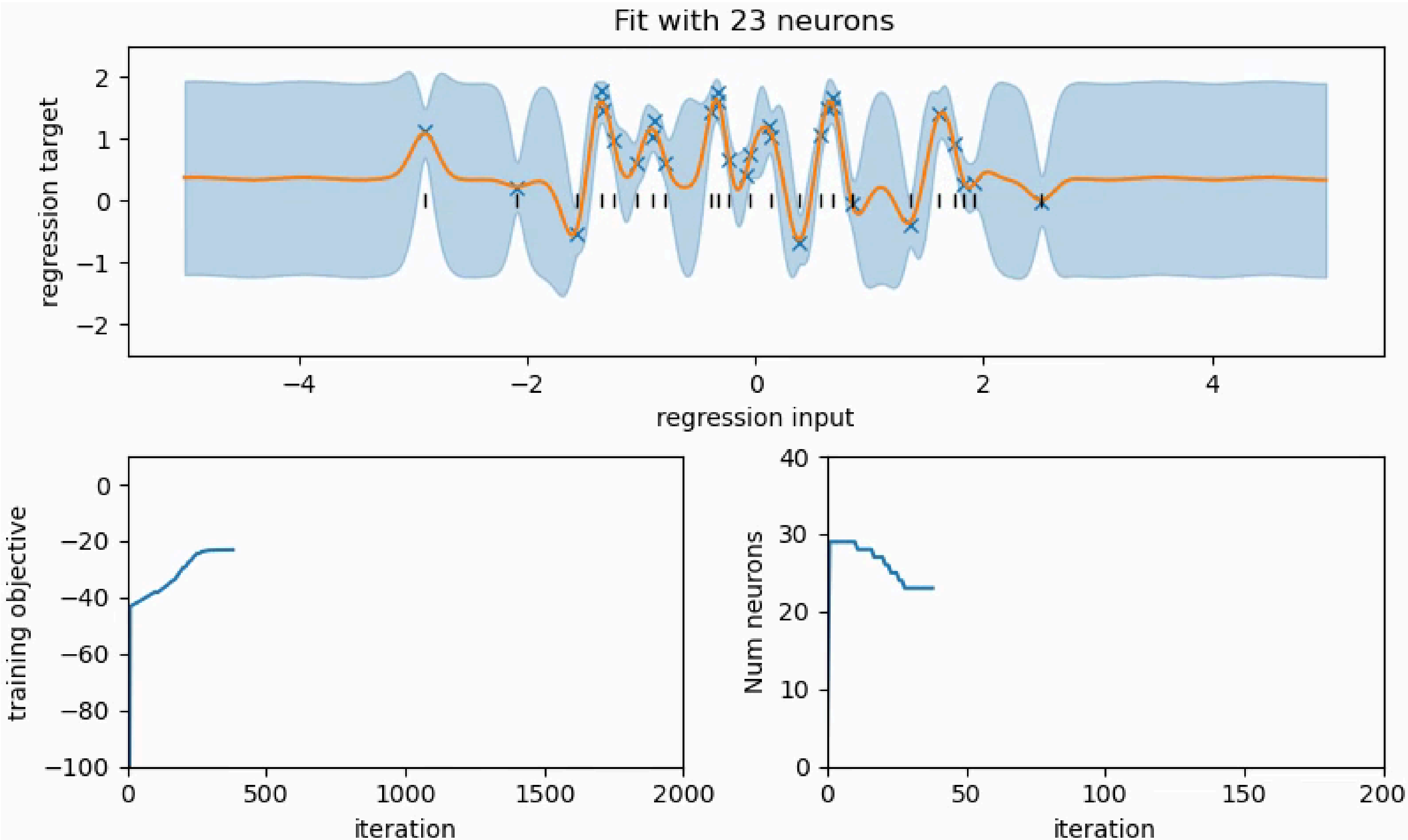

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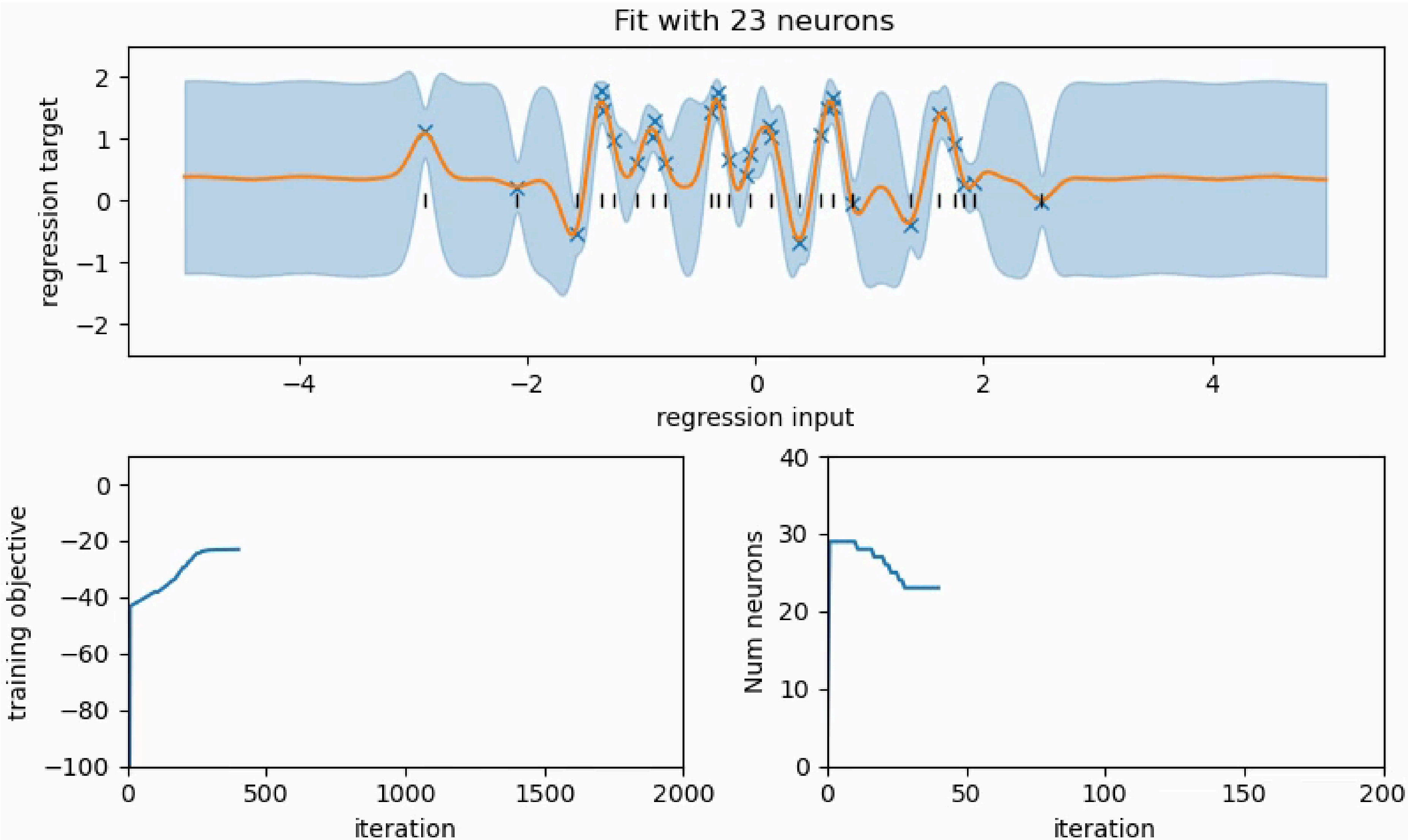

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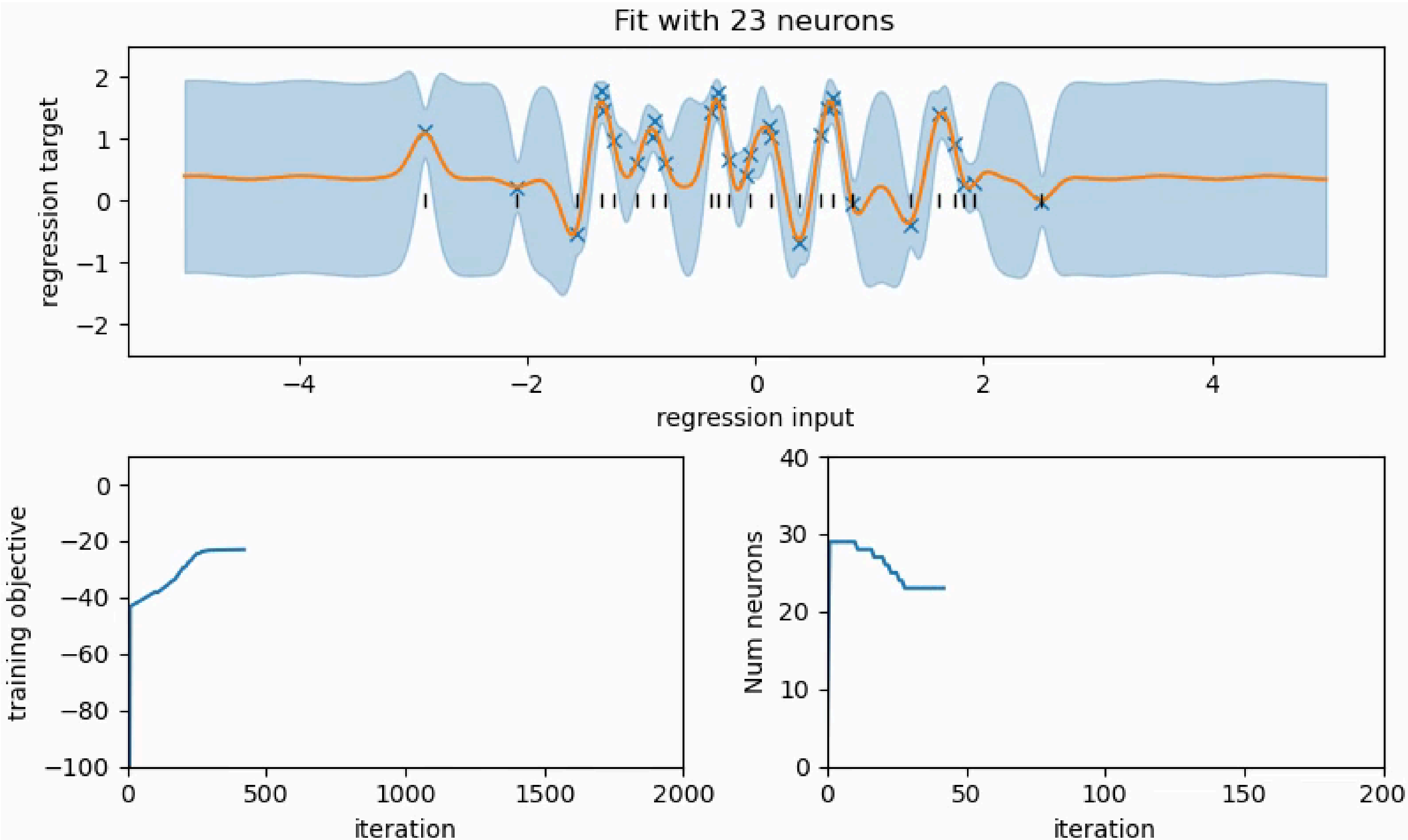

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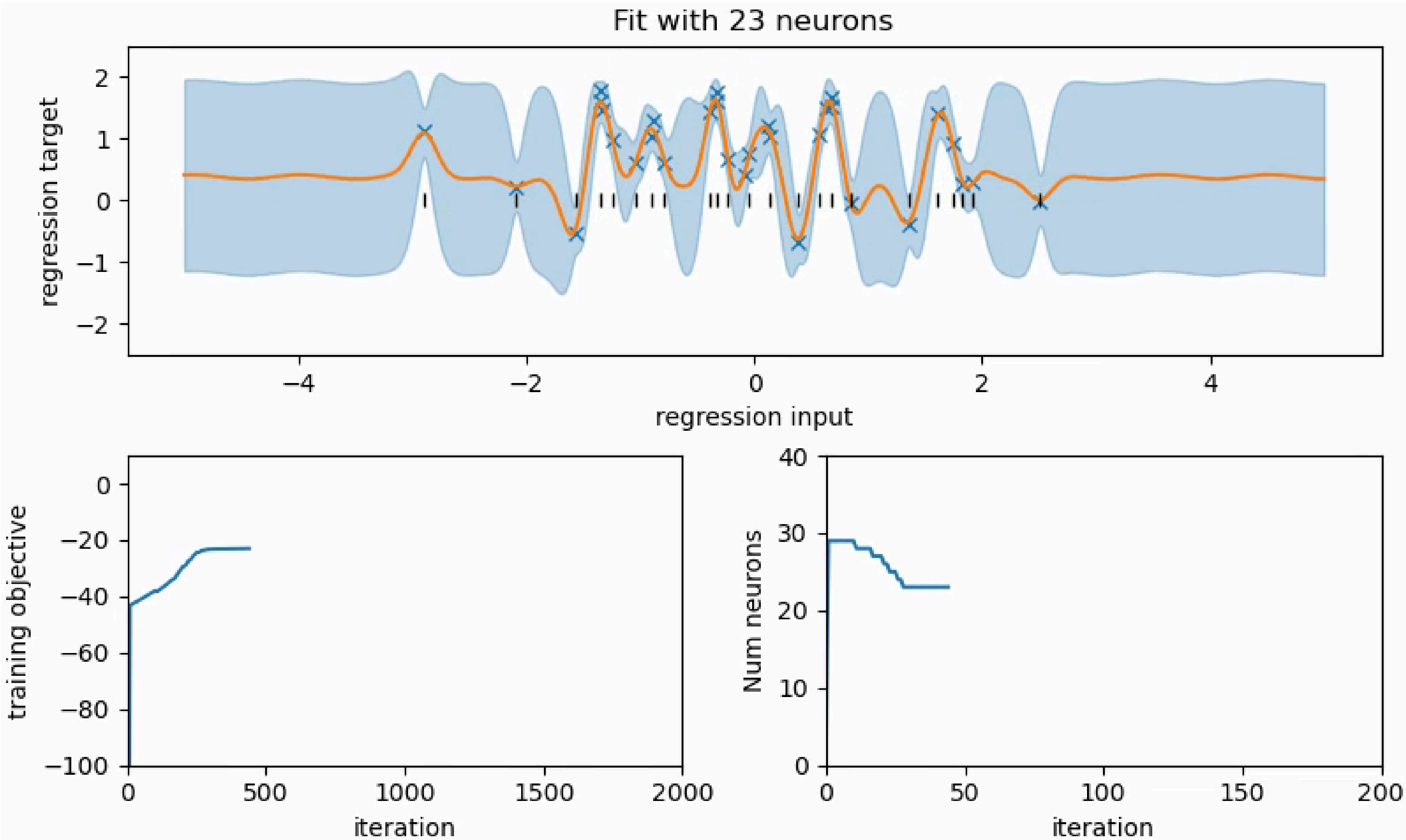

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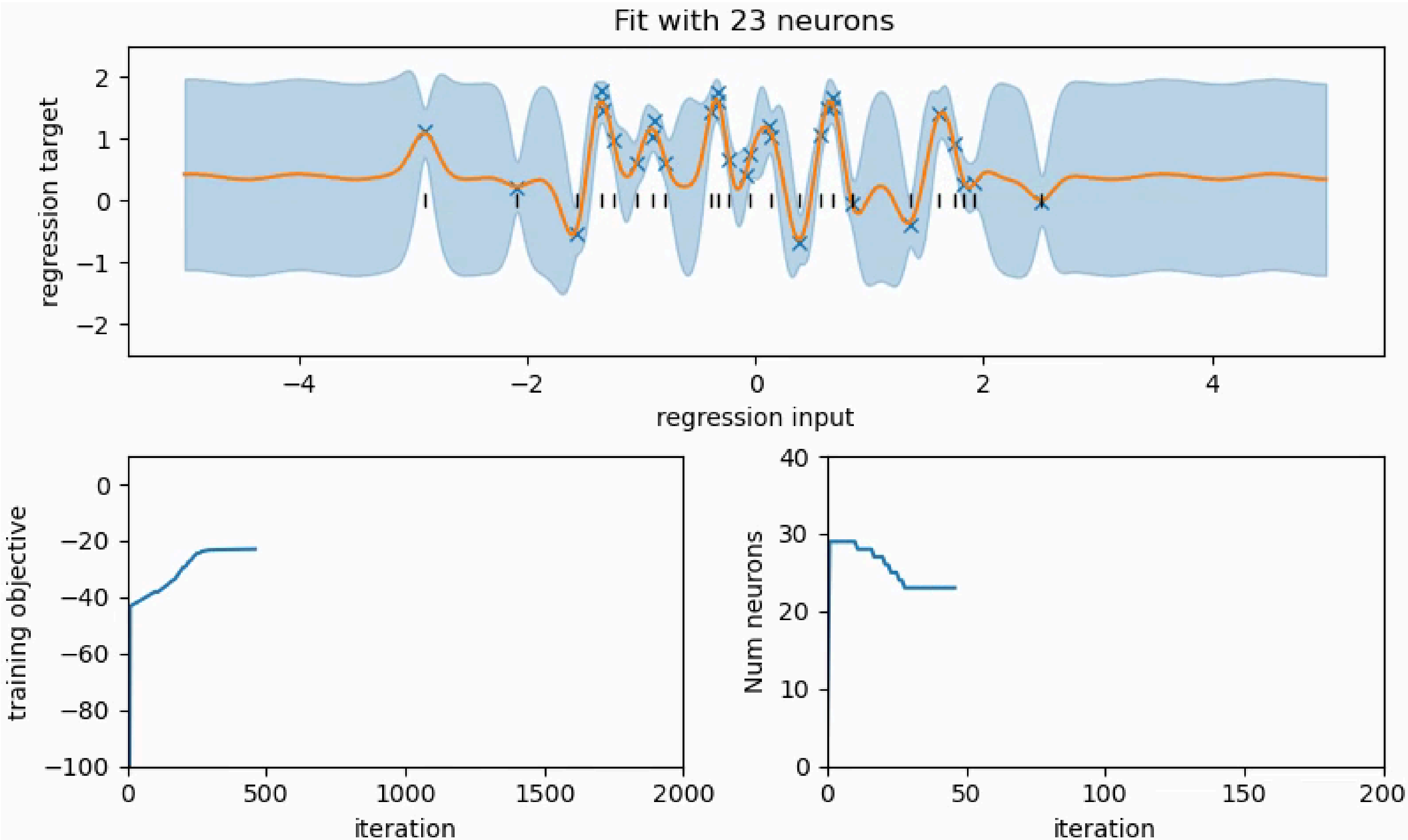

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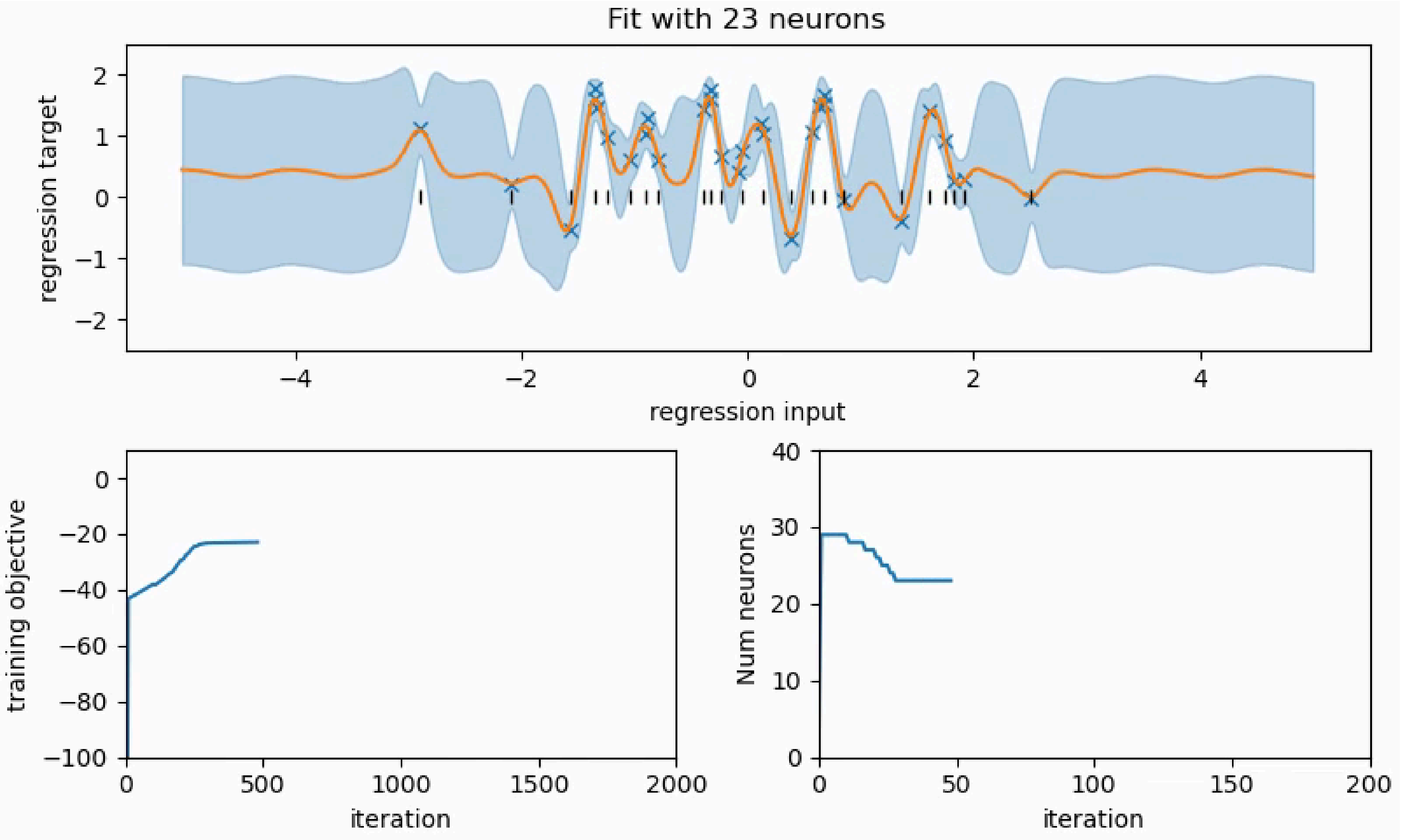

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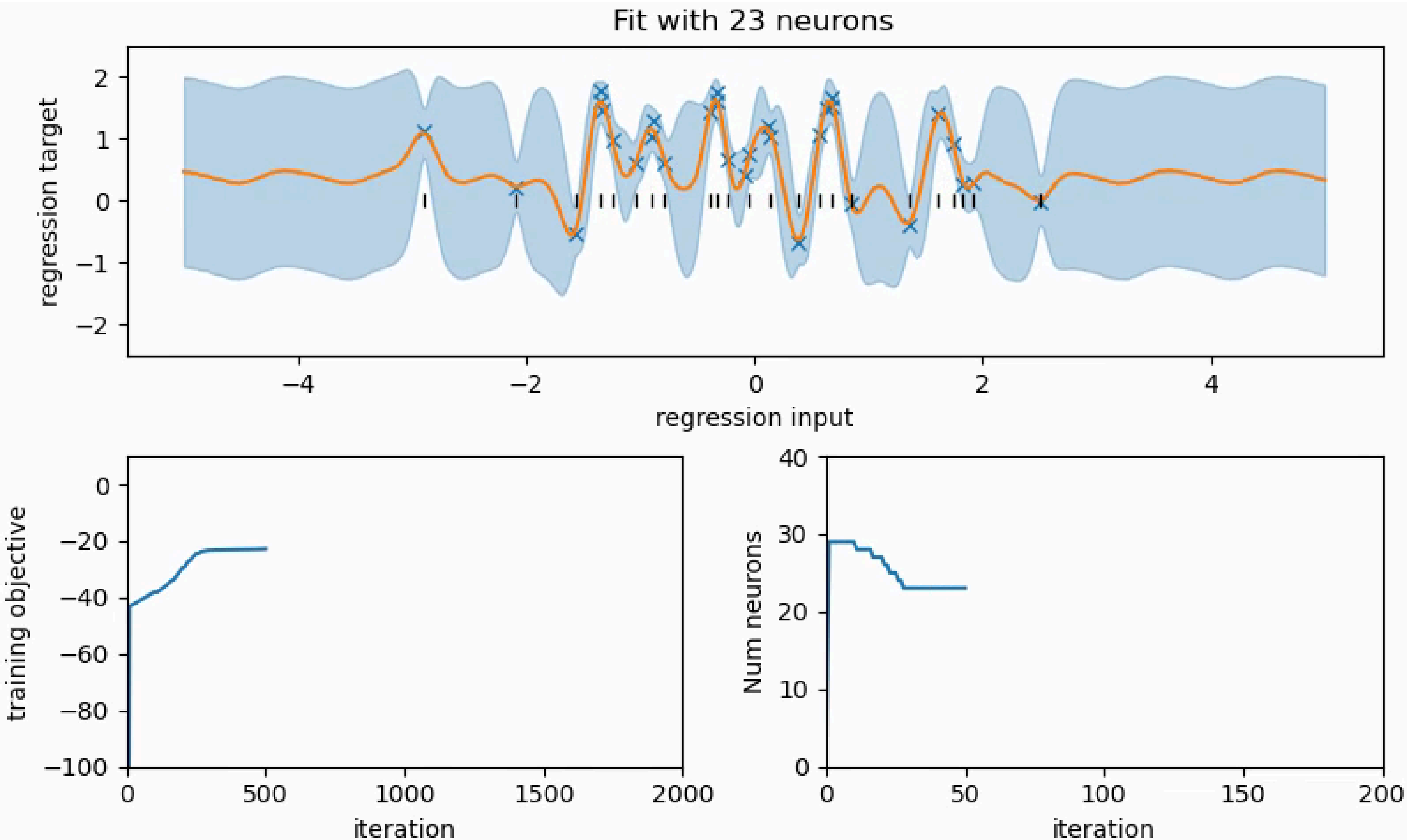

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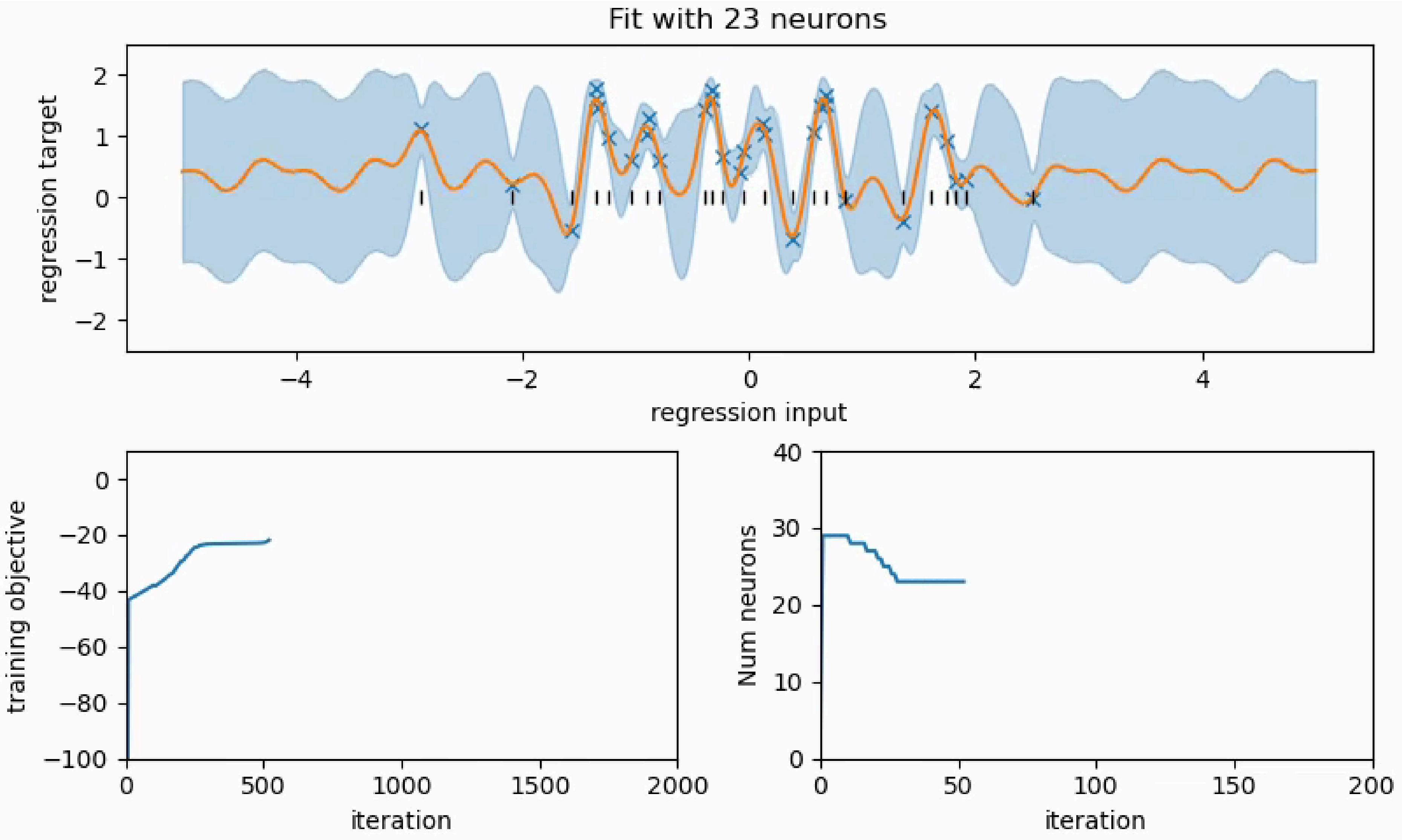

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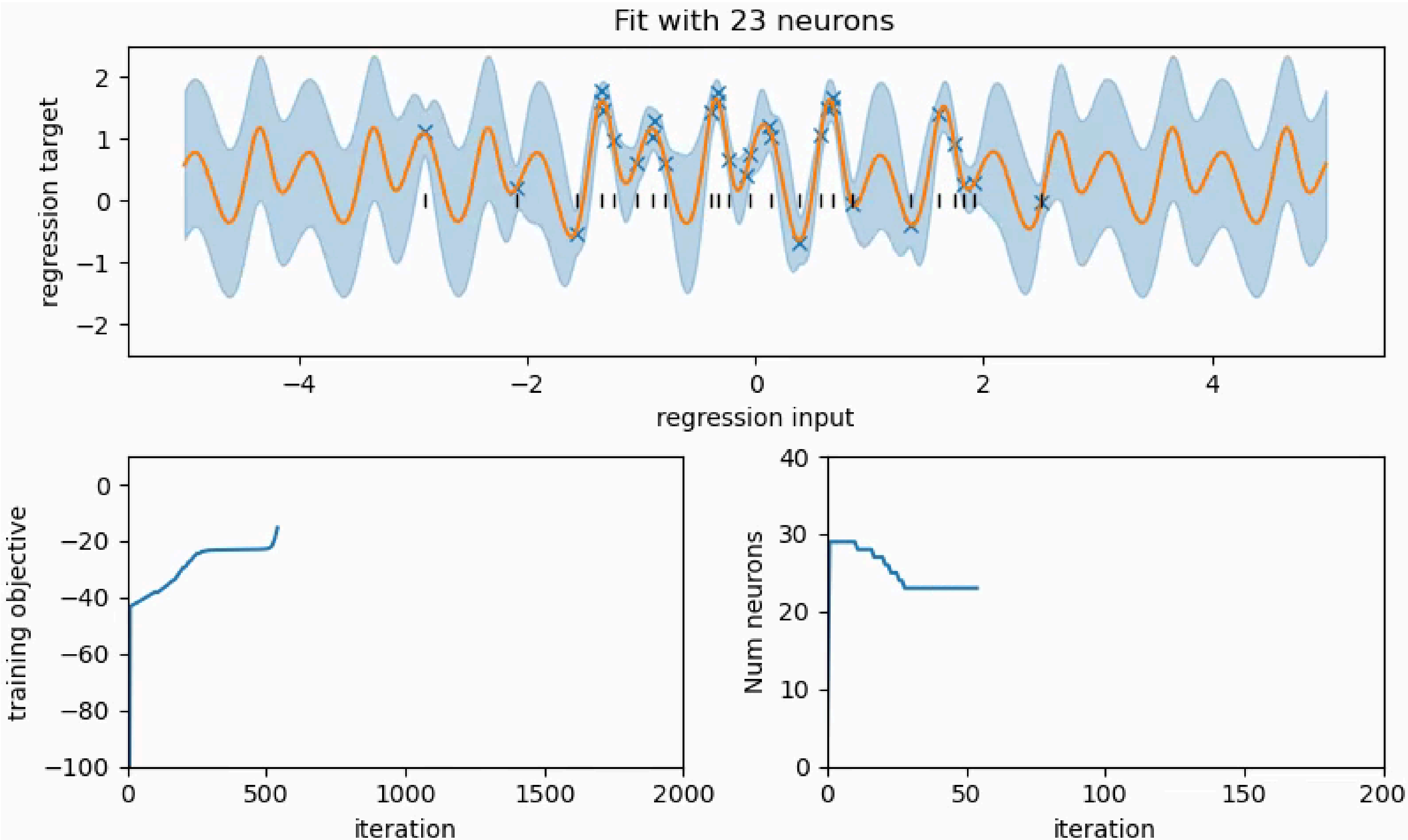

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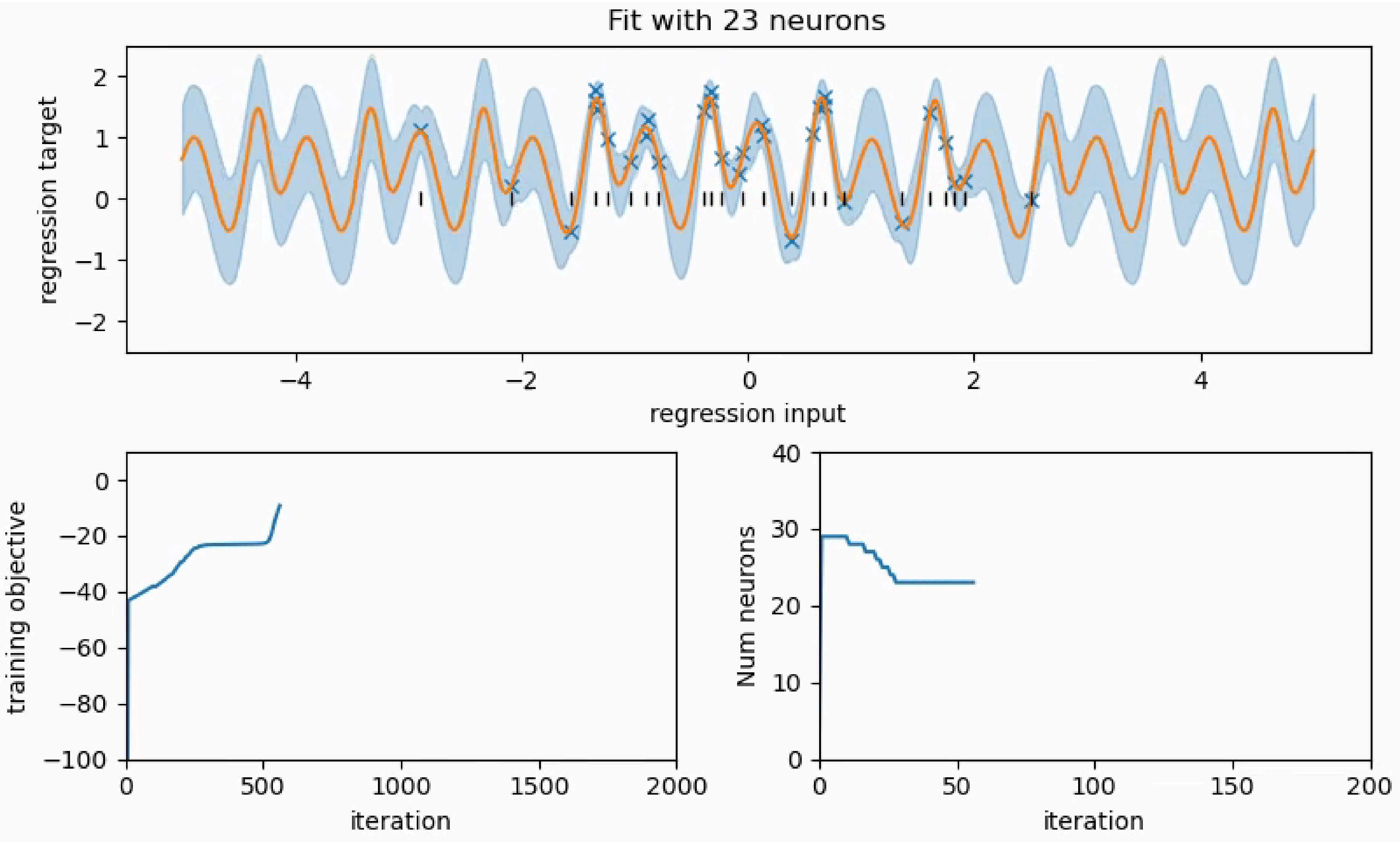

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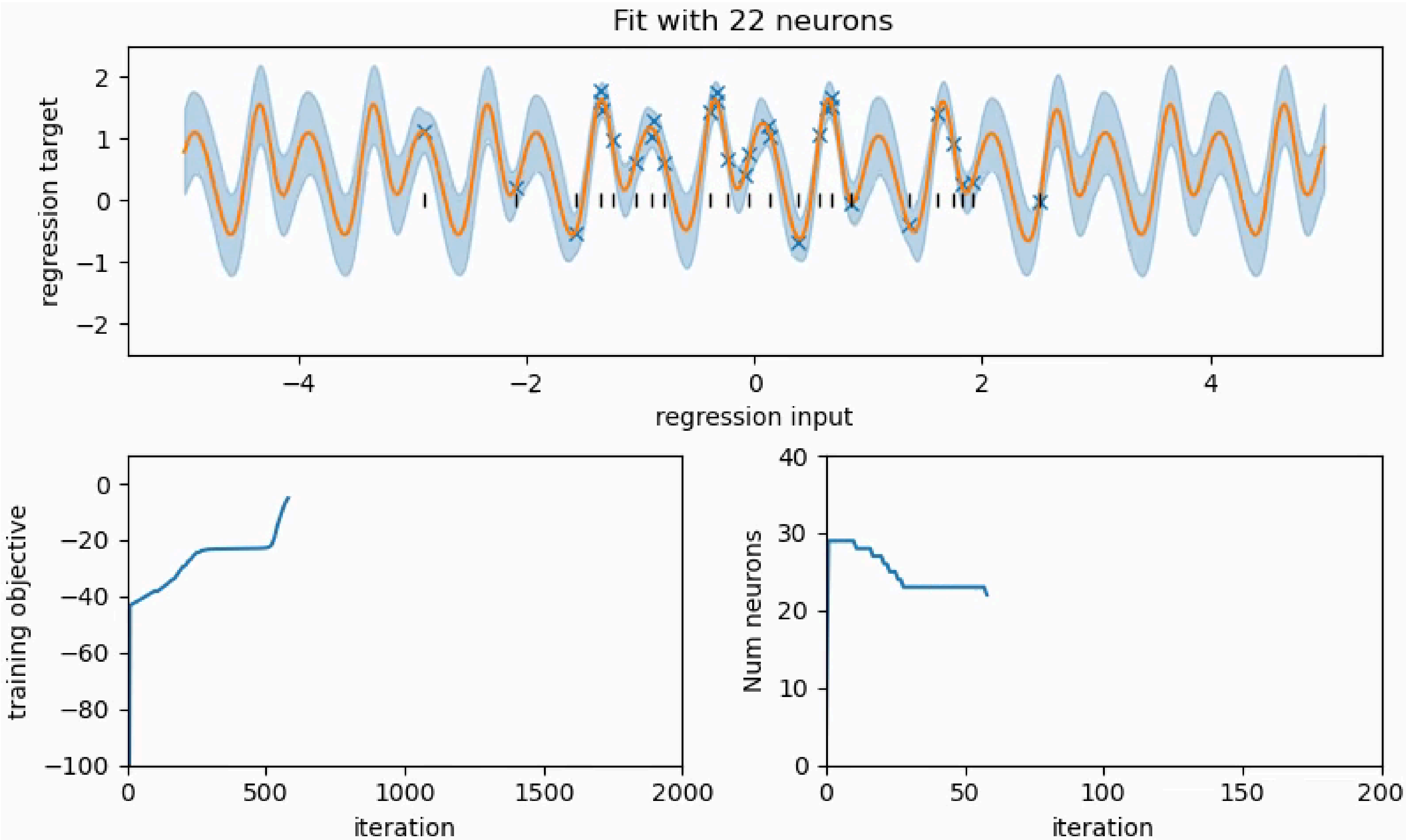

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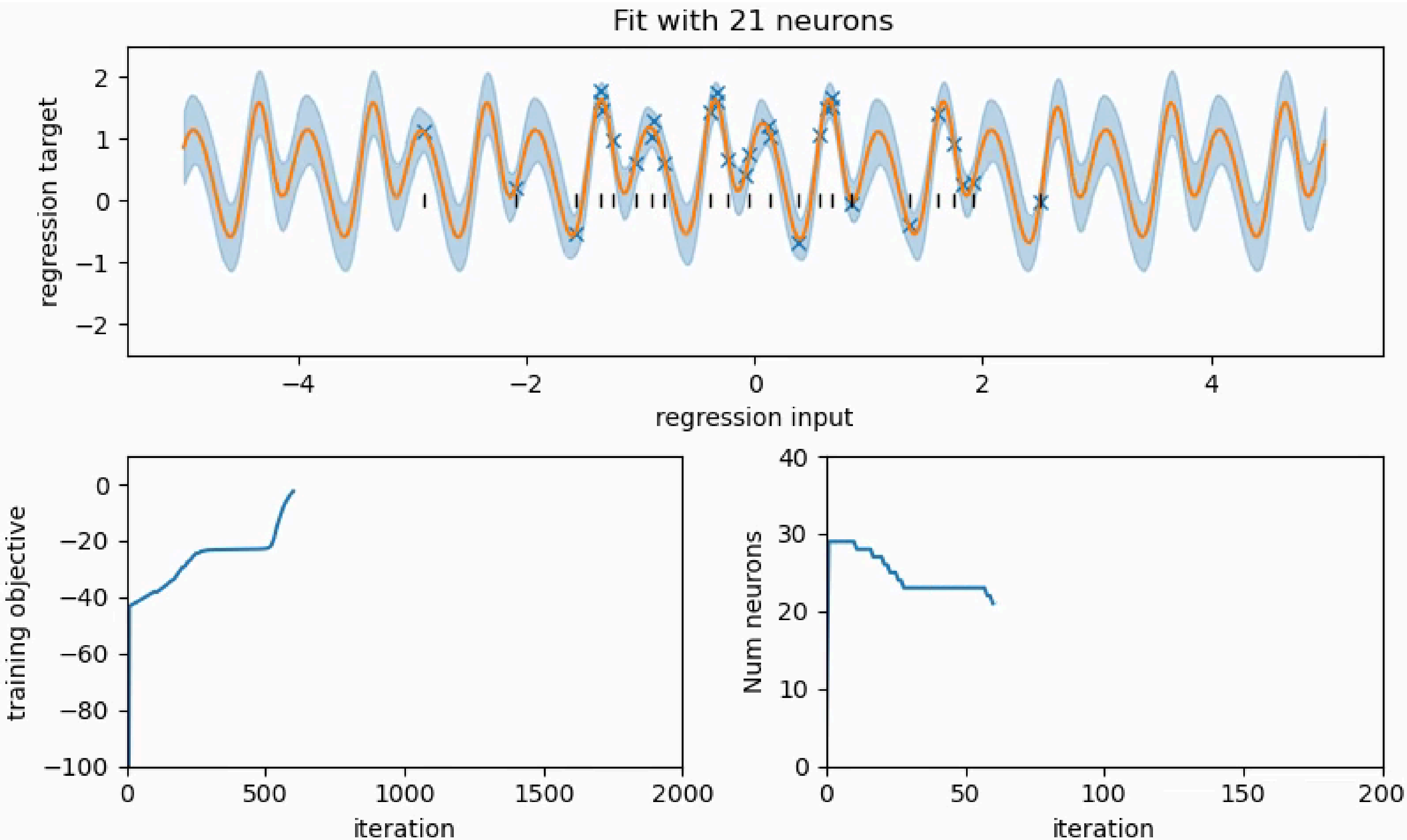

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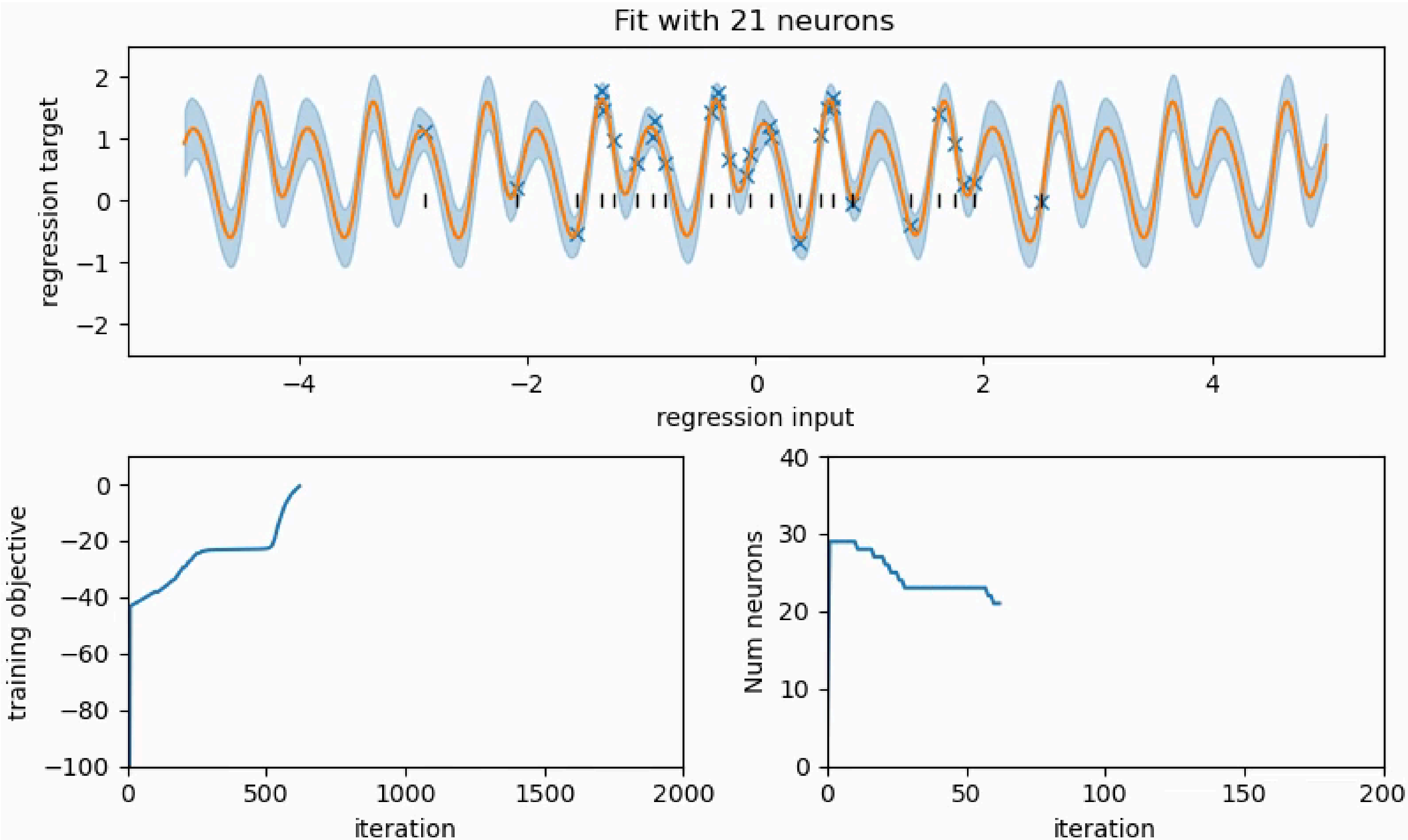

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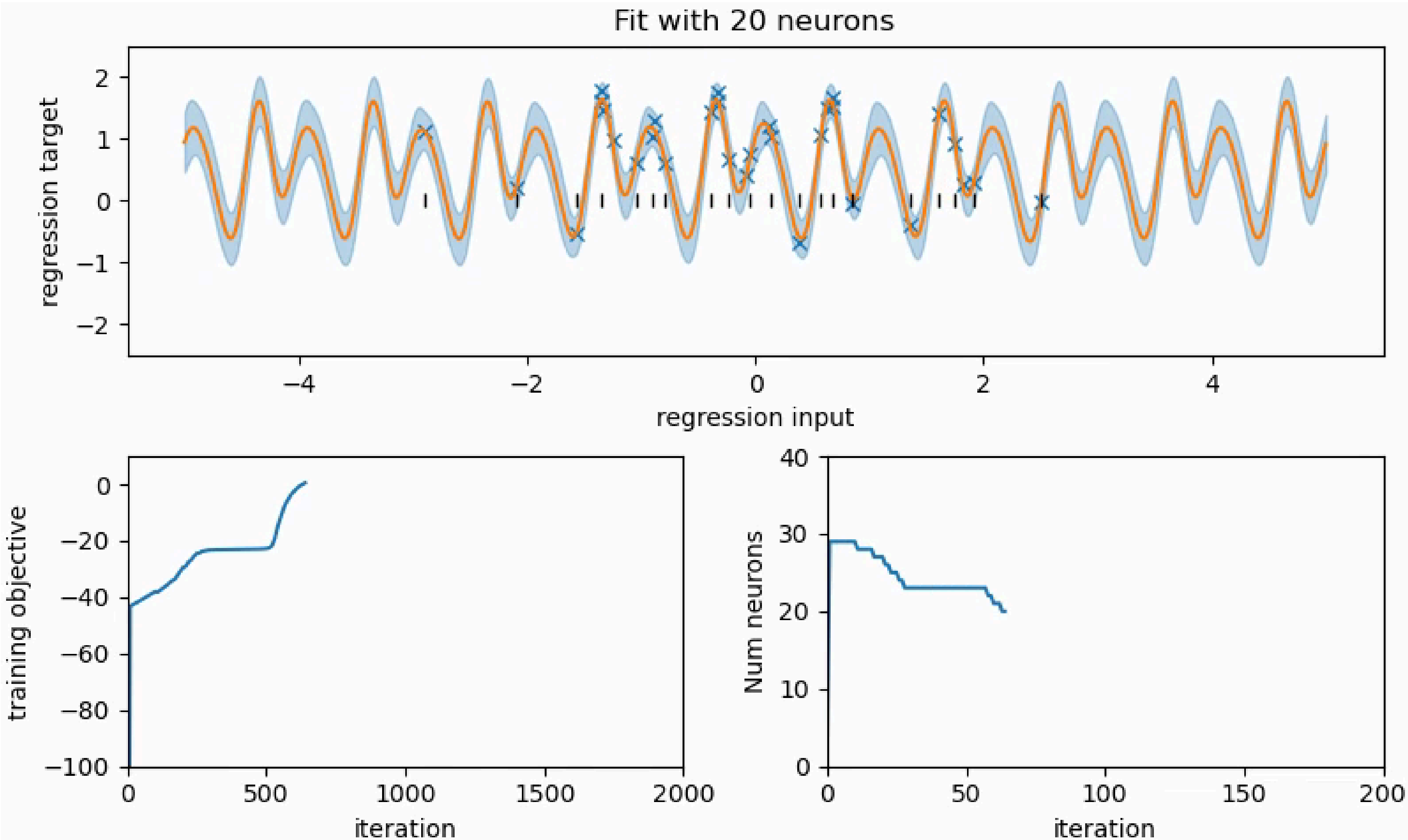

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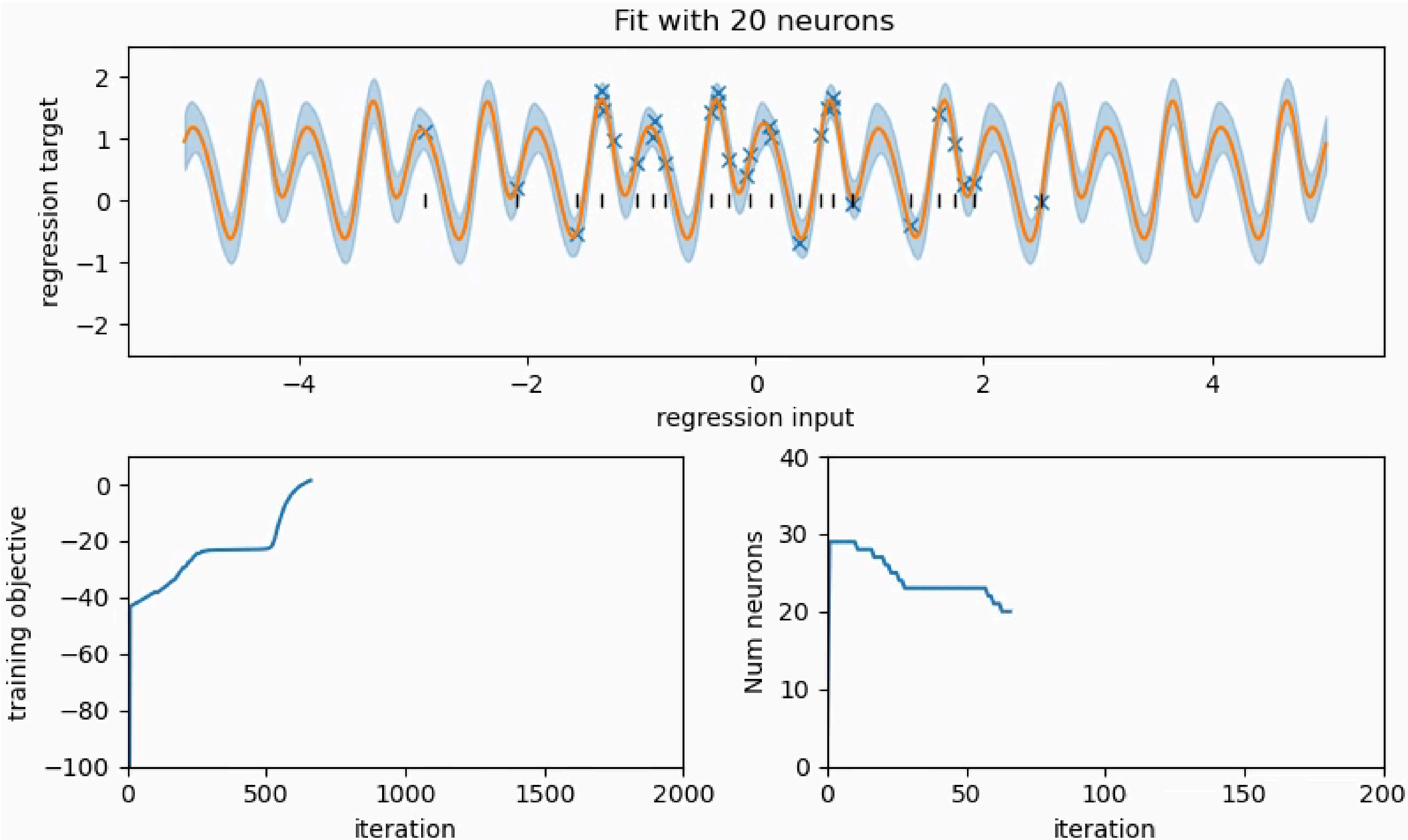

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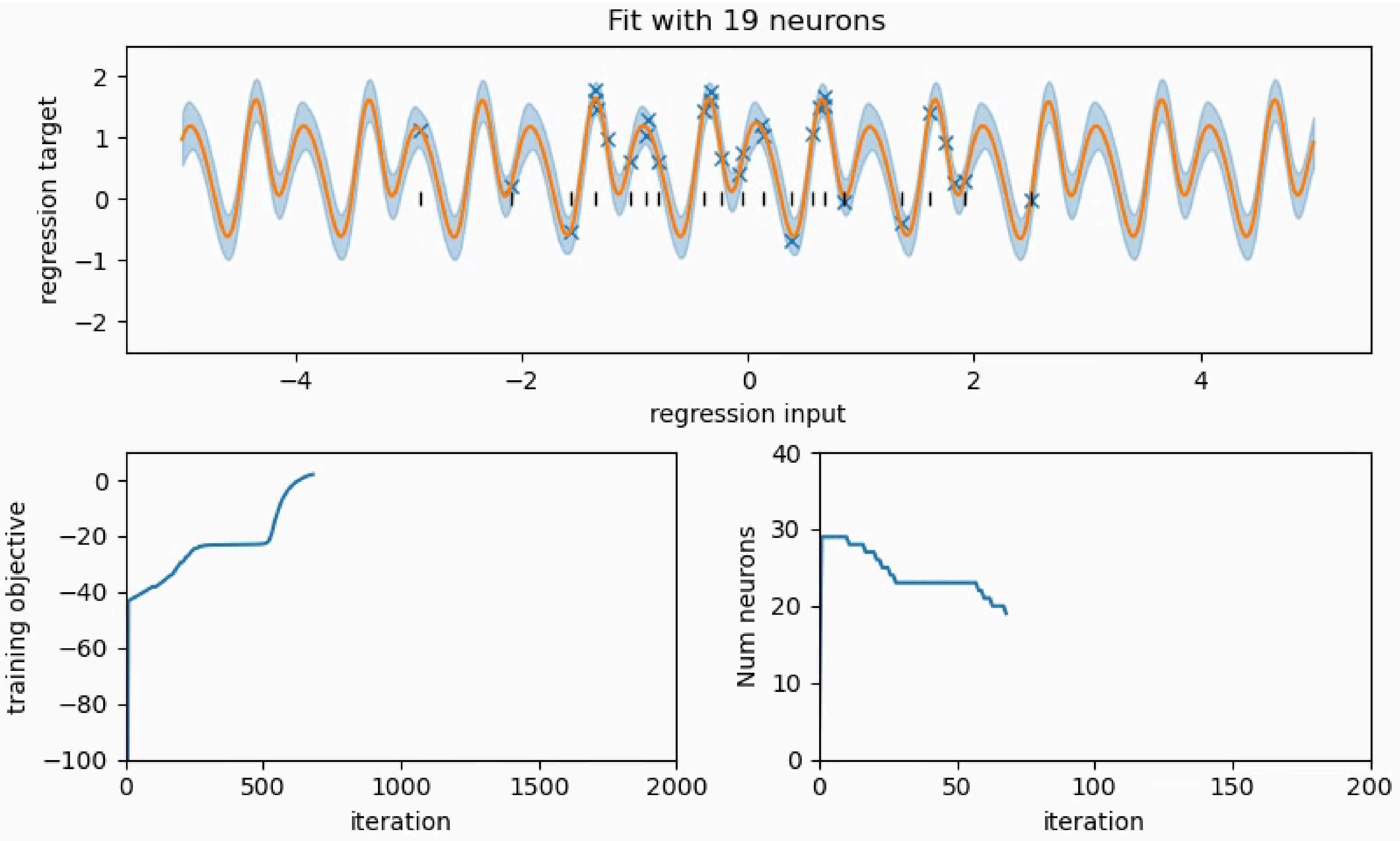

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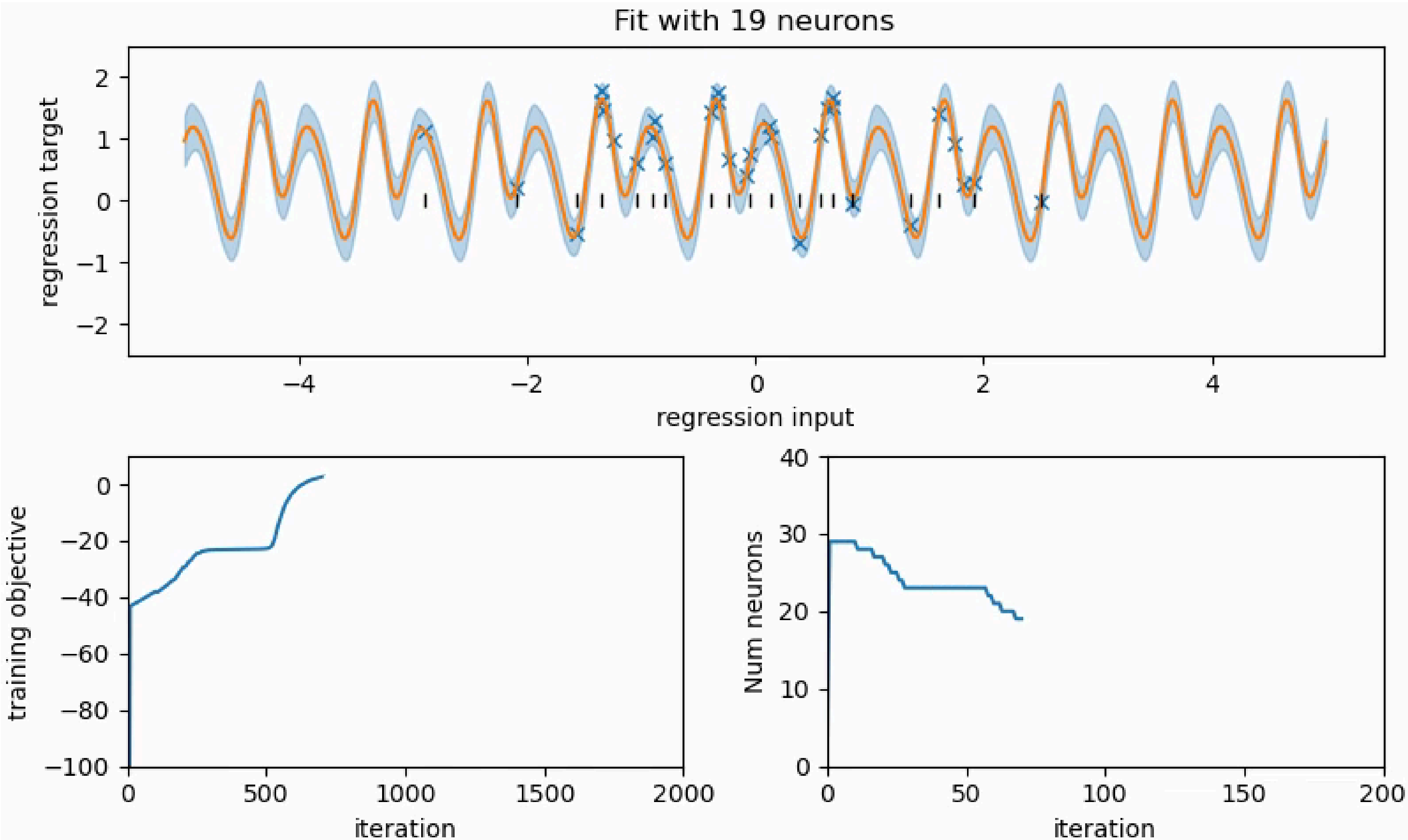

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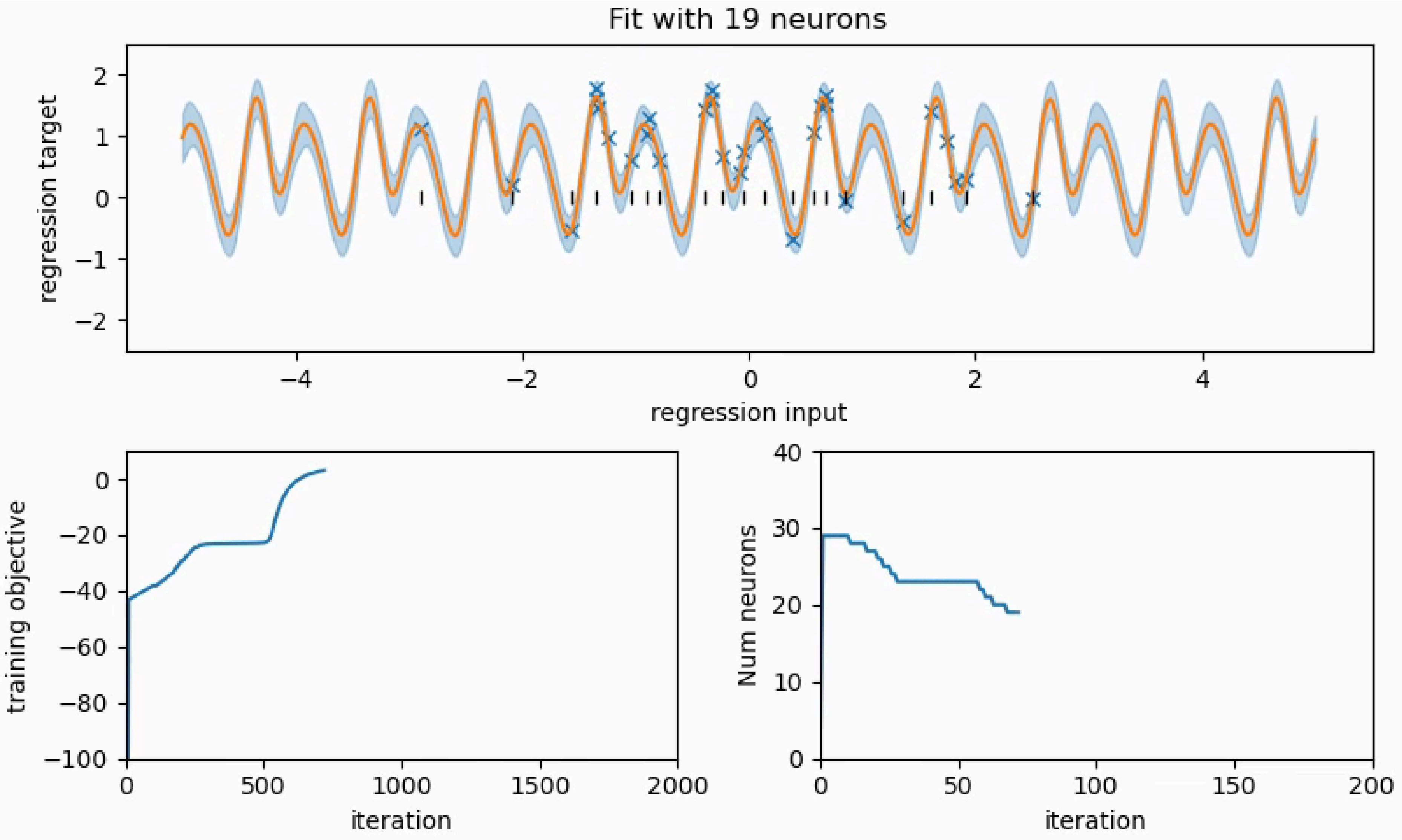

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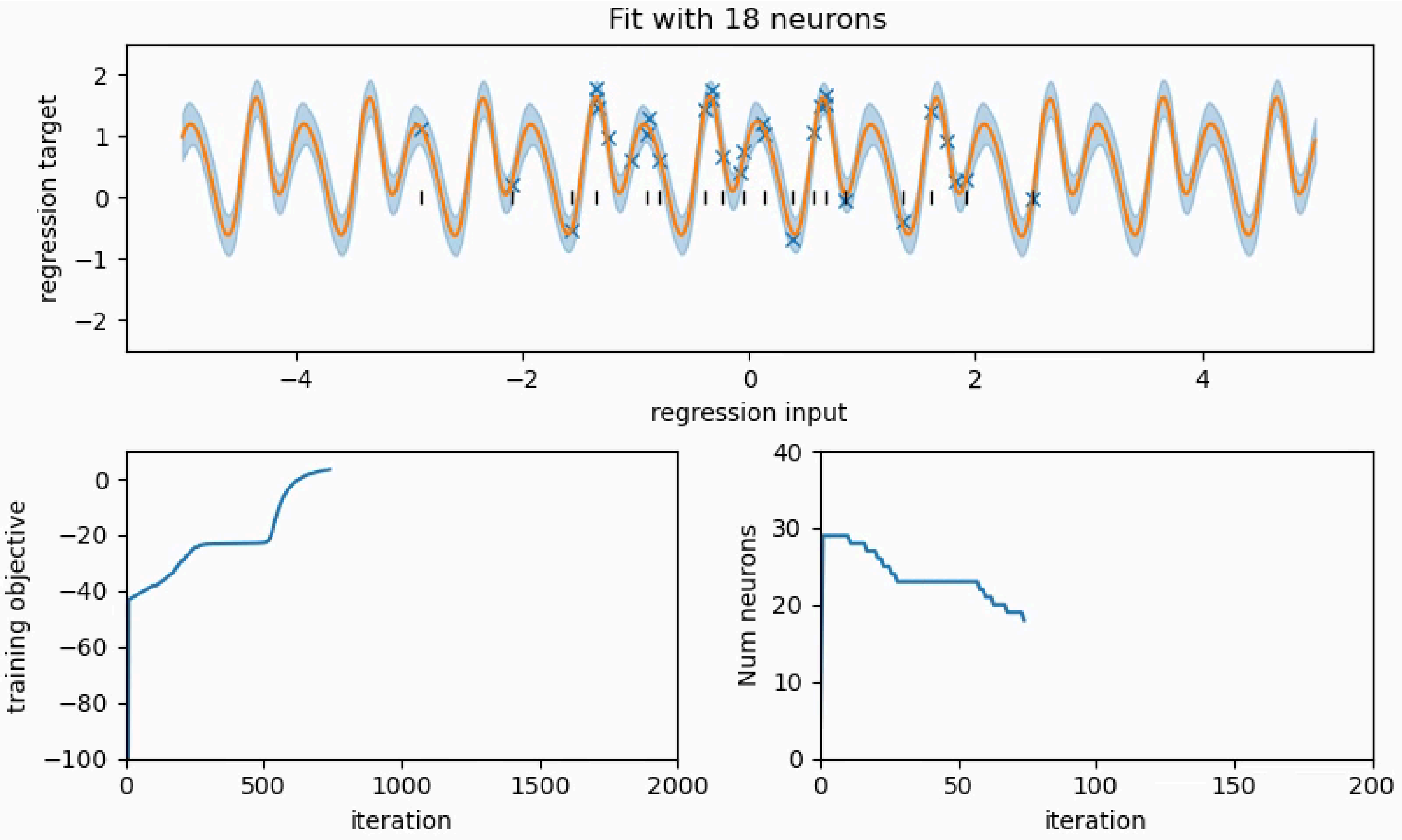

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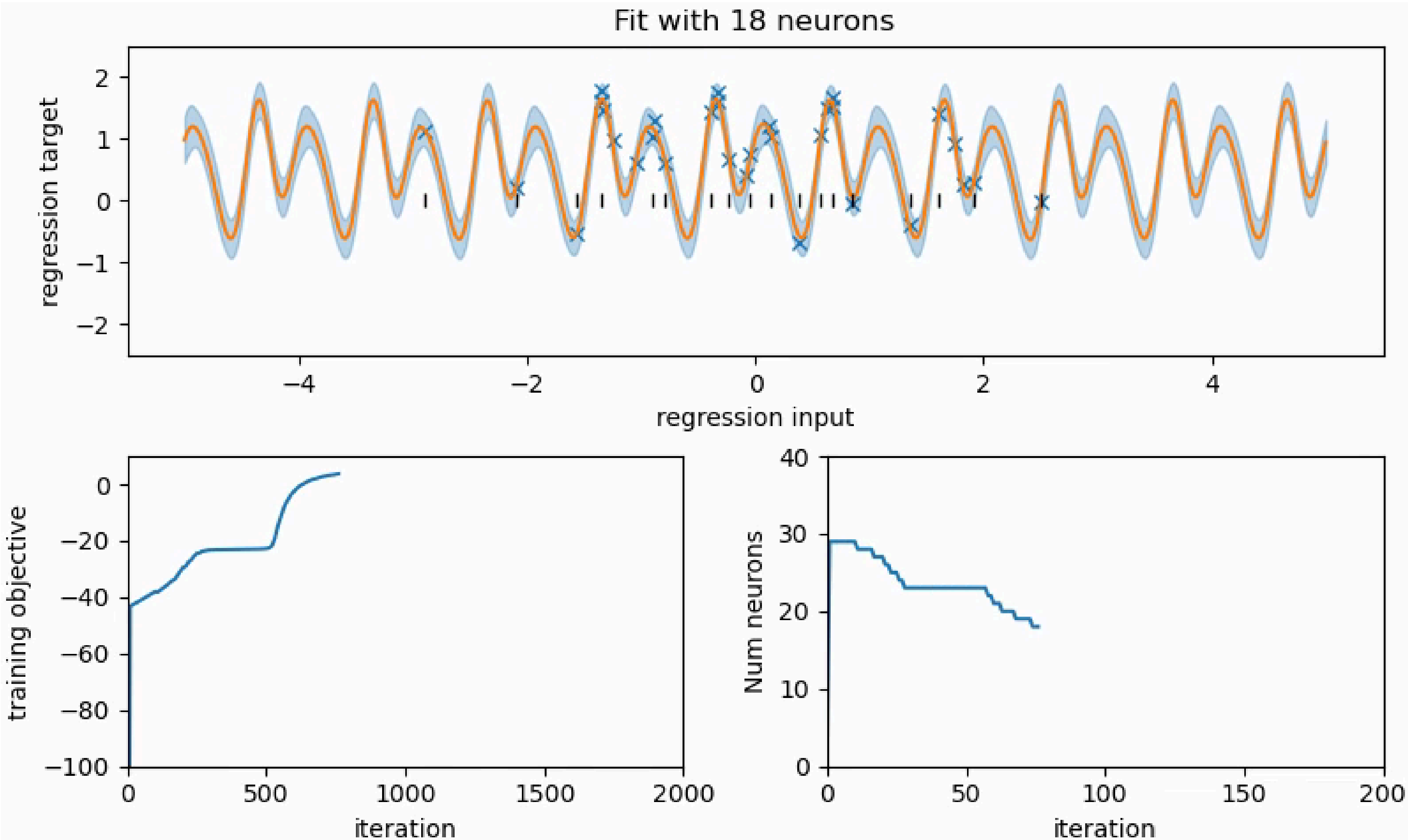

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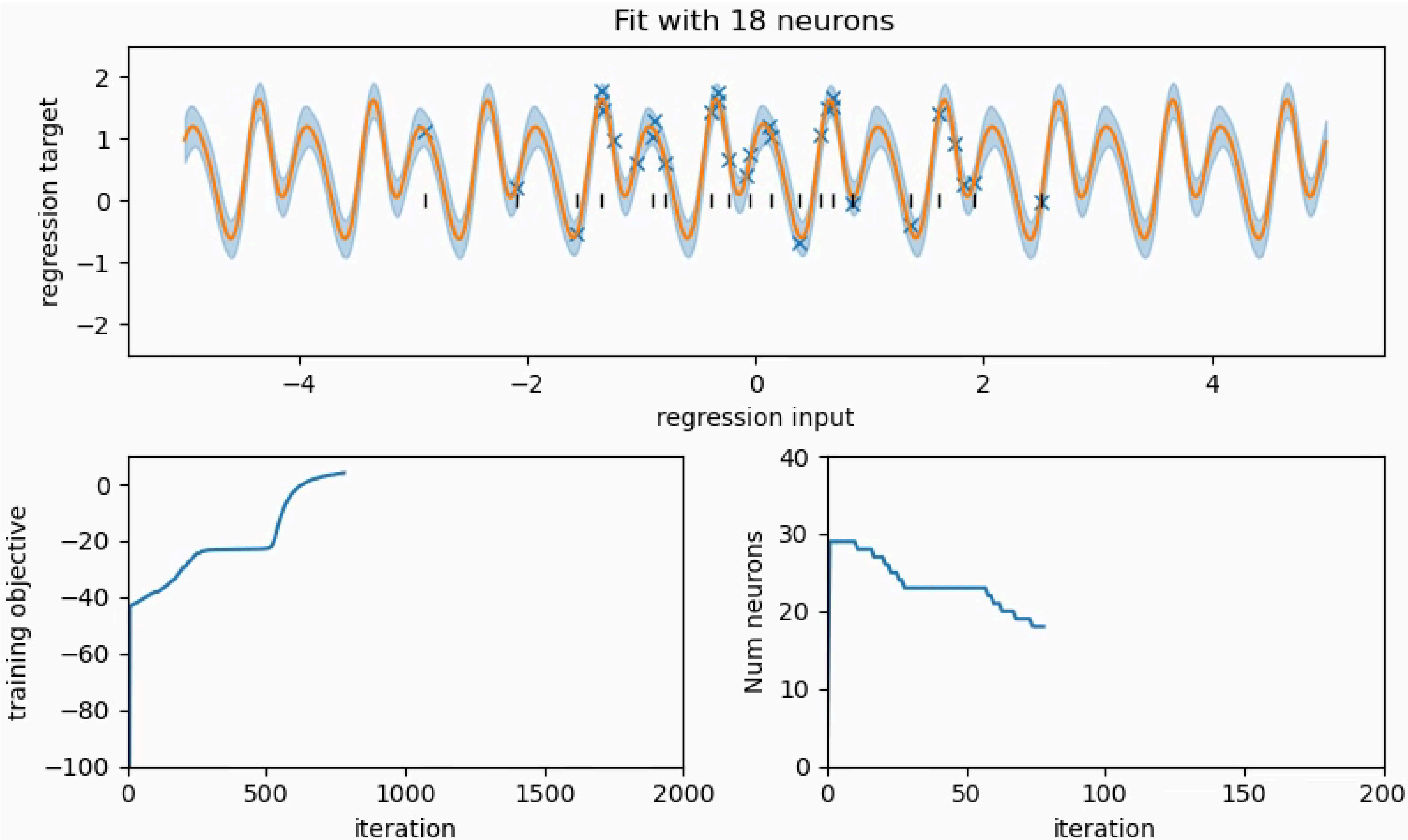

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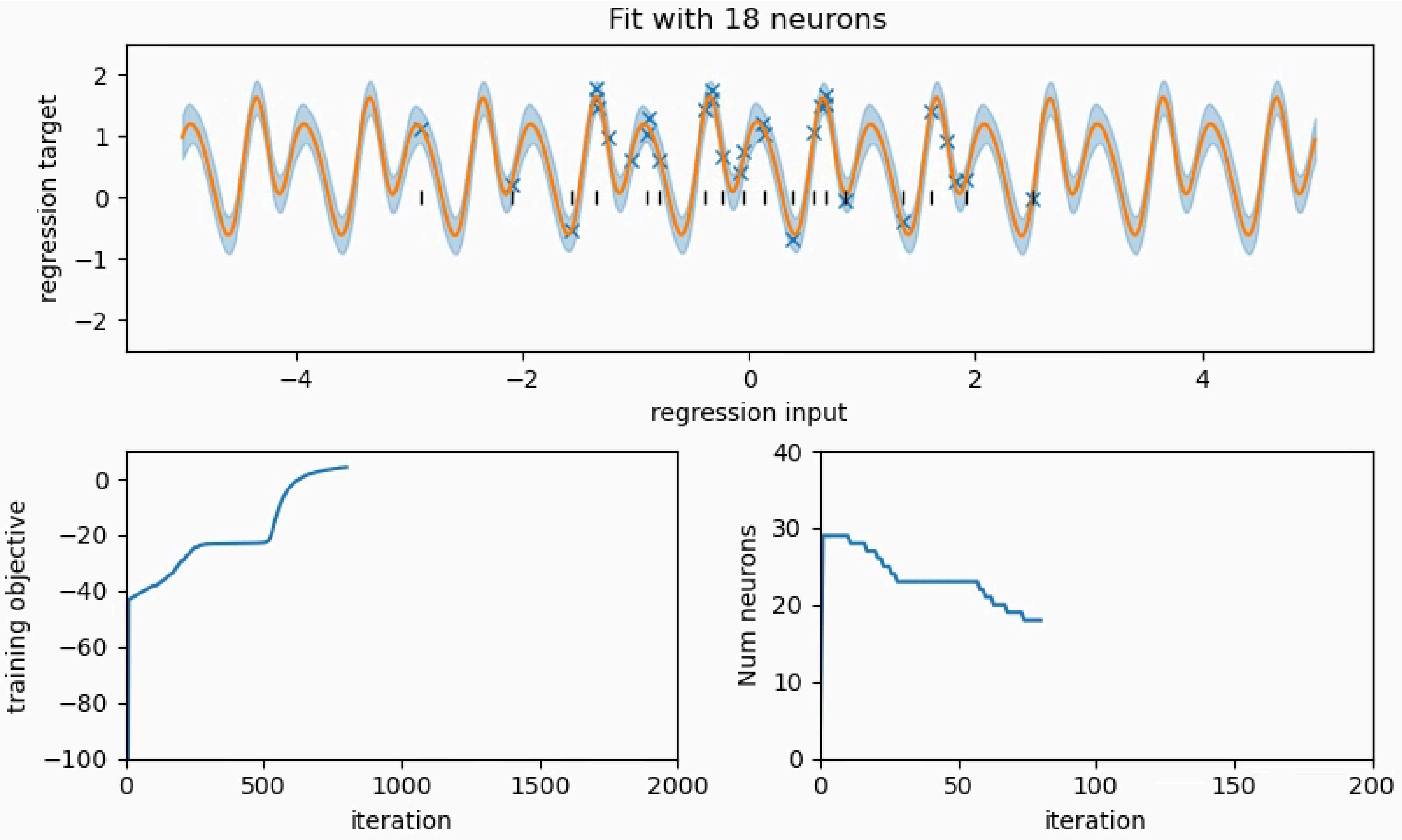

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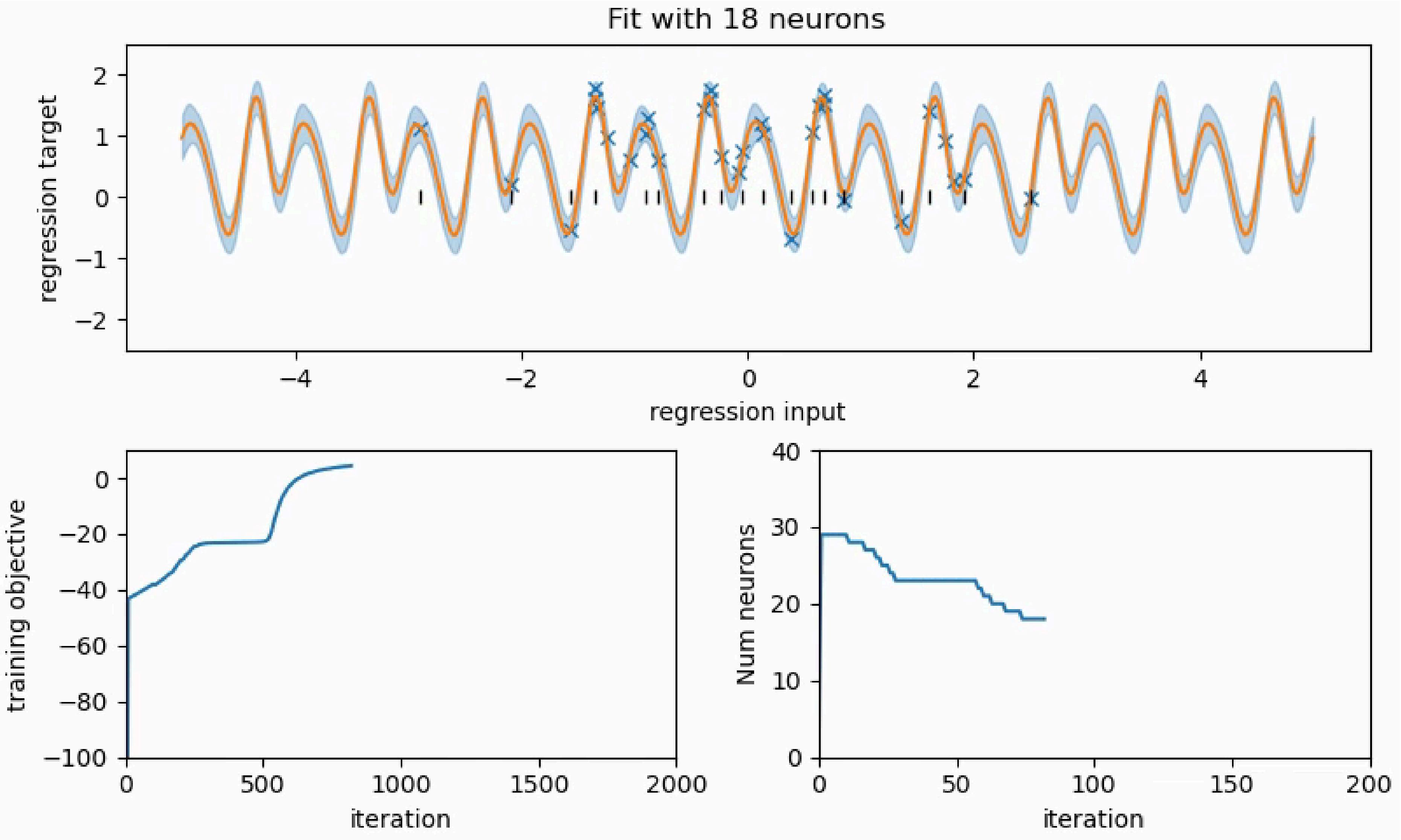

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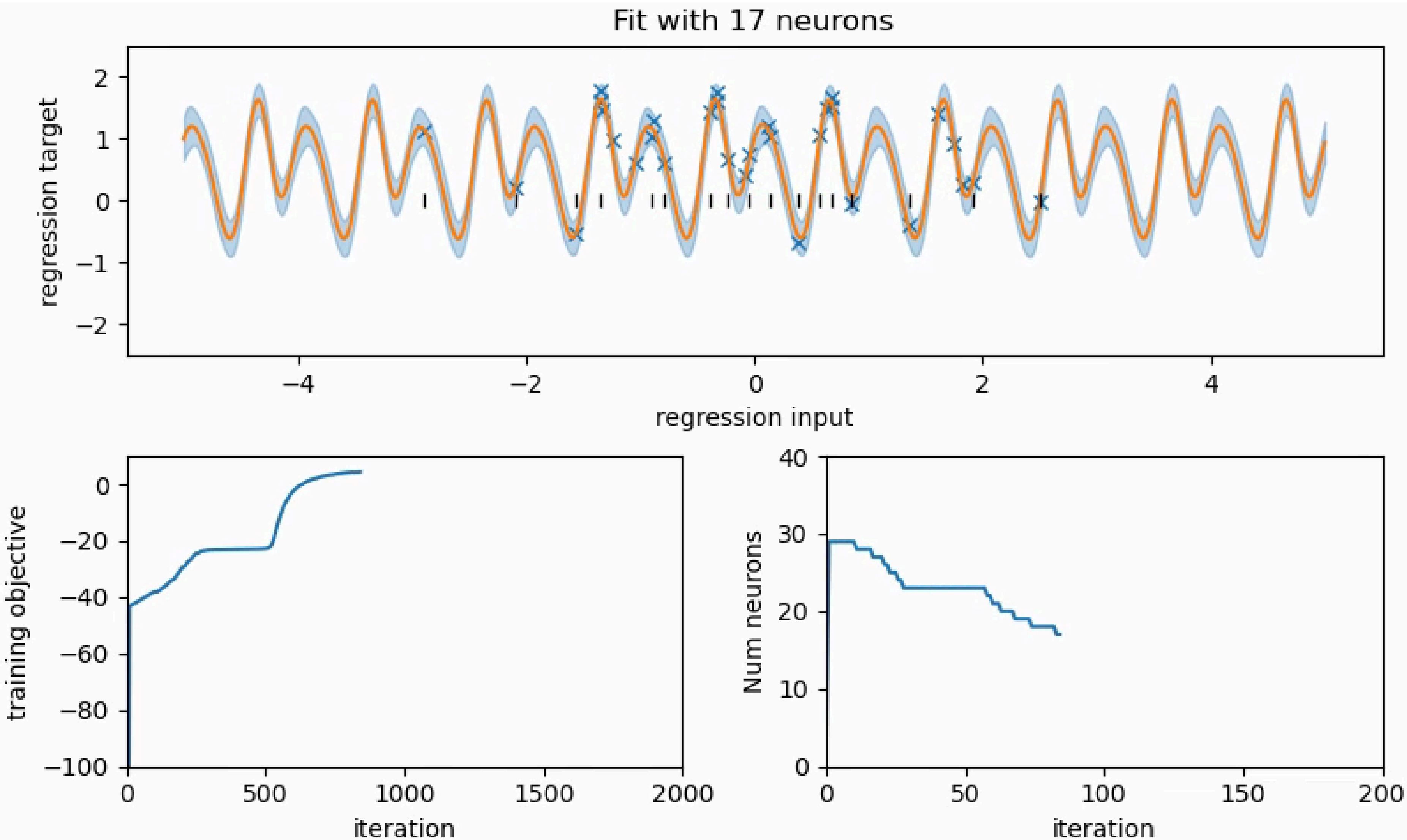

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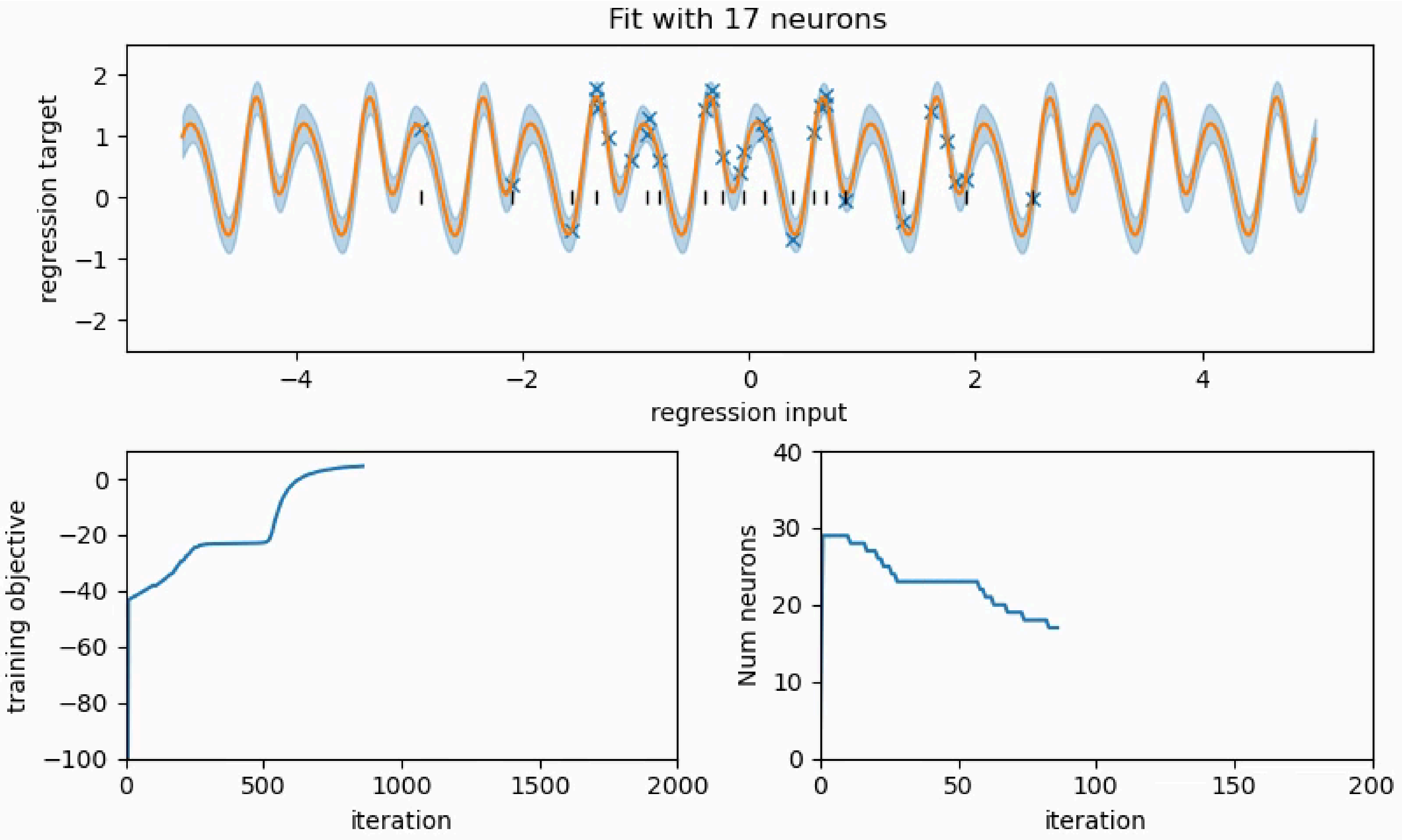

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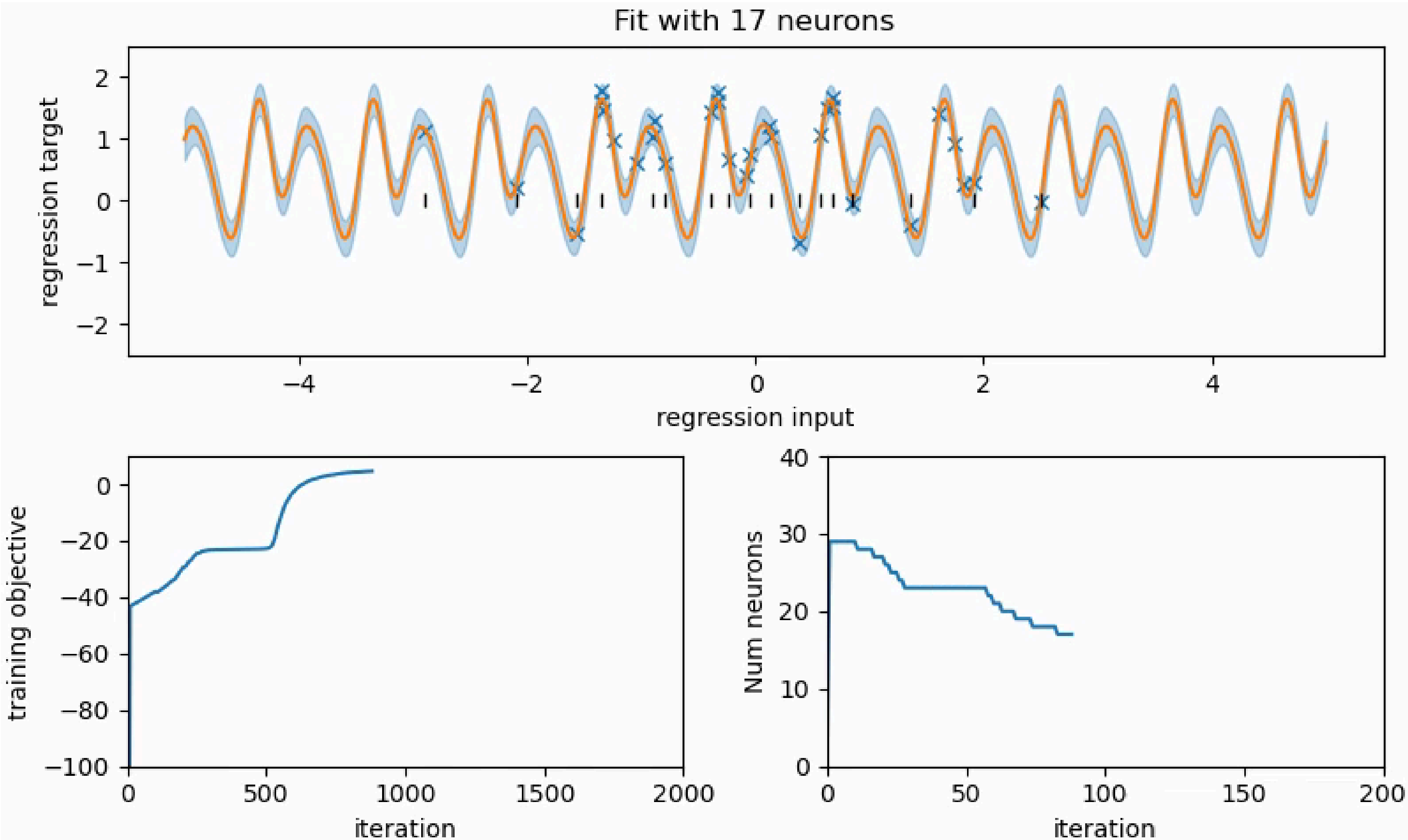

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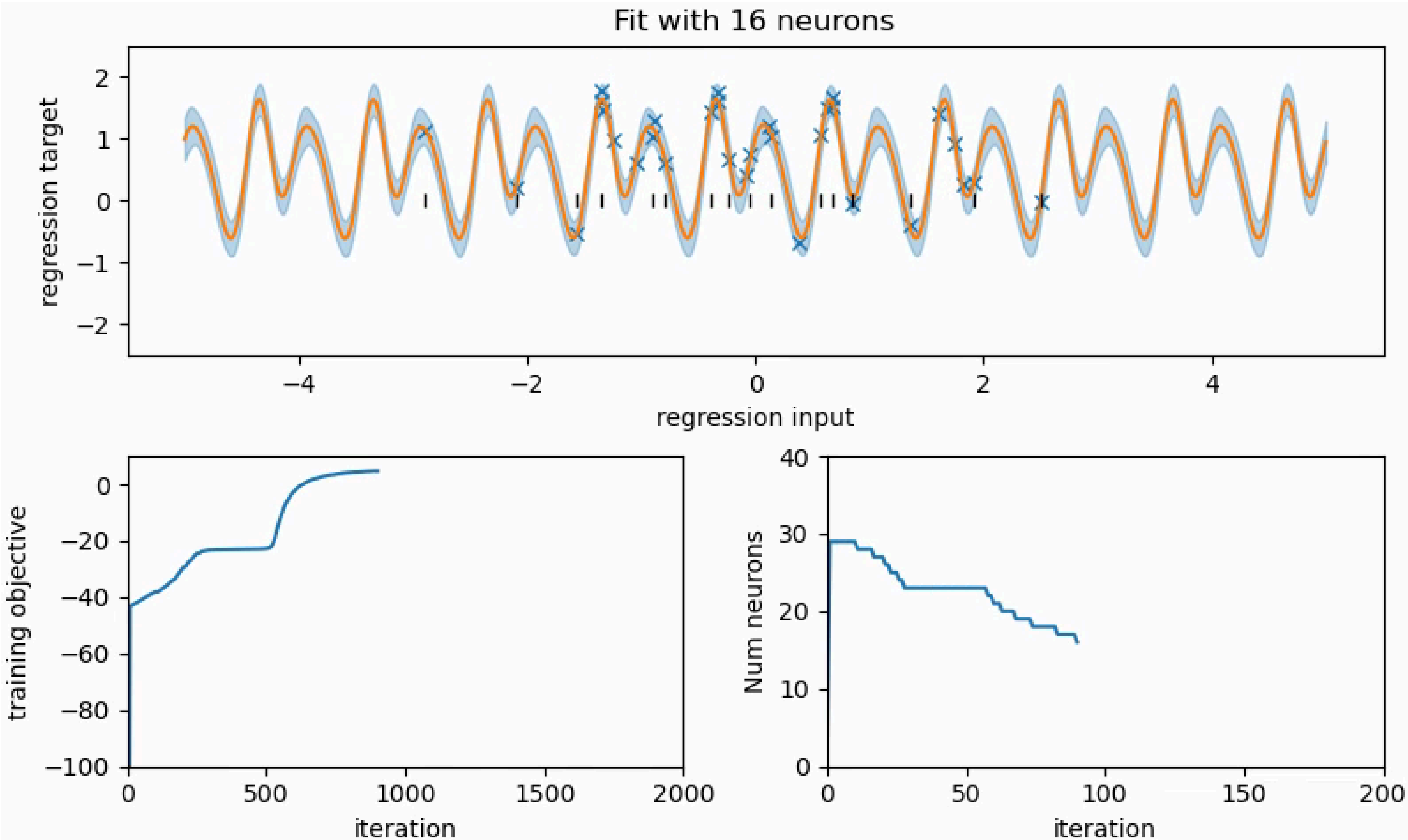

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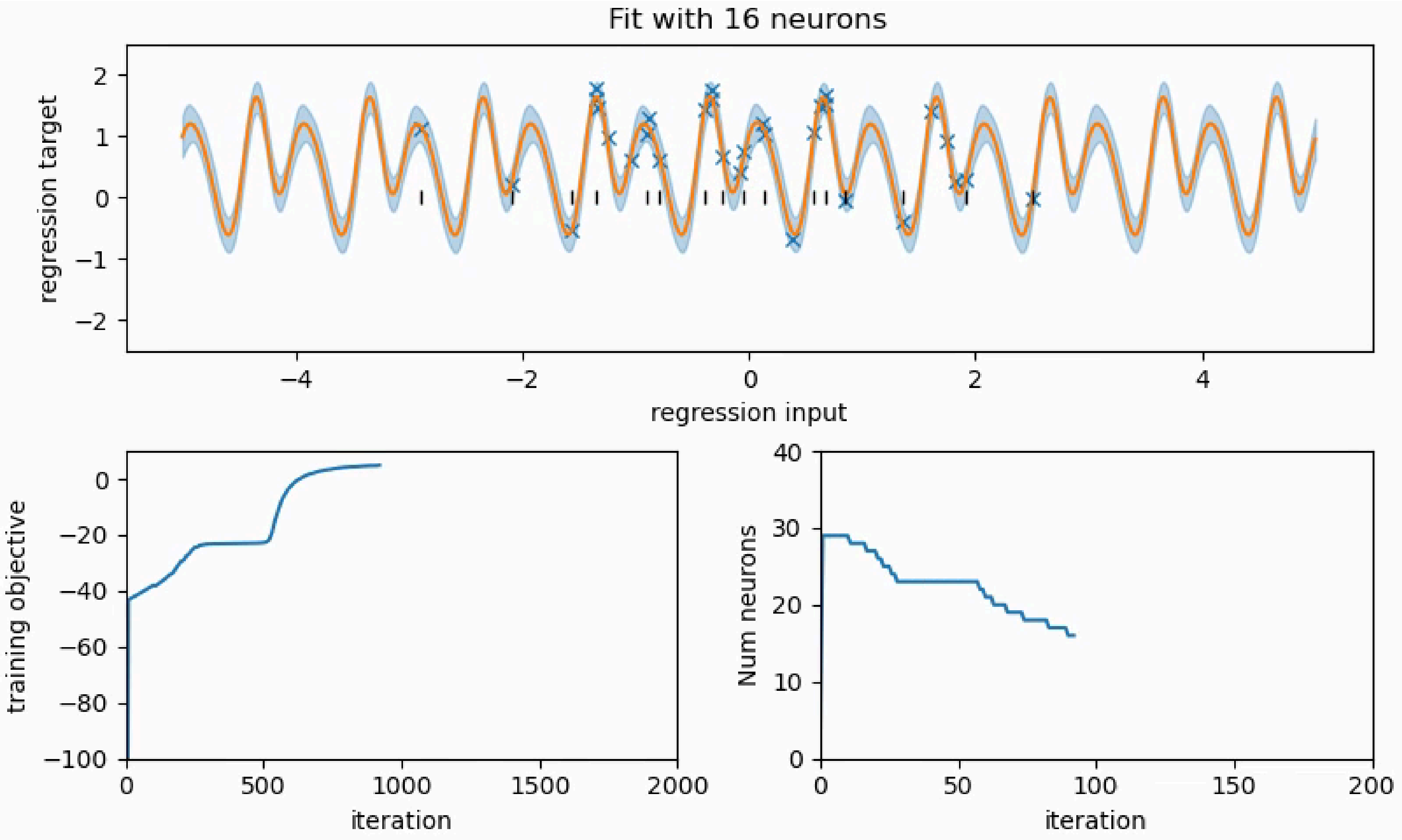

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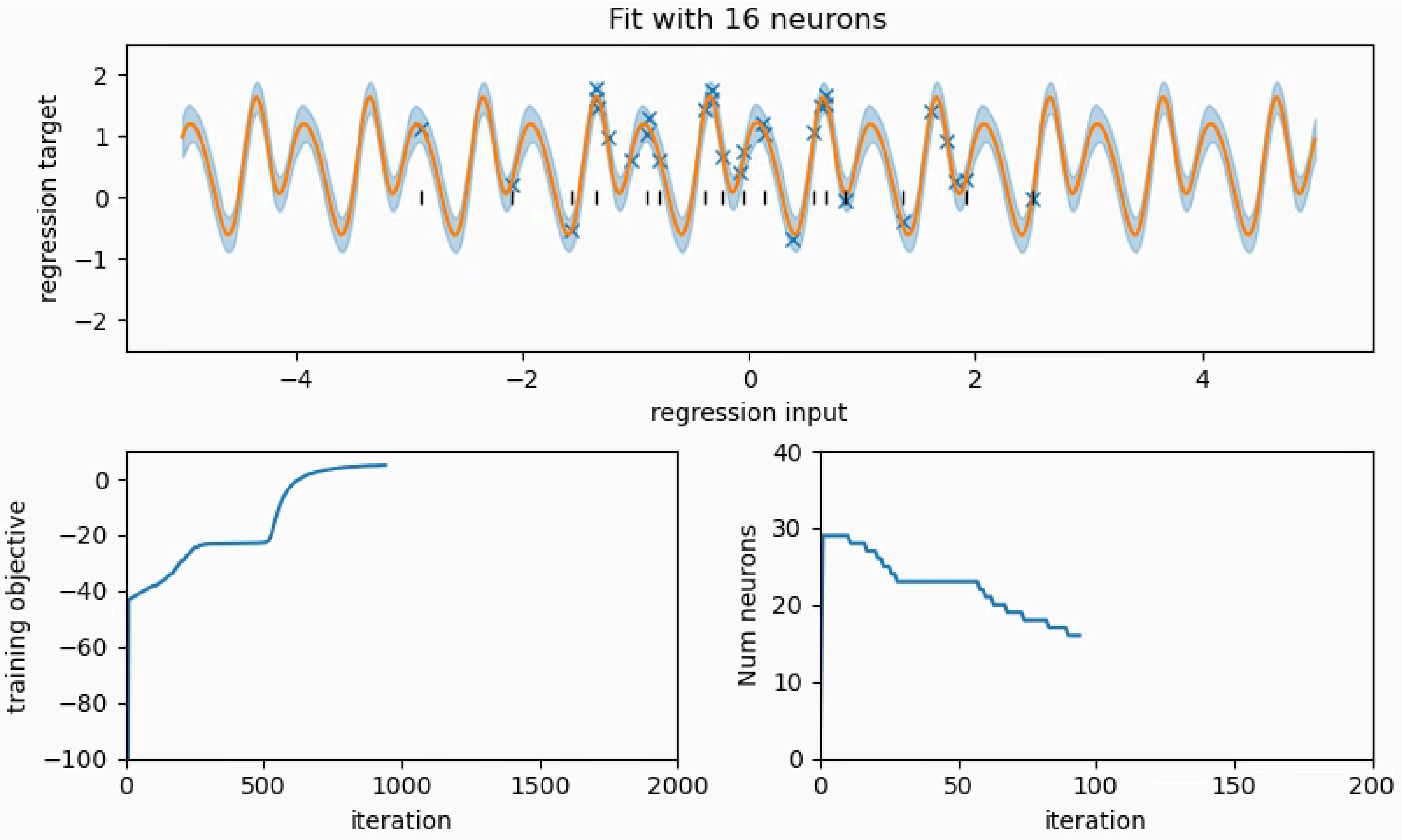

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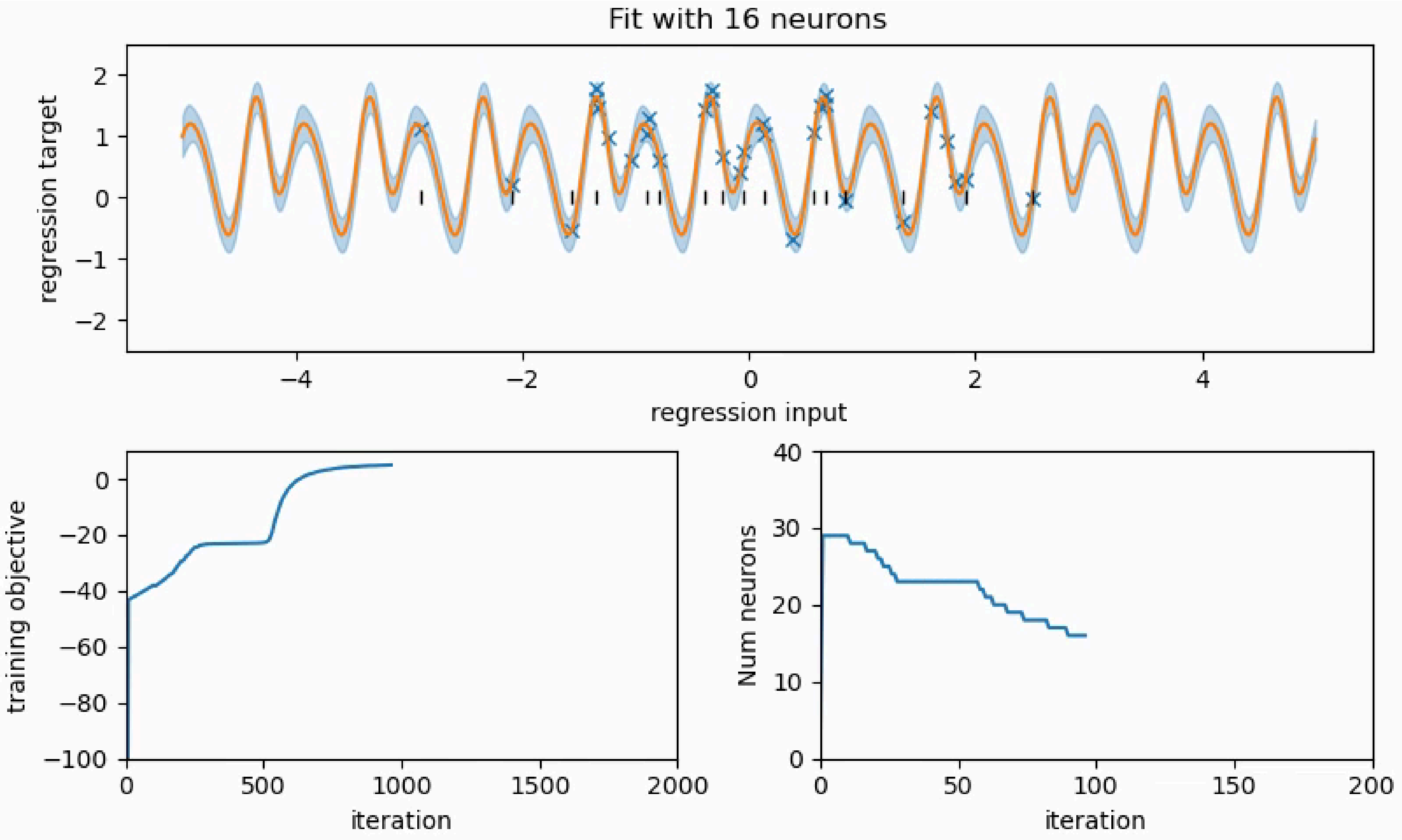

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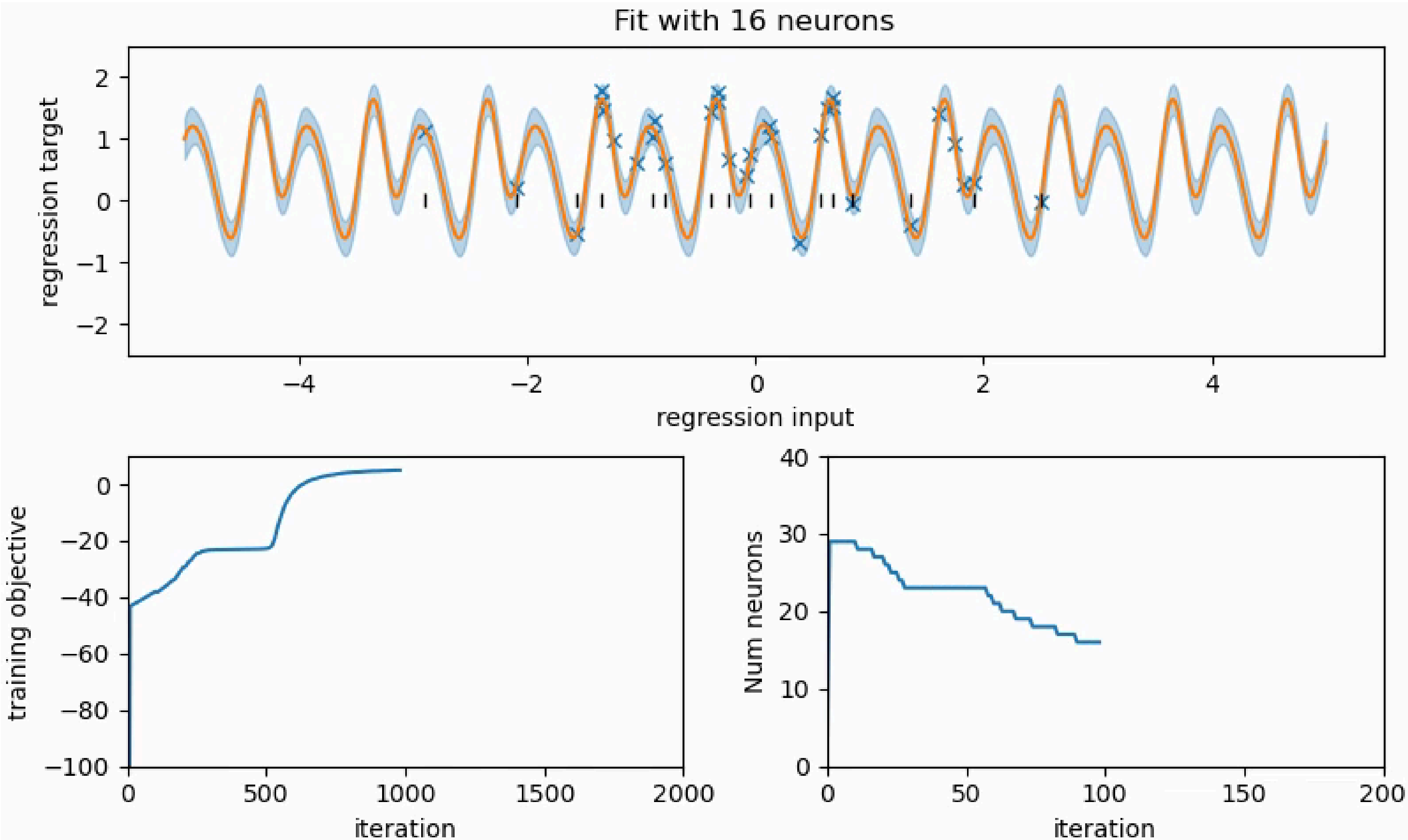

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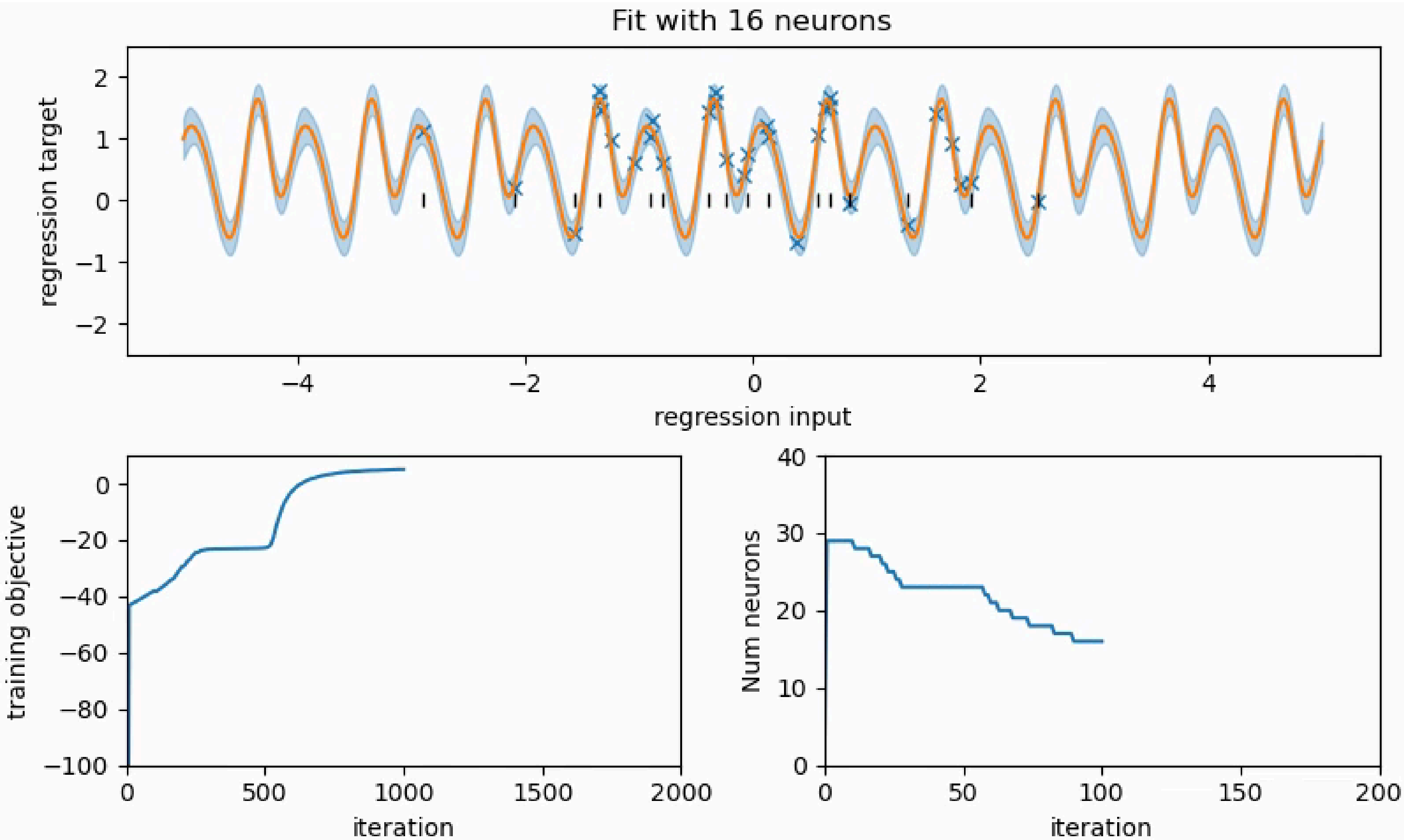

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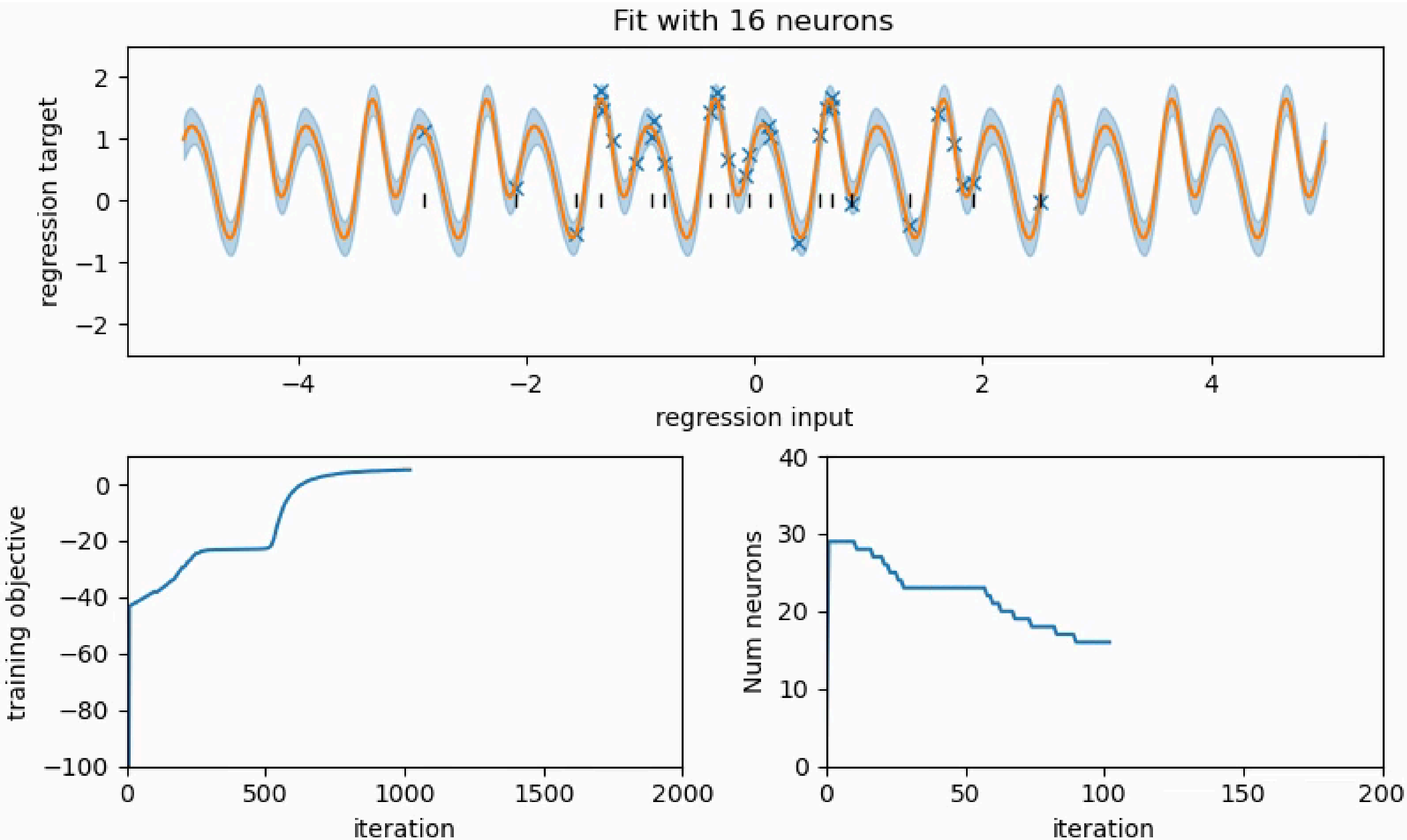

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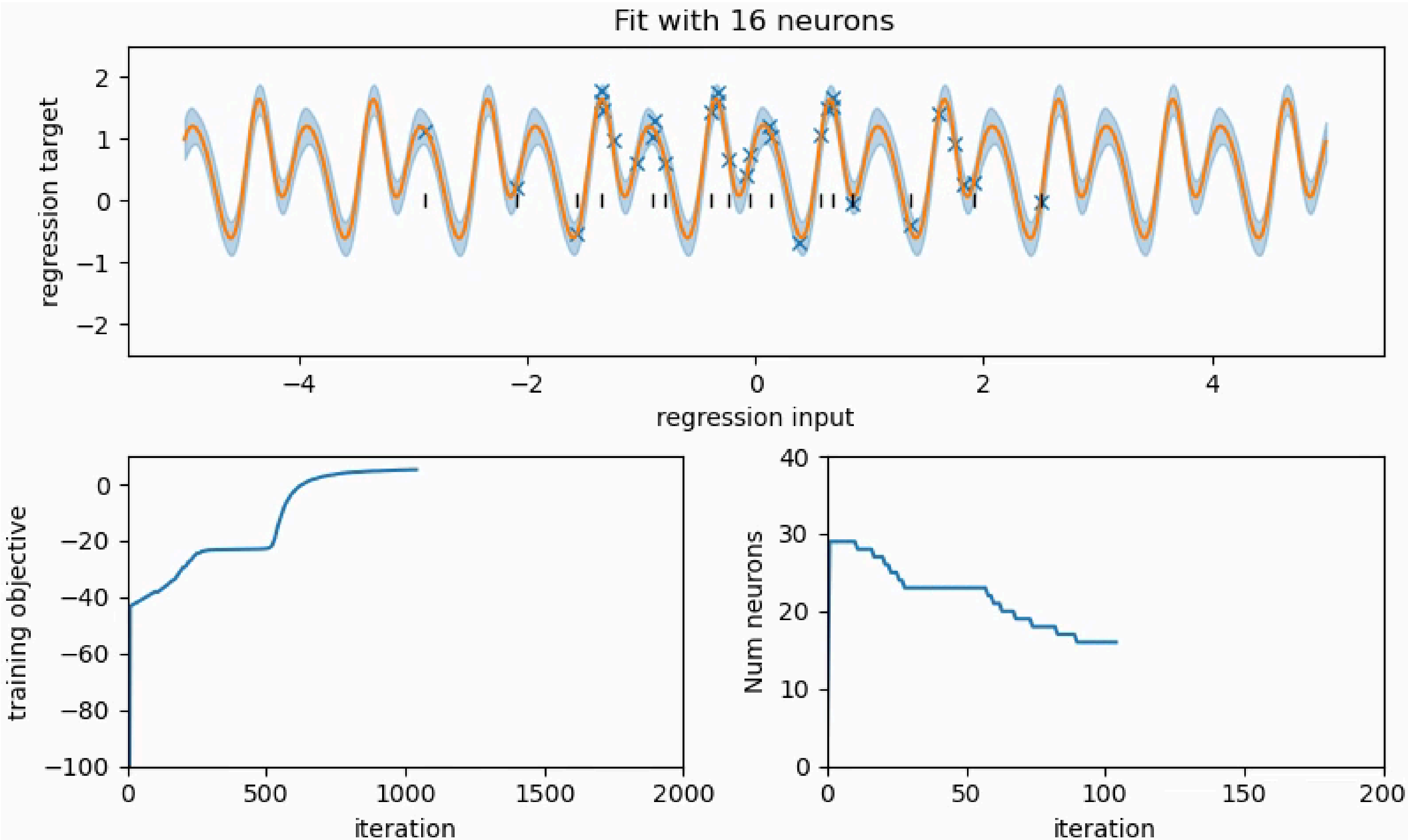

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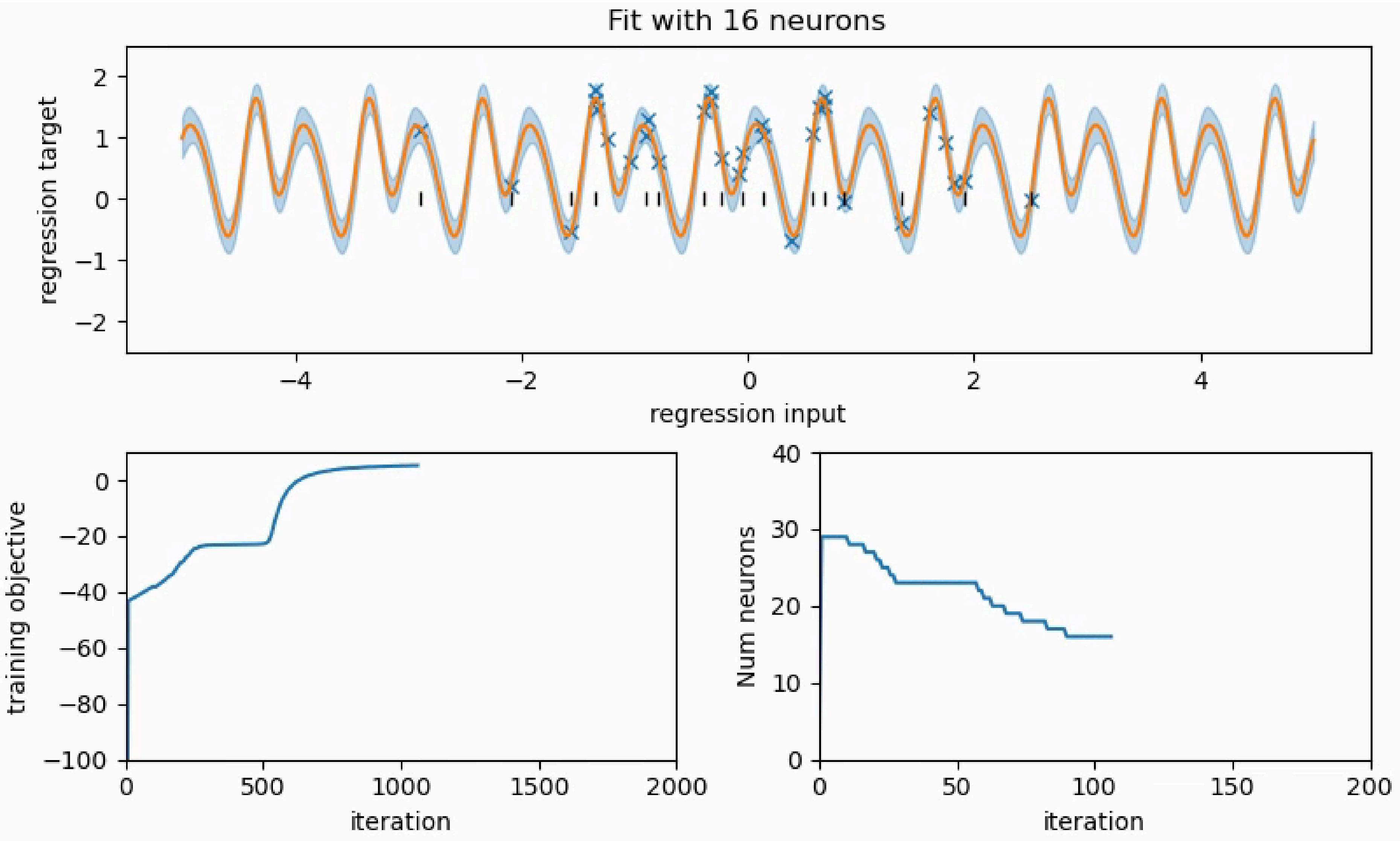

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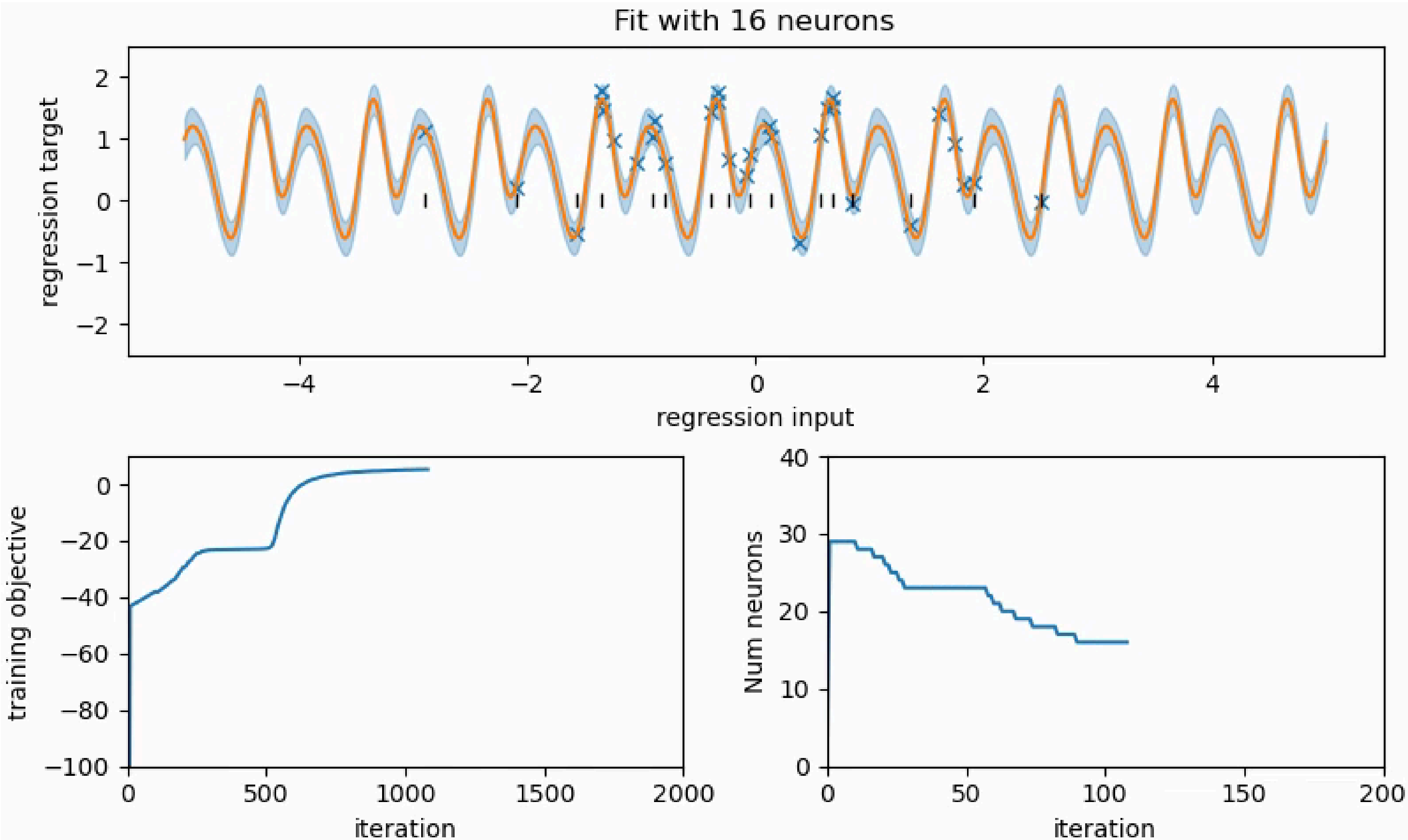

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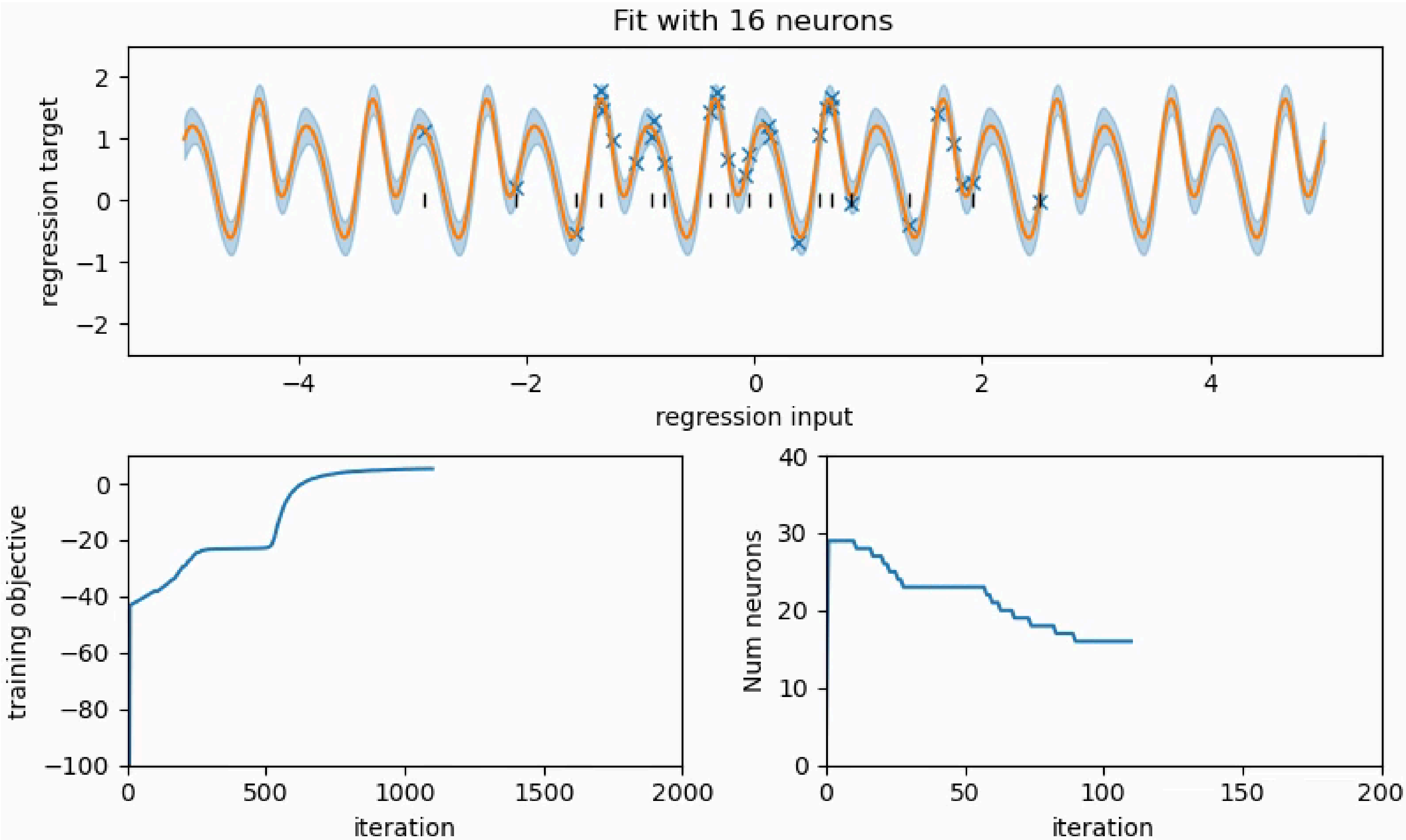

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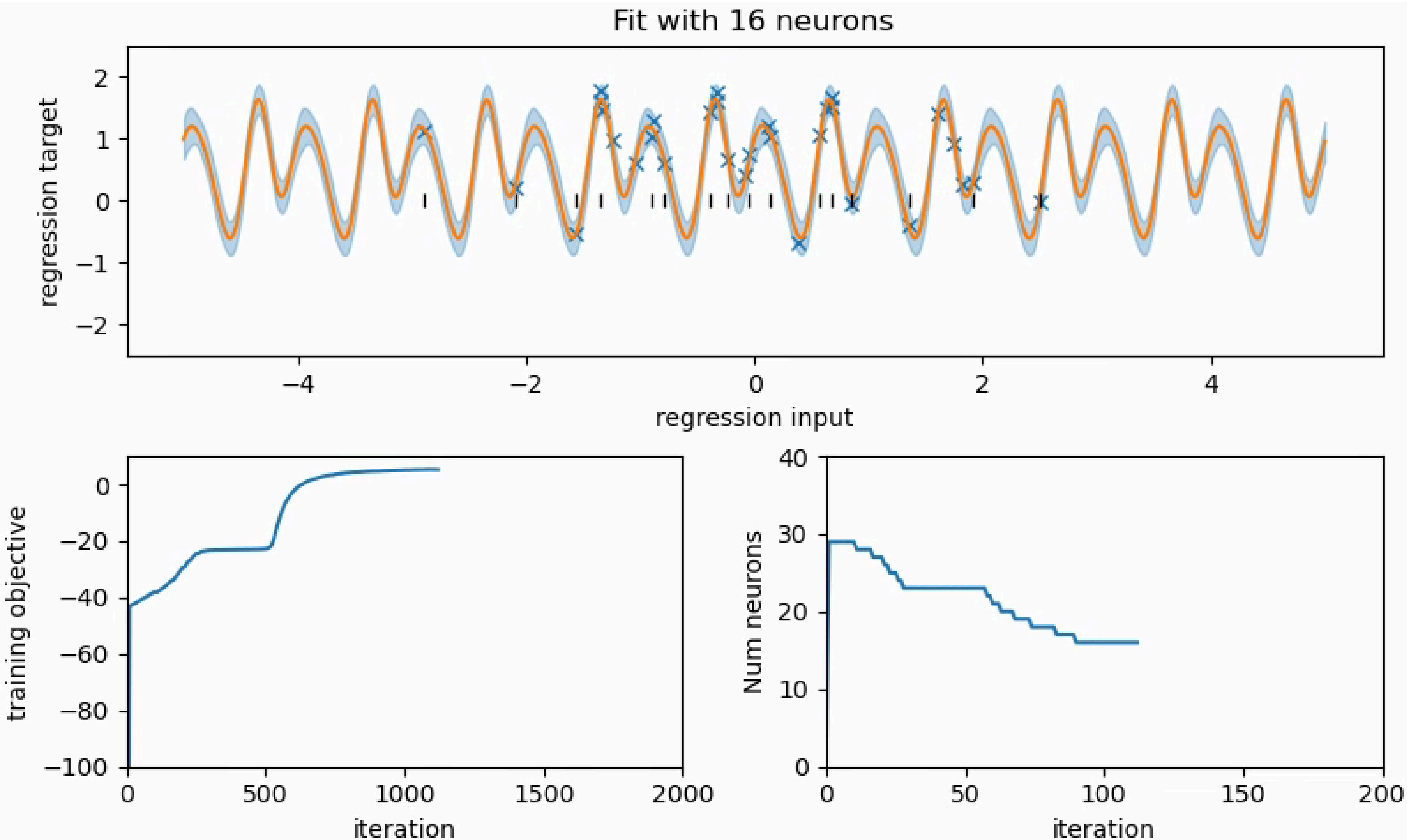

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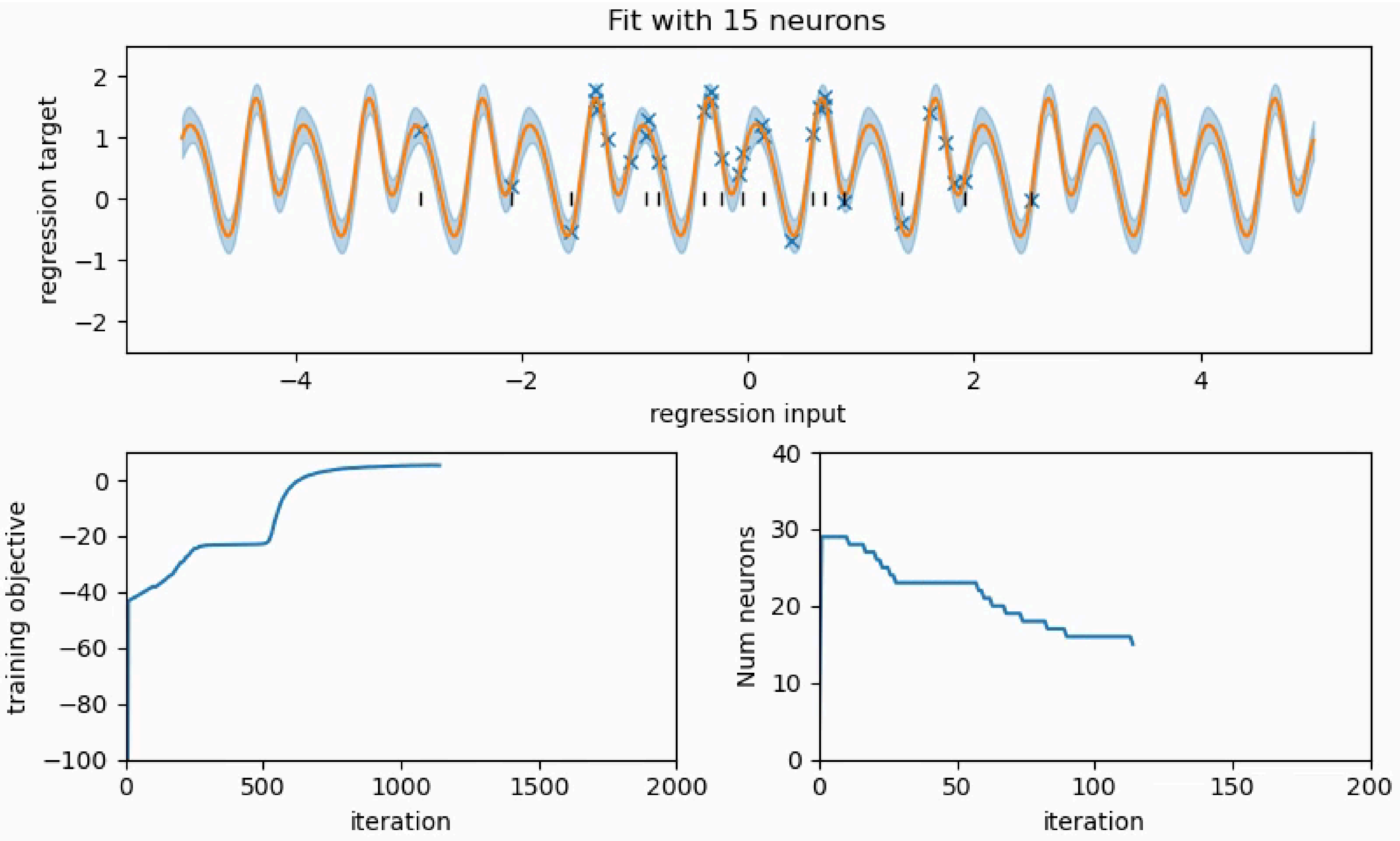

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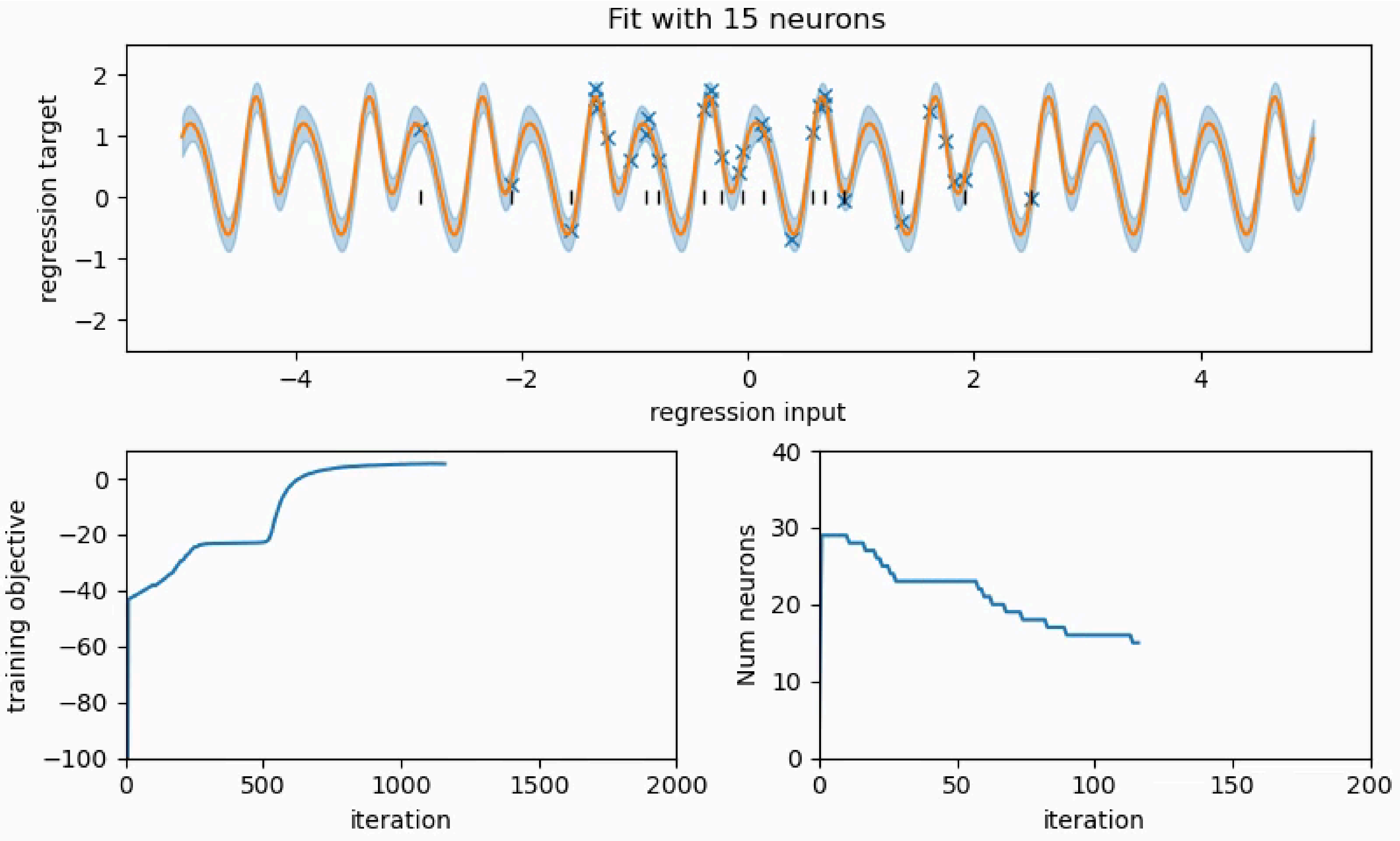

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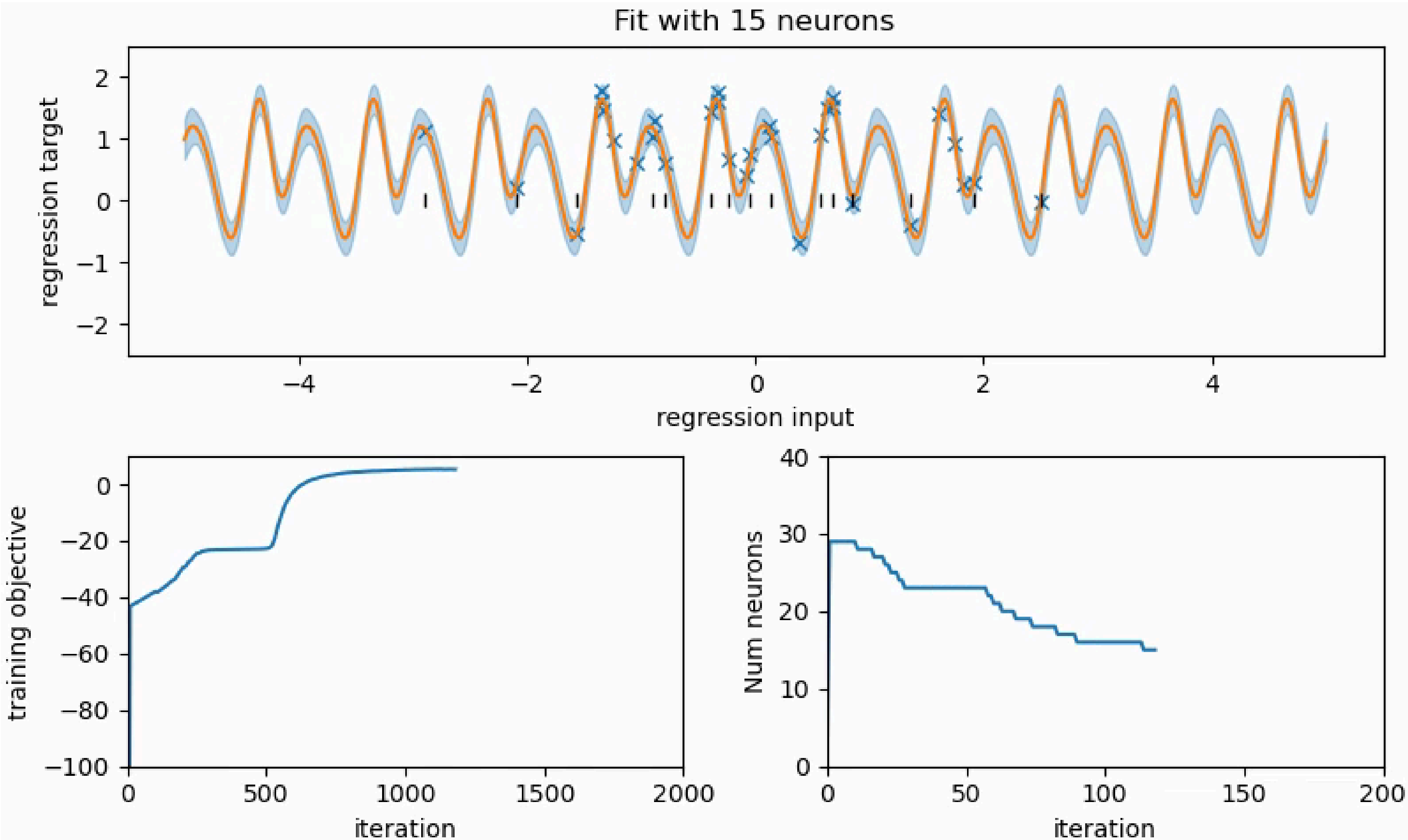

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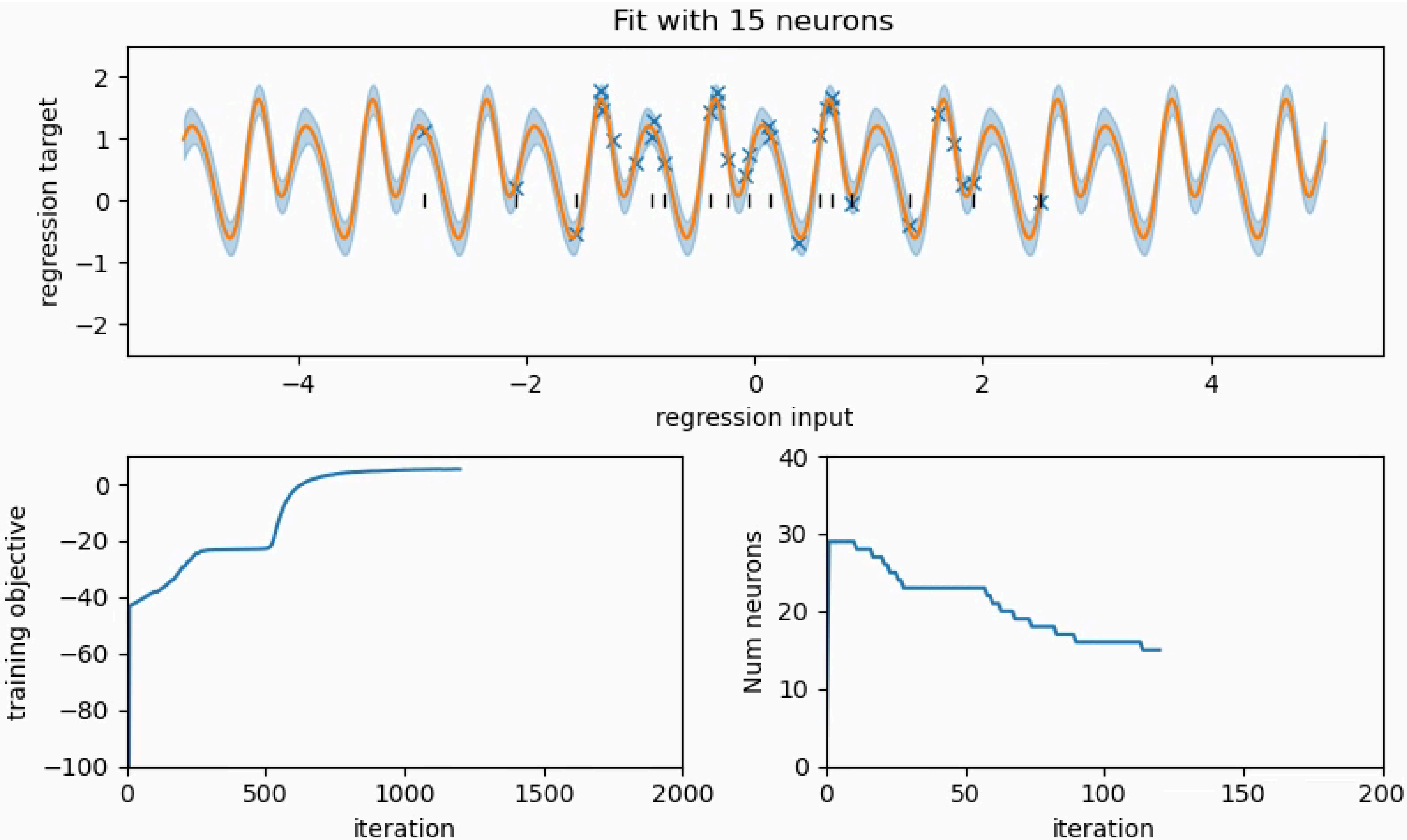

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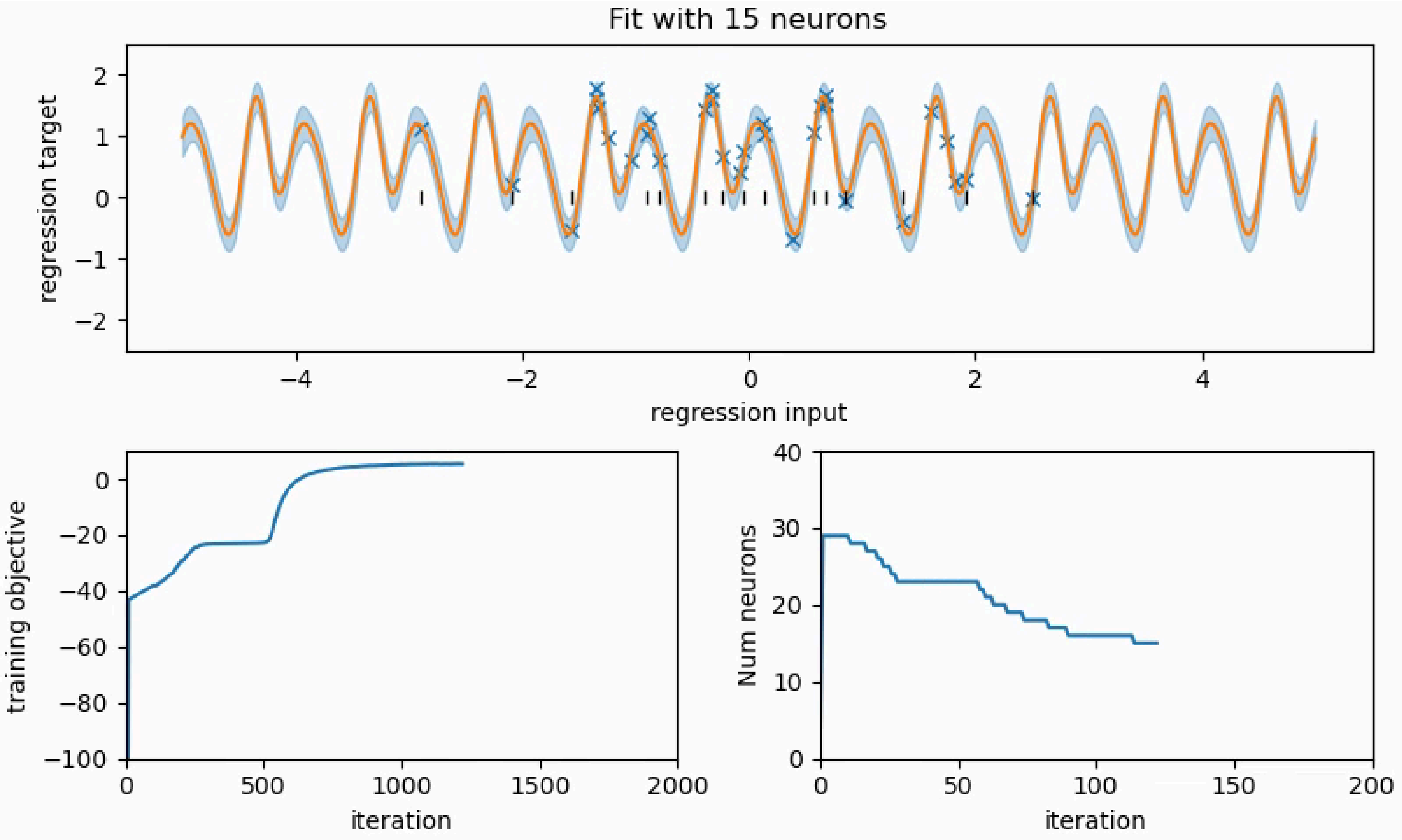

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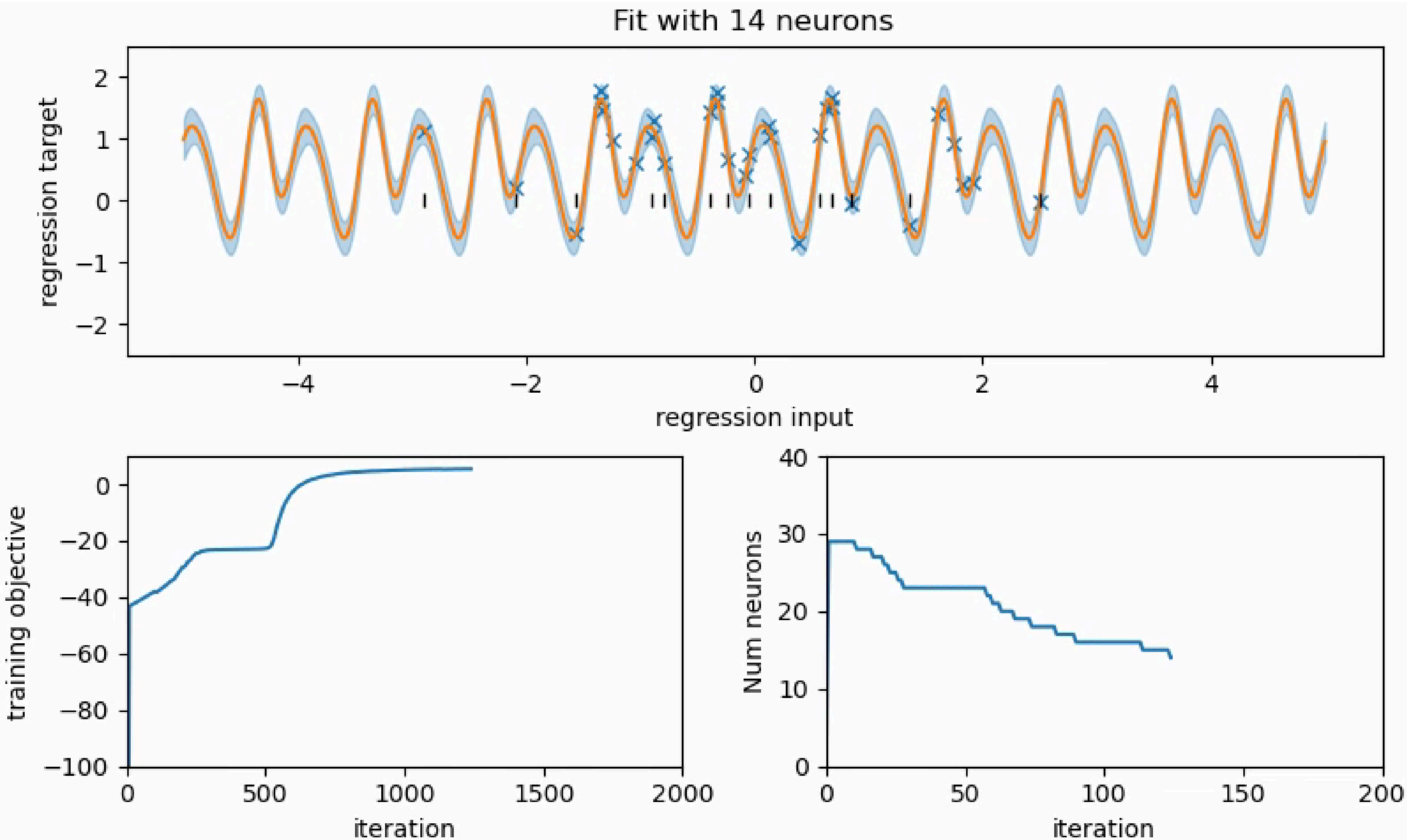

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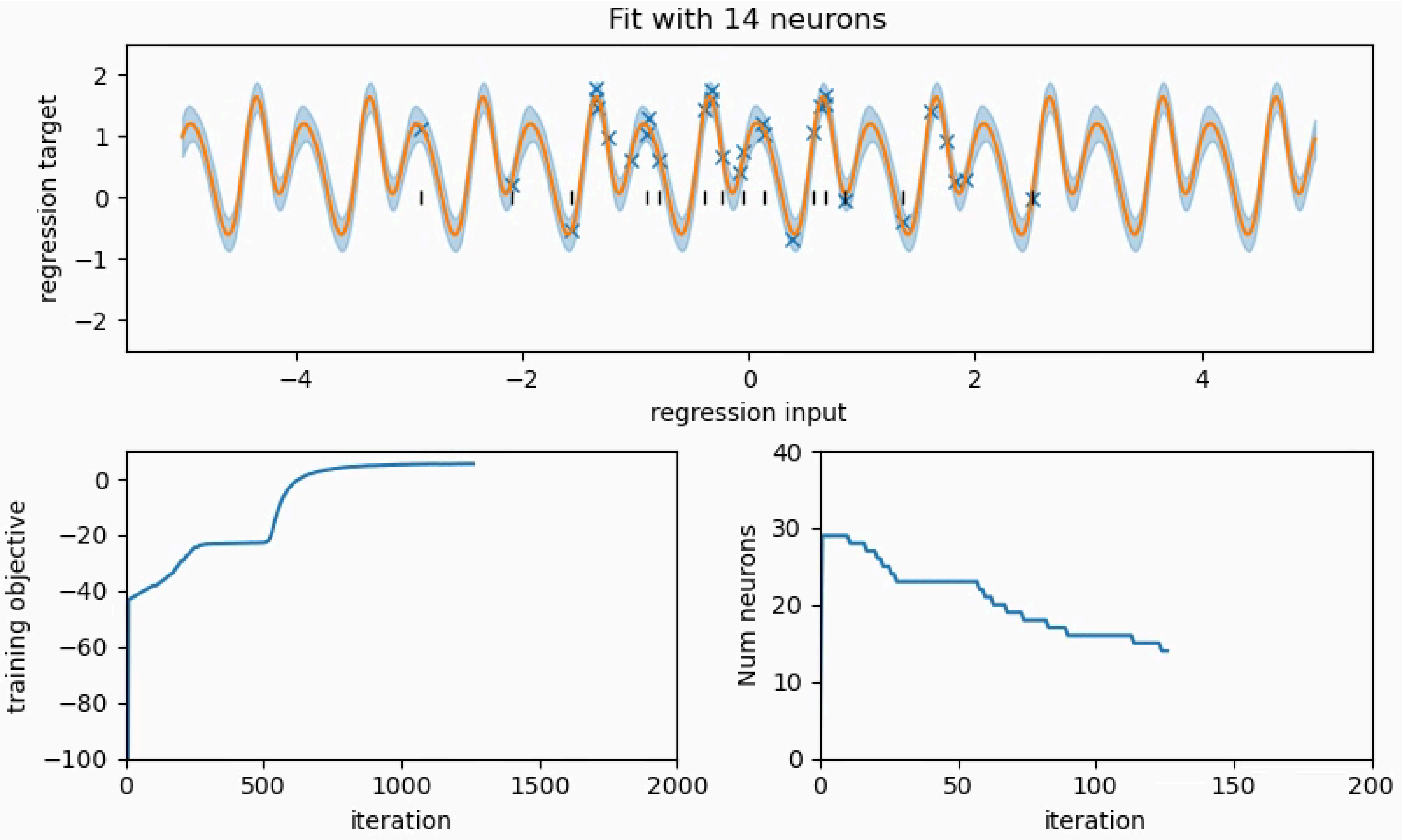

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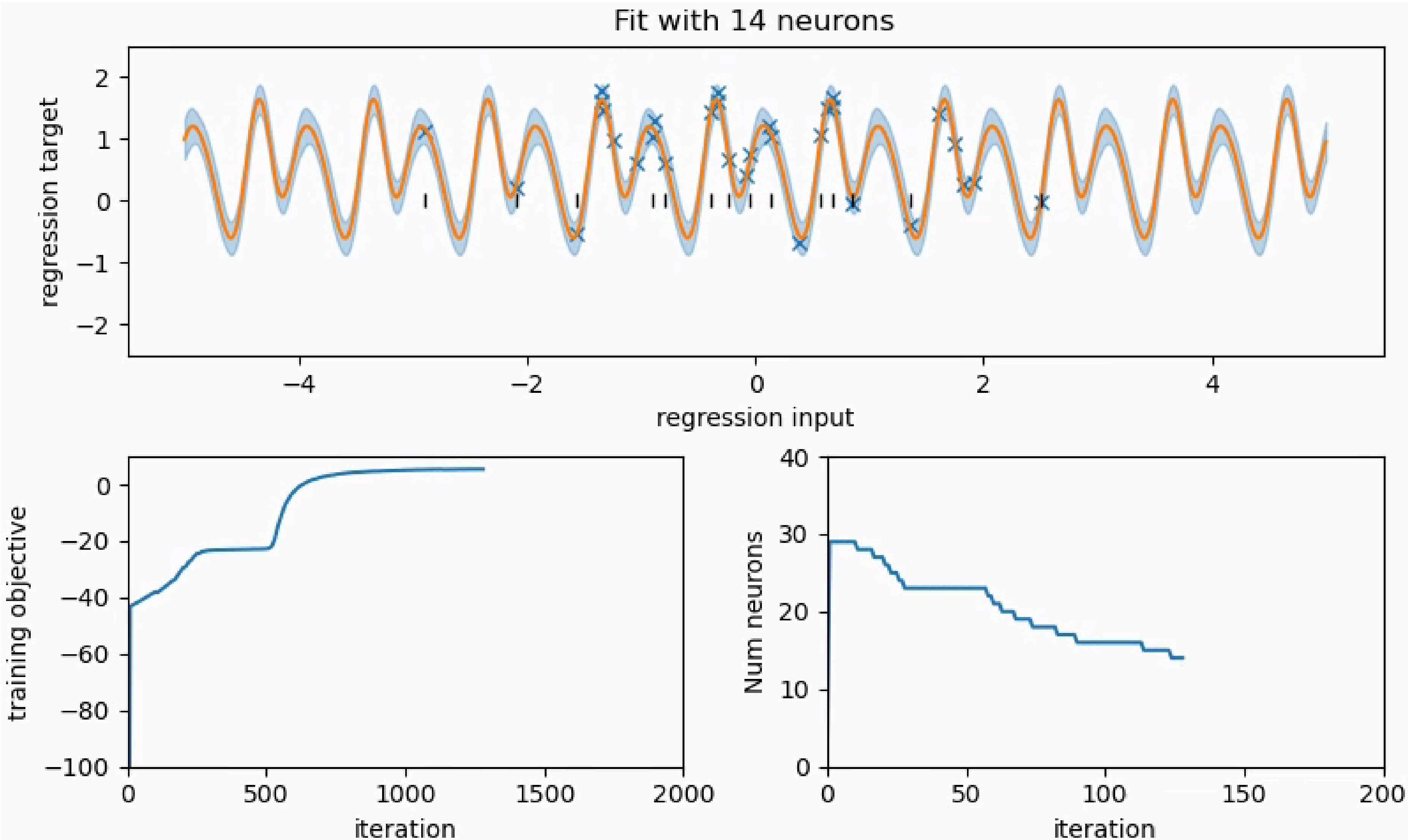

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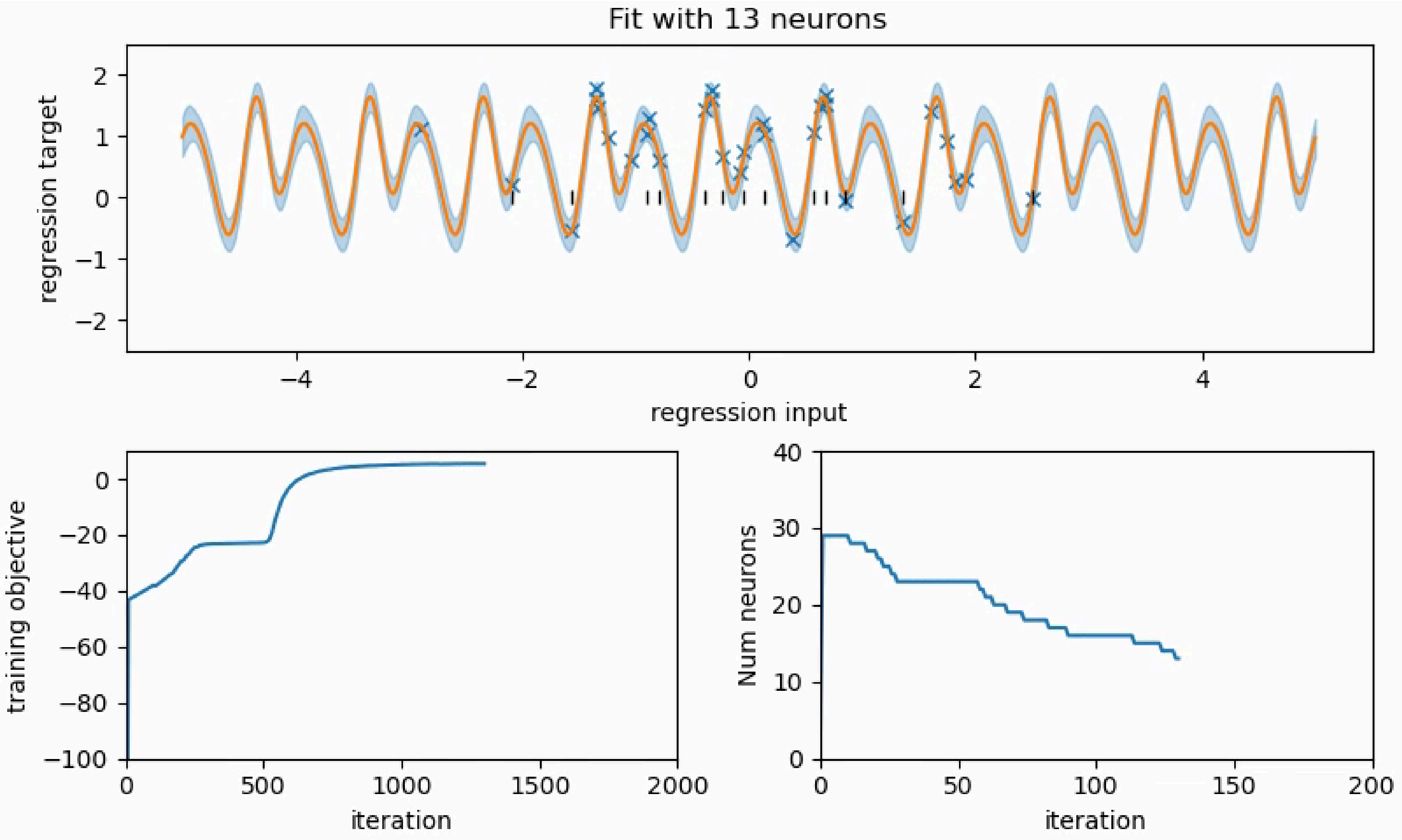

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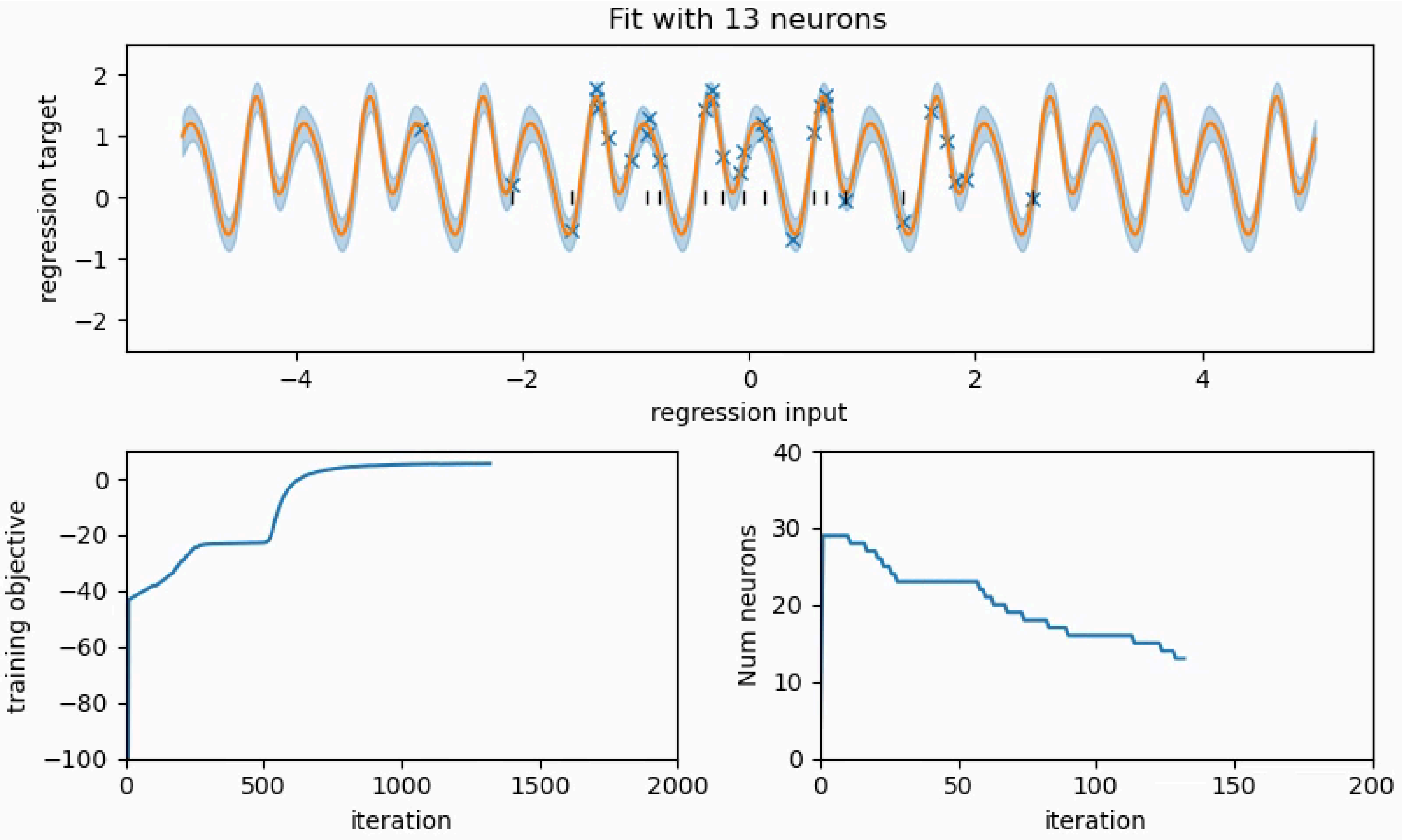

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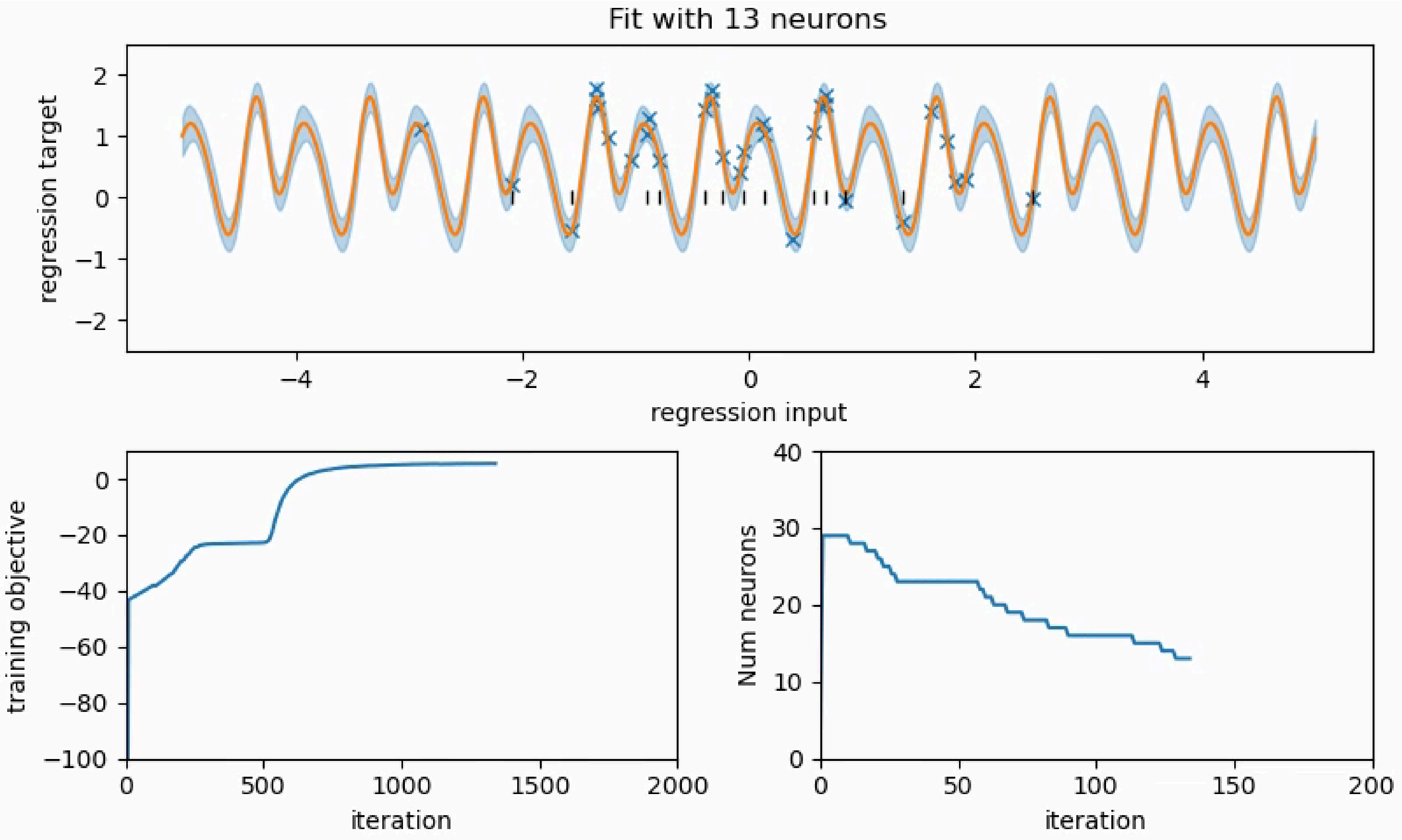

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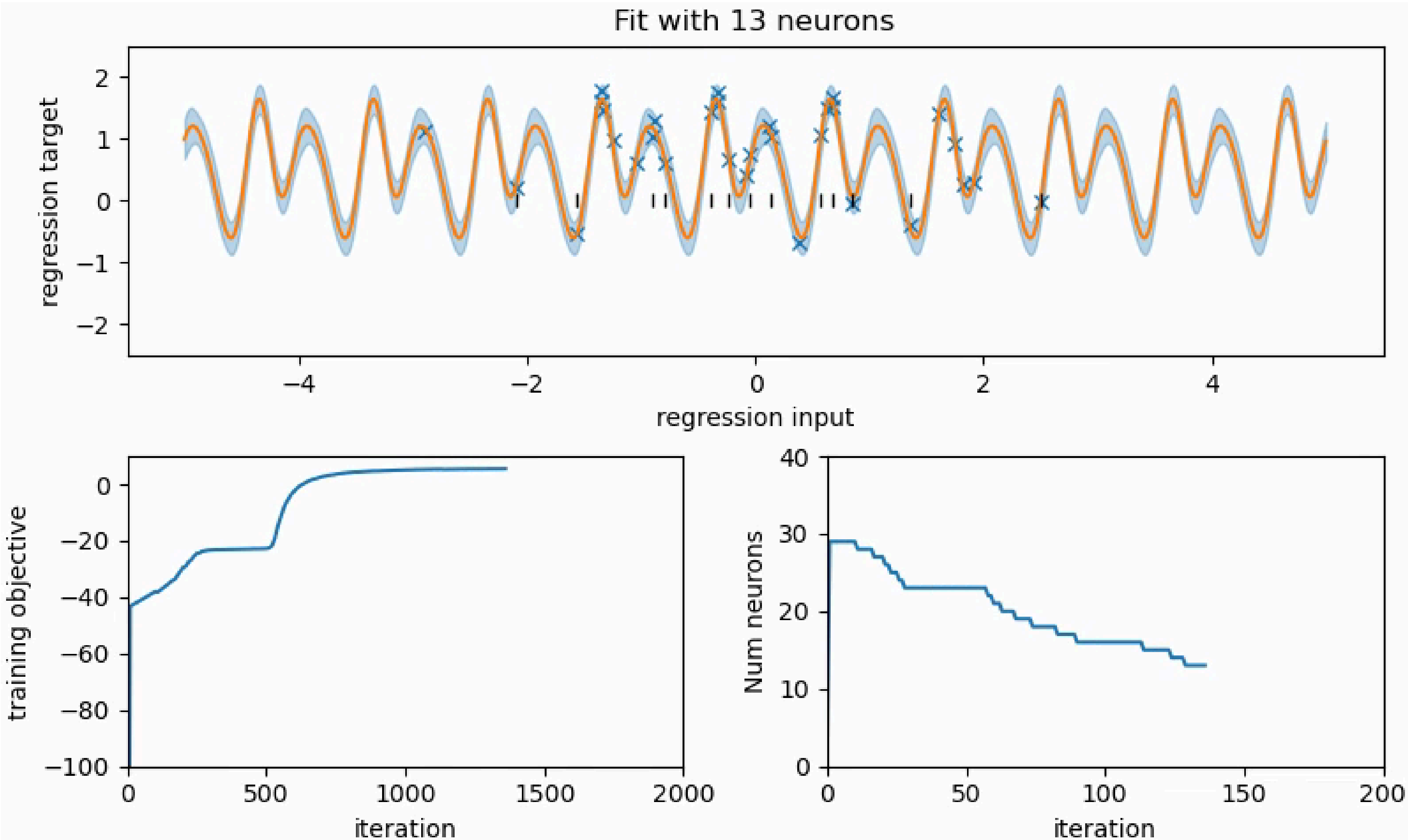

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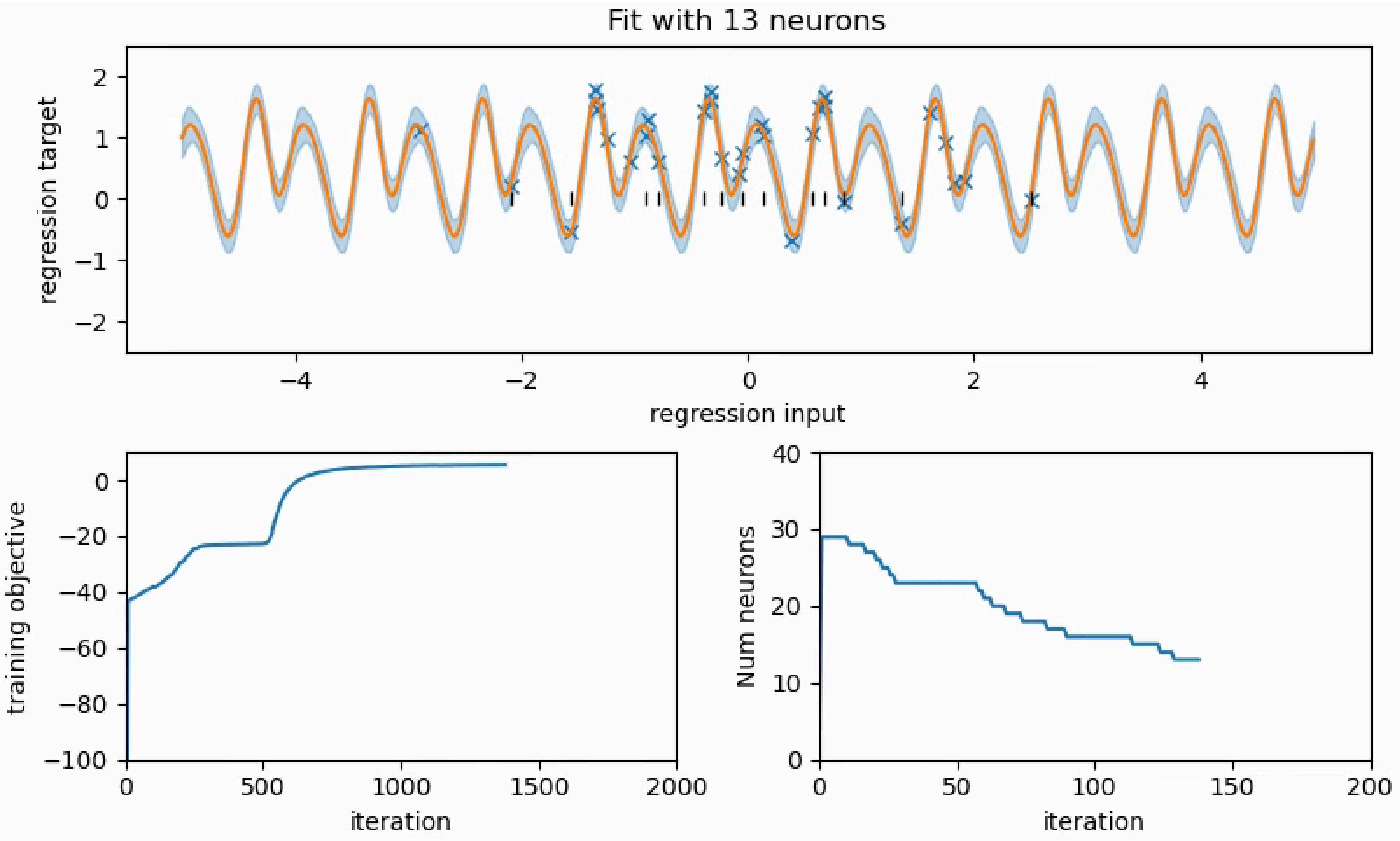

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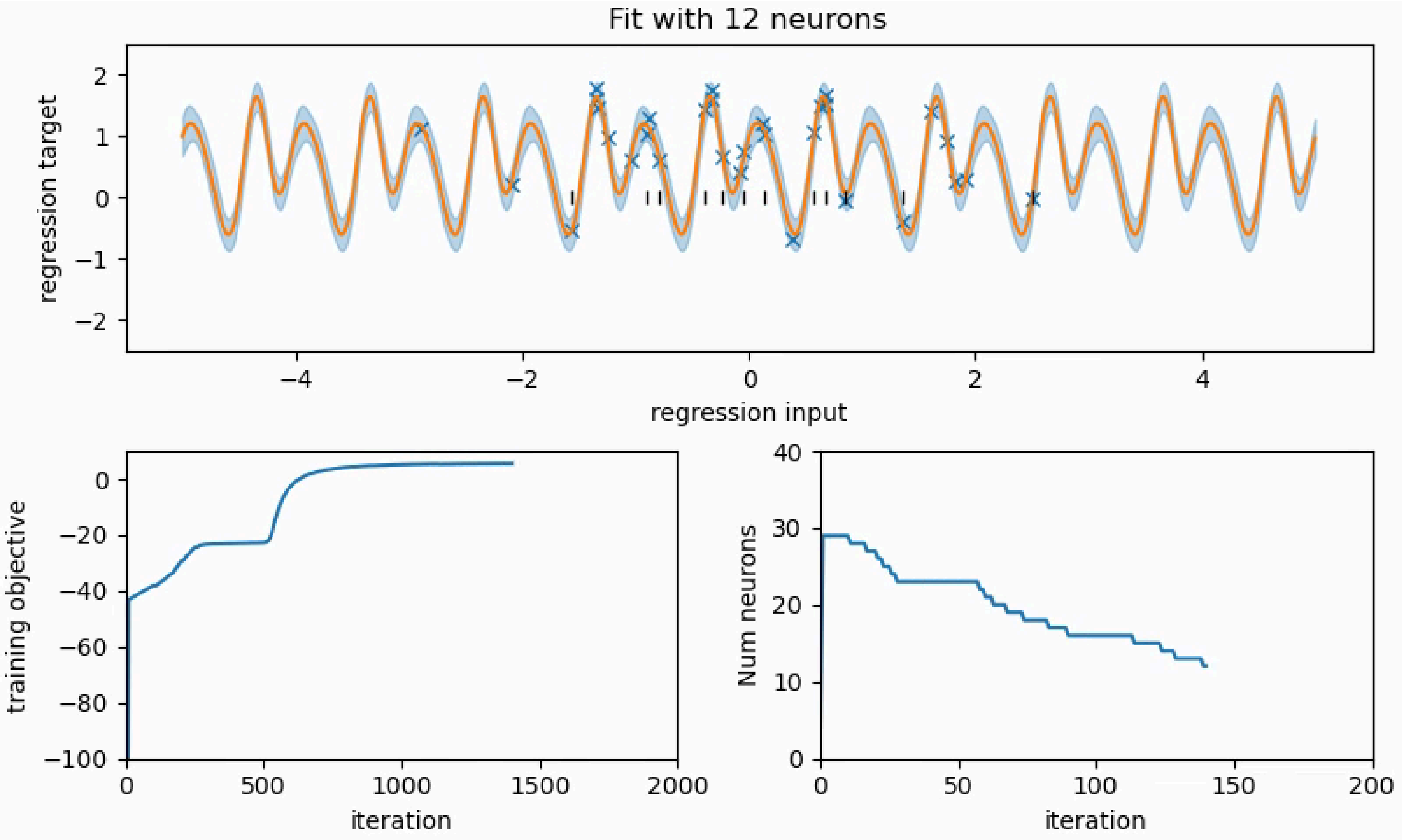

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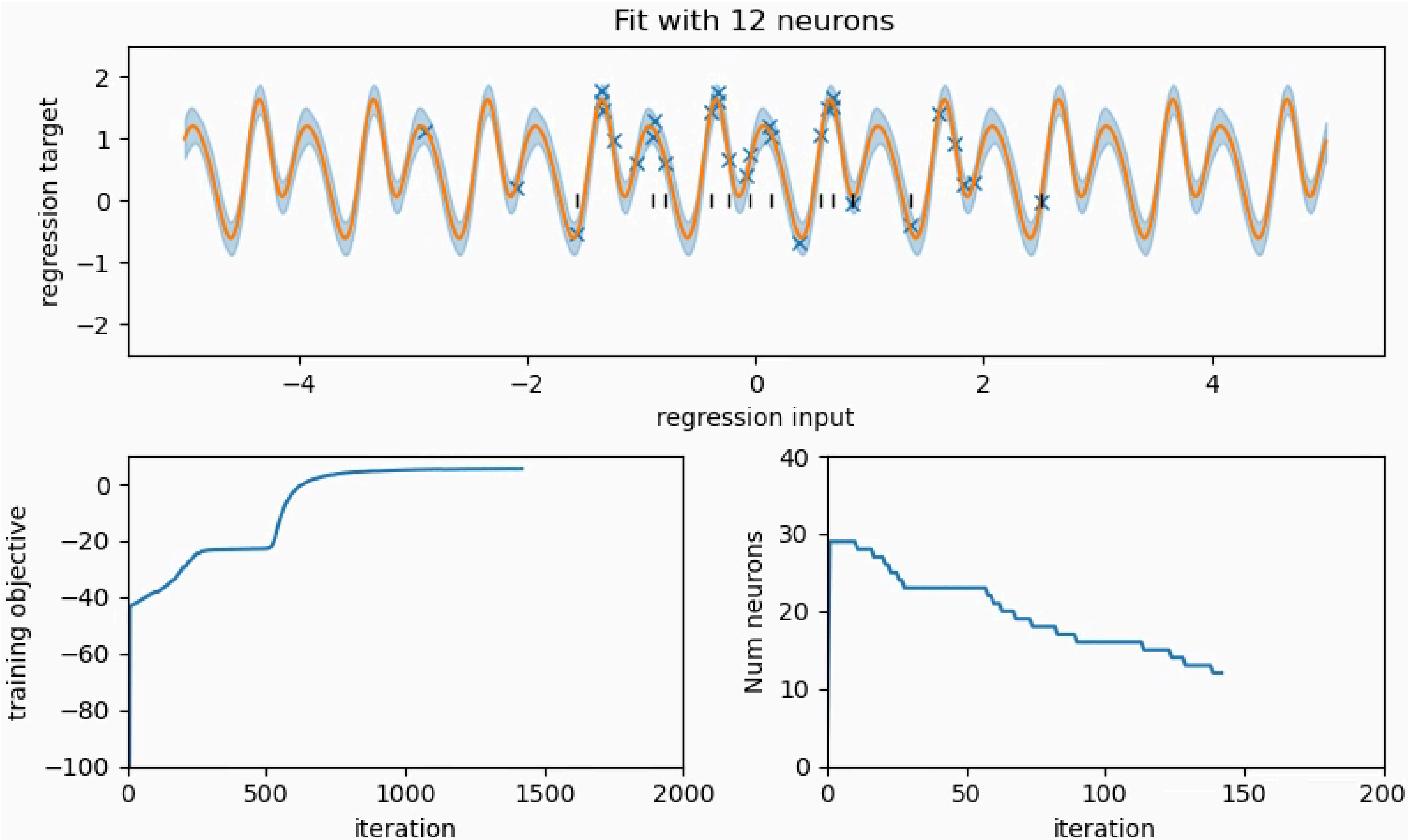

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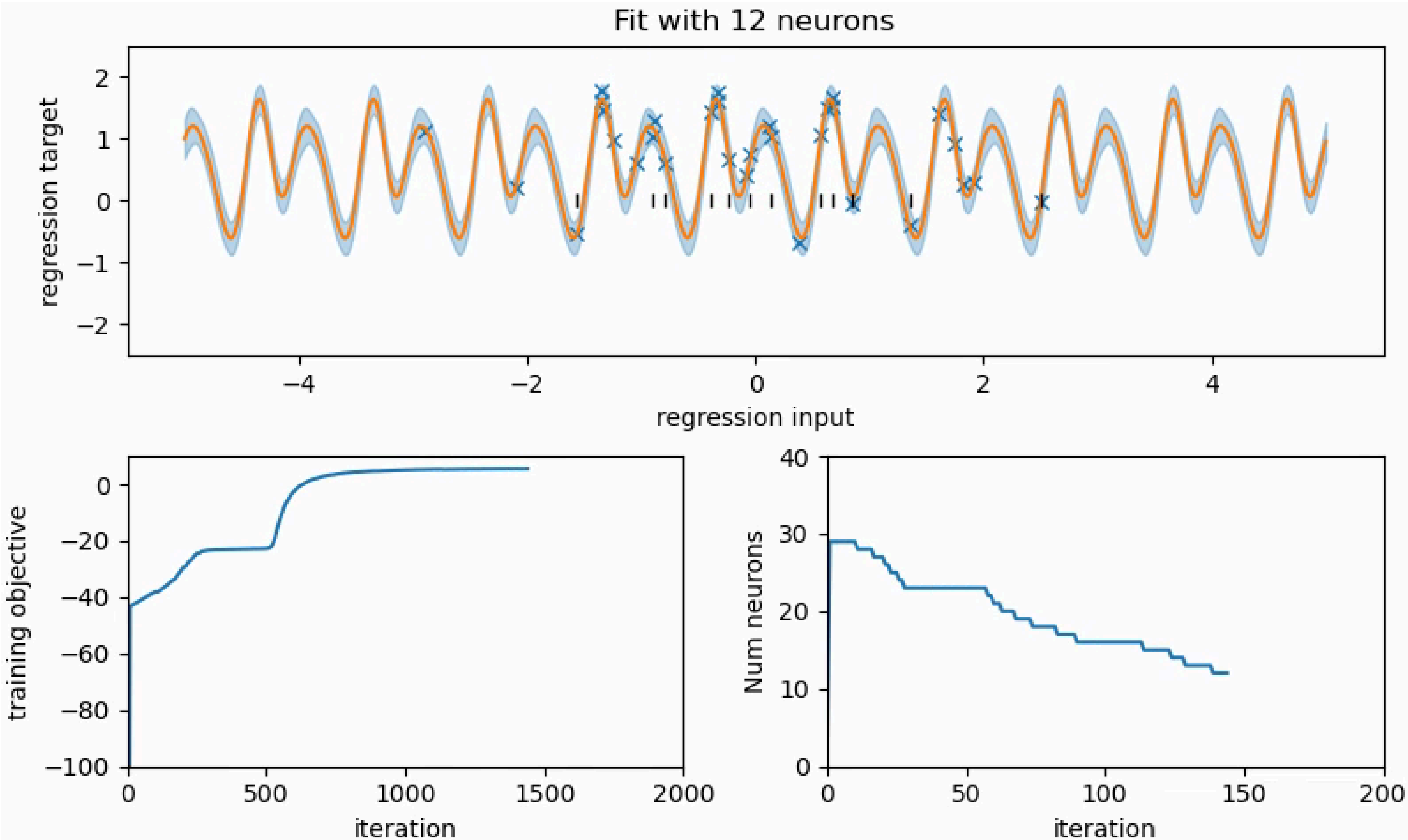

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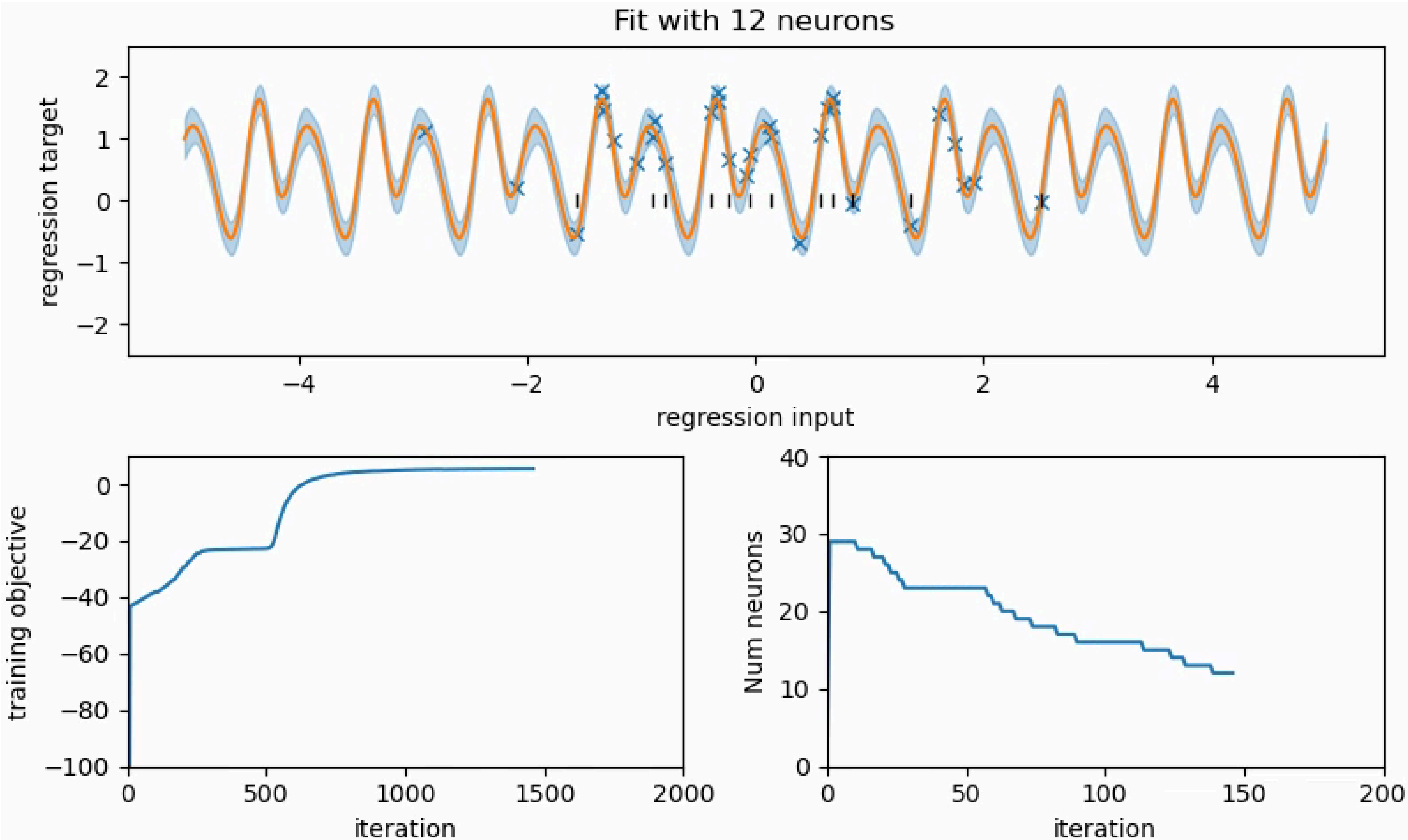

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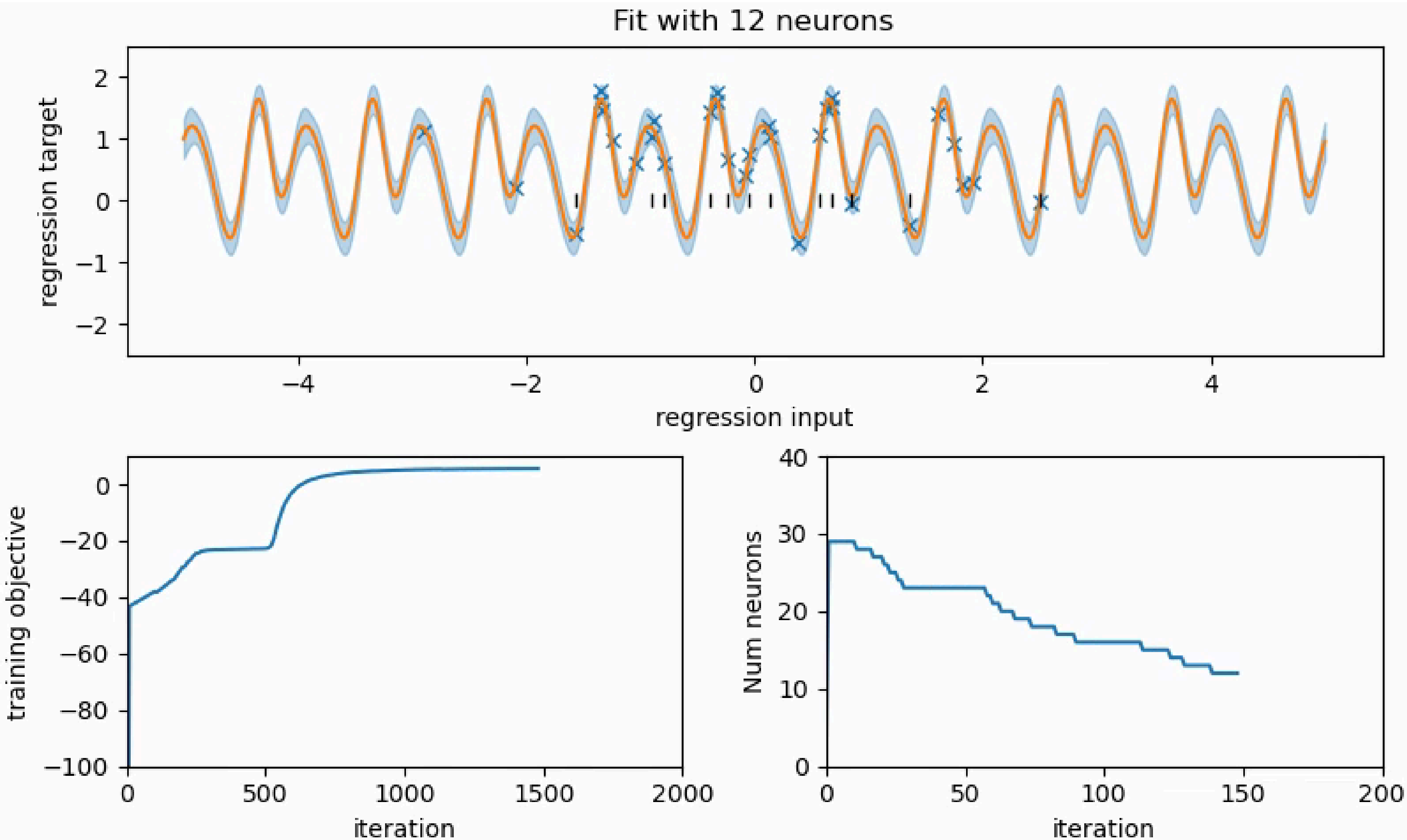

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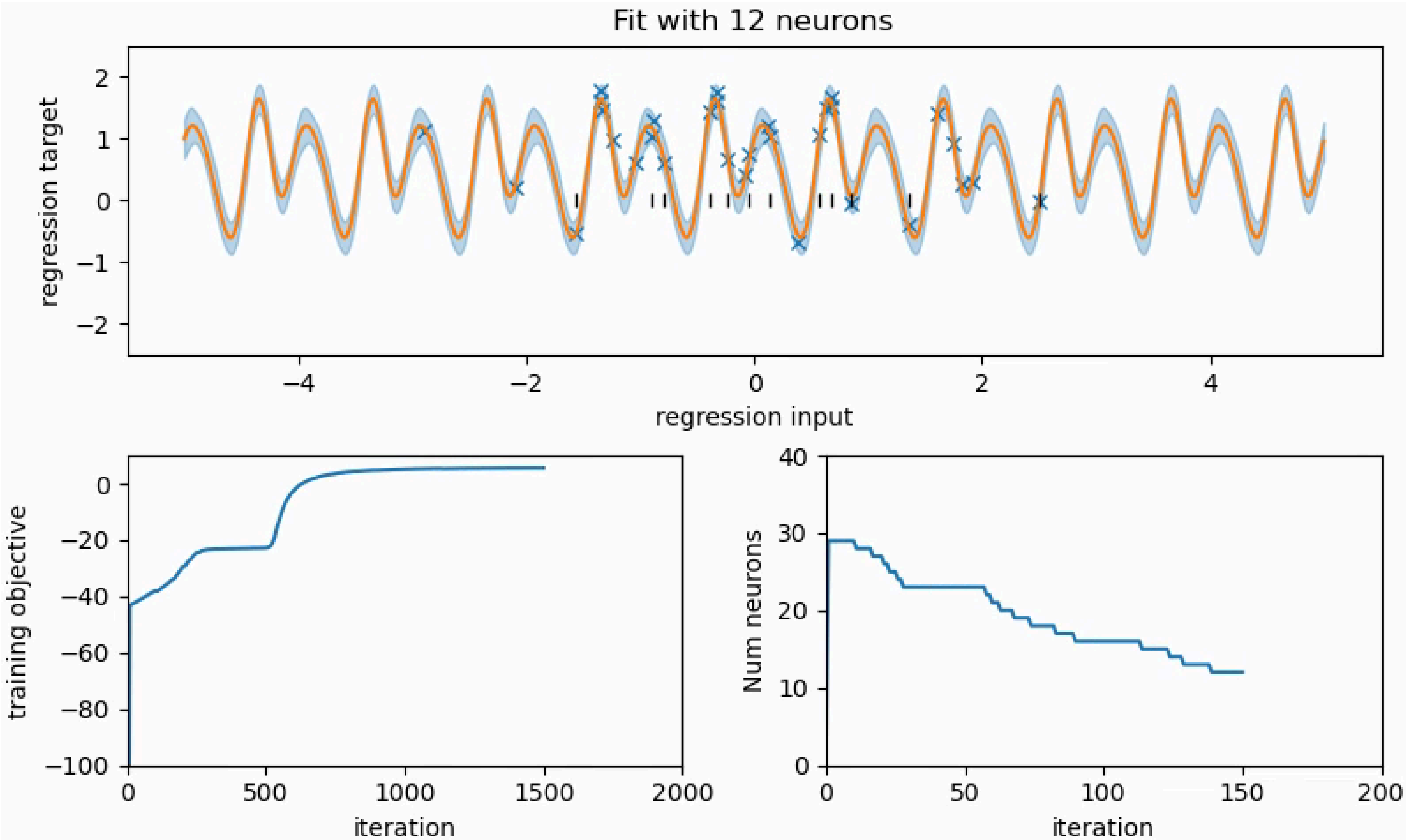

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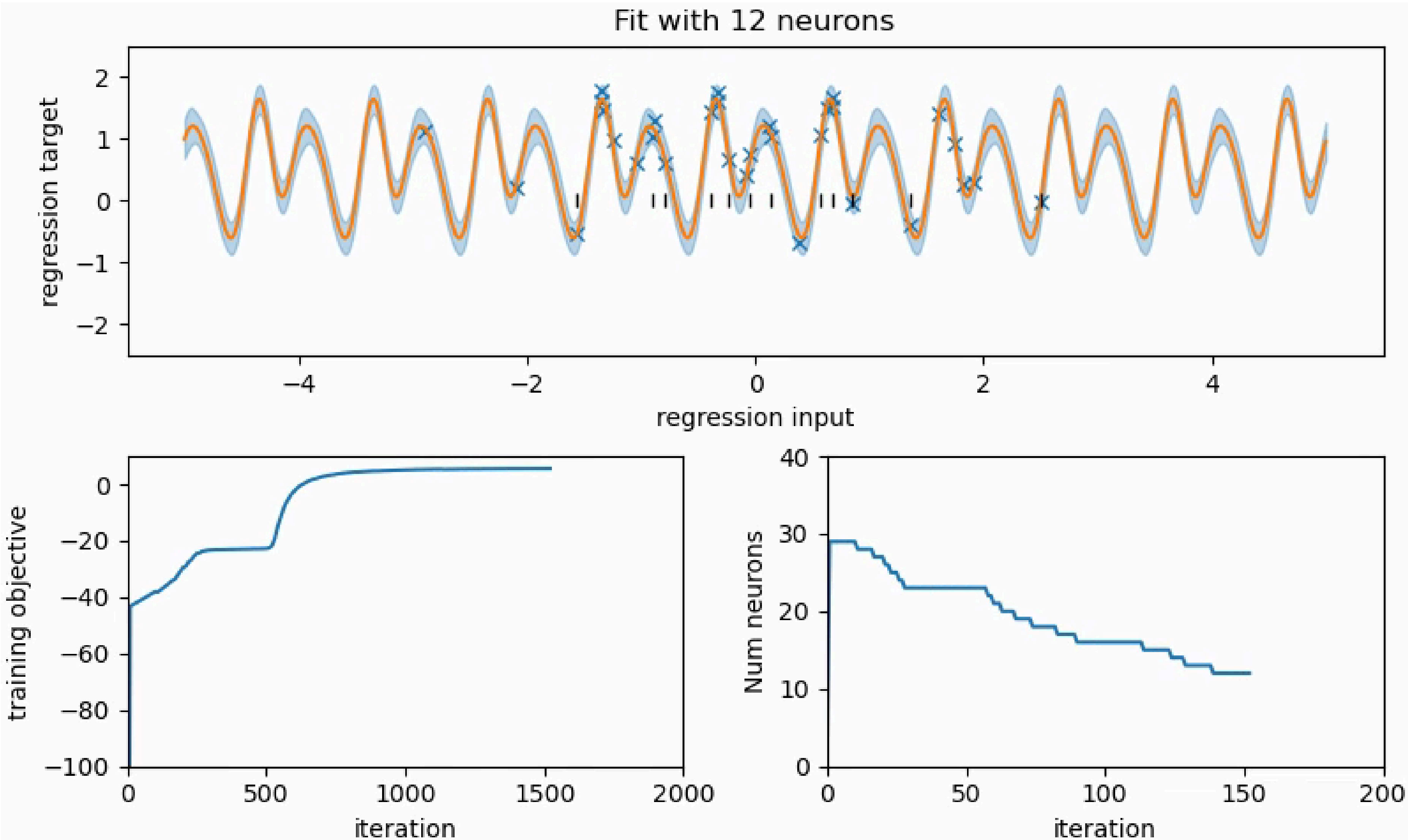

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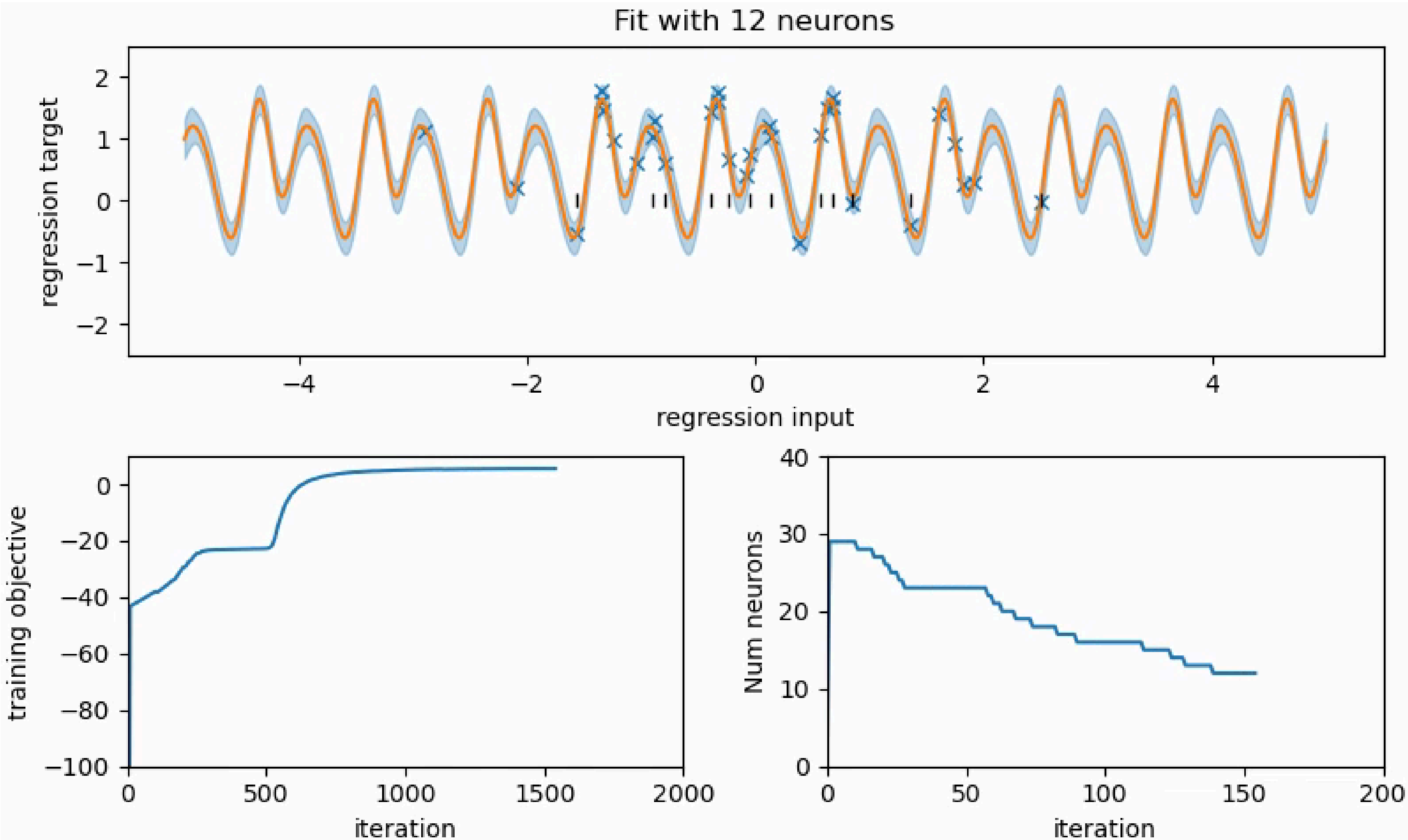

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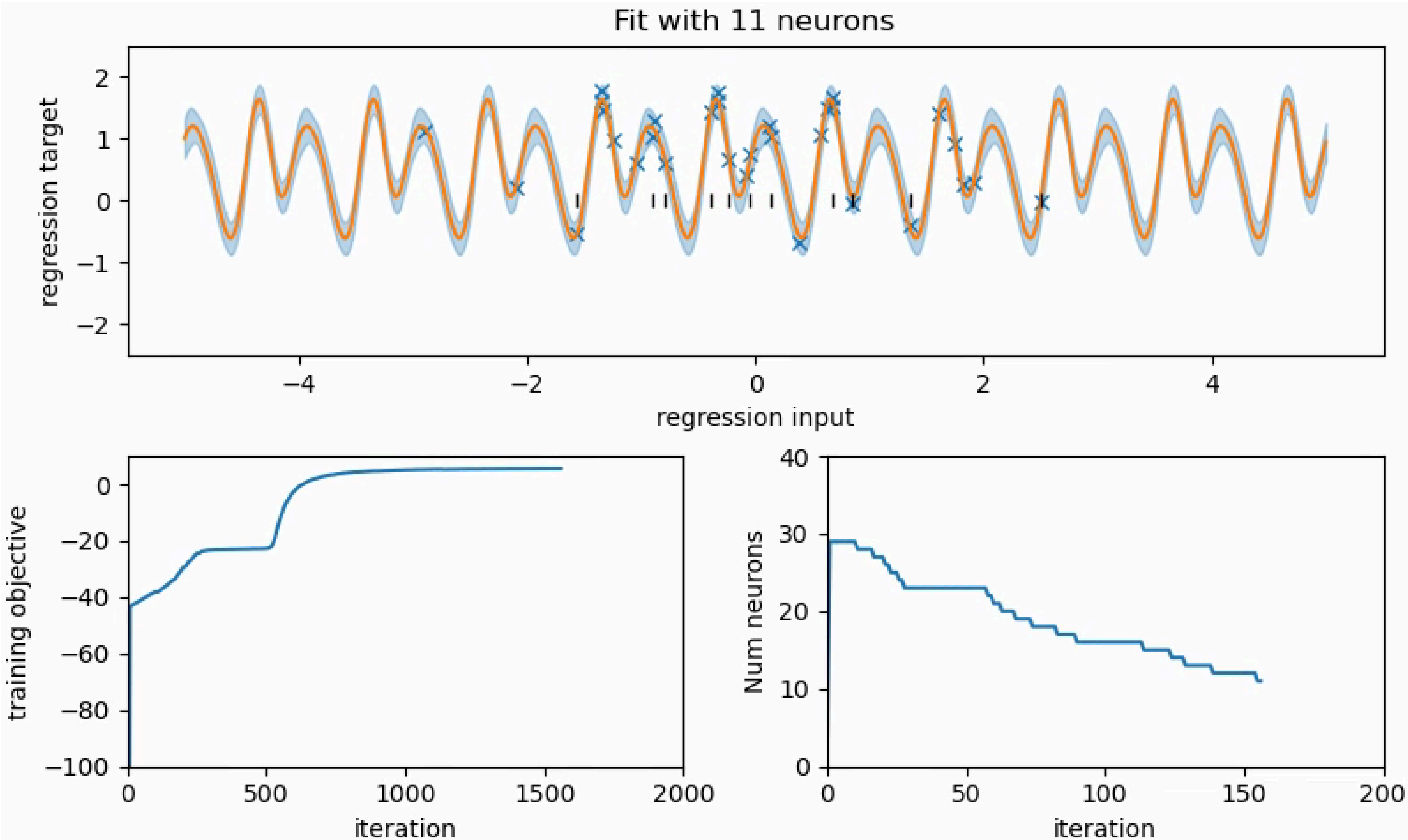

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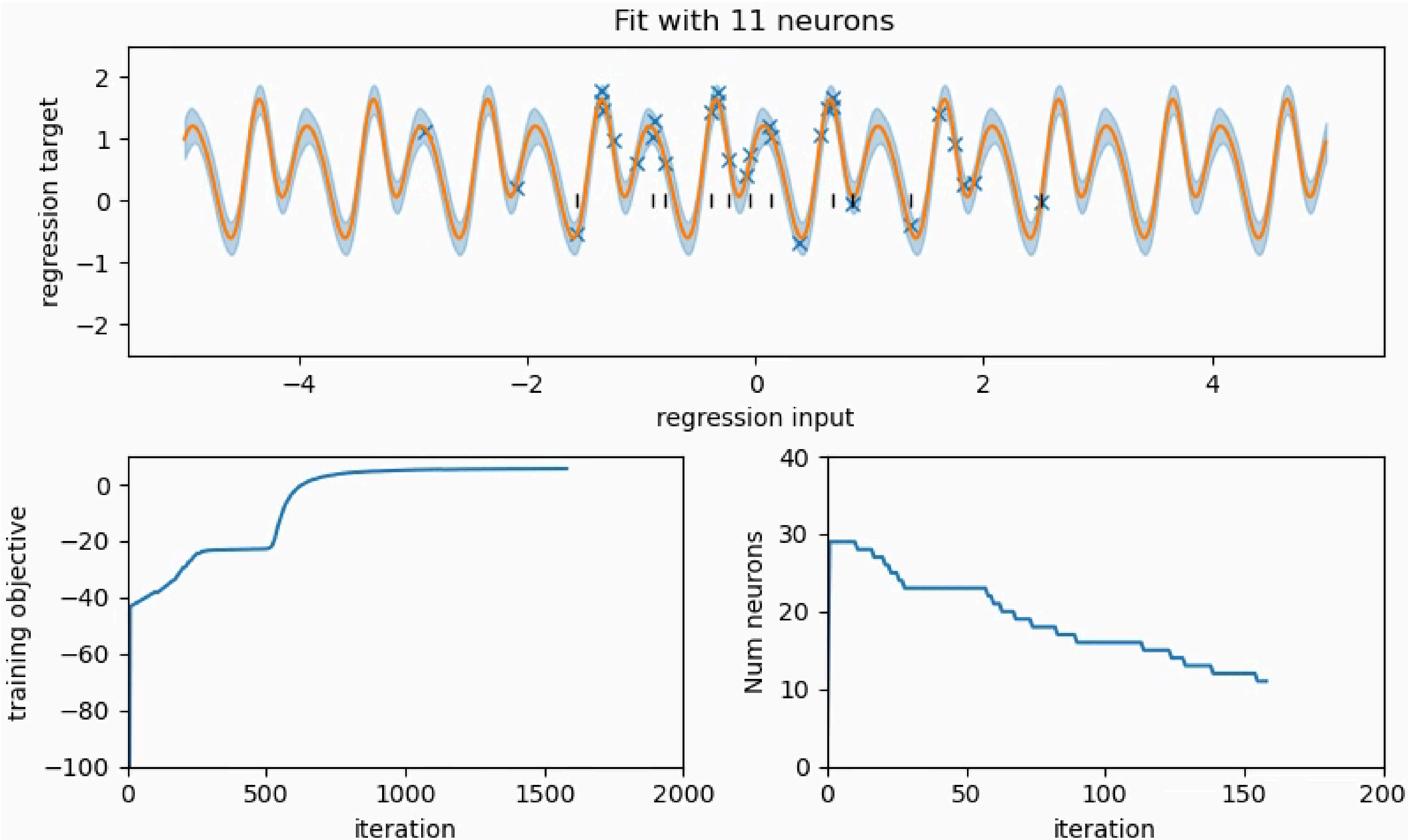

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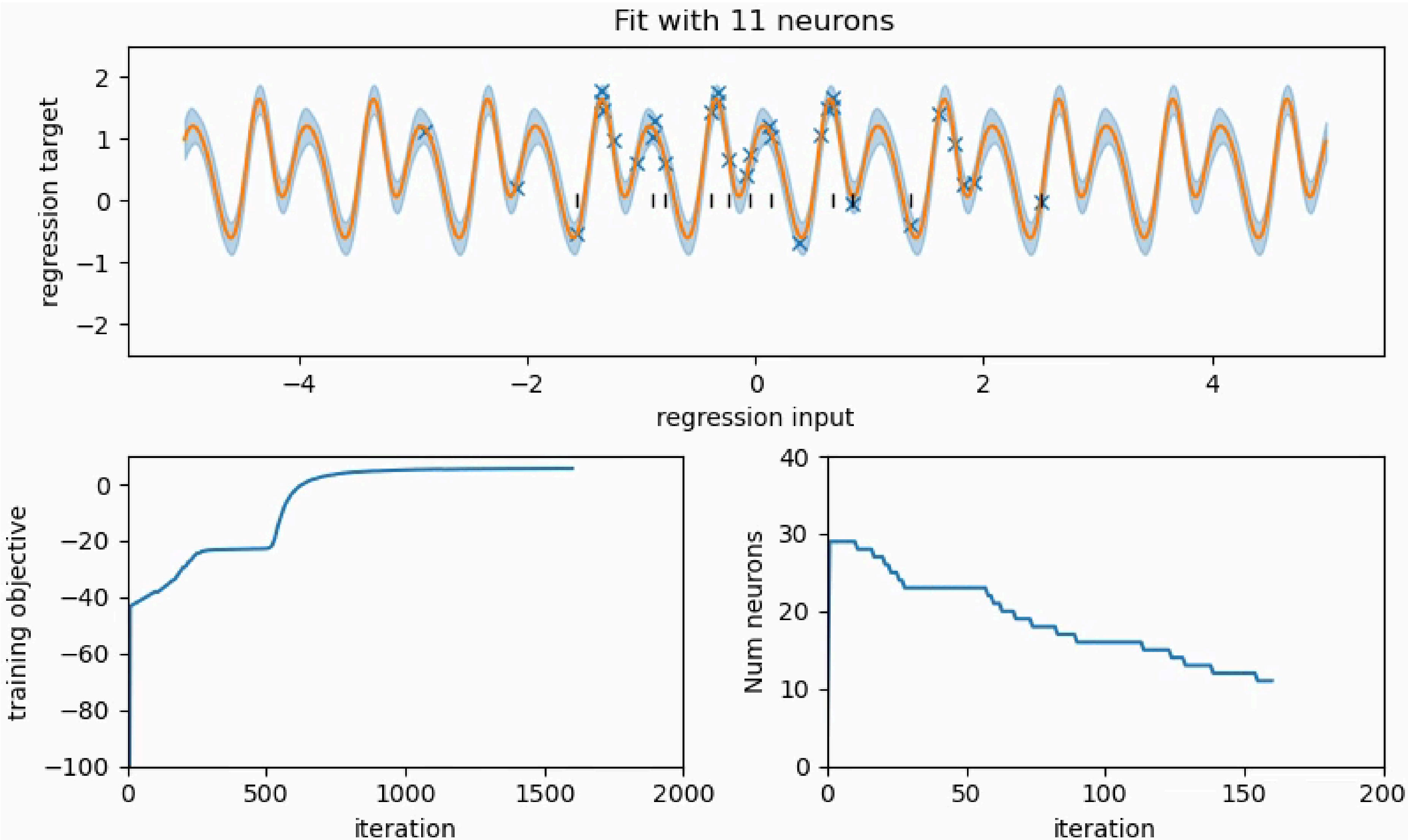

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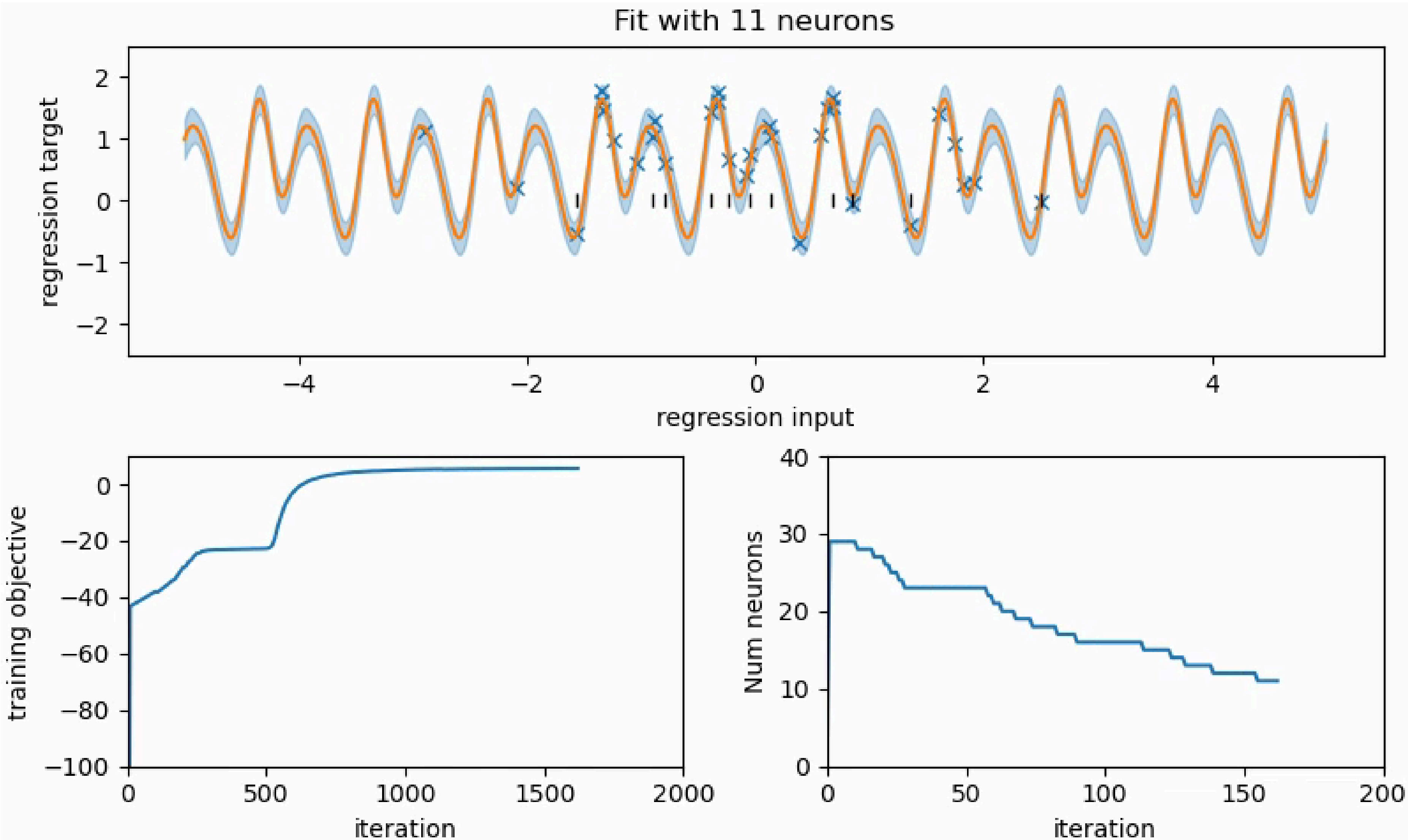

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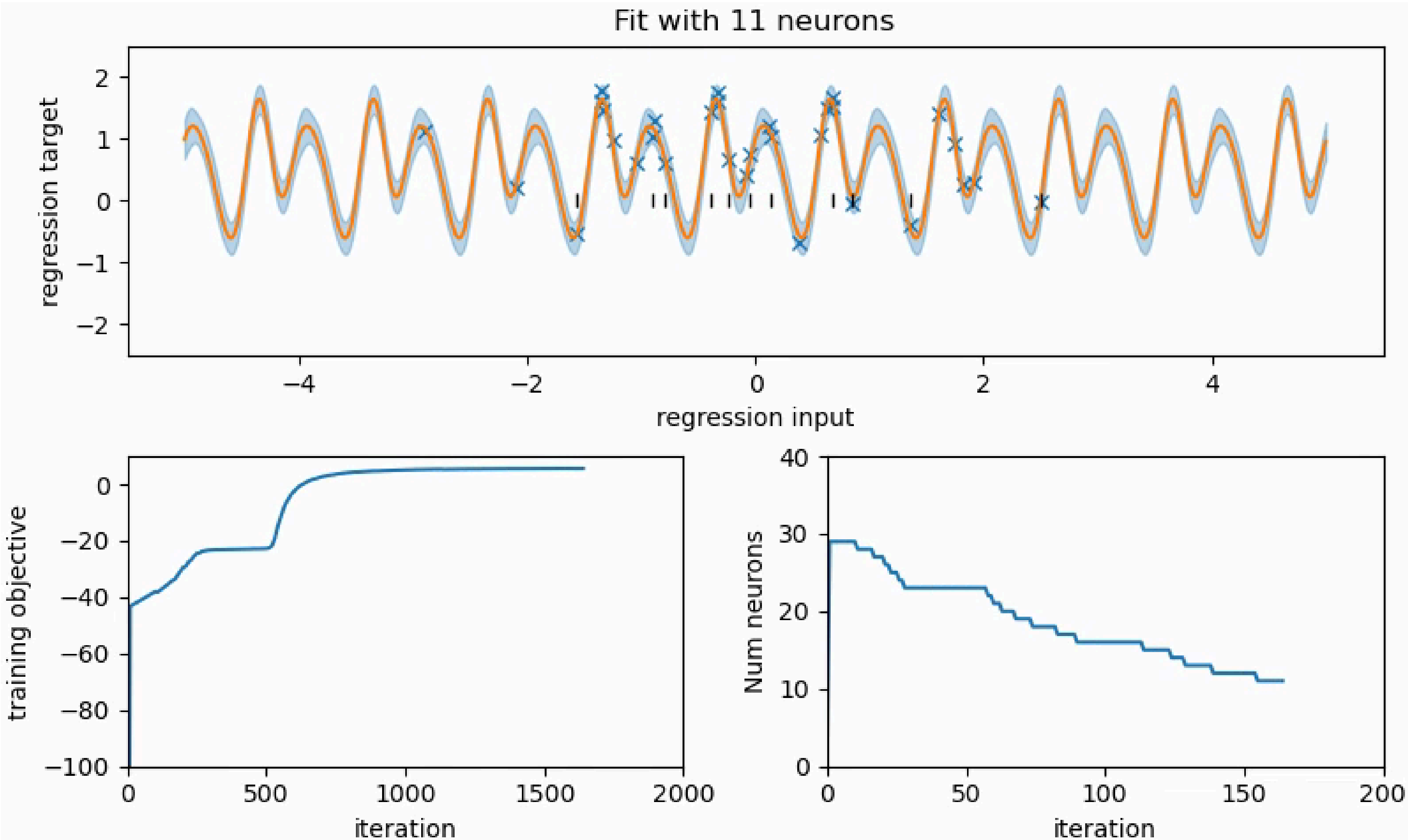

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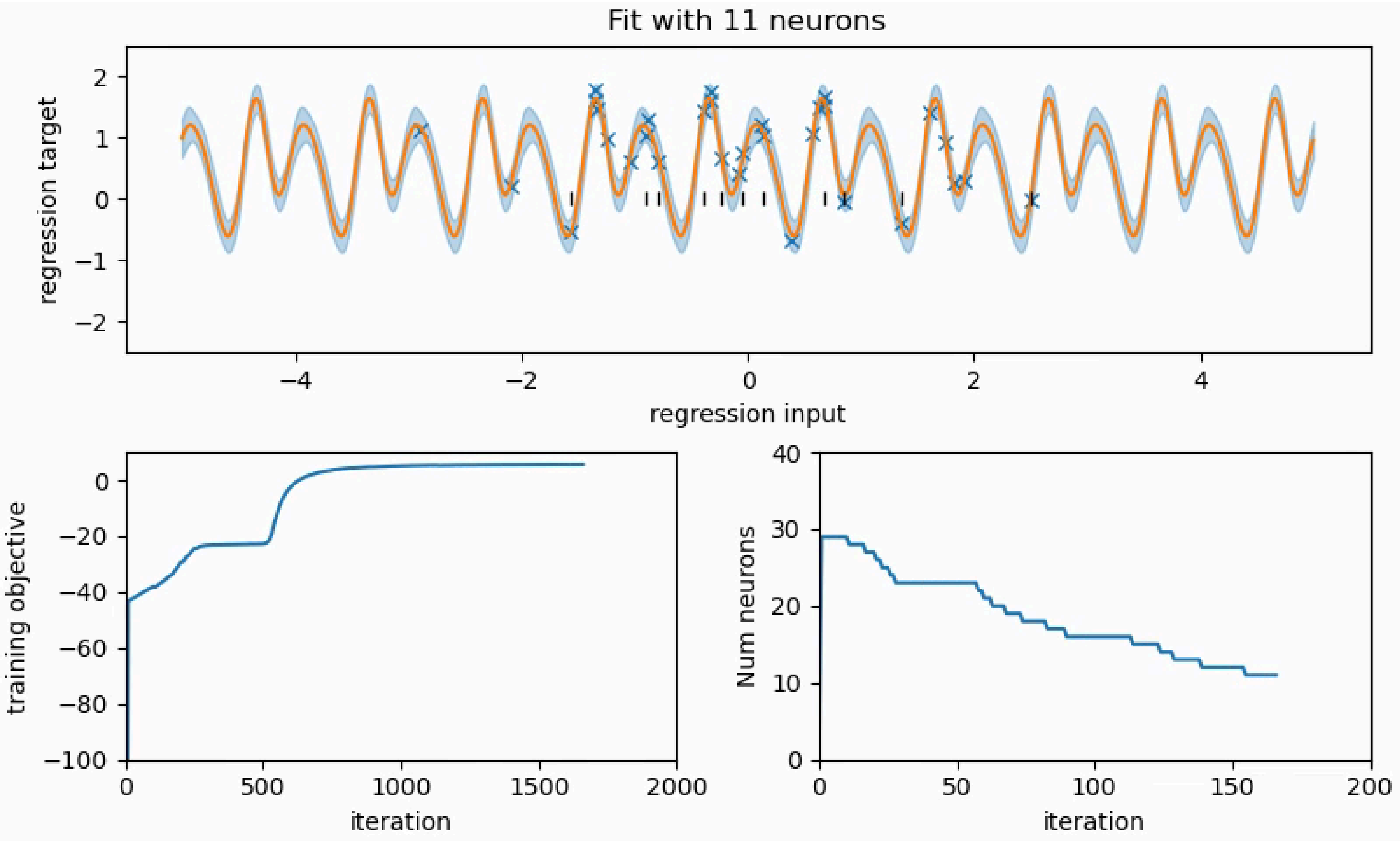

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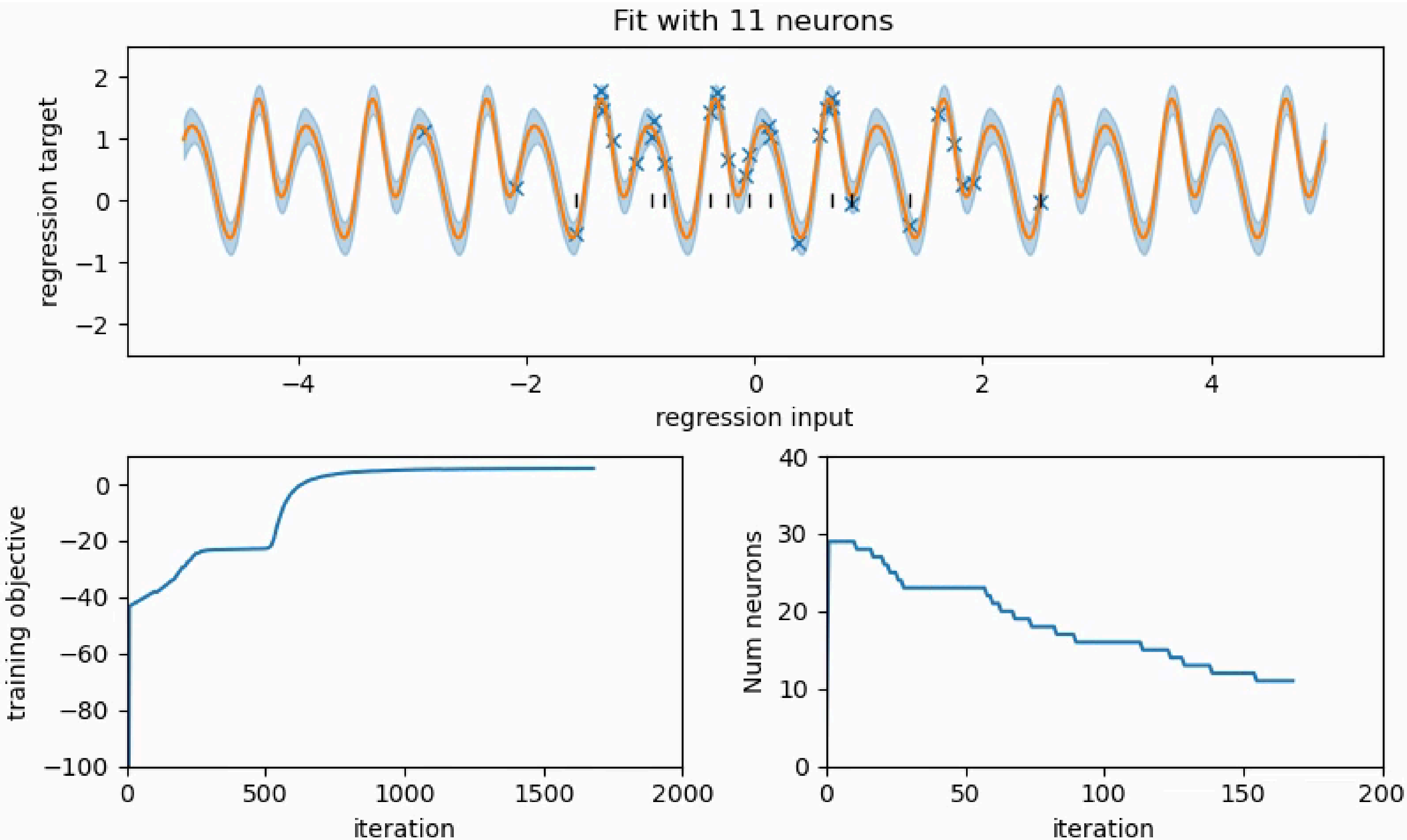

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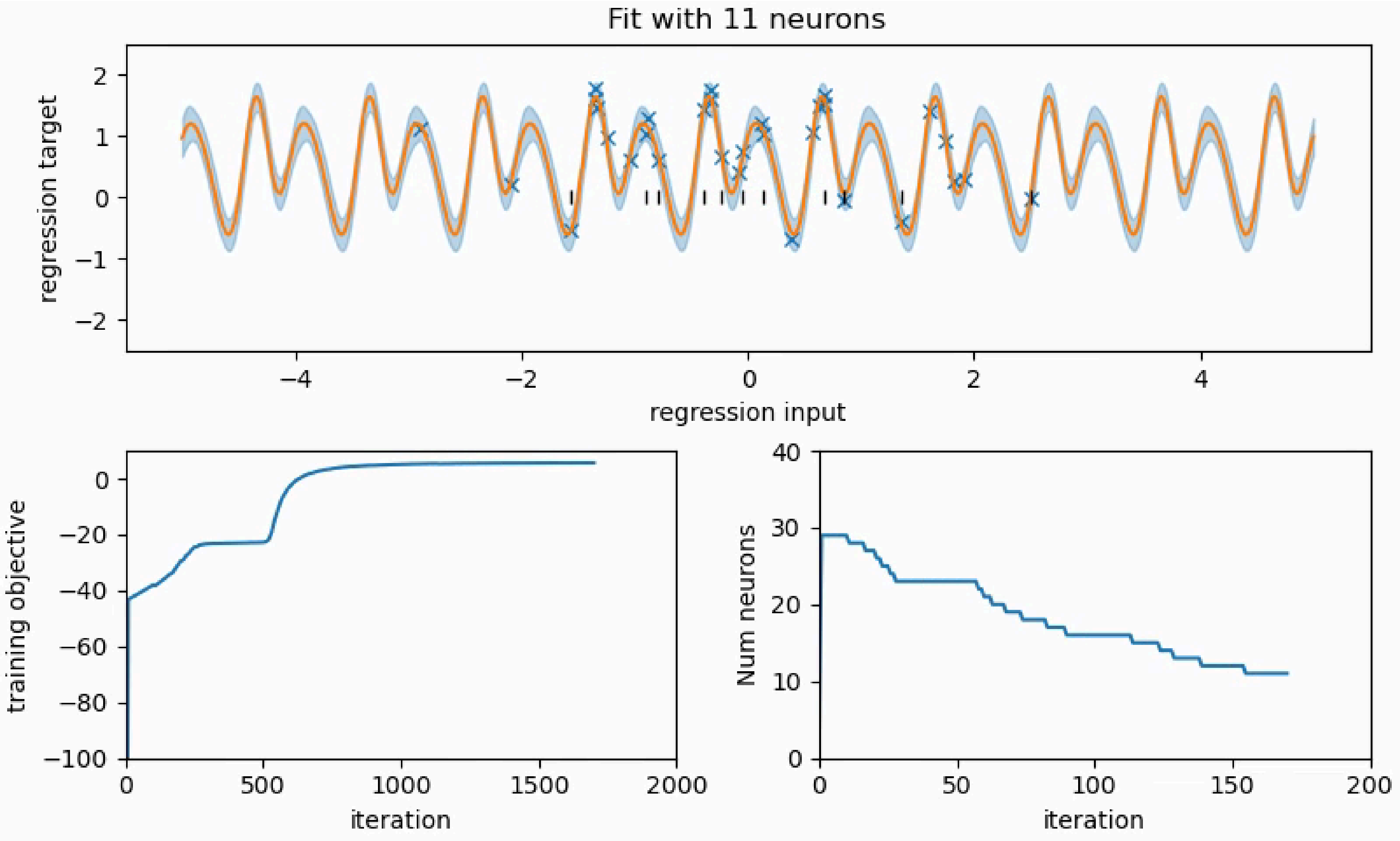

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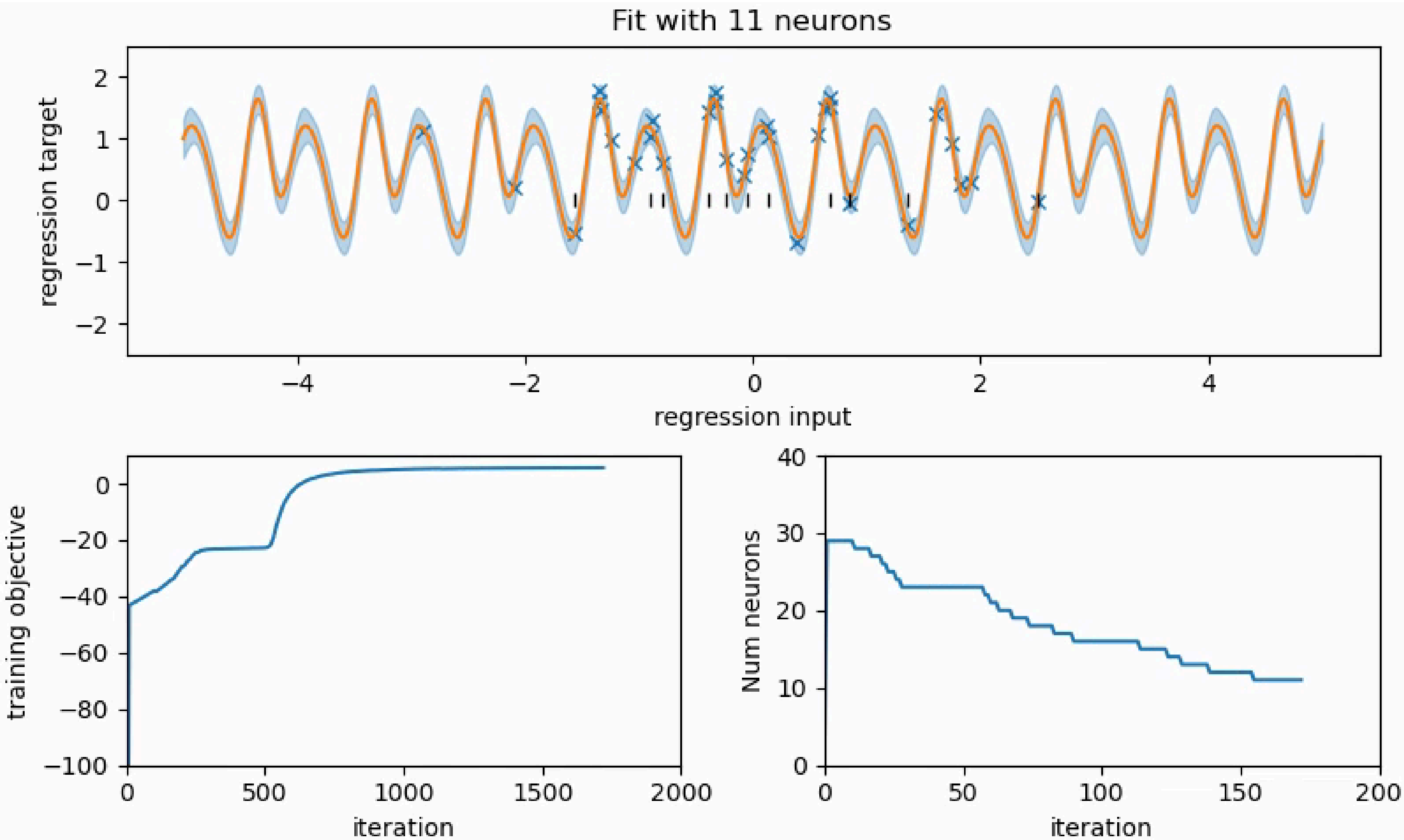

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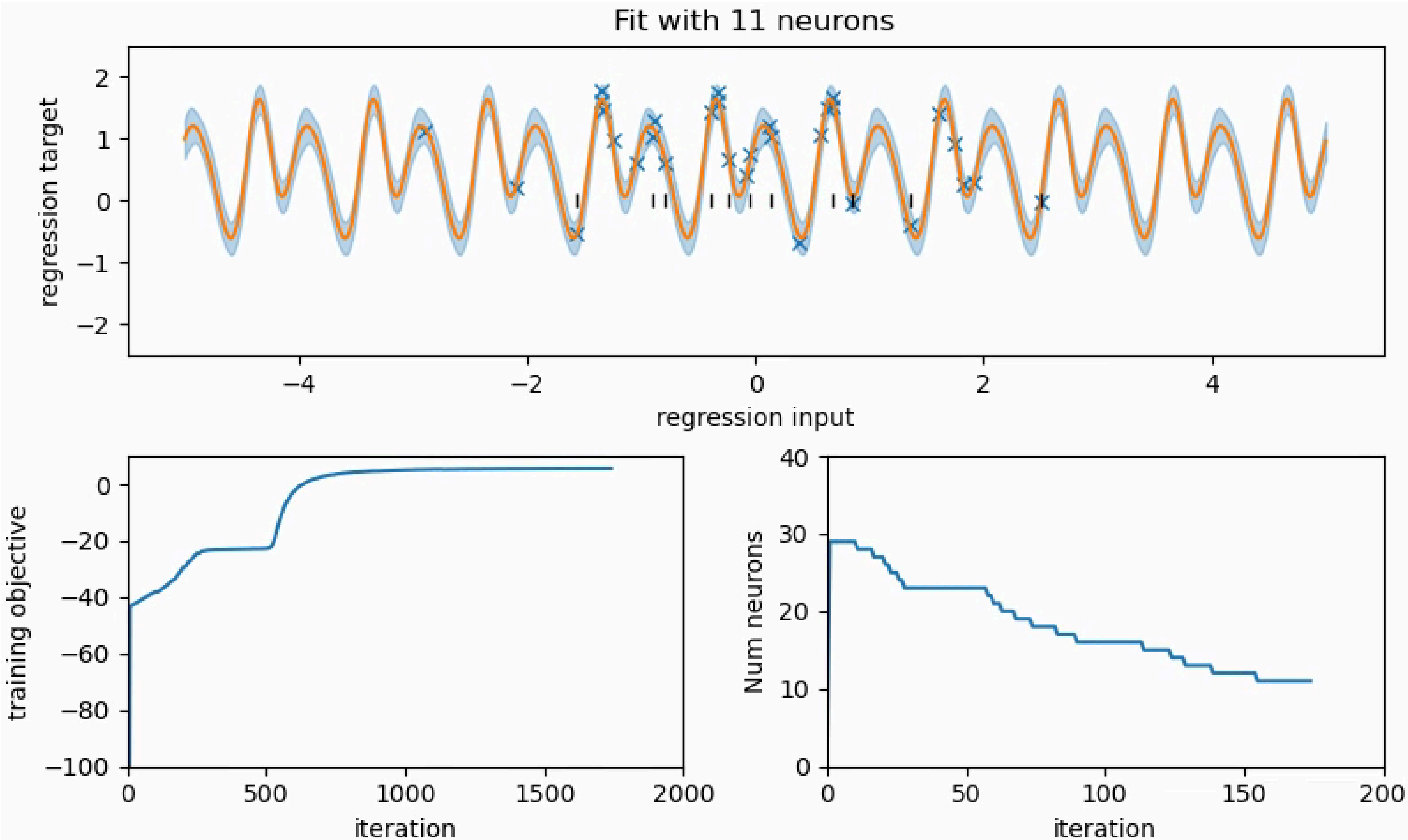

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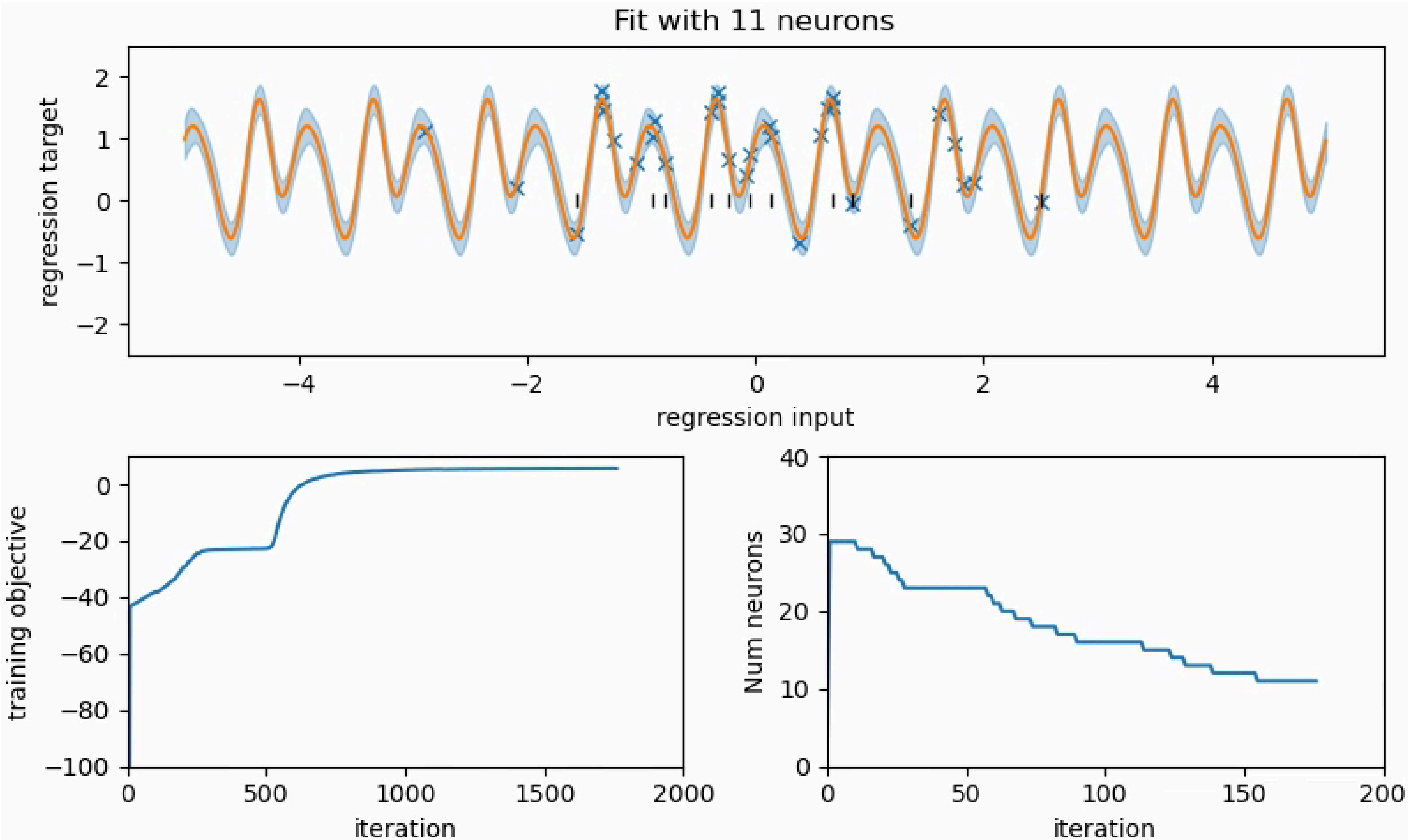

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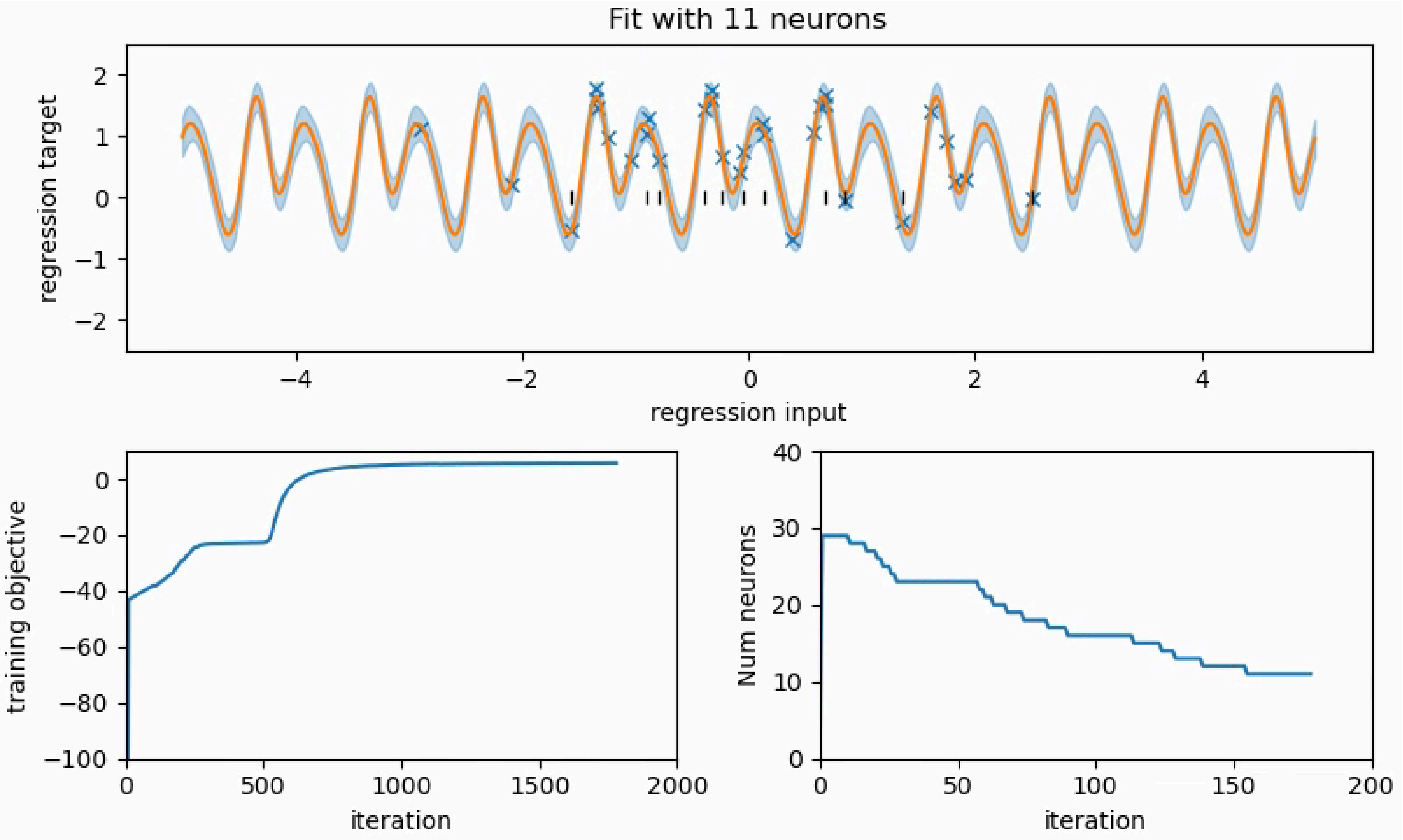

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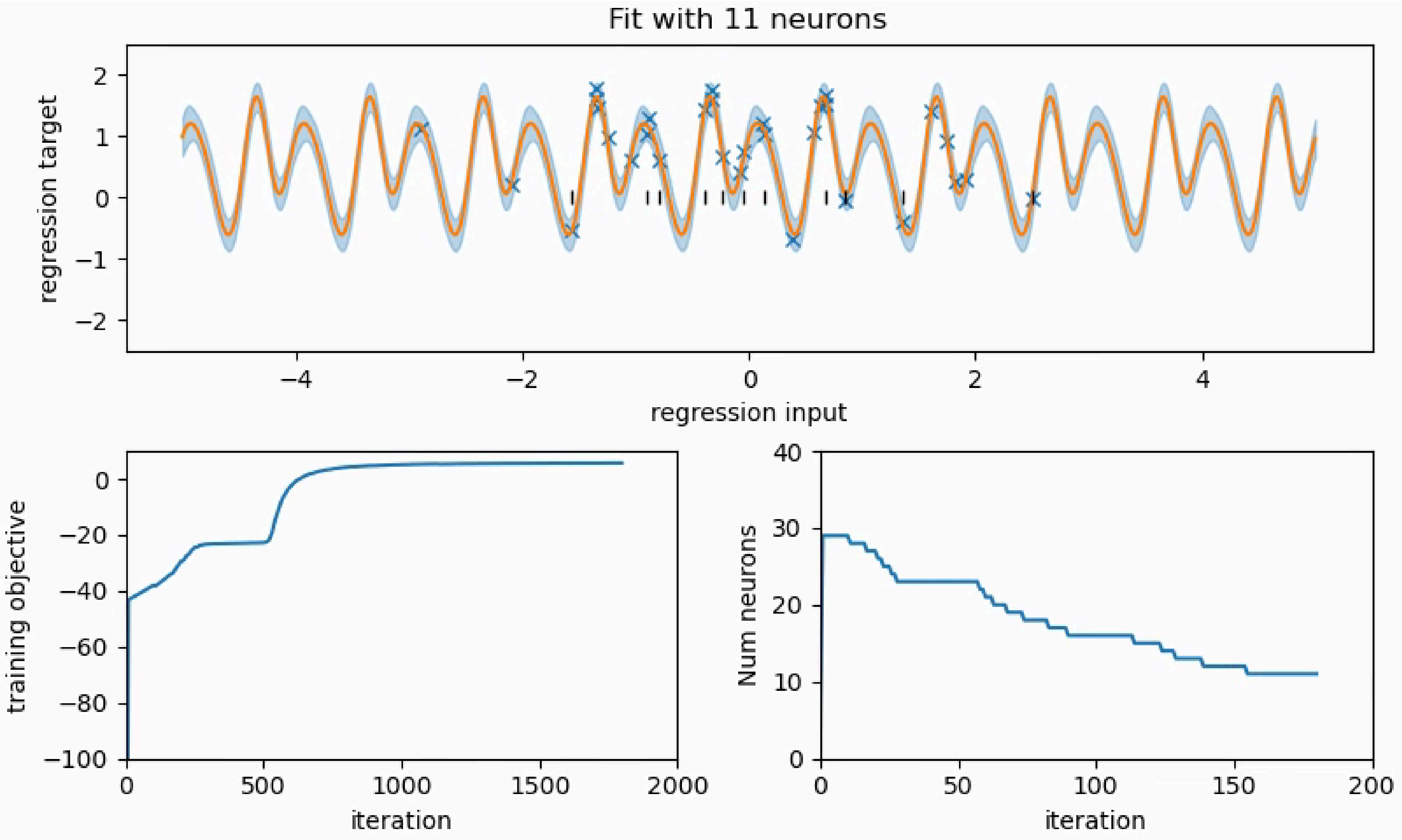

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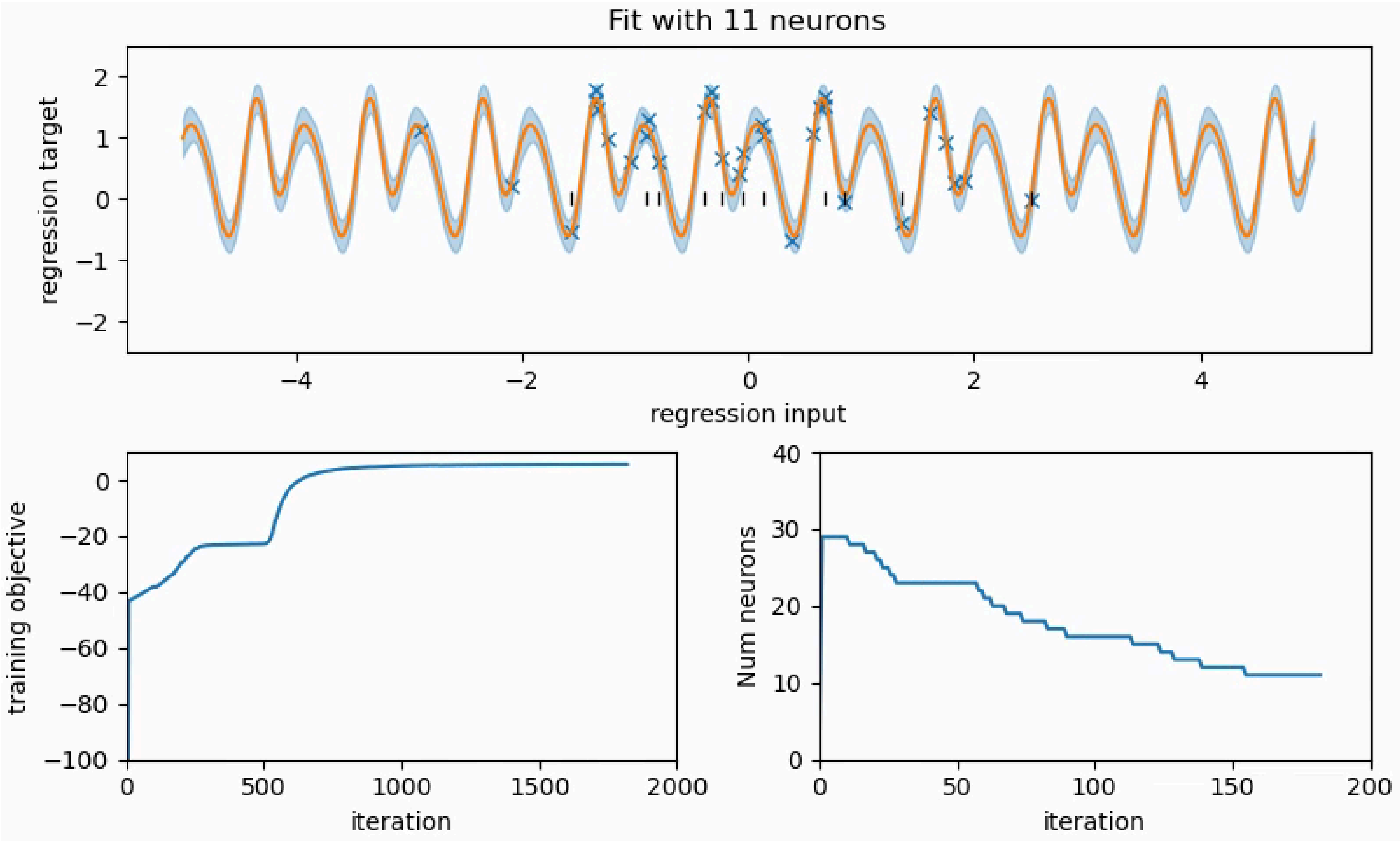

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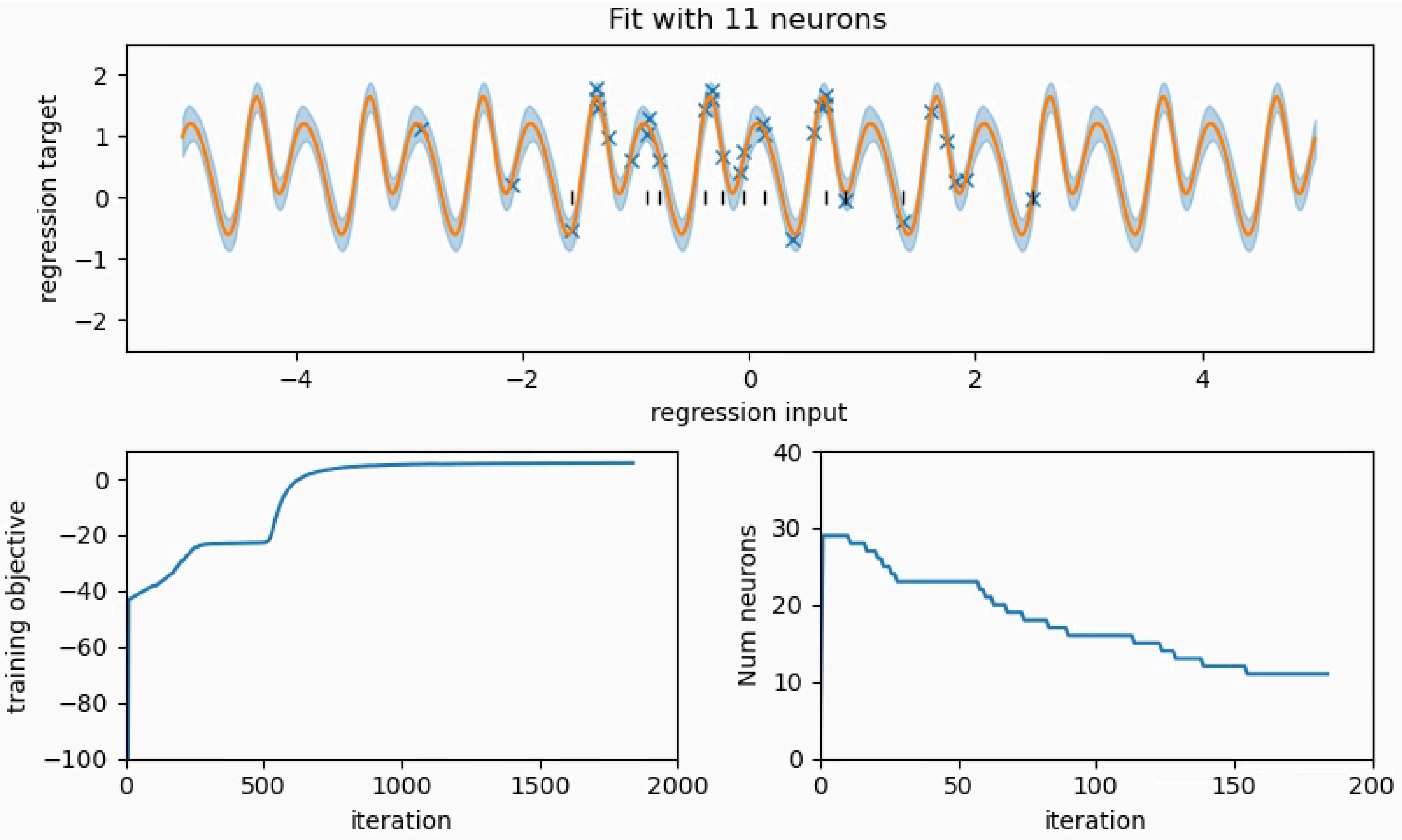

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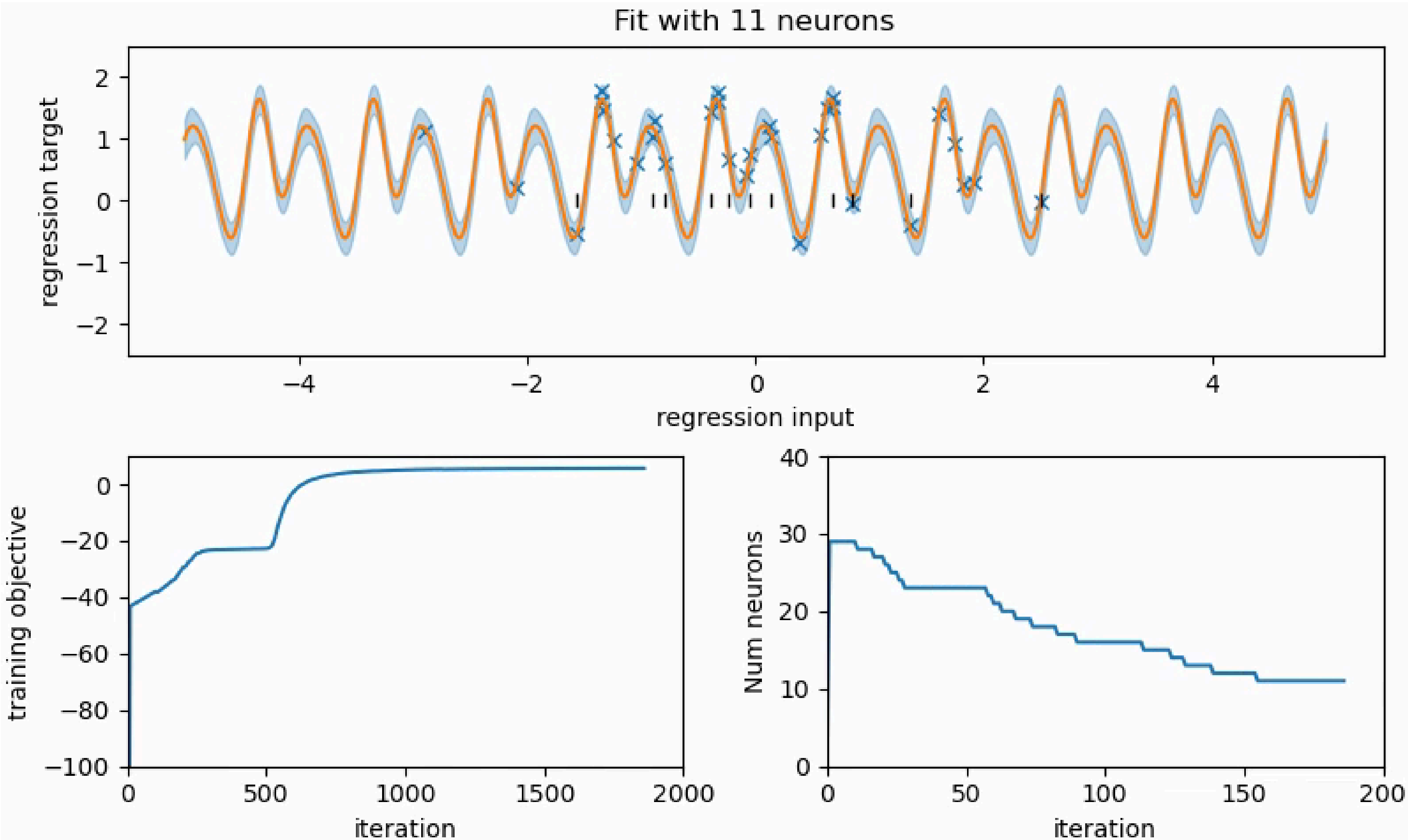

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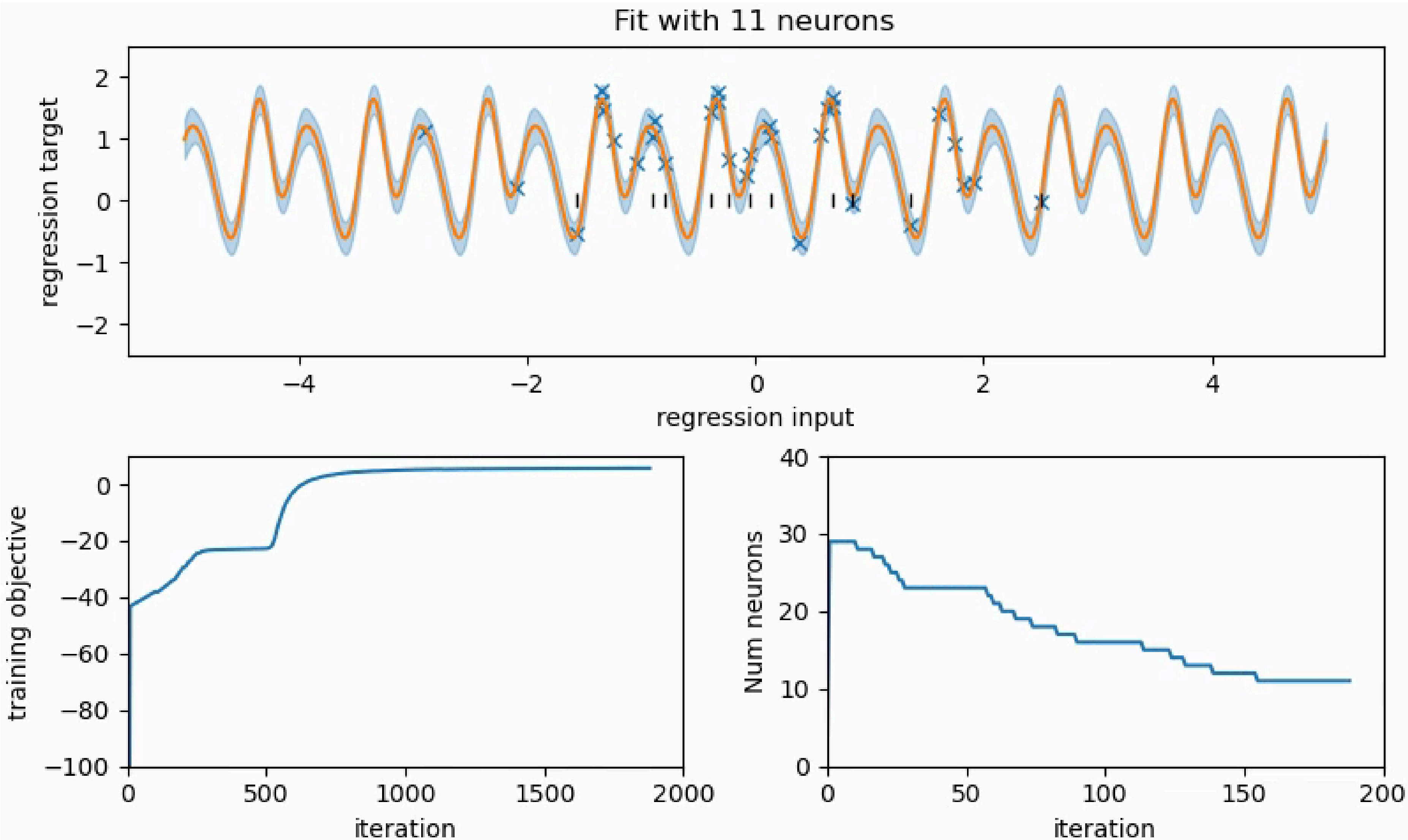

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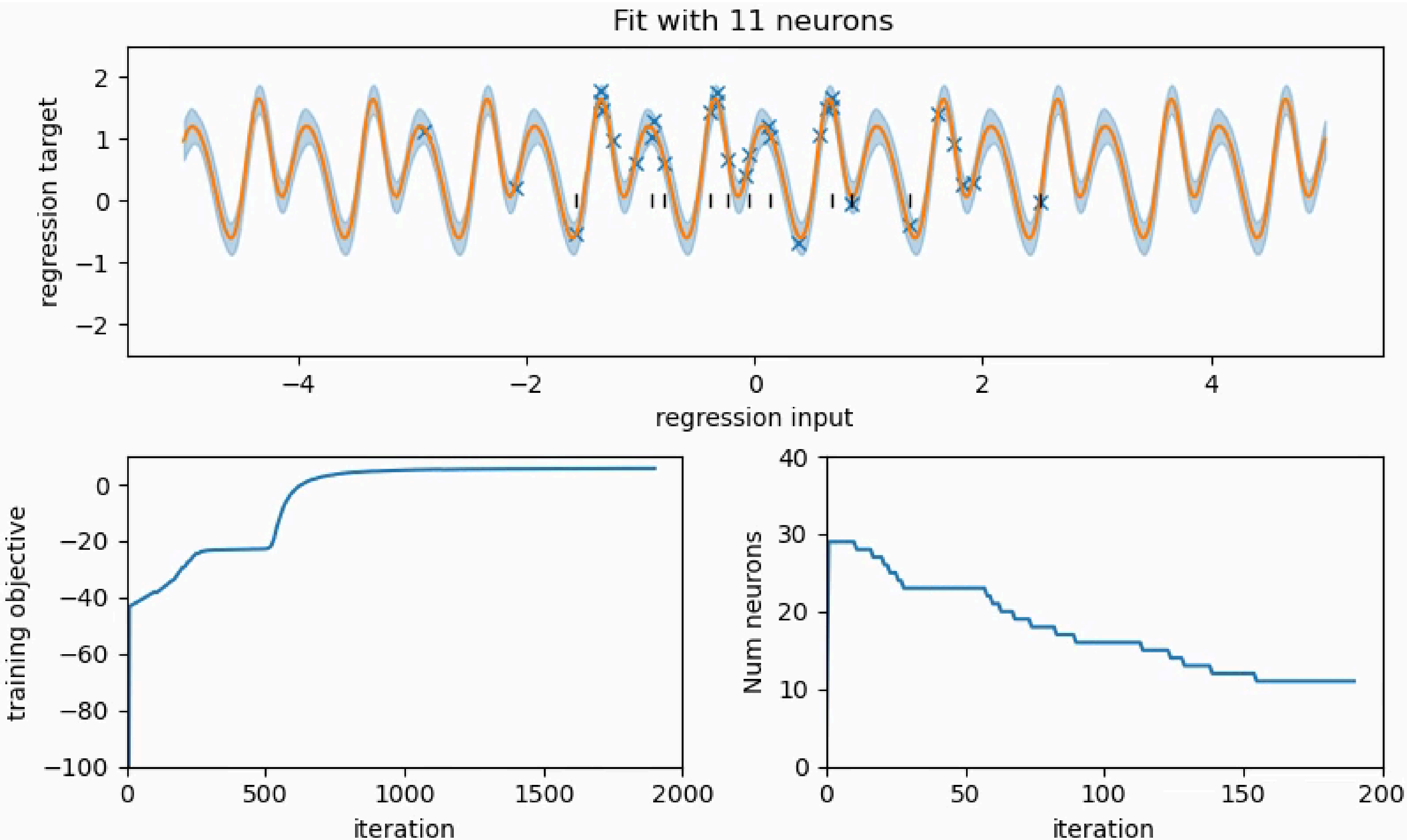

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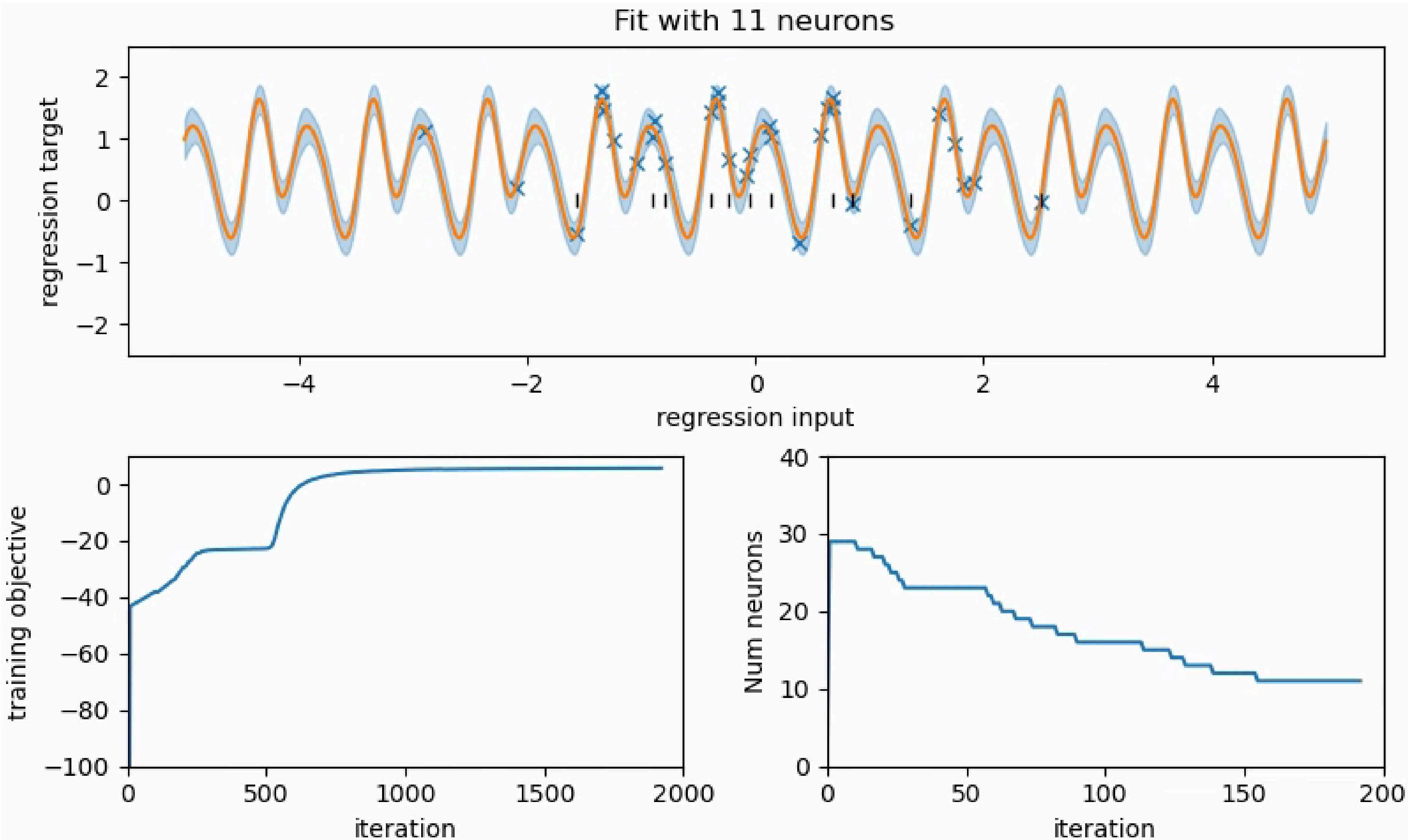

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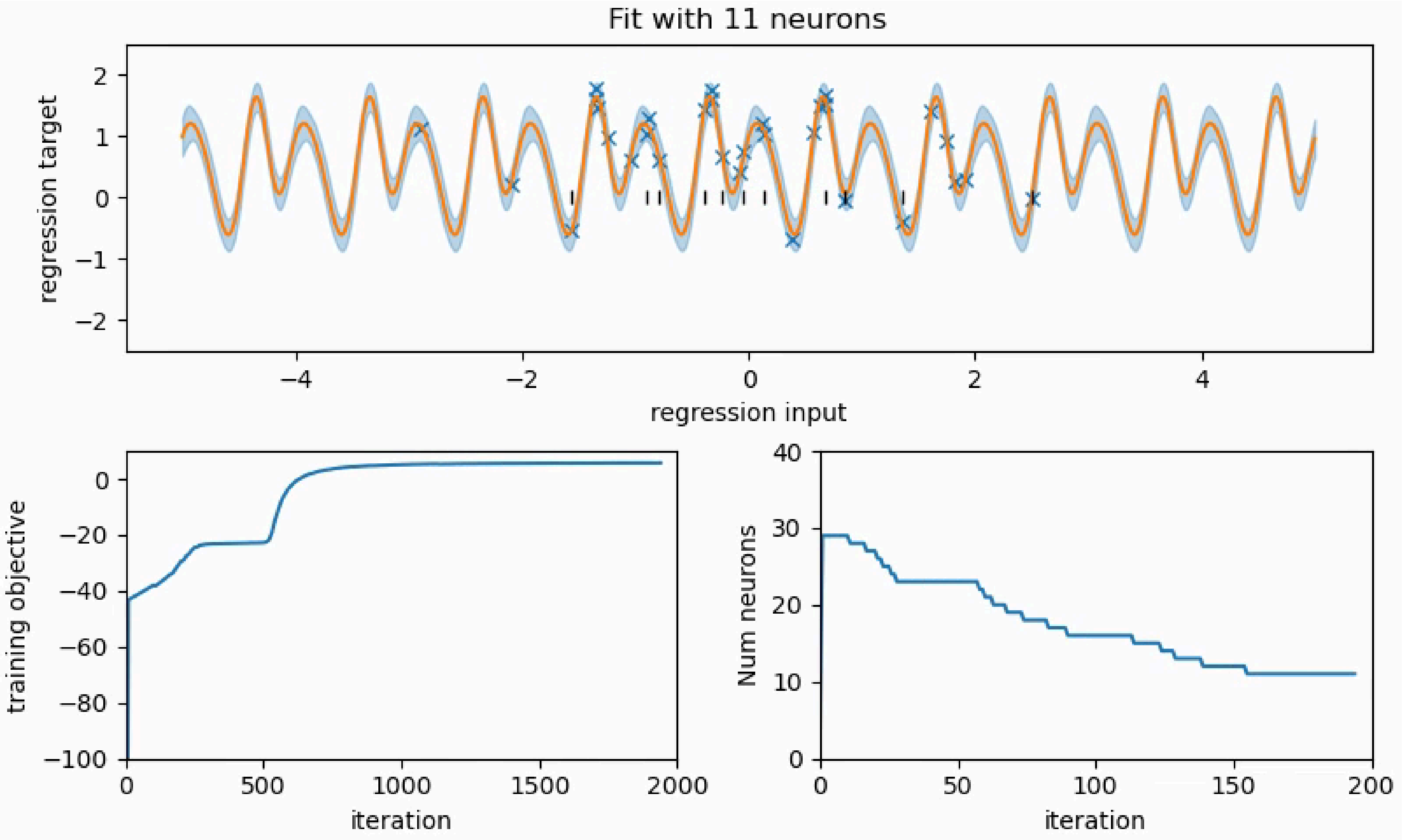

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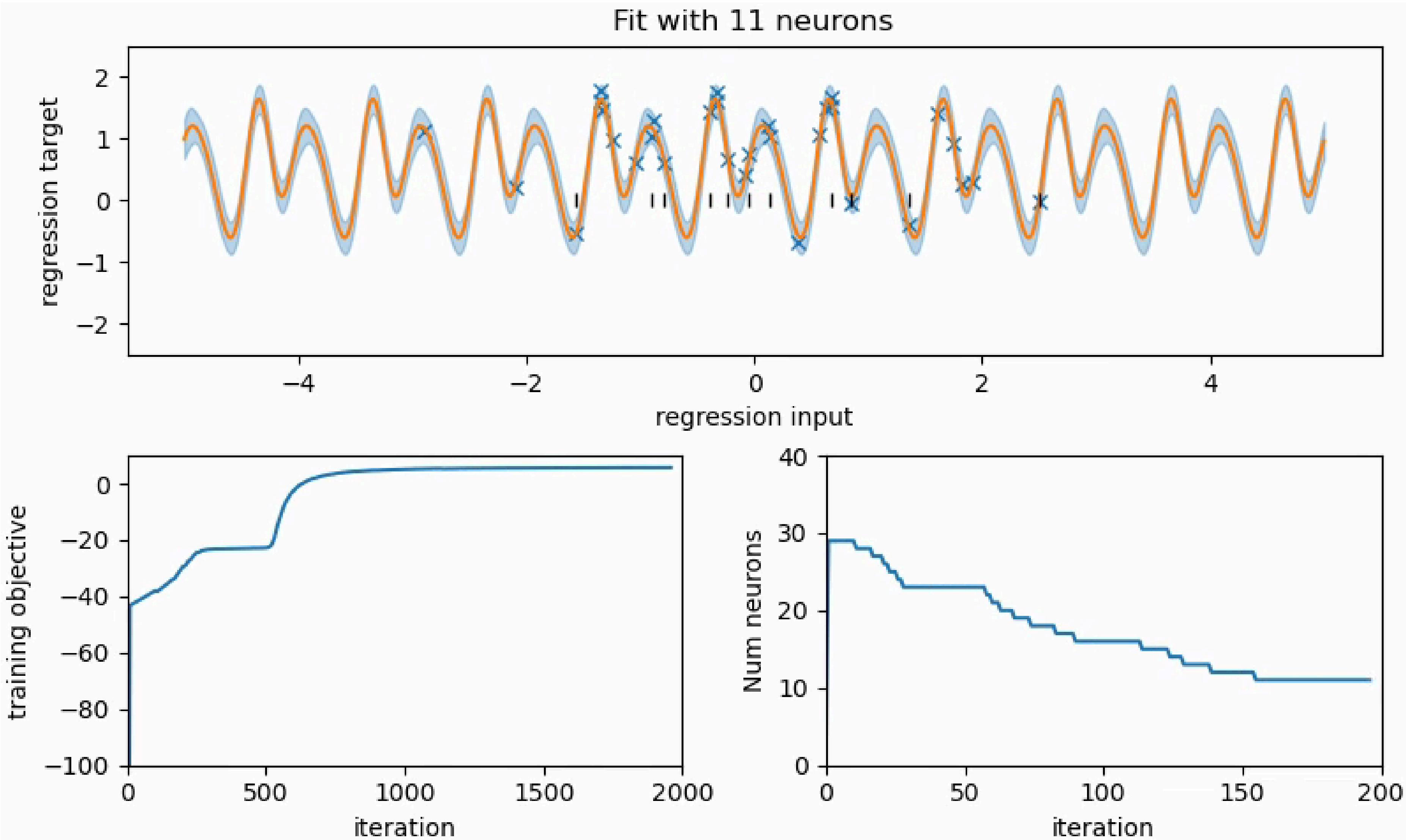

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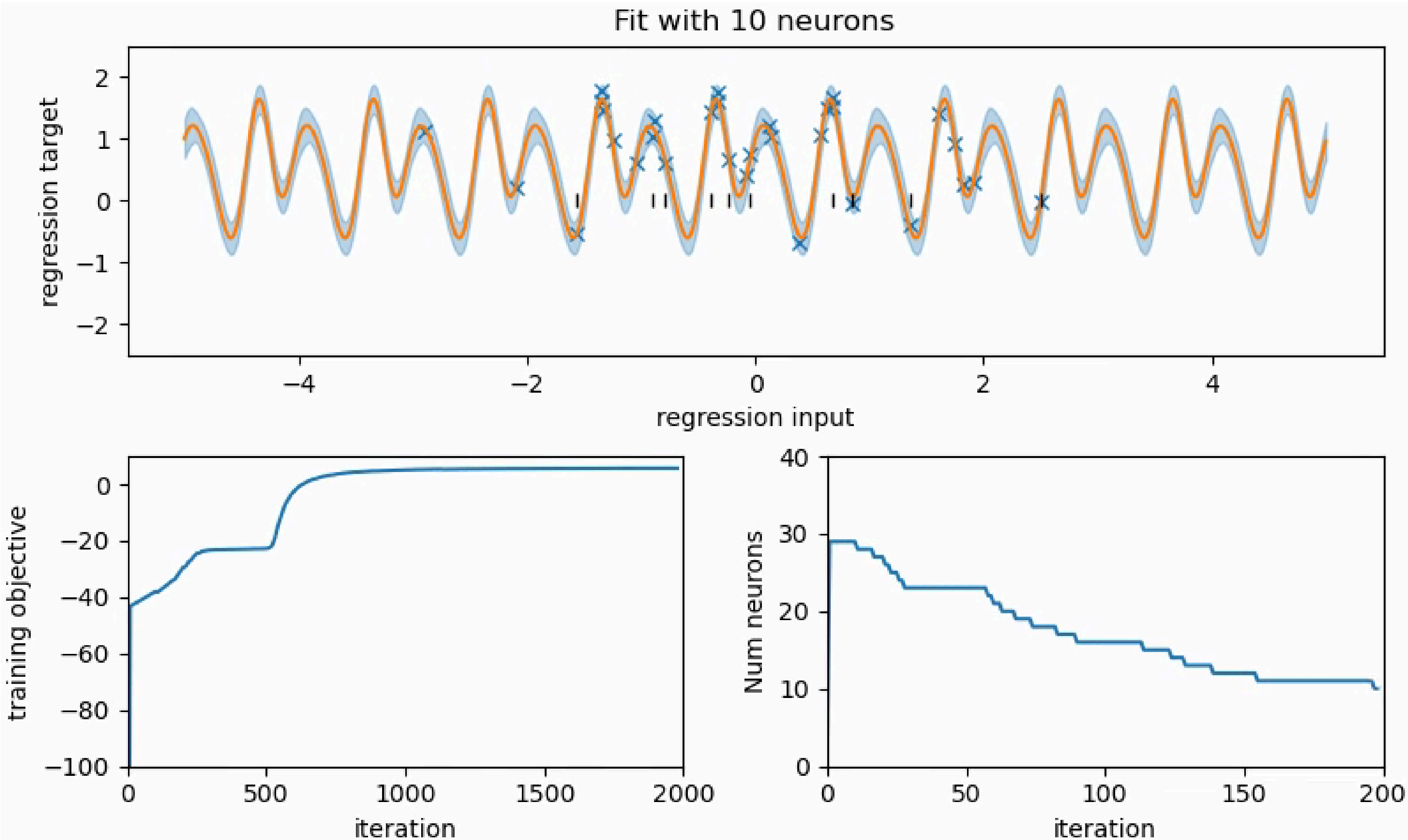

# Conclusion

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We saw:

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- Approximate GPs can give all benefits of nonparametric models, but with *decoupled model size*.
- Bounding the approximation error, gives a principle for determining model size.
- This leads to *adaptive* behaviour of the size of the network, to the problem.

> **We can have our cake and eat it**
>
> We can *define* an infinite-sized model, but near-perfectly approximate it with *just* the right amount of computational resources!

# Designing a Neural Network Training Procedure

🎯 **New procedures for training neural networks!**

Can we automatically find:

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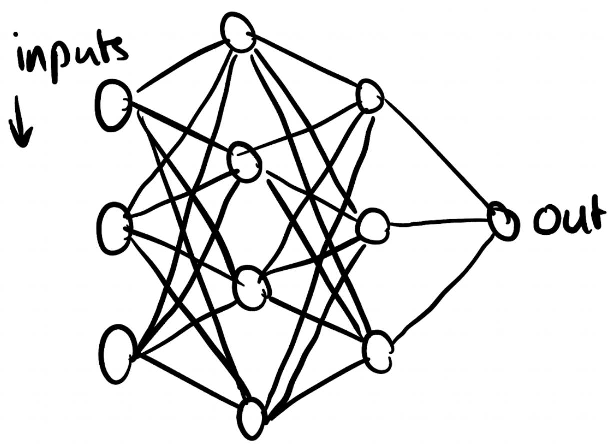
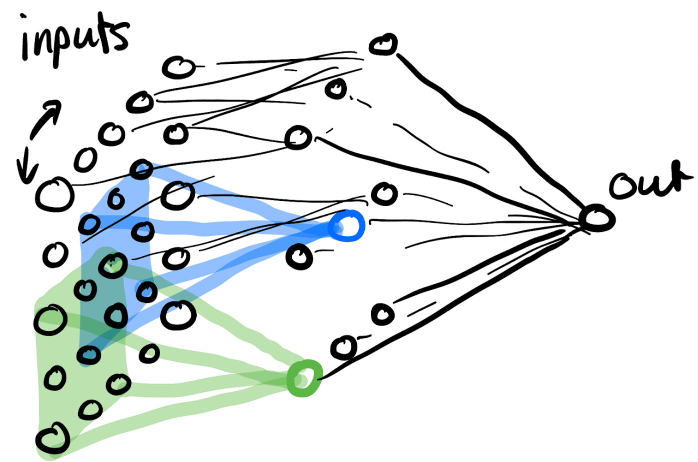
- Inductive bias / **connectivity structure** / architecture




Fully connected




Convolutional

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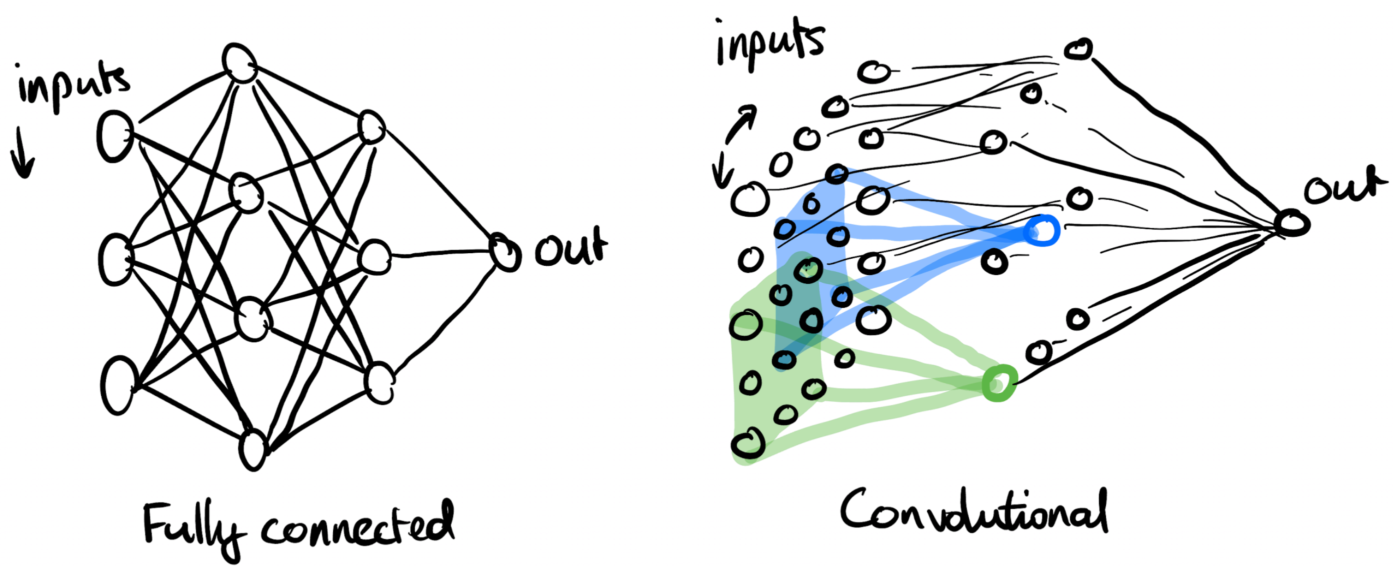


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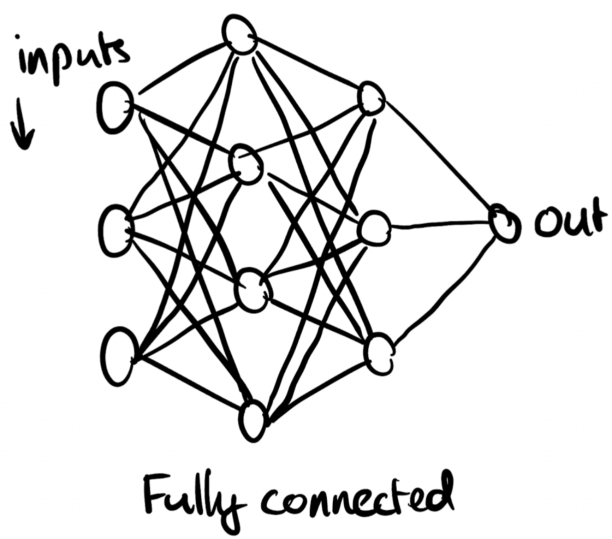


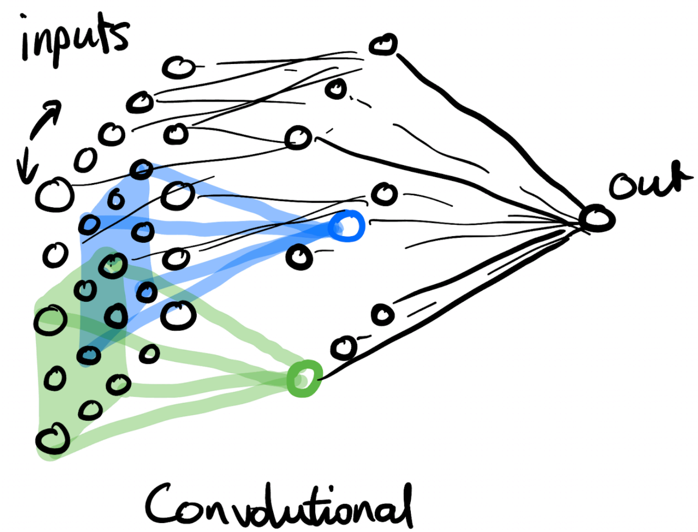


- Choose network **size** (how *many* neurons)

💡 **More efficient, more adaptive, more automatic!**

# Papers

Gaussian processes:

- For an overview of Titsias/Hensman's (Hensman et al., 2013; Titsias, 2009) method for VI in GPs, see my thesis (van der Wilk, 2019)
- Proof of accuracy of variational approximation (basis for when to stop adding inducing variables / basis functions) (Burt et al., 2019; 2020)
- Adaptive model size for continual learning (Pescador-Barrios et al., 2024)
- Overall narrative of this talk (online soon!)

Bayesian Model Selection in Neural Networks:

- Bayesian Model Selection (Laplace approximation) *recovers* ResNets, without explicit human design (Ouderaa et al., 2023)
- See more by Tycho van der Ouderaa!

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